

1 Results

My partner for this experiment was fellow classmate Charles Herder. We shared in all parts of the measurement process, but this report was completed individually. For parts (3) and (4) of this exercise, we used a $.47\mu\text{F}$ capacitor. We find it also interesting to note that throughout this lab, the motor often entered oscillations. In order to avoid such behavior, we used small drive signals, on the order of tenths of a volt.

1. We measured an approximate 100% overshoot ($P_0 \approx 2$) and a damped frequency of $\omega_d = \frac{2\pi}{T} = 26.7\text{Hz}$. We can deduce ζ by the relation,

$$P_0 = 1 + \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right). \quad (1)$$

Unfortunately, our limited resolution in the measurement of P_0 only allows us to claim $\zeta = 0$. This is to be compared with the calculated value of $\zeta = 0.0372$.

For this value of the damping ratio, the damped frequency ω_d can be related to the natural frequency ω_n by:

$$\omega_d = \omega_n \sqrt{1-\zeta^2} = \omega_n \quad (2)$$

which yields $\omega_n = 26.7\text{Hz}$. This is contrasted with the calculated frequency of $\omega_n = 25.5\text{Hz}$.

2. We measured the peak frequency to be $\omega_p = 2\pi \cdot 4.7 = 29.5\text{Hz}$. At this frequency, the system gain was $M_p = 5$. From these values, we can deduce the parameters ζ and ω_n as follows:

$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \quad (3)$$

which can be rearranged for the relevant ζ :

$$\zeta = \sqrt{\frac{1 - \sqrt{1 - M_p^{-2}}}{2}}. \quad (4)$$

This yields $\zeta = 0.1$. And, by using $\omega_p = \omega_n \sqrt{1-2\zeta^2}$, we find $\omega_n = 29.8\text{Hz}$. These values are sufficiently different from the previously mentioned calculated values, indicating erroneous measurements.

3. We measured $P_0 = 1.1$. It was not possible to measure the ring frequency of the step response, because the oscillations decayed so rapidly, as to prevent a valid measurement of the oscillation period. This situation was made even more difficult by the small signals that we used in order to avoid motor oscillation.

On the other hand, we obtained the following in the frequency domain measurements: $\omega_p = 47.1\text{Hz}$, $M_p = 1.13$. Using the same procedures as in part (2), this yields the following: $\zeta = 0.5$ and $\omega_n = 69\text{Hz}$. This is not consistent with the predicted values from pre-lab: $\zeta = 0.5$ and

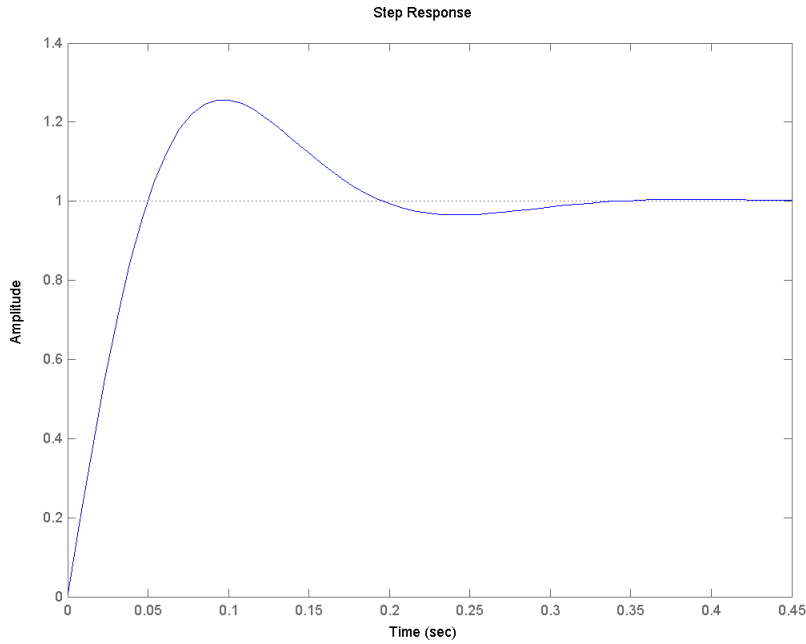


Figure 1: Step response (oriented to give positive step in output) of the closed-loop system with PD controller. The M_p is roughly consistent with the measured value, and the rapid decay of the oscillations is also qualitatively consistent with our observations.

$\omega_n = 25.5\text{Hz}$. We can ascribe the discrepancy in bandwidth to the near pole-zero cancellation in the closed loop system. (The PD controller introduces a zero.)

See Figures 1 and 2 for the numerical step-response and the Bode plot of the system respectively. I have attached the Matlab code as an appendix. Please refer to the individual figure captions for the comparisons between numerical results and measurements.

4. At a ramp rate of $R = 0.8$ V per second, we measured a steady state error of $e \approx 17\text{mV}$. Based on a simple linear scaling, we then expect an error of $e = 21\text{mV}$ at unit ramp. This is roughly an order of magnitude off of the predicted result.

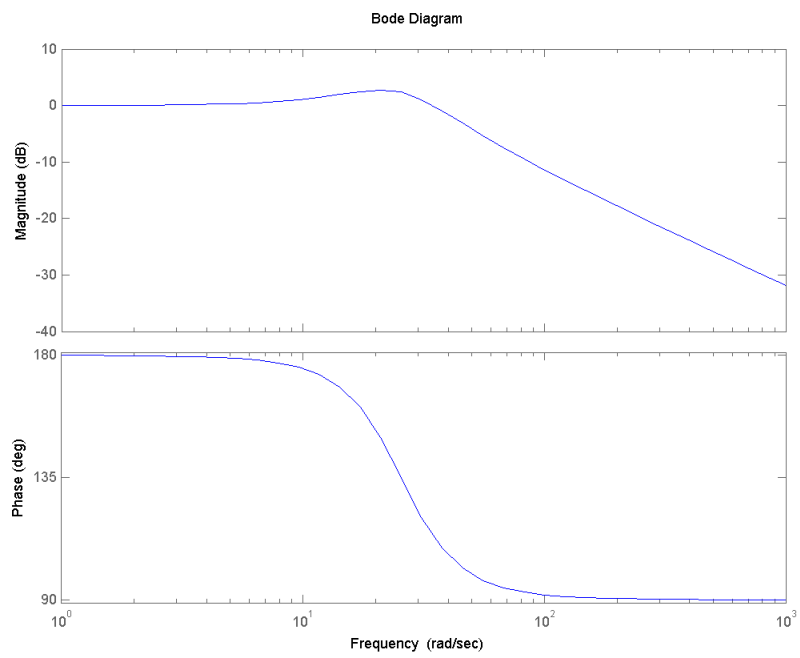


Figure 2: Frequency response. The location of ω_p and height M_p are roughly consistent with our measurement.