

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.003: Signals and Systems—Spring 2008

Quiz 1

Thursday, March 13, 2008

Directions: The exam consists of 5 problems on pages 2 to 19. Please make sure you have all the pages. **Enter all your work and your answers directly in the spaces provided on the printed pages of this booklet. Please make sure your name is on all sheets. DO IT NOW!** All sketches must be adequately labeled. Unless indicated otherwise, **answers must be derived or explained**, not just simply written down. This examination is closed book, but students may use one $8\frac{1}{2} \times 11$ sheet of paper for reference. Calculators may not be used. **Note that the problems are not in the order of difficulty. Solve the problems that you can first.**

NAME: _____

Check your section	Section	Time	Room	Rec. Instr.	TA
<input type="checkbox"/>	1	10-11	36-155	Berthold K P Horn	Yajun Fang
<input type="checkbox"/>	2	11-12	36-155	Luca Daniel	Shay Maymon
<input type="checkbox"/>	3	12-1	36-155	Luca Daniel	Shay Maymon
<input type="checkbox"/>	4	12-1	24-307	Berthold K P Horn	Yajun Fang
<input type="checkbox"/>	5	1-2	24-307	Cardinal Warde	Sanquan Song
<input type="checkbox"/>	6	2-3	24-307	Cardinal Warde	Sanquan Song

Please leave the rest of this page blank for use by the graders:

Problem	No. of points	Score	Grader
1	12		
2	20		
3	21		
4	21		
5	26		
Total	100		

Problem 1 (12 Points)

Consider the following three systems:

- **System A:**

$$y(t) = \text{Re}\{x(t)\}$$

- **System B:**

$$y(t) = x(t) + \frac{dx(t+1)}{dt}$$

- **System C:**

$$y[n] = e^{j\omega_0(n+1)}x[n] \quad \omega_0 \neq 0$$

where x and y are the input and output of each system, respectively.

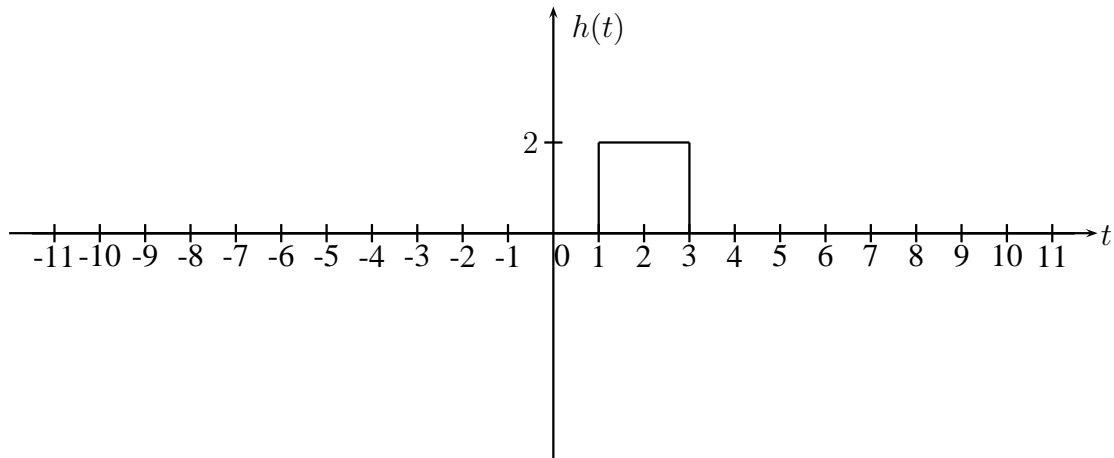
Circle YES or NO for each of the following questions for each of these systems (no explanation is required).

	System A	System B	System C
Is the system linear ?	YES NO	YES NO	YES NO
Is the system time invariant ?	YES NO	YES NO	YES NO
Is the system causal ?	YES NO	YES NO	YES NO
Is the system stable ?	YES NO	YES NO	YES NO

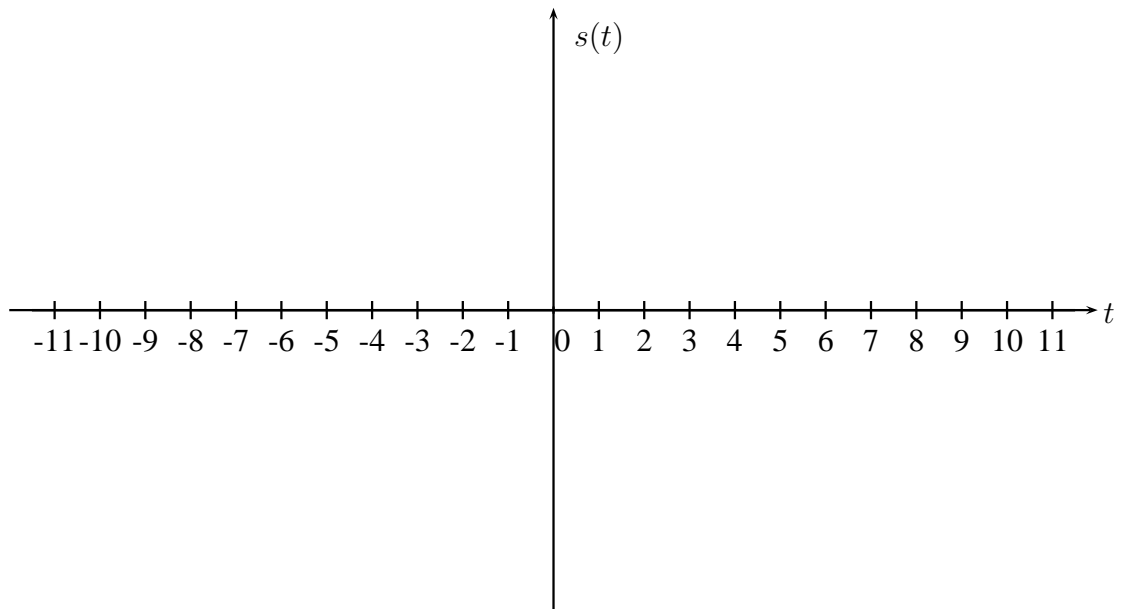
Work Page

Problem 2 (20 Points)

The impulse response of an LTI system is given by

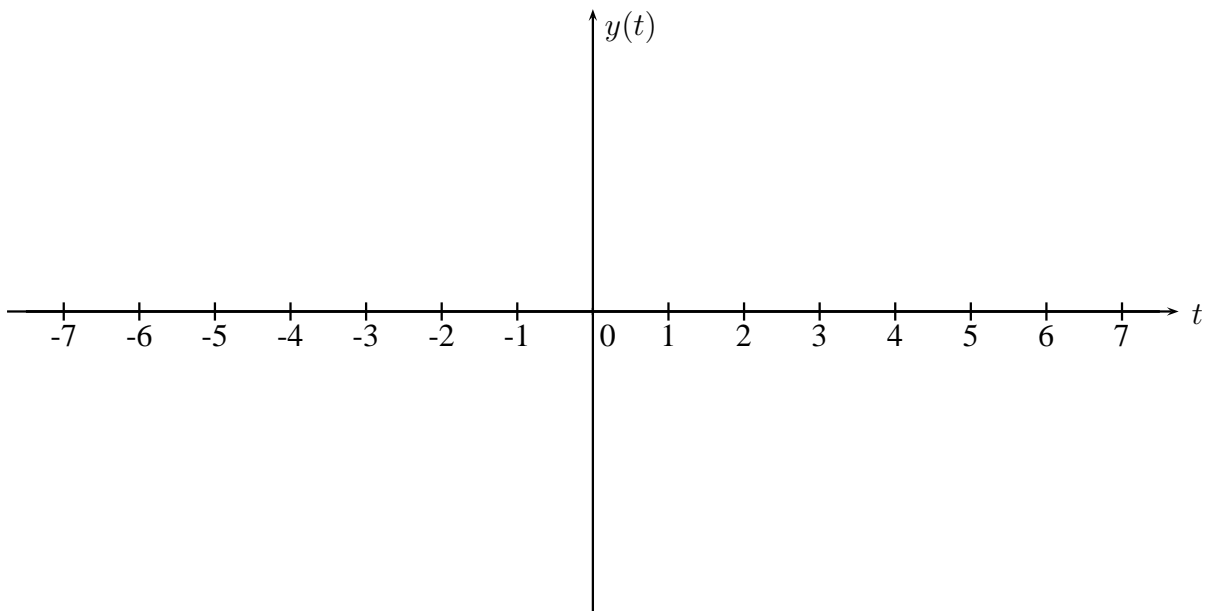
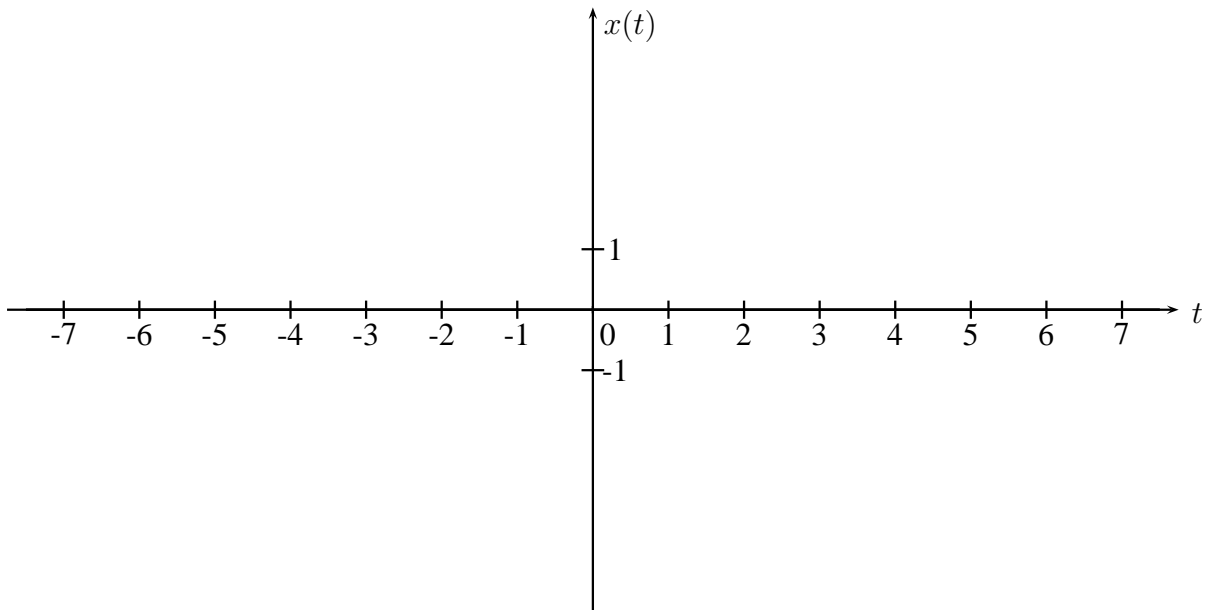


- (a) Compute and sketch below the step response $s(t)$ of the system (*i.e.* the output when the input is the unit step $u(t)$).



Work Page

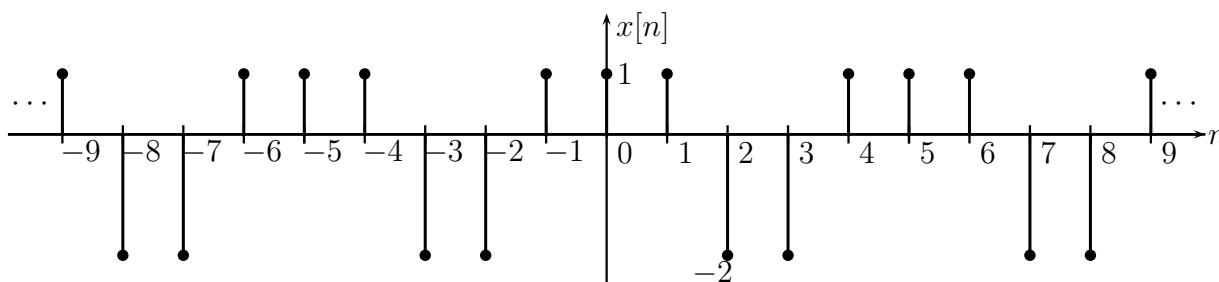
- (b) One cycle of a sine wave $x(t)$ (shown below) is the input to the system. Compute and sketch the corresponding output $y(t)$.



Work Page

Problem 3 (21 Points)

Consider the signal $x[n]$ shown below:



- (a) What is the maximum number of independent Fourier series coefficients that this signal can have?

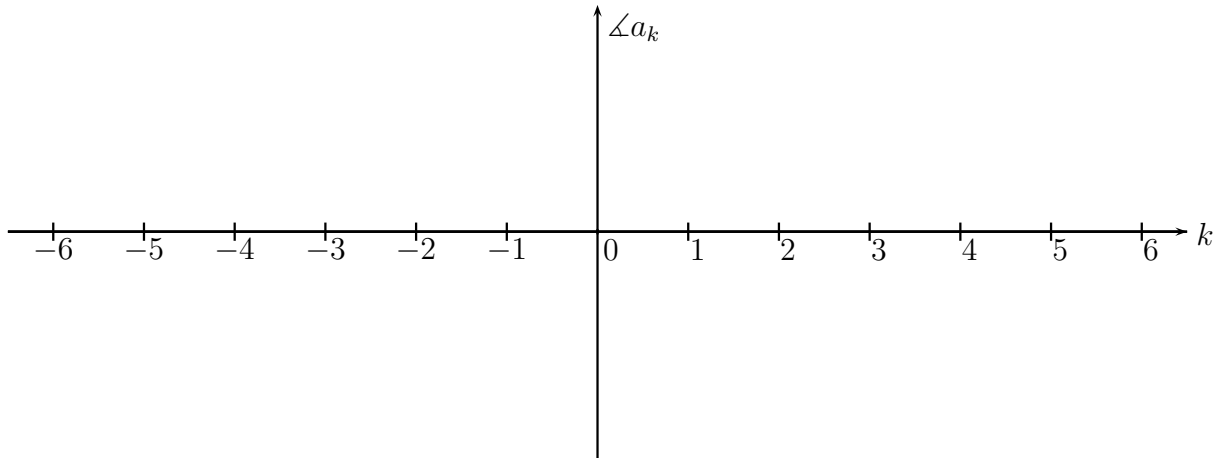
Maximum number =

Brief explanation:

- (b) Derive the Fourier series coefficients a_k of $x[n]$.

Work Page

(c) On the graph below, plot the phase of a_k .

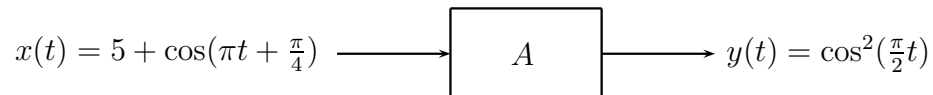


Work Page

Problem 4 (21 Points)

For each of the following systems, determine whether an LTI system exists that produces $y(t)$ for the input $x(t)$.

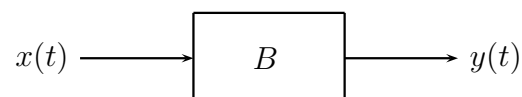
(a)



System A **CAN / CANNOT** be an LTI system. (**circle the right answer**)

Brief explanation:

(b)



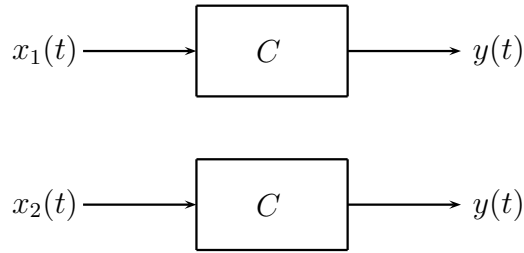
Where $x(t)$, $y(t)$ are periodic signals with fundamental periods T and $\frac{T}{2}$, respectively.

System B **CAN / CANNOT** be an LTI system. (**circle the right answer**)

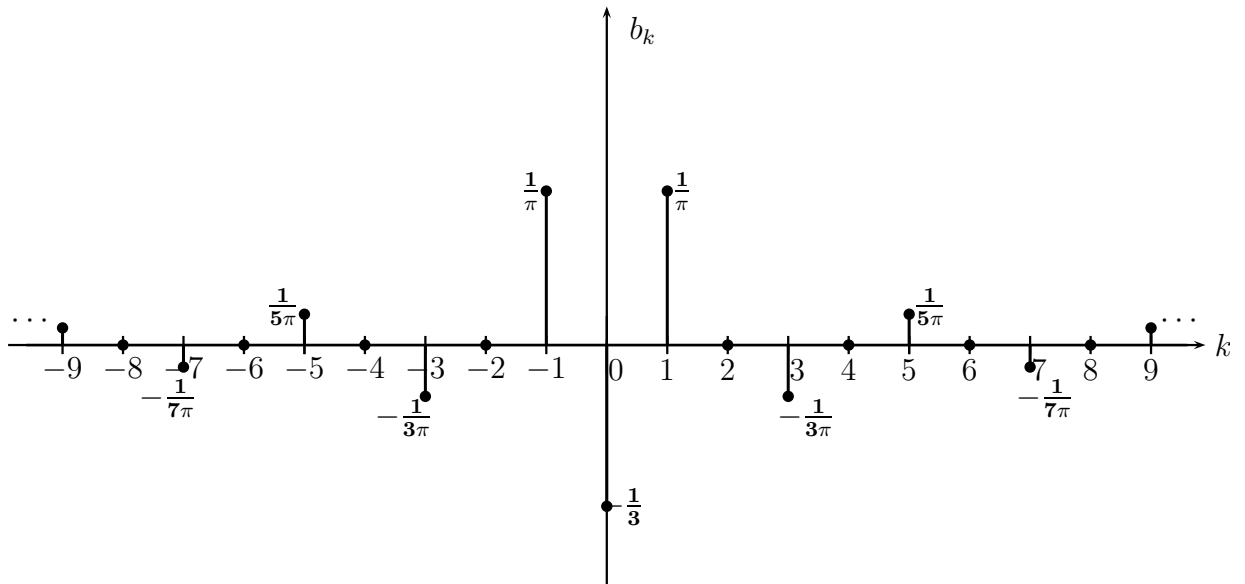
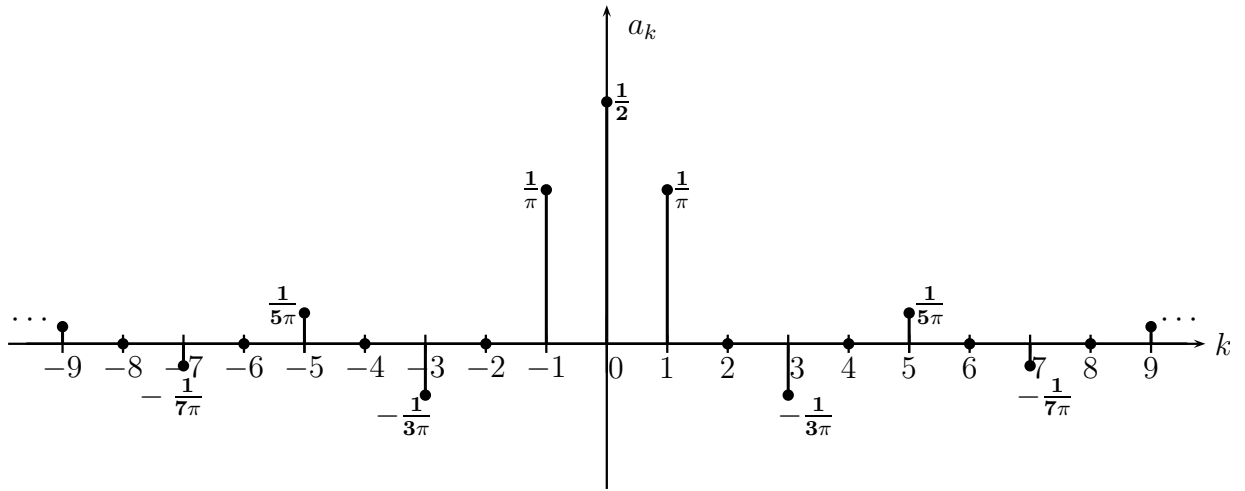
Brief explanation:

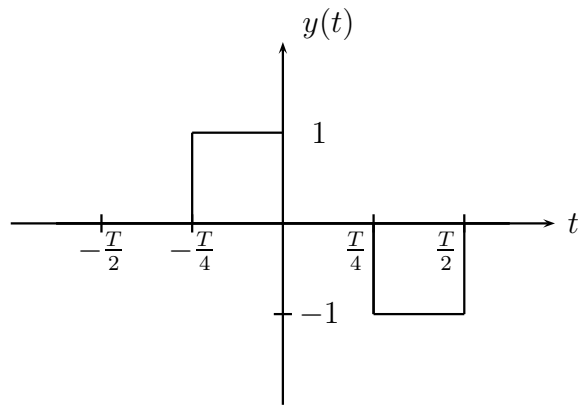
Work Page

(c)



Where $x_1(t), x_2(t)$ and $y(t)$ are periodic signals with fundamental period T . The Fourier series coefficients a_k of $x_1(t)$ and b_k of $x_2(t)$ as well as **one period** of $y(t)$ are given below.



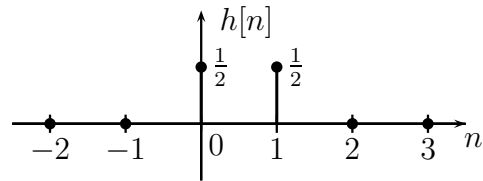


System C **CAN** / **CANNOT** be an LTI system. (circle the right answer)

Brief explanation:

Problem 5 (26 Points)

Suppose we have an LTI system with an impulse response $h[n]$



- (a) Find the frequency response $H(e^{j\omega})$ of this system.

$$H(e^{j\omega}) =$$

- (b) Given the following input signal $x_b[n]$, find out the output signal $y_b[n]$.

$$x_b[n] = 1$$

$$y_b[n] =$$

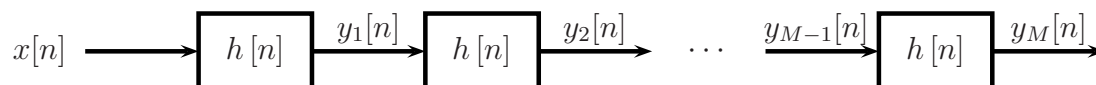
- (c) Given the following input signal $x_c[n]$, find out the output signal $y_c[n]$.

$$x_c[n] = (-1)^n$$

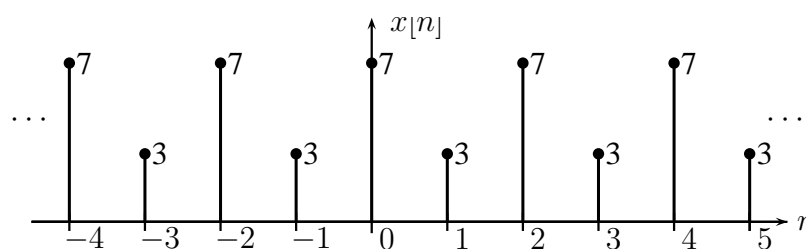
$$y_c[n] =$$

Work Page

(d) Consider the following system:



Given the following input signal $x[n]$:



Evaluate this expression: $\lim_{M \rightarrow +\infty} y_M[n]$.

$$\lim_{M \rightarrow +\infty} y_M[n] =$$

(e) For a DT periodic signal $x[n]$ with **arbitrary** period N , evaluate $\lim_{M \rightarrow +\infty} y_M[n]$.

$$\lim_{M \rightarrow +\infty} y_M[n] =$$

Work Page