Hydrogen wavefunctions in spherical coordinates

Here, $a = 0.53\mathring{A}$ is the Bohr radius.

$$|1,0,0\rangle \quad : \quad \psi = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$$

$$|2,0,0\rangle \quad : \quad \psi = \frac{1}{\sqrt{8\pi a^3}} \left(1 - \frac{r}{2a}\right) e^{-r/2a}$$

$$|2,1,-1\rangle \quad : \quad \psi = \frac{1}{8\sqrt{\pi a^3}} \left(\frac{r}{a}\right) e^{-r/2a} \sin(\theta) e^{-i\phi}$$

$$|2,1,0\rangle \quad : \quad \psi = \frac{1}{4\sqrt{2\pi a^3}} \left(\frac{r}{a}\right) e^{-r/2a} \cos(\theta)$$

$$|2,1,1\rangle \quad : \quad \psi = \frac{-1}{8\sqrt{\pi a^3}} \left(\frac{r}{a}\right) e^{-r/2a} \sin(\theta) e^{+i\phi}$$

$$|3,0,0\rangle \quad : \quad \psi = \frac{1}{\sqrt{27\pi a^3}} \left(1 - \frac{2}{3}\frac{r}{a} + \frac{2}{27}\left(\frac{r}{a}\right)^2\right) e^{-r/3a}$$

$$|3,1,-1\rangle \quad : \quad \psi = \frac{2}{27\sqrt{\pi a^3}} \left(1 - \frac{1}{6}\frac{r}{a}\right) \left(\frac{r}{a}\right) e^{-r/3a} \sin(\theta) e^{-i\phi}$$

$$|3,1,0\rangle \quad : \quad \psi = \frac{4}{27\sqrt{2\pi a^3}} \left(1 - \frac{1}{6}\frac{r}{a}\right) \left(\frac{r}{a}\right) e^{-r/3a} \cos(\theta)$$

$$|3,1,1\rangle \quad : \quad \psi = \frac{-2}{27\sqrt{\pi a^3}} \left(1 - \frac{1}{6}\frac{r}{a}\right) \left(\frac{r}{a}\right) e^{-r/3a} \sin(\theta) e^{+i\phi}$$

$$|3,2,-2\rangle \quad : \quad \psi = \frac{1}{162\sqrt{\pi a^3}} \left(\frac{r}{a}\right)^2 e^{-r/3a} \sin^2(\theta) e^{-2i\phi}$$

$$|3,2,-1\rangle \quad : \quad \psi = \frac{1}{81\sqrt{\pi a^3}} \left(\frac{r}{a}\right)^2 e^{-r/3a} \sin(\theta)\cos(\theta) e^{-i\phi}$$

$$|3,2,0\rangle \quad : \quad \psi = \frac{1}{81\sqrt{6\pi a^3}} \left(\frac{r}{a}\right)^2 e^{-r/3a} \left(3\cos^2(\theta) - 1\right)$$

$$|3,2,1\rangle \quad : \quad \psi = \frac{-1}{81\sqrt{\pi a^3}} \left(\frac{r}{a}\right)^2 e^{-r/3a} \sin(\theta)\cos(\theta) e^{+i\phi}$$

$$|3,2,2\rangle \quad : \quad \psi = \frac{1}{162\sqrt{\pi a^3}} \left(\frac{r}{a}\right)^2 e^{-r/3a} \sin^2(\theta) e^{+2i\phi}$$