

Optical Spectroscopy of Hydrogenic Atoms:

Rydberg fits of ^1H and ^2H , and Determination of the mass ratio

Tony Hyun Kim
(Partner: Connor McEntee)
10/31/2008
8.13 MW2-5 Prof. Roland

Topics to be discussed

1. Introduction

1. Historical context
2. Quantum mechanics of atomic emission

2. Experimental setup

3. Analysis and Results

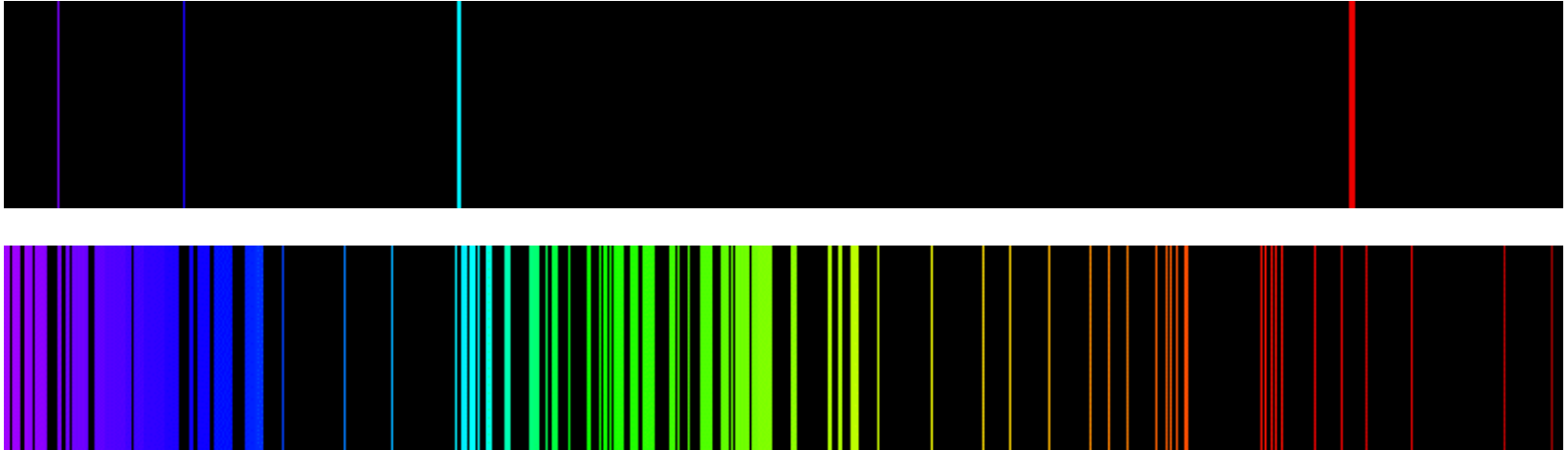
1. Fitting the lineshape
2. Mercury calibration
3. Hydrogen and deuterium fits

4. Sources of Error; Possible improvements

5. Conclusions

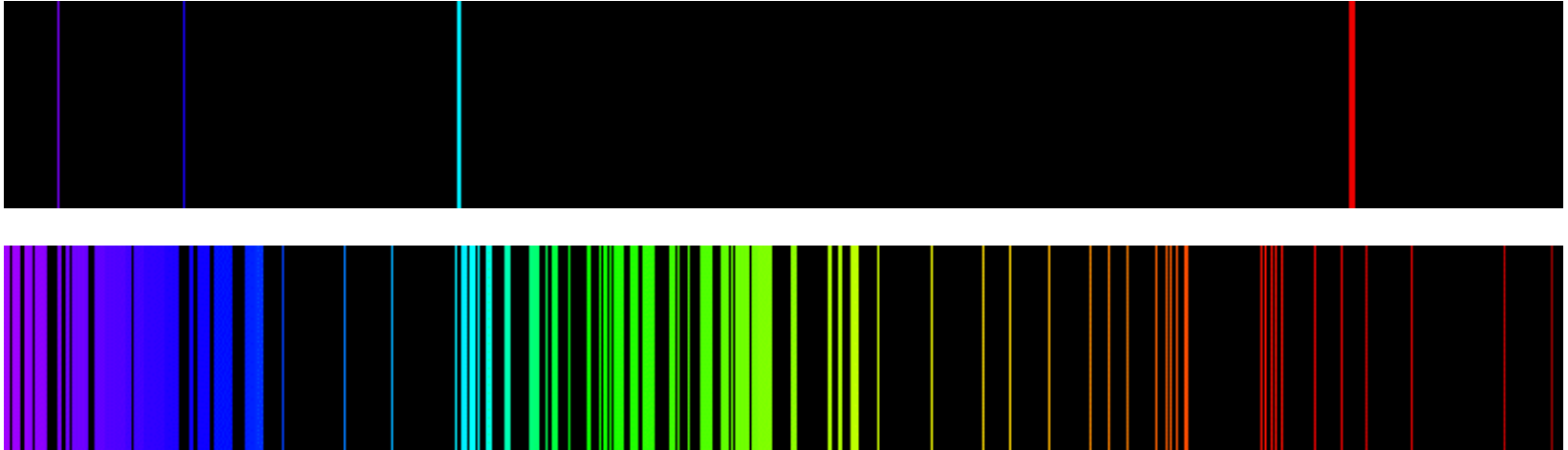
1. Verification of the hydrogen Rydberg
2. Estimate of the hydrogen-deuterium mass ratio

Atomic spectra: a historical context



- By 19th century, tremendous amounts of atomic spectral data collected.
- **What are the underlying mathematical patterns and the physical explanations?**

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$$\frac{1}{\lambda} = R \cdot \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$$

QM of the optical electron

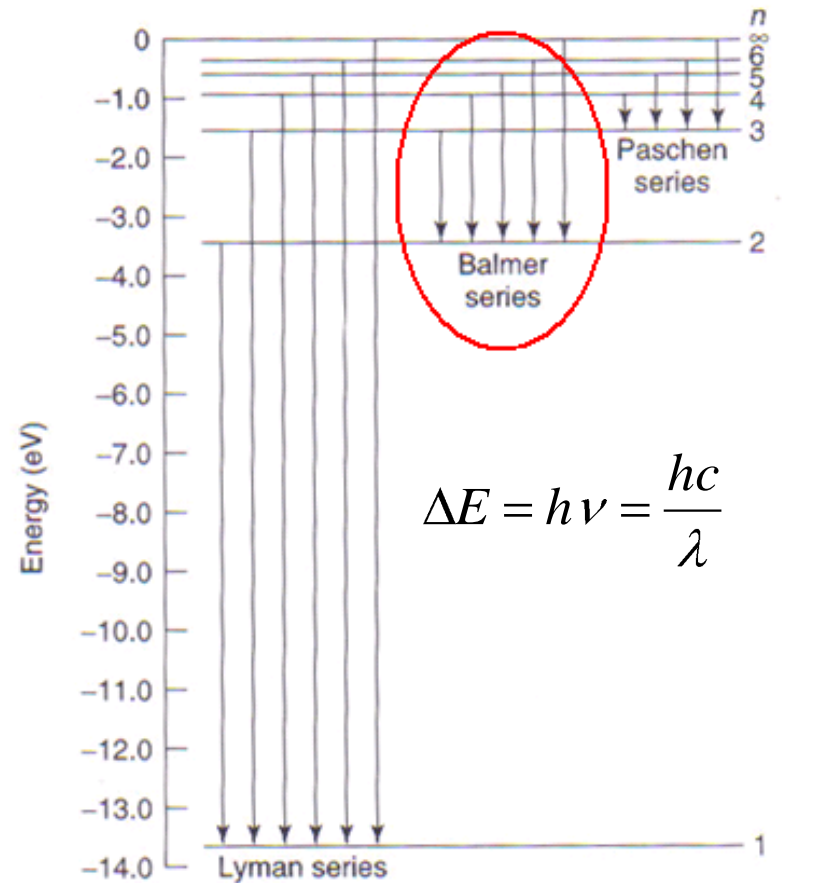
- Reduced, one-body SE yields eigenenergies:

$$E_n = - \left[\frac{\mu}{2\hbar^2} \left(\frac{e^2}{4\pi\epsilon_0} \right)^2 \right] \cdot \frac{1}{n^2}$$

- Light emitted when electron undergoes transition: $n_i \rightarrow n_f$

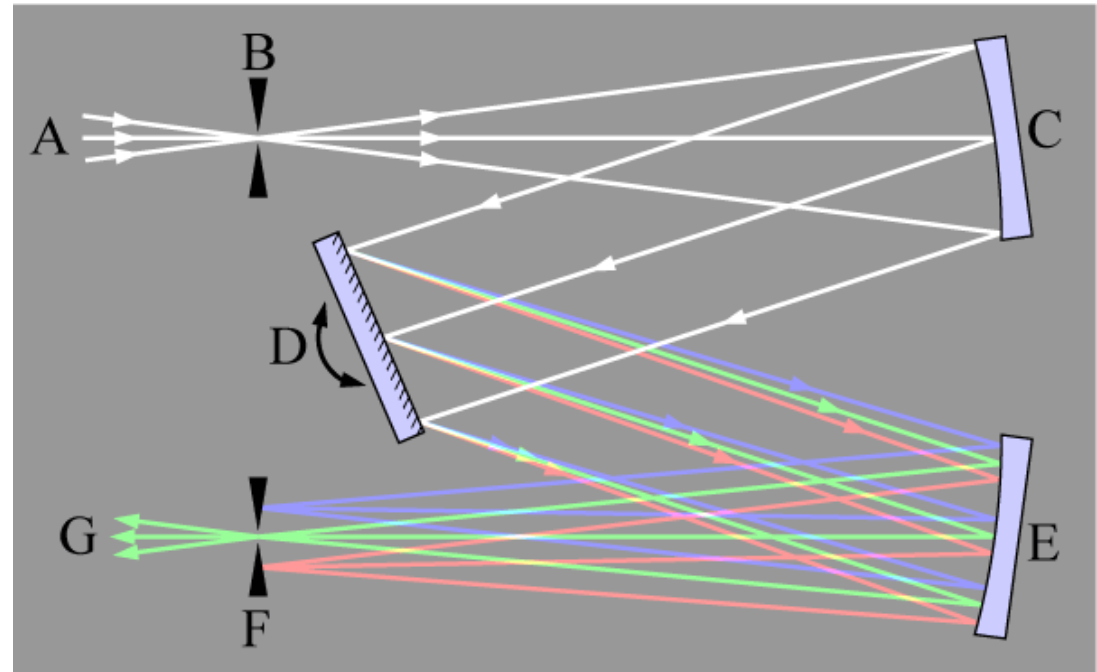
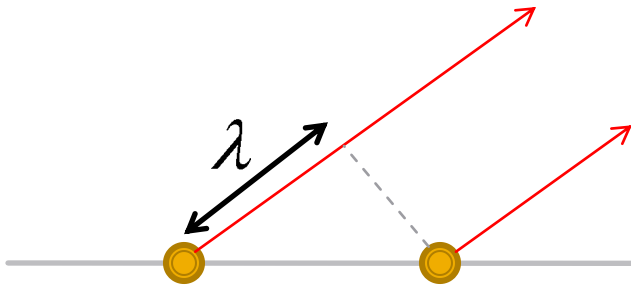
$$\frac{1}{\lambda} = \left[\frac{\mu}{4\pi c \hbar^3} \left(\frac{e^2}{4\pi\epsilon_0} \right)^2 \right] \cdot \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$R = \frac{\mu}{4\pi c \hbar^3} \left(\frac{e^2}{4\pi\epsilon_0} \right)^2$$



Energy levels and transitions in the spectrum of hydrogen.

Experimental setup: Basic principles



- Used JY1250M monochromator ($R \sim 10^4 \rightarrow 0.03 \text{ \AA}$ step size!)
- Counter indicates the orientation of grating

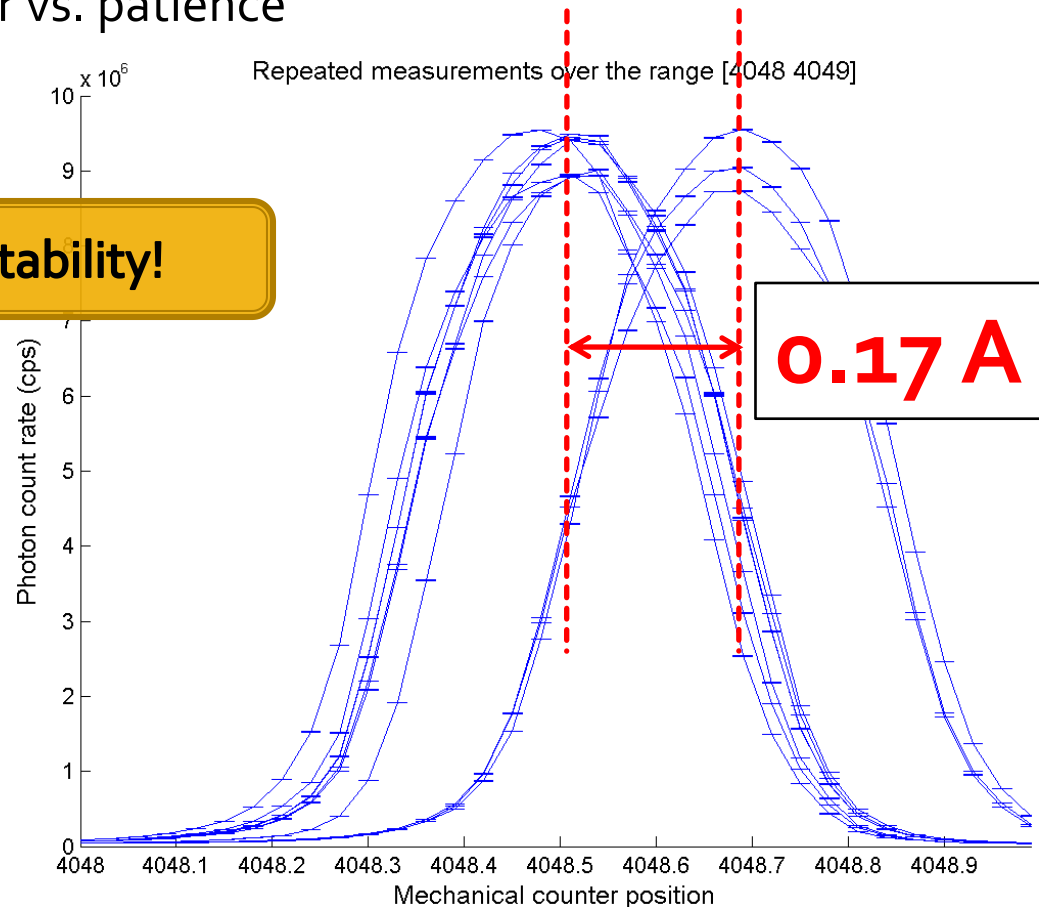
Experimental setup parameters

- Slit sizes: quality of lineshape vs. signal size
- Integration time: \sqrt{N} -error vs. patience

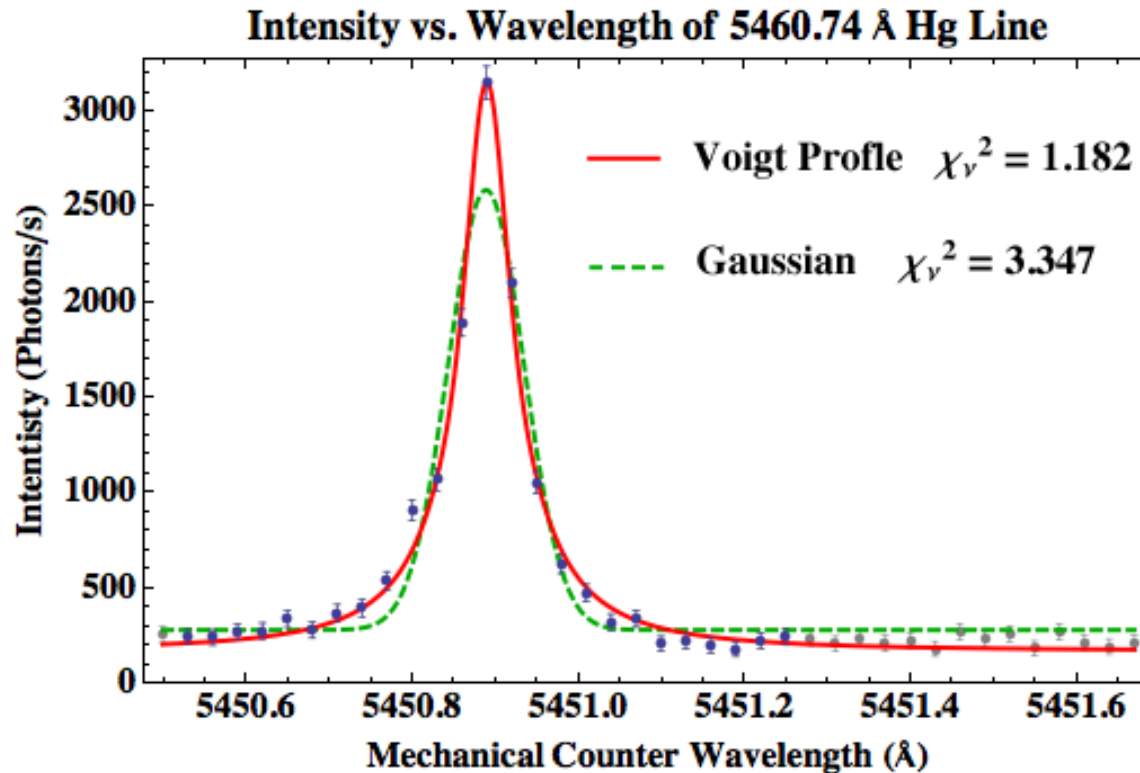
Experimental setup parameters

- Slit sizes: quality of lineshape vs. signal size
- Integration time: \sqrt{N} -error vs. patience
- Most important:

Shot-to-shot repeatability!



Typical monochromator data



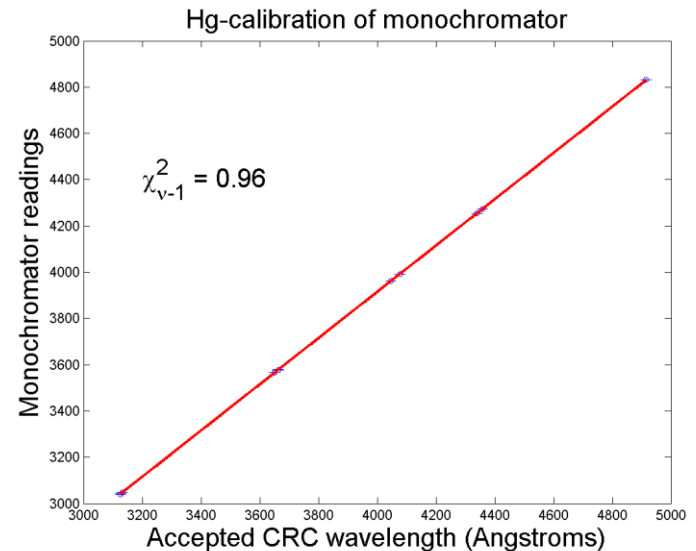
- Voigt gives better fit than Gaussian, but the means agree!
- Typical errors in fitted mean: ~ 0.001 Å

Mercury calibration

- Monochromator's counter is not physically accurate

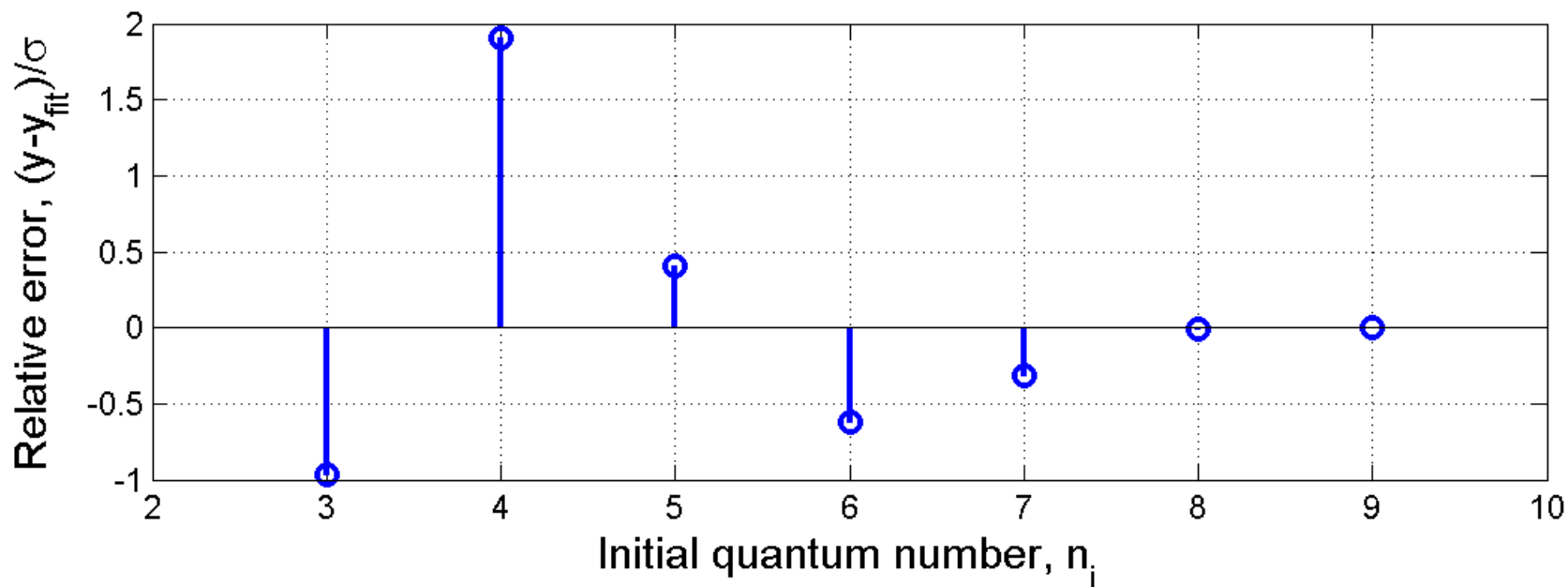
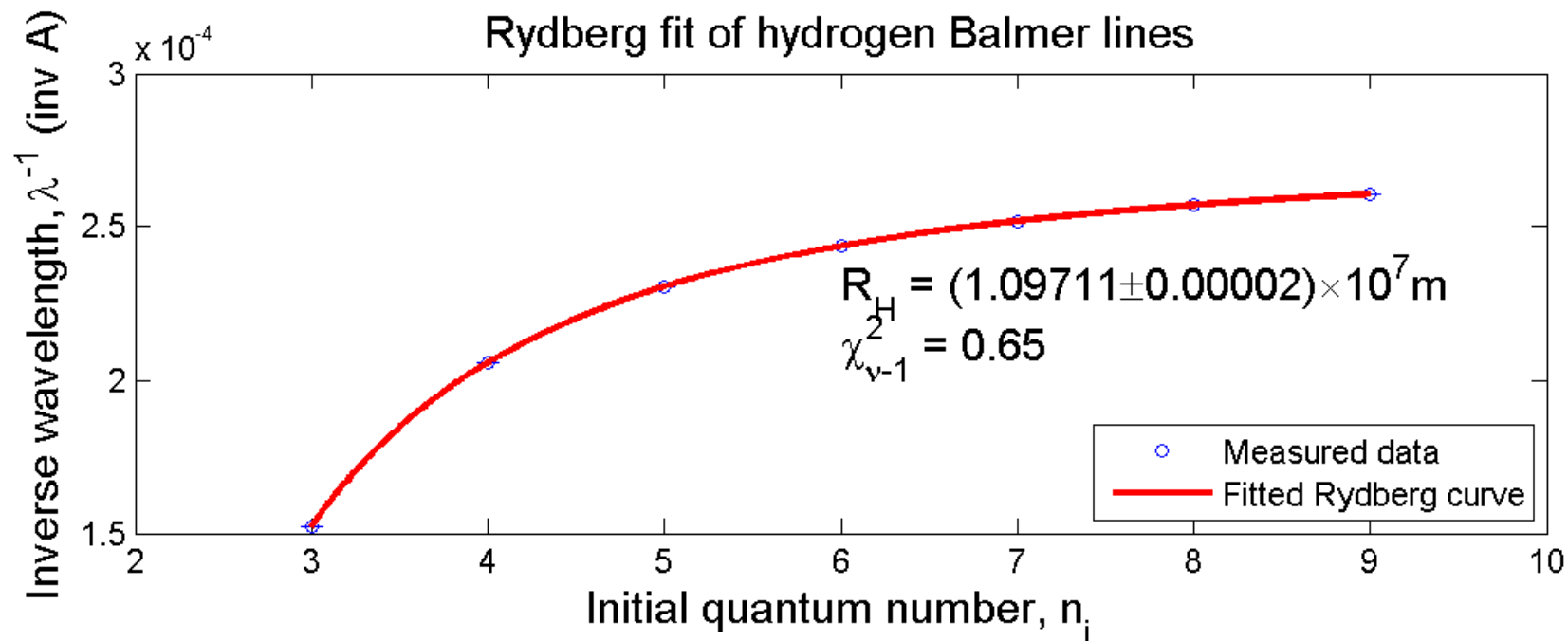
TABLE I: Mercury calibration of the 3600g/mm grating. AW denotes “Accepted wavelength”; and MC denotes “Mechanical counter”

AW (Å)	MC (Å)	AW (Å)	MC (Å)
3125.67	3040.78	4046.56	3961.34
3131.55	3046.64	4077.83	3992.61
3131.84	3046.94	4339.22	4253.93
3650.15	3465.07	4358.33	4253.93
3662.88	3577.98	4916.07	4831.23
3663.28	3578.30		



- Produced quadratic conversion function:

$$\lambda_M = (3.5 \pm 1.6) \times 10^{-7} \cdot \lambda_A^2 + (0.997 \pm 0.001) \cdot \lambda_A + (-79.3 \pm 2.4)$$



Results: Hydrogen Rydberg

- Published value of Rydberg constant:

$$R_{H,NIST} = 1.096776 \times 10^7 \text{ m}^{-1}$$

- Our measured value:

$$R_{H,ex} = (1.09711 \pm 0.00002) \times 10^7 \text{ m}^{-1}$$

- Correct for index of refraction of air ($n = 1.0003$)

$$R_{H,ex} \rightarrow R_{H,ex} / n = (1.09678 \pm 0.00002) \times 10^7 \text{ m}^{-1}$$

Results: $^1\text{H}/^2\text{H}$ mass ratio

- From $R_H / R_D = \mu_H / \mu_D \dots$
 - Compute m_D using known values of m_e , m_p (NIST)

$$\left(\frac{m_D}{m_H} \right)_{ex} = 1.87 \pm 0.17$$

- Published value:

$$\left(\frac{m_D}{m_H} \right)_{NIST} = 1.999$$

An improved optical setup

- Since we regard the Hg-lines as “ruler”...
- Scheme for circumventing the **0.17Å mechanical error**:

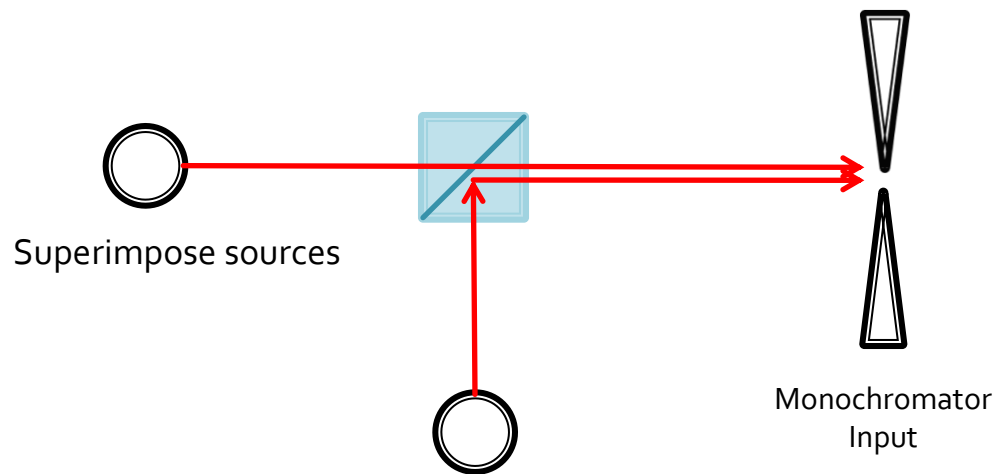


TABLE II: Observed wavelengths of $H = {}^1H$ and $D = {}^2H$ samples. All measurements have been converted to physical wavelengths by the Hg-based conversion formula. Results are given in angstroms.

n_i	λ_H	λ_D	$\Delta\lambda = \lambda_H - \lambda_D$
3 ^a	$6562.94 \pm .18$	$6561.60 \pm .18$	$1.34 \pm .26$
4	$4860.91 \pm .20$	$4859.83 \pm .21$	$1.08 \pm .29$
5	$4340.38 \pm .18$	$4339.17 \pm .18$	$1.21 \pm .26$
6	$4101.83 \pm .18$	$4100.73 \pm .18$	$1.11 \pm .26$
7	$3970.12 \pm .18$	$3969.07 \pm .18$	$1.04 \pm .26$
8	$3889.04 \pm .18$	$3888.05 \pm .18$	$0.99 \pm .26$
9	$3835.38 \pm .18$	$3834.54 \pm .20$	$0.83 \pm .26$

- Especially useful for isotope shift measurement

Conclusions

- Performed spectroscopy of $^1\text{H}/^2\text{H}$
- The Rydberg formula for hydrogen was confirmed. Remarkable agreement with published values:

$$R_{H,ex} = (1.09678 \pm 0.00002) \times 10^7 \text{ m}^{-1}$$

$$R_{H,NIST} = 1.096776 \times 10^7 \text{ m}^{-1}$$

- Ratio of Rydberg constants was used to deduce mass ratio:

$$\left(\frac{m_D}{m_H} \right)_{ex} = 1.87 \pm 0.17$$

$$\left(\frac{m_D}{m_H} \right)_{NIST} = 1.999$$