Poisson statistics: Measurement of γ-radiation from ^{137}Cs source

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We present the results of a counting experiment involving the detection of gamma-rays from a radioactive ^{137}Cs source. With numerical simulations in Matlab, we evaluate whether the measured data is reasonably described by the Poisson distribution, taking into statistical fluctuations.

1. INTRODUCTION: THE POISSON DISTRIBUTION

We mean by a set of "statistically independent" events two or more events such that the occurrence of one does not affect the likelihood of the others. Everyday examples of such processes include the successive results of a coin toss, or the likelihood of getting a particular hand of cards from a well-shuffled deck. In this report, we investigate the radioactive emission of 663 KeV photons by a sample of ^{137}Cs. We show that the emission behavior is well described by a particular probability distribution that assumes the independence of events (i.e. the Poisson distribution). Hence, in an indirect way, the radioactive decay is shown to be consistent with a model in which the atoms in the sample undergo decay and emit photons independently of the others.

The Poisson distribution $P(x;\mu)$ is a discrete probability distribution that describes the number of times that some independent event will be observed during an observation period T. In our case, that independent event is the detection of γ-rays. The distribution takes as a parameter μ, which describes the average number of counts one expects in repeated observation experiments. Then, the distribution is specified by the following equation:

$$P(x;\mu) = \frac{\mu^x e^{-\mu}}{x!} \tag{1}$$

where a nonnegative integer x is the number of times that the event is observed.

In basic texts on probability[1], it is shown that the standard deviation of the Poisson distribution is not an additional independent parameter, but is given by $\sigma = \sqrt{\mu}$. This relation is one way by which we will assess whether the measured data is in fact described by the Poisson distribution.

Lastly, we note the relationship between μ and the observation period T. Suppose we observe the number of decays over a long period of time τ, and record X number of decays within that period. Then, it is reasonable to define

$$\lambda = \lim_{\tau\to\infty} \frac{X}{\tau} \tag{2}$$

as the "mean rate" λ of the process. For a shorter observation time T, we then argue that $\mu = \lambda T$. In fact, this was the manner in which we configured the experiment for specific expectations μ: first we determined the rate λ from a longer run, then scaled by the period T.

2. EXPERIMENTAL SETUP

The experimental apparatus involves a stick containing radioactive ^{137}Cs, which was directed at the NaI scintillator as shown in Figure 1. The flash from the scintillator was converted into an electrical signal by the means of a photomultiplier tube (PMT), which was powered at approximately 1kV throughout the experiment.

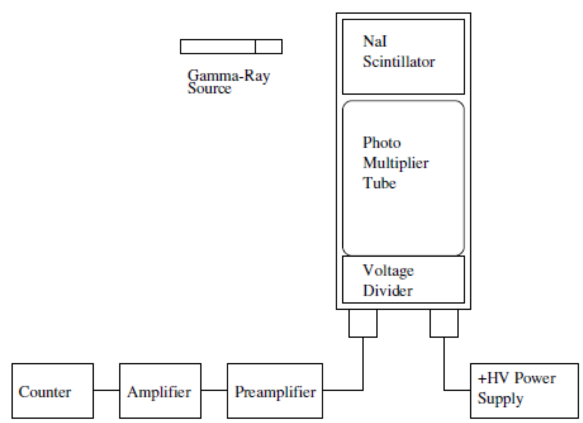

FIG. 1: The gamma-ray source and the scintillator/PMT constitute the basic experimental setup for detecting gamma emission. A counter circuit was used to electrically record the number of counts in a fixed interval of time. Diagram taken from [2]

The signal from the PMT (pulse height $\approx$ 50mV) was amplified to a 5V pulse, which was beyond the comparator point of the counter. The amplified pulse duration was approximately 10 to 20 μs, which are very short intervals with respect to the overall γ-detection rates that we configured for. Hence, it is highly unlikely that multiple pulses could overlap as to give erroneous counting by the electronic counter.

The above setup offered two simple methods with which we could vary the overall count rate. First, there was a dramatic dependence of the count rate to the

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source-scintillator distance. Secondly, we could also vary the gain of the amplifier with respect to the comparator setpoint, which yielded a fine control over the count rate.

The electronic counter offered a means to pre-configure the observation period. As discussed in the introduction, we used a long 100 second run to determine the average rate λ. After setting the rate to a desired value (we targeted $\lambda \approx 1, 4, 10, 100$ sec^{-1}), we collected the counts for one hundred 1-second measurements. (Note that since $T = 1$ sec, the expectation μ and the rate λ are numerically equal. Therefore, in the following discussion, we will use λ and μ interchangeably to refer to particular data sets.)

In addition to the measurements in the lab, we also performed simulations of the Poisson distribution on Matlab, using the scripts that were provided by the 8.13 staff. With the simulations, we generated many Poisson data sets, from which we could measure the statistical fluctuations in the data set characteristics (i.e. mean, standard deviation). From this information, it was possible to evaluate whether our experimental data was a reasonable Poisson data set. The scripts were obtained from /mit/8.13/matlab/ on the MIT server.

3. RESULTS AND DISCUSSION

The four sets of 100-sample data were arranged into histograms as to be easily compared to the theoretical results given by the Poisson distribution. In this paper we will mainly discuss the results for the $\mu \approx 100$ data set. The results here are representative of the other sets. The histogram for $\mu \approx 100$ is given in Figure 2, which also displays the least squares fit of the properly-scaled Poisson distribution function (Eq. 1). We also calculated the mean and the standard deviation directly, which were found to be $\mu = 86.6$ and $\sigma = 8.34$ respectively.

While the χ^2/dof of 0.8121 in Figure 2 indicates a good fit, the rather large relative uncertainties due to low count numbers makes it difficult to claim that the distribution is indeed Poisson. Hence, we compared the experimental data to simulated data sets (which are by construction derived from the Poisson distribution), to see whether the conclusion is appropriate.

To use the simulator, we had to deduce the underlying mean ($\overline{\mu}$) for our $\mu \approx 100$ data set. By considering the long 100-second run, we calculated a mean rate of $\lambda = 87.5$ sec^{-1} which in turn gives $\overline{\mu} = \lambda T = 87.5$. In addition, we were able to verify this value by considering the cumulative average of the individual 1-second runs. That is, we computed the quantity:

$$r_c(j) = \frac{\sum_{i=1}^{i=j} x_i}{\sum_{i=1}^{i=j} t_i} \tag{3}$$

for $j = 1, 2, \cdots, 100$ where x_i is the number of counts detected in time t_i. In the introduction, we claimed that $\overline{\mu} = \lambda \cdot (1sec) = 87.5$ should be the average number of counts one expects in repeated observation experiments. In computing the cumulative averages (Eq. 3) we verified that the averages do in fact tend towards the mean as determined by the long-period experiment. This convergence is demonstrated in Figure 3.

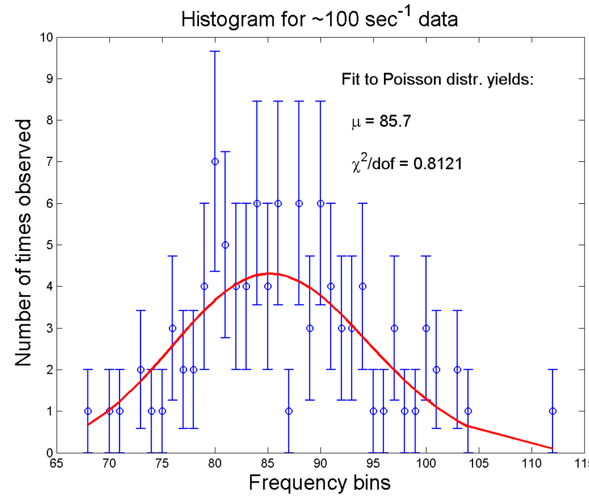

FIG. 2: Histogram for the $\lambda \approx 100$ sec^{-1} experiment. Bins that had zero counts were removed from the fit, so that the χ^2 value of the fit would be finite.

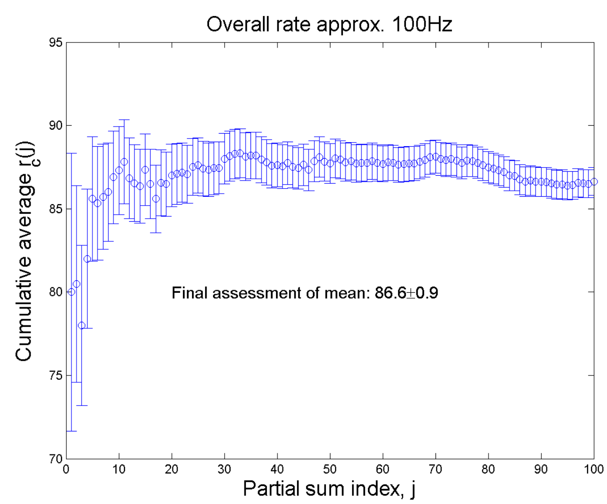

FIG. 3: Cumulative averages $r_c(j)$ (Eq. 3) for the 1-second measurements. The final mean is 86.6 ± 0.9 which contains the long-run mean $\overline{\mu}$ of 87.5 within its error bounds.

Having determined the likely parent mean to be 87.5, we then generated in Matlab ten 100-element distributions with this parameter, in order to observe the statistical fluctuations of Poisson data sets. Their means and standard deviations are tabulated in Table I. We obtained a set whose mean and standard deviations were characterized by: $\mu = 87.5 \pm 1.0$ and $\sigma = 9.0 \pm 0.8$. The characteristics of the measured data fall within these ranges.

TABLE I: The means and standard deviations of the 100-element Poisson data sets generated by Matlab simulation. The mean of the parent distribution was 87.5, which is derived from the long 100-second measurement.

Set #	Mean	Std. dev.	Set #	Mean	Std. dev.
1	87.03	8.09	6	87.87	8.49
2	87.58	8.30	7	88.28	9.95
3	87.89	9.97	8	88.15	8.96
4	86.15	9.87	9	88.32	9.27
5	85.37	7.93	10	88.85	8.81

As an additional check, we also considered the relation $\sigma = \sqrt{\mu}$ which holds for the theoretical Poisson distribution. For each data set, we defined the quantity $\sigma/\sqrt{\mu}$ which gives an indication of how well the relation is satisfied. For the measured data, $\sigma/\sqrt{\mu} = 0.90$; whereas the simulated set yields $\sigma/\sqrt{\mu} = 0.95 \pm 0.08$. It then follows that the measured data set can be regarded as a Poisson distribution up to typical statistical fluctuations.

4. BRIEF ERROR ANALYSIS

In this experiment, we did not intend to precisely quantify any specific physical process. It could have very well been the case (and *was* the case) that sources other than the ^{137}Cs were contributing to the measurements. However, as long as these other sources also satisfy the conditions for the Poisson distribution (i.e. independent, has mean rate), then the effect of these additional sources would simply be to modify the overall count rate of our experiment. But since our task was simply to produce final apparent counting rates of 1 sec^{-1}, 4 sec^{-1}, 10 sec^{-1} and 100 sec^{-1}, the experiment was insensitive to the actual source of each γ-ray. The results could be compromised if these background levels fluctuated appreciably throughout the course of the experiment. However, we have no reason to suspect such behavior.

5. CONCLUSIONS

We performed counting experiments of γ-ray emission from a ^{137}Cs sample, configured for four different rates. By a direct fit of the data, we found that the Poisson distribution adequately describes the results.

In addition, the measured data set was compared against similar sized samples that were *known* to be generated from the Poisson distribution. It was observed that the mean and the standard deviation of our measured data set was in accordance with these Poisson samples. We also considered whether the relation $\sigma = \sqrt{\mu}$, true for the theoretical Poisson distribution, was valid for our measured data. In comparison to the simulations, we concluded that the measured data satisfied the relation within statistical limits.

The theory of the Poisson distribution, in particular its assumption of the independence of events, then suggests that the decay process of each atom is independent of the other atoms in the sample.

[1] P. Bevington and D. Robinson, *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, 2003).

[2] J. lab staff, *Poisson statistics lab guide.*

Acknowledgments

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