Multi-Sourcing and Miscoordination in Supply Chain Networks

Kostas Bimpikis

Graduate School of Business, Stanford University, kostasb@stanford.edu

Douglas Fearing

McCombs School of Business, The University of Texas at Austin, shayane@stanford.edu

Alireza Tahbaz-Salehi

Columbia Business School, Columbia University, alirezat@columbia.edu

This paper studies sourcing decisions of firms in a multi-tier supply chain when procurement is subject to disruption risk. We argue that features of the production process that are commonly encountered in practice (including differential production technologies and financial constraints) may result in the formation of inefficient supply chains, owing to the misalignment of the sourcing incentives of firms at different tiers. We provide a characterization of the conditions under which upstream suppliers adopt sourcing strategies that are sub-optimal from the perspective of firms further downstream. Our analysis highlights that a focus on optimizing procurement decisions in each tier of the supply chain in isolation may not be sufficient for mitigating risks at an aggregate level. Rather, we argue that a holistic view of the entire supply network is necessary to properly assess and secure against disruptive events. Importantly, the misalignment we identify does not originate from cost or reliability asymmetries. Rather, firms' sourcing decisions are driven by the interplay of the firms' risk considerations with non-convexities in the production process. This implies that bilateral contracts that could involve under-delivery penalties may be insufficient to align incentives.

Key words: Multi-sourcing, disruption risk, supply chain networks.

1. Introduction

Despite their vital role in the production process in any modern economy, supply chain linkages have been increasingly recognized as a source of propagation and amplification of risk. Such a role was highlighted by two recent natural disasters in Asia, the 2011 Tōhoku earthquake (and the subsequent tsunami) in Japan and the severe flooding during the monsoon season in Thailand. As documented by several articles,[1] these events caused severe disruptions in a wide range of industries (most notably in the automotive and electronics industries), raising questions about firms' understanding of the architecture of their supply networks and the extent of their exposure to disruption risks. In addition, these disasters highlighted that many firms' efforts to hedge risk by

diversifying their supplier base and forming arborescent-like production chains were circumvented by the choices of other firms further upstream the chain, resulting in structures that featured significant overlaps. For instance, as observed by Automotive News (2012), Japanese "automakers thought they had hedged risk by diversifying Tier 1 and Tier 2 suppliers, assuming the supply base is in the form of a tree's roots, spreading out further down the line. But the supply chain was actually more diamond shaped, with rival suppliers turning to the same subsuppliers for parts."

Motivated by these observations, this paper studies the sourcing decisions of firms in a multi-tiered supply chain in the presence of supplier disruption risk. We argue that features of the production process that are commonly encountered in practice (including differential production technologies and financial constraints) coupled with the presence of disruption risk may result in the formation of inefficient supply chains. In particular, we show that upstream firms may find it optimal to adopt sourcing strategies that are sub-optimal from the points of view of firms further downstream. Identifying such potential causes of misalignment provides guidance on possible actions that downstream firms can take in order to alleviate the resulting inefficiencies.

We present the above insights in the context of a a three-tier supply chain model, consisting of a pair of suppliers at the top two tiers and a single downstream manufacturer. Procurement from the top tier firms is subject to disruption risk. To capture the chain's production process in a general and parsimonious manner, we model the mapping between the total number of units a firm receives from its upstream suppliers (the firm's inputs) and the volume of the intermediate good it produces (the firm's output) by a general, potentially non-linear, function h.

Our first result establishes that if h is concave or convex everywhere (e.g., due to economies or diseconomies of scale in the production technology, respectively), then the optimal level of supplier diversification from the points of view of the middle tier firms and the manufacturer coincide. In other words, in the absence of any non-convexities in the production process, the sourcing preferences of the firms throughout the chain are fully aligned.

Such an alignment of incentives, however, may break down if the production function h exhibits non-convexities. In fact, our main results show that depending on the shape of h, the optimal sourcing decisions of the suppliers may be sub-optimal from the downstream firm's perspective. We provide a characterization of whether and how such incentive misalignments may manifest themselves. In particular, we present conditions under which the manufacturer considers the middle tier firms' optimal sourcing profiles insufficiently or excessively diversified.

The divergence of preferences over the chain in the presence of non-convexities is a consequence of the fact that the joint sourcing decisions of the upstream firms not only affect their own profits, but also have a first-order impact on the risk profile of the procurement channels available to the downstream firms. More specifically in the context of our model, the correlation between the

number of units the manufacturer obtains from the two middle tier firms depends on the structure of the supply chain further upstream.

The intuition underlying our general results is most transparently understood by focusing on a special case in which the production function h is S-shaped, in the sense that it is convex for small input sizes, but is otherwise concave everywhere else (and hence, on average). As a way of insuring themselves against disruption risks, middle tier firms find it optimal to source from all potential upstream suppliers. Multi-sourcing from the same set of suppliers, however, makes the firms' sourcing profiles less diverse (and hence, more correlated). Consequently, the convexity of h for small input values implies that, under multi-sourcing, a large negative yield shock at *either* upstream firm may reduce the output of *both* middle tier firms simultaneously, hence significantly reducing the total quantity delivered to the manufacturer. The negative impact of such simultaneous disruptions on the manufacturer's bottom line may be large enough to outweigh any gains in reducing the probability of individual failures, making the chain sub-optimal from the manufacturer's perspective.

As a way of illustrating the relevance of the effect we study, we present two examples based on realistic features of the supply process that result in non-convexities in the production functions of upstream suppliers. First, following the recent work on the role of financial distress and bankruptcy in operational decisions (e.g., Yang, Birge, and Parker (2015), Yang and Birge (2017)), we explore how bankruptcy risk may induce suboptimal sourcing decisions from upstream suppliers. Furthermore, we show how the choice between production technologies with different yield ratios and fixed start-up costs, may elicit a similar effect.

Importantly, the inefficiencies we identify arise due to the interaction of the firms' risk considerations with their non-convex production functions. In particular and in contrast to most of the prior literature, the sub-optimality of the equilibrium supply chain is not driven by asymmetries in the suppliers' characteristics (such as procurement costs, disruption profiles, or information) or by competition effects. Rather, it is the consequence of the fact that the optimal levels of risk diversification from the points of view of firms at different tiers of the supply chain may be significantly different in the presence of non-convexities. We also remark that even though we illustrate our main insights by incorporating non-convexities into the production process, the non-trivial interaction between multi-sourcing and supply chain miscoordination is potentially present in any environment that features non-convexities and moral hazard.

In summary, our analysis highlights that strategies that focus on optimizing sourcing decisions in each tier in isolation may not be sufficient for mitigating risks at an aggregate level. Rather, a thorough understanding of the entire structure of the supply network is necessary to properly assess and secure against disruptive events. We also discuss a number of actions that firms can undertake

in order to address the misalignment of incentives between the different tiers of the supply chain, including segmenting or regionalizing the chain, standardizing component parts, or offering better financial terms in their bilateral procurement contracts. Furthermore, our results suggest that, in certain scenarios, downstream firms may need to explicitly include the sourcing decisions of their suppliers as part of the terms of their contracts or resort to (potentially more expensive) sourcing strategies, such as *direct sourcing.* This adds yet another dimension to the question of whether a firm should delegate or control its procurement process. Even in the absence of any information asymmetries, downstream firms may find it optimal to control their procurement process when the costs from potential disruptions are high.

Related Literature. Several recent papers study the management of disruptions in supply chains via multi-sourcing, mostly focusing on models that involve a two-tier setting. For example, Tomlin (2006) focuses on a model in which a firm employs two suppliers of different reliabilities, and explores the effectiveness of various disruption management strategies (such as, carrying excess inventory, sourcing from the more reliable supplier, or passive acceptance). Relatedly, Dada, Petruzzi, and Schwarz (2007) explore the procurement problem of a newsvendor with unreliable suppliers and find that supplier unreliability reduces the service level experienced by the customers.

Hopp and Yin (2006) consider arborescent supply chain networks and suggest guidelines on how to protect them against catastrophic failures. Their analysis suggests that an optimal policy makes use of safety stock inventory or backup capacity in at most one node in each path to the end costumer. More recently, Qi, Shen, and Snyder (2010) consider an integrated supply chain design problem in which the goal is to determine where to place retailers so that the cost of meeting customer demand while protecting against supplier disruptions is minimized. Furthermore, Federgruen and Yang (2009) study the question of optimal supply diversification when firms differ from one another in terms of their yield distributions, procurement costs, and capacity levels, whereas DeCroix (2013) considers an assembly system when component suppliers may be subject to disruption risk and provides efficient heuristic policies to compute the optimal order quantities.[2] Unlike these papers, which mainly study risk mitigation strategies from the point of view of a single firm, our objective is to examine the sourcing incentives of firms that belong to different tiers of a supply chain and identify general conditions under which the endogenously formed structures are (sub)optimal for the firms further downstream.

Also related is the recent work of Ang, Iancu, and Swinney (2017) who study optimal sourcing and disruption risk in multi-tier supply chains. As in Tomlin (2006), they focus on an environment with asymmetric supplier cost structures and reliabilities and provide a comparison of the performance of different supply chain structures. In contrast to these works, the divergence in the firms' sourcing incentives in our paper is not driven by supplier asymmetries. Rather, we identify a novel effect

that arises due to the interplay between the firm's risk considerations and the non-convexities in the production process, an effect that is present even in the absence of asymmetries. As we further elaborate in Section 5, this has implications with regards to how a firm can induce optimal sourcing decisions: when the misalignment originates from cost/reliability asymmetries at upstream tiers, simple under-delivery penalties may be sufficient to induce an optimal chain as they can be used to offset the advantage that an upstream supplier has over the rest. In contrast, we argue that bilateral contracts are inherently insufficient to deal with the types of misalignments we identify.

Our paper is also related to Babich, Burnetas, and Ritchken (2007) and Tang and Kouvelis (2011) who study competition in the presence of exogenously correlated disruption risks. Their results establish that in the presence of competition, a firm may be willing to forgo the benefits of dual-sourcing to differentiate itself from its competitor and benefit in the event of a favorable realization of uncertainty. In contrast to these works, we analyze how the incentives of firms at higher tiers would *endogenously* determine not only the structure of the supply chain, but also the extent of supplier risk correlation faced by downstream firms.

In addition to the above mentioned papers, a different strand of literature, including Bernstein and DeCroix (2004, 2006), studies decentralized and modular assembly systems in which final products are assembled from modules produced by higher level tiers in the presence of demand uncertainty. Belavina and Girotra (2015), on the other hand, study the effect of the supply network configuration on the efficacy of relational sourcing in ensuring socially responsible behavior.

Our paper also contributes to the smaller literature that focuses on the interplay between the structure of the supply chain and its performance. Corbett and Karmarkar (2001) focus on entry decisions and post-entry competition in multi-tier serial supply chains. They study how the cost structures of firms in different tiers determine the number of entrants and the level of competition, and as a result, affect prices and quantities in each tier. Majumder and Srinivasan (2008) focus on contracting in large acyclic supply chains and show that contract leadership affects the performance of the entire supply chain. More recently, Acemoglu et al. (2012, 2017) study whether and how supply chain linkages can function as a mechanism for propagation and amplification of disruptions in general supply networks, while assuming that all firms are competitive.

Also related is the recent work of Federgruen and Hu (2016), which studies sequential multi-market price competition in general supply networks. They show the existence and equivalence (in the sense of firms' profits) of equilibria under linear price-only contracts while abstracting away from yield or other types of uncertainty. We, on the other hand, mainly focus on the propagation of yield shocks in the chain and study the implications of multi-sourcing strategies of the firms.

The paper is also related to the works of Stiglitz (2010a,b) who studies the implications of financial market integration and argues that liberalization of capital markets may be undesirable

from a social welfare perspective. Finally, our work is related to a recent series of papers in the finance literature, such as Ibragimov, Jaffee, and Walden (2011), Wagner (2011) and Bimpikis and Tahbaz-Salehi (2014), which explore the trade-off between diversification and diversity of portfolios of financial institutions. The present work focuses on an entirely different context, sourcing in multi-tier supply chain when procurement is subject to disruption risk, and provides a comprehensive characterization of when inefficiencies may arise as a function of the underlying production process.

To summarize, our framework is among the very few recent papers that go beyond the standard two-tier supply chain and explore the implications of disruption risk in the formation of multi-tier chains. More importantly, and in contrast to most of the prior work that focuses on asymmetries between firms as the main source of any inefficiencies, we identify a novel effect that originates from the interplay between the firms' risk considerations and non-convexities in the production process: mitigating risk at an individual level may actually lead to a higher aggregate risk for the firms further downstream. Although simple amendments to bilateral contracts may be effective in restoring the distortions due to supplier asymmetries, e.g., under-delivery penalties, they fail to induce optimal supply chain structures in the presence of non-convexities.

2. Model

Consider a three-tier supply network, depicted in Figure 1, consisting of a single manufacturer, two identical tier 1 component suppliers (denoted by A and B and referred to as *suppliers* for short) and a pair of tier 2 part fabricators (denoted by 1 and 2 and referred to as *fabricators*).

The Manufacturer. The manufacturer has access to a technology that can transform each unit of an intermediate component (sourced from the component suppliers) into a unit of a final good that is subsequently sold to a downstream market. It purchases the required components at a unit price of p, which we assume to be fixed and exogenously given. Note that the supply chain illustrated in Figure 1 does not describe an assembly operation but rather the manufacturer's sourcing of one key component. In the presence of disruption risks, the manufacturer may find it optimal to source this component from multiple suppliers (as opposed to the potentially simpler option of sole-sourcing) in order to guarantee a sufficient number of units for its own customers.

The manufacturer's revenue depends on the total number of units delivered by its two suppliers. Specifically, conditional on the delivery of y units of the intermediate good, it obtains a revenue equal to $\phi(y)$, where ϕ is a (weakly) increasing function. The manufacturer's net profit is thus given by $\psi(y) = \phi(y) - py$. Given its general form, function ϕ is essentially a reduced-form mapping that can capture different features of the manufacturer's technology (e.g., its production cost or efficiency) or those of the downstream market (e.g., the price elasticity of demand). For example, if
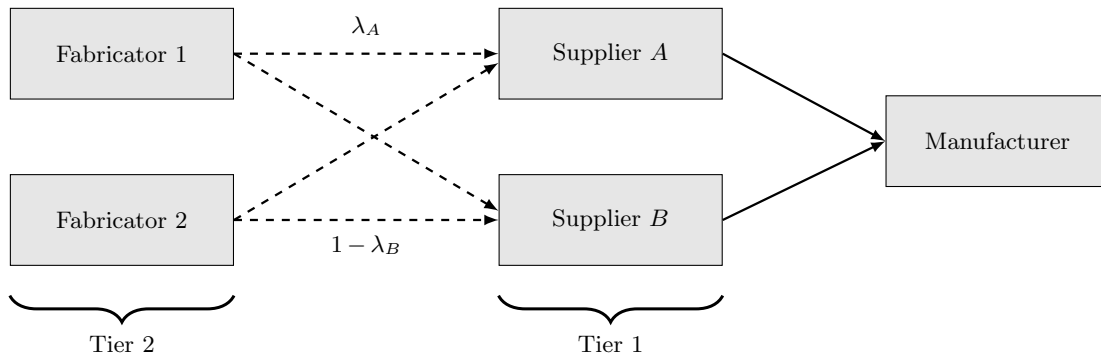
Figure 1   A three-tier supply network with a single manufacturer, a pair of tier 1 component suppliers, and a pair of tier 2 part fabricators.

demand for the final good is deterministic, inelastic, and equal to D and the manufacturer incurs quadratic production costs to convert the suppliers' output to the final good, then

$$\phi(y) = p_f \min\{y, D\} - c \cdot \min\{y, D\}^2,$$

where p_f denotes the price in the downstream market. If, on the other hand, demand in the downstream market is linear, in which case the final good's price as a function of the quantity sold is given by $p_f(y) = \alpha_f - \beta_f y$ for some constants $\alpha_f, \beta_f > 0$, and the manufacturer's production costs are negligible, then the revenue function ϕ takes the following form: $\phi(y) = (\alpha_f - \beta_f y)y$. Throughout the paper, we restrict attention to the revenue function ϕ being concave. Apart from being an assumption that is widely adopted in the literature (both examples above involve concave revenue functions), we view this as being practically relevant since the marginal revenue from selling an additional unit to the market is arguably often decreasing in the total volume of sales.

The Suppliers. Tier 1 suppliers produce the intermediate components by transforming parts received from the fabricators at tier 2. Each supplier contracts with the tier 2 firms to deliver up to a pre-specified number of units at a given price q. The key decision made by each supplier is how to allocate its order between the two fabricators. More specifically, supplier $j \in \{A, B\}$ chooses the fraction $\lambda_j \in [0, 1]$ of its total order that it places with fabricator 1. Thus, the case in which $\lambda_j \in \{0, 1\}$ corresponds to the supplier single-sourcing, whereas at the other extreme, $\lambda_j = 1/2$ corresponds to the case in which it divides its orders equally between the two fabricators.

We denote the suppliers' common production function by a general increasing function h, which represents the technology that transforms input parts into components delivered to the manufacturer. Thus, conditional on receiving x units from the fabricators, the net profit of a supplier is equal to $\pi(x) = ph(x) - qx$.

Even though we refer to h as a production function, it does not have to be interpreted as being tied to a physical production process. Rather, similar to the manufacturer's revenue function ϕ, function h simply represents a reduced-form mapping from the number of parts available to the supplier to the number of components it sells to the manufacturer. Thus, for instance, it may capture yield loss due to quality issues, or as we show in Section 4, it may represent other forms of technological, financial, or operational constraints.

The Fabricators. At the top of the supply chain, the part fabricators are responsible for producing parts to be used by the component suppliers. The key underlying assumption in our model is that the interaction between the fabricators and the suppliers is subject to disruption risk in terms of the quantity delivered (e.g., due to production yield uncertainty at the fabricator).

Formally, we let the random variable $z_i \in [0,1]$ capture the fraction of the total order quantity delivered by fabricator $i \in \{1,2\}$ which is proportionally allocated to the two suppliers. Thus, the total number of parts delivered to supplier j is equal to $x_j = \lambda_j z_1 + (1-\lambda_j)z_2$, where λ_j captures j's sourcing decision. Throughout the paper, we assume that z_1 and z_2 are independently distributed with a common probability density function f that has support over the unit interval.

As a final remark, we emphasize that we have deliberately restricted our attention to a fully symmetric environment, both in terms of the suppliers' production functions and the fabricators' (un)reliability. This assumption is made to ensure that our results are driven by the interplay between the firms' production functions and risk mitigation considerations, as opposed to any form of asymmetries between different firms. Clearly, the presence of such asymmetries may induce additional distortions in the firms' sourcing incentives.

3. Supply Chain Miscoordination

In this section, we study how the sourcing decisions of the suppliers affect the profits of the manufacturer and the performance of the supply chain. In particular, we show that depending on the shape of functions ϕ and h, the sourcing preferences of the suppliers may not align with those of the manufacturer. To exhibit the potential wedge in the firms' preferred sourcing strategies in the most transparent manner, we restrict the suppliers to choose between either sourcing from a single tier 2 firm or dividing their orders equally between the two fabricators, an outcome to which we simply refer as dual-sourcing. Furthermore, for the results we state in Section 3, we abstract away from the possibility of over-ordering in the chain as summarized in Assumption 1 below.

ASSUMPTION 1. *We assume that neither the downstream manufacturer nor the tier 1 suppliers over-order, i.e., the manufacturer's total order quantity from tier 1 is equal to its optimal order quantity in the absence of disruption risk.*

For example, for a differentiable revenue function ϕ, the manufacturer's total order quantity y is such that $\phi'(y) = p$, i.e., equal to the optimal order quantity in the absence of risk. Moreover, for expositional simplicity, we normalize the manufacturer's total order quantity y to two, which is split equally between the two tier 1 suppliers.

Assumption 1 allows for tractable analysis in this quite general framework in which we do not impose any assumptions on the suppliers' production function h or the fabricators' yield distribution (as captured by f). We relax this assumption both in the context of the Examples in Section 4 (that impose some structure on h) and in a setting we study in the Electronic Companion (that imposes realistic assumptions on the yield distribution) and show that our findings remain robust.

We start by deriving conditions on function h under which both tier 1 suppliers prefer to dual-source. Let Π_s and Π_d denote the expected profits of a component supplier under single-sourcing and dual-sourcing, respectively. It is immediate that the expected profit of the firm under single-sourcing is equal to

$$\Pi_s = \int_0^1 \left[ph(z) - qz \right] f(z) dz,$$

whereas if the supplier sources equally from both fabricators, its expected profit is

$$\Pi_d = \int_0^1 \int_0^1 \left[ph\left(\frac{z_1 + z_2}{2} \right) - q(z_1 + z_2)/2 \right] f(z_1) f(z_2) dz_1 dz_2.$$

Comparing the two expressions above implies that tier 1 suppliers strictly prefer dual-sourcing to single-sourcing if and only if

$$\Pi_s - \Pi_d = p \int_0^1 \int_0^1 \Delta_{z_1 z_2} f(z_1) f(z_2) dz_1 dz_2 < 0, \tag{1}$$

where

$$\Delta_{zz'} = \frac{1}{2} \left(h(z) + h(z') \right) - h\left(\frac{z + z'}{2} \right).$$

Thus, inequality (1) provides a necessary and sufficient condition under which tier 1 suppliers dual-source in equilibrium. Given the definition of $\Delta_{zz'}$, it is immediate that the curvature of h plays a central role in the optimal level of diversification from the points of view of the suppliers. For instance, the suppliers strictly prefer to source from both fabricators as long as h is strictly concave, whereas they are indifferent between single- and dual-sourcing when h is linear.

We next determine the conditions under which the manufacturer obtains a higher profit if the suppliers dual-source. Let Φ_s and Φ_d denote the expected profits of the manufacturer when both tier 1 suppliers employ single-sourcing and dual-sourcing strategies, respectively.[3] We have,

$$\Phi_s = \int_0^1 \int_0^1 \psi(h(z_1) + h(z_2)) f(z_1) f(z_2) dz_1 dz_2$$

and

$$\Phi_d = \int_0^1 \int_0^1 \psi(2h((z_1 + z_2)/2))f(z_1)f(z_2)dz_1 dz_2,$$

where recall that $\psi(y) = \phi(y) - py$. Therefore, the manufacturer would strictly prefer the tier 1 suppliers to dual-source if and only if

$$\Phi_s - \Phi_d = \int_0^1 \int_0^1 \left[\psi\Big(h(z_1) + h(z_2) \Big) - \psi\Big(h(z_1) + h(z_2) - 2\Delta_{z_1 z_2} \Big) \right] f(z_1)f(z_2)dz_1 dz_2 < 0. \quad (2)$$

As in the case of the suppliers, the above inequality captures the fact that the optimal level of diversification from the point of view of the manufacturer depends on the shapes of functions h and ϕ. More importantly, however, comparing inequality (2) with (1) enables us to determine whether the sourcing incentives of the firms at different tiers of the supply chain coincide with one another. In particular, it implies that if $\Delta_{z_1 z_2}$ has the same sign for all pairs $z_1 \neq z_2$, then inequality (2) is satisfied if and only if (1) holds, regardless of the shape of ϕ, leading to the following result:

PROPOSITION 1. *If the production function h is concave, then the suppliers and the manufacturer are better off when tier 1 firms dual-source. On the other hand, if h is convex, then all parties find it optimal for the suppliers to single-source.*

In other words, there is no wedge between the firms' optimal degree of diversification as long as the production function is concave or convex everywhere. Such an alignment of sourcing incentives, however, may not necessarily hold when function h does not satisfy the conditions of Proposition 1. In fact, as our next result illustrates, the optimal supply chain structure from the manufacturer's perspective may not coincide with the one that the suppliers prefer.

PROPOSITION 2. *Suppose that the following conditions are satisfied:*
(a) The production function h is strictly concave on average, that is,

$$\int_0^1 \int_0^1 \Delta_{z_1 z_2} f(z_1)f(z_2)dz_1 dz_2 < 0.$$

(b) There exists $\bar{x} > 0$ such that h is strictly convex for all values below $\bar{x}$, that is,

$$\Delta_{z_1 z_2} > 0 \ \ for \ all \ \ z_1, z_2 \leq \bar{x}.$$

Then, there exists a concave revenue function ϕ such that tier 1 suppliers find it optimal to dual-source, whereas the manufacturer would be better off if they employ a single-sourcing strategy.

Thus, even though the suppliers find it optimal to source equally from each fabricator, the manufacturer prefers a less diversified sourcing strategy at the higher tiers. The intuition underlying Proposition 2 can be understood by comparing the costs and benefits of dual-sourcing by the tier

1 suppliers from the point of view of the manufacturer. When the suppliers diversify their own sources of input (that is, when $\lambda = 1/2$), the likelihood that each single supplier faces a shortage is reduced. This is clearly beneficial not only from the point of view of the suppliers themselves, but also from that of the manufacturer: such a diversified strategy would also increase the expected number of units delivered to the manufacturer from each supplier.

The manufacturer, however, also bears an implicit cost when tier 1 firms decide to dual-source. Given the concavity of the revenue function ϕ, the manufacturer's expected profit is not additively separable in the number of units it obtains from its two suppliers. Hence, it is critical for the manufacturer to obtain enough components from at least one supplier. Yet, when tier 1 firms choose $\lambda_A = \lambda_B = 1/2$, the likelihood that their outputs are low simultaneously increases due to the *overlap* in their procurement channels. Specifically, under dual-sourcing, a large disruption at *either of the fabricators* may reduce the output of *both* suppliers significantly. In contrast, had a supplier used a single-sourcing strategy, it would have only been exposed to the risk of a severe disruption at the sole fabricator it sources from.

The assumptions underlying Proposition 2 have straightforward interpretations. Condition (a), which is the same as inequality (1), essentially implies that the production function h exhibits enough concavity on average. This assumption guarantees that tier 1 suppliers find it optimal to dual-source, as such a sourcing strategy reduces the disruption risk they face. In other words, assumption (a) captures the standard benefit of risk diversification for the suppliers. Condition (b), on the other hand, states that even though dual-sourcing reduces risk on average, that is not the case if the fabricators face severe shocks. In other words, under dual-sourcing, the realization of a small enough z would lead to severe drops in the output of both suppliers. This assumption plays a significant role in creating the wedge between the incentives of firms at different tiers. Note that, as already shown in Proposition 1, if $\Delta_{z_1 z_2} < 0$ for all $z_1 \neq z_2$, then the sourcing preferences of the suppliers would be fully aligned with those of the downstream manufacturer.

The concavity of the revenue function ϕ guarantees that the manufacturer finds simultaneous severe disruptions at both suppliers costly. More specifically, even though dual-sourcing reduces the risk faced by the suppliers, the resulting benefit for the manufacturer does not justify the cost associated with the risk of simultaneous severe disruptions at both channels. Note that if ϕ is linear, then the preferred structure of the supply chain from the point of view of the manufacturer would coincide with that of the suppliers, regardless of the form of the production function h.

To summarize, Proposition 2 illustrates that depending on the shape of function h, tier 1 suppliers (when they have access to the same tier 2 fabricators) may choose a sourcing strategy that is too diversified from the point of view of the manufacturer, and thus result in overlapping procurement channels as opposed to an arborescent supply chain.[4] As we show in Subsection 3.1, the conditions

of the proposition are satisfied by a wide array of production functions. This, however, is not the only way that misalignments of incentives across the chain may manifest themselves. Rather, as our next result illustrates, under a different set of conditions, the manufacturer may prefer the suppliers to dual-source, even though the suppliers find it optimal to source from a single fabricator.

PROPOSITION 3. *Suppose that the following conditions are satisfied:*
(a) The production function h is strictly convex on average, that is,

$$\int_0^1 \int_0^1 \Delta_{z_1 z_2} f(z_1) f(z_2) dz_1 dz_2 > 0.$$

(b) There exists $\bar{x} > 0$ such that h is strictly concave for all values below $\bar{x}$, that is,

$$\Delta_{z_1 z_2} < 0 \quad \text{for all} \ \ z_1, z_2 \leq \bar{x}.$$

Then, there exists a convex revenue function ϕ such that tier 1 suppliers find it optimal to single-source, whereas the manufacturer would be better off if they employ a dual-sourcing strategy.

The intuition for the above result is similar to that of Proposition 2. In particular, the assumption that the production is convex on average guarantees that the suppliers find it optimal to rely on a single fabricator. However, the concavity of h for small values of z, alongside with the assumption that the revenue function is also concave, implies that the manufacturer would benefit from risk-diversification at higher tiers.

3.1. Non-Convexities and Miscoordination

The juxtaposition of Propositions 1–3 highlights that not only the presence of non-convexities in the supply chain can create a wedge between the incentives of firms at different tiers, but also the exact nature of such non-convexities plays a first-order role in determining the type of misalignments that may arise.

In order to further clarify how the shape of the production function affects the optimal structure of the supply chain from the points of view of firms at different tiers, it is instructive to focus on specific functional forms. In particular, we assume that the yield shocks are uniformly distributed over the unit interval and focus on production functions that are symmetric around their unique inflection point $x^* \in (0, 1)$.[5]

Figure 2 depicts one such function: it is strictly convex for $x < x^*$ and has a negative curvature if $x > x^*$. Such an S-shaped h may, for example, represent a production technology where quality/yield improves with scale, but these benefits are overcome by diminishing returns associated with management complexity at large volumes (e.g., Williamson (1967), McAfee and McMillan (1995)). Given the symmetry of such a convex-concave production function around its inflection
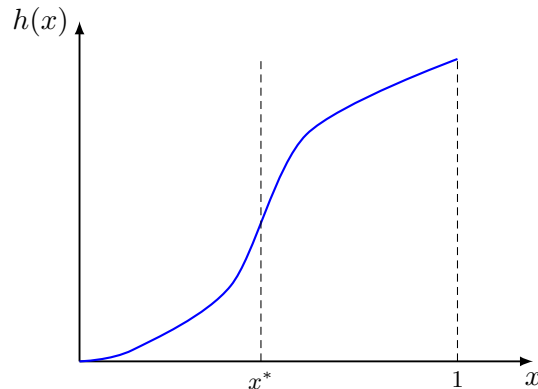
Figure 2   An example of a convex-concave function.

point, a value of $x^* < 1/2$ means that $h(x)$ is concave on average, even though it is strictly convex at small input sizes. Hence, such a function satisfies the conditions of Proposition 2, leading to the following result:

COROLLARY 1. *Suppose that h is convex-concave with inflection point $x^* \in (0,1)$. There exists a concave function ϕ such that*

  (i) *If $x^* < 1/2$, then the suppliers find it optimal to dual-source, whereas the manufacturer would be better off if the suppliers employ a single-sourcing strategy.*
  (ii) *If $x^* > 1/2$, then all parties find it optimal for the suppliers to single-source.*

Part (i) states that when the concave segment of the production function is larger, the effect of dual-sourcing for tier 1 suppliers is unambiguously positive. However, the convexity of h for small values of x implies that, under dual-sourcing, the manufacturer's loss from an increase in the likelihood of simultaneous disruptions on its procurement channels outweigh the benefits of diversification. In other words, the supply chain would exhibit excessive diversification from the point of view of the manufacturer. Part (ii) then shows that, for such a convex-concave function h, this is the only type of misalignment that can arise. Note that similar arguments can characterize the nature of preference misalignment when the curvature of the production function changes sign from negative to positive at x^* (akin to Proposition 3).

Table 1 below summarizes our findings in terms of the shapes of h and ϕ. In particular, it states whether and how these functions' curvature properties can create a wedge between the optimal levels of diversification from the points of view of firms at different tiers of the supply chain.

We end this discussion by arguing that even though our results were presented in the context of a three-tier supply chain consisting of a pair of firms at the top two tiers, the model's underlying insights remain valid if each tier consists of multiple firms. As long as firms in each tier need to rely on a common set of potential upstream firms, the presence of non-convexities can lead to misalignments between their desired sourcing decisions and those of their downstream customers.

	ϕ: concave	ϕ: convex
h: convex	perfect alignment (single-sourcing)	perfect alignment (single-sourcing)
h: concave	perfect alignment (dual-sourcing)	perfect alignment (dual-sourcing)
h: convex-concave	potential misalignment (excessive diversification)	potential misalignment (insufficient diversification)
h: concave-convex	potential misalignment (insufficient diversification)	potential misalignment (excessive diversification)

Table 1 Alignment or misalignment of sourcing preferences as a function of h and ϕ's curvatures. The directions of potential misalignments are indicated from the manufacturer's perspective.

4. Examples

In this section, we present two examples to illustrate how different features of the procurement process may result in non-convexities and subsequently lead to a misalignment of diversification preferences between the suppliers and the manufacturer. The first example examines sourcing in the presence of bankruptcy risk whereas the second considers the choice between production technologies with different yields and start-up costs.[6] Each of the examples extends beyond the model analyzed in Section 3 to demonstrate that the insights developed are more broadly applicable.

4.1. Sourcing in the Presence of Bankruptcy Risk

Consider the model presented in Section 2 and suppose that the suppliers can transform each unit of the input sourced from the fabricators to a unit of the intermediate good. However, in order to initiate production, each supplier has to pay a fixed cost of v. If it is unable to cover v, the firm has to cease production. On the other hand, once the supplier pays v, it remains in business and obtains a continuation value equal to R. This value — which is obtained in addition to any revenue the firm obtains from selling the output to the manufacturer — captures the value of staying in business or that of the firm's other operations.[7]

Thus, even though tier 1 suppliers can transform their inputs to the intermediate good one-for-one, their effective production function is given by

$$h(x) = \begin{cases} 0 & \text{if } x < x_d \\ x & \text{if } x_d \le x, \end{cases} \tag{3}$$

where $x_d = v/(p-q)$ is the minimum number of parts required from the fabricators to allow for production. Figure 3 depicts this function.

Given the shape of the above production function, one can show the following result (even when the manufacturer may over-order, i.e., when Assumption 1 is relaxed).
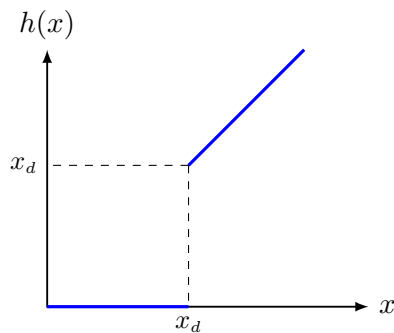
Figure 3   The suppliers' effective production function in the presence of bankruptcy risk.

PROPOSITION 4. *Suppose that $x_d < 1/2$ and that the suppliers can choose any sourcing strategy $\lambda \in [0,1]$. Then, there exists $\bar{R}$ and a concave ϕ such that for $R > \bar{R}$, the suppliers choose a sourcing strategy that is too diversified from the manufacturer's perspective.*

The intuition underlying the above proposition is similar to our earlier results (and specifically Proposition 2): by dual-sourcing, each supplier can reduce the likelihood of bankruptcy, an event that it finds very costly. A side effect of such a strategy, however, is that the two suppliers' procurement channels exhibit more overlap and their outputs become more correlated; hence, the likelihood of the event that they cease production simultaneously increases. This means that the manufacturer would have obtained higher expected profits had the suppliers chosen $\lambda \neq 1/2$.[8]

One would expect the inefficiency identified above to be a first-order concern in industries where production involves sizable fixed costs and/or firms operate with high leverage or under high levels of financial uncertainty. In such environments, downstream firms could follow a number of steps to alleviate the adverse effects associated with their suppliers' sourcing decisions. First, it is straightforward to see that the benefit upstream firms derive from diversifying their sourcing strategy (and thus forming an inefficient supply chain on aggregate) is decreasing with x_d. In turn, this suggests that actions which aim at reducing x_d weaken the suppliers' incentives to follow a dual sourcing strategy (which is potentially harder to manage and monitor). Concretely, the manufacturer can directly subsidize the suppliers' fixed costs or renegotiate the terms of their contractual arrangements to offer higher margins (at least for low quantity levels) and, thus, indirectly incentivize its suppliers to follow sourcing strategies that are optimal from its point of view.

As an alternative to (directly or indirectly) subsidizing the suppliers' fixed operating costs to reduce their risk of bankruptcy, the manufacturer can formally or informally commit to offer financial assistance to the suppliers in the event of a looming bankruptcy in the form of a low-interest loan or bailout funds. The consequence of such a commitment would be to reduce the

supplier's insurance motive in applying a dual-sourcing strategy, and thus result in a supply chain structure that is preferred by the manufacturer.

The discussion above may help explain GM's recent (and quite uncharacteristic) strategy reversal: according to reports in the Wall Street Journal (Wall Street Journal (2016a), GM pledged to give the opportunity to a subset of its parts makers that operate under rising material costs and unstable financial environments to renegotiate their contract terms periodically to "avoid production disruptions by ensuring suppliers don't succumb to financial strain."

4.2. Production Technology Choice

In this example, we assume that each supplier has access to two distinct production technologies, which we call *manual* and *automated* respectively. The manual production is less efficient than the automated production in the sense that it has a lower yield, for instance due to quality losses. Formally, we assume that $\mu_m < \mu_a$, where μ_t is the production yield for technology $t \in \{a, m\}$. A prototypical example of the yield impact of technology decisions is in semiconductor manufacturing, where yield is highly sensitive to contamination risks (and automation can significantly improve yields). Despite the fact that the automated technology is more efficient than the manual technology, it requires a fixed setup cost of v_a. In order to focus on the non-convexities introduced by technology choice, we assume that both technologies have identical marginal costs of production c that we normalize to zero. We also assume that the automated technology is capacity constrained with capacity denoted by w_a. Finally, we assume that $p\mu_m > q$, which guarantees that production is always justified, even if the suppliers only employ the manual production technology.

First, we characterize the production level of each component supplier as a function of the aggregate quantity it receives from the fabricators. Let x denote the total number of units that is delivered to a supplier. Given the setup cost of the automated technology, there exists a quantity x_m such that for $x \leq x_m$, the supplier would only employ manual production. Indeed,

$$x_m = \frac{v_a}{p(\mu_a - \mu_m)}.$$

On the other hand, given the higher yield of the automated production technology, the supplier finds it optimal to utilize it as soon as paying its fixed setup cost v_a is justified. In particular, for any $x \in (x_m, x_a]$, the supplier would only use the automated technology, where $x_a = w_a/\mu_a$ is the number of inputs that would make the capacity constraint of the automated technology bind. Finally, beyond this point, the supplier utilizes both production technologies side-by-side. Note that in view of the fact that $p\mu_m > q$, the supplier employs all available inputs to meet the

manufacturer's order. Hence, to summarize, the optimal production function of a supplier that has access to the two technologies discussed above is given by

$$h(x) = \begin{cases} \mu_m x & \text{if } x \le x_m \\ \mu_a x & \text{if } x \in (x_m, x_a] \\ \mu_m(x - x_a) + \mu_a x_a & \text{if } x > x_a, \end{cases} \tag{4}$$

To simplify the analysis and exposition, we normalize the manufacturer's optimal order to each supplier *in the absence of disruption risk* to be equal to one, i.e., $\mu_m(1 - x_a) + \mu_a x_a = 1$ (this is without loss of generality, since we allow the manufacturer to order more than one to mitigate the supply uncertainty). Further, we assume that $x_a \le 1/2$. Note that as depicted in Figure 4, the production function $h(x)$ is linear for $x < x_m$, whereas it is concave for $x > x_m$. As discussed in Section 3, this non-convexity of the production function plays a crucial role in our results.
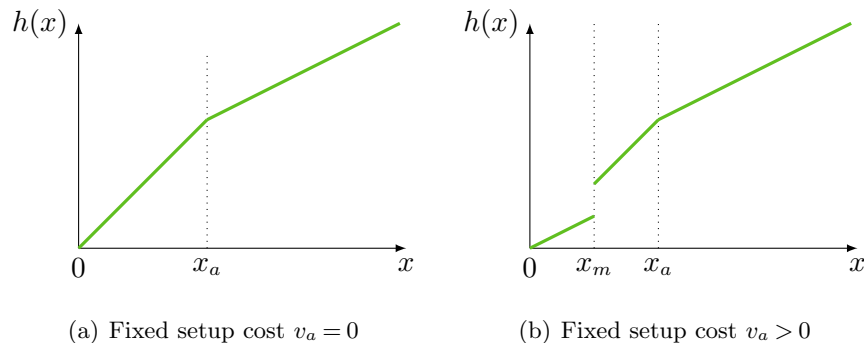

(a) Fixed setup cost $v_a = 0$ (b) Fixed setup cost $v_a > 0$

Figure 4 The output of a given supplier as a function of the number of units delivered by the fabricators.

As in the general model, we assume that the disruption risk in the supply chain is represented by independent and identical distributions on the fraction of the total order quantities delivered by the fabricators, z_1 and z_2 respectively. More specifically, we assume that z_1 and z_2 are uniformly distributed over $[0,1]$. Recall that each supplier receives a payment of p for any unit delivered to the manufacturer. Hence, if a supplier sources fractions λ and $1 - \lambda$ of its inputs from fabricators 1 and 2, respectively, its expected profit is equal to

$$\Pi(n, \lambda) = -qn\mathbb{E}[z] - v_a \mathbb{P}(\lambda n z_1 + (1 - \lambda)n z_2 > x_m) + p \int_0^1 \int_0^1 h(\lambda n z_1 + (1 - \lambda)n z_2) dz_1 dz_2, \tag{5}$$

where n is the total number of units ordered by the supplier. We allow the manufacturer to over-order in anticipation of potential disruptions, i.e., set $n > 1$, but we assume that the suppliers do not over-order. Although the latter assumption is necessary for analytical tractability (as we already allow the manufacturer to over-order), it is plausible when the margins for upstream firms are small and, thus, over-ordering is relatively expensive for them. Then, the first term on the right-hand side of (5) is simply the expected cost of production, whereas the second term captures the expected

setup cost of the automated technology. Finally, the last term represents the expected revenue of the supplier. As equation (5) shows, the fact that the fabricators are subject to disruptions means that the expected profit of each supplier depends on its sourcing strategy. In particular, suppliers choose $\lambda \in [0,1]$ in order to maximize Π. We have the following result:

PROPOSITION 5. *The suppliers prefer to dual-source regardless of the total order size and the fixed setup cost; that is, $\Pi(n, 1/2) \geq \Pi(n, \lambda)$ for all n and all $\lambda \in [0,1]$.*

Thus, both suppliers find it optimal to source equally from the two fabricators. This is simply again a consequence of the standard benefits of diversification: dual-sourcing reduces the aggregate risk that each supplier is exposed to. More specifically, it maximizes the likelihood that the supplier obtains enough inputs from the fabricators to employ the more productive automated technology. However, this does not necessarily imply that dual-sourcing by the suppliers is optimal from the point of view of the manufacturer. In fact, as our next result shows, the incentives of the manufacturer and the suppliers are not always fully aligned.

The manufacturer's expected profit as a function of the suppliers' sourcing decisions is equal to

$$\Phi(n, \lambda_A, \lambda_B) = \mathbb{E}\psi\Big(h(\lambda_A n z_1 + (1 - \lambda_A) n z_2) + h(\lambda_B n z_1 + (1 - \lambda_B) n z_2) \Big), \qquad (6)$$

where λ_A and λ_B are the sourcing decisions of the tier 1 firms and the expectation is with respect to the supply shocks z_1 and z_2. We have the following result:

PROPOSITION 6. *Let the manufacturer's total order size to its tier 1 suppliers be equal to $2n$. Then,*

  (i) *If there is no cost involved in setting up the automated technology, i.e., $v_a = 0$, the suppliers dual-sourcing is optimal from the manufacturer's perspective, as long as $1 \leq n \leq 2 - x_a$;*

  (ii) *On the other hand, if $v_a > 0$, complete dual-sourcing by the suppliers is not optimal for the manufacturer; that is, for all n, there exists $\lambda \in [0,1]$ such that $\Phi(n, \lambda, 1 - \lambda) > \Phi(n, 1/2, 1/2)$.*

The two parts of Proposition 6 that are directly analogous to Propositions 1 and 2 illustrate the insights we developed in Section 3: in the absence of non-convexities in the suppliers' production functions, i.e., when $v_a = 0$ (Figure 4(a)), the sourcing incentives of firms at different tiers are fully aligned. On the other hand, this no longer holds in the presence of non-convexities (as in Figure EC.2) and as a result, the optimal supply network structure from the points of view of the suppliers does not coincide with that of the manufacturer. Again, as is the case for the model of Section 2, even though the likelihood of under-delivery by each supplier is reduced, dual-sourcing implies that each supplier under-delivers at the same states of the world at which the other one cannot meet its obligations. The resulting increase in the likelihood of such simultaneous "failures" reduces the profits of the manufacturer to the extent that it may outweigh the benefits of dual-sourcing. It is

the presence of this implicit cost that lies behind the discrepancy between the incentives of firms at different tiers: even though the manufacturer finds the simultaneous under-delivery by both suppliers very costly, the suppliers do not fully internalize the effect of their sourcing decisions.

As in the previous example, it is straightforward to see that the suppliers' incentive to choose a dual-sourcing strategy that is suboptimal from the point of view of the manufacturer weakens as the fixed setup cost v_a of the automated technology decreases. In addition, manufacturers could directly invest in their suppliers' R&D efforts to increase the productivity of their technologies (captured in the context of this example by μ_a) or decrease their setup costs. Such investments could not only result in short-term cost savings (due to a productivity increase) but also incentivize suppliers to follow the manufacturer's preferred sourcing strategy.

5. The Role of Contracts

Our results in the previous sections establish that the presence of non-convexities in the production process can result in misaligned preferences at different tiers, and thus leading to the formation of sub-optimal supply chains from the point of view of the manufacturer. In this section, we discuss the role that contracts play in our results.

In particular, in addition to the presence of non-convexities in the production function h, the emergence of a wedge between the sourcing preferences of different firms in our model relies on two key assumptions on the types of contracts that firms can write with one another.

First, note that the interaction between the manufacturer and its suppliers in our model is subject to a form of moral hazard: even though the sourcing decisions of the suppliers have a first-order bearing on the manufacturer's profits, these decisions are assumed to be non-contractable. Such moral hazard problems would arise if, for example, the upper echelons of the chain are not fully observable to the downstream firm. If the downstream firm can observe, verify, and contract on the sourcing decisions of its suppliers (that is, λ_A and λ_B), it can induce them to adopt manufacturer-optimal sourcing decisions via λ-contingent contracts.

The mere presence of moral hazard, however, does not necessarily imply that the resulting supply chain structure is suboptimal from the manufacturer's perspective. Note that even though the manufacturer might not be able to extract all the surplus from the suppliers, in general — and in the presence of complete contracts — it may still be able to induce the right sourcing decisions by providing them with enough rents. However, the key observation is that the optimal profit-maximizing contract may require cross-contingencies, in the sense that the per unit price paid to supplier i should not only depend on the number of units delivered by that supplier, but also on the number of units delivered by supplier $j \neq i$. In other words, simply employing *bilateral* non-linear (no matter how complex) contracts may not induce the right sourcing decisions by the suppliers at

minimal cost. To see this, note that in the absence of cross-contingent contracts, the manufacturer needs to make deductions about the sourcing decisions of supplier i based solely on the value of x_i, the number of units delivered by that supplier. However, given that the suppliers source from a common set of fabricators, the correlation between x_A and x_B provides the manufacturer with valuable information about the underlying sourcing decisions λ_A and λ_B. Including this information in the contracts enables the manufacturer to align the suppliers' incentives with its own at a potentially lower cost (however, the resulting contracts may be very difficult to implement or enforce).[9] Thus, it is the presence of moral hazard coupled with the incompleteness of contracts that leads to the divergence of incentives between firms at different tiers.[10]

More generally, this observation illustrates the potential insufficiency of bilateral contracts for mitigating risks throughout the chain. In particular, contracts whose terms are contingent only on individual outputs may be insufficient for dealing effectively with supply risk at higher tiers. Contrast this with the case when suboptimal supply chains originate from asymmetries in the suppliers' cost or reliability profiles. Then, *bilateral* penalty functions allow for supply chain coordination (see also Ang, Iancu, and Swinney (2017)). This further implies that the presence of non-convexities in the production process may necessitate the use of different and potentially more complex and expensive sourcing strategies as it is typically challenging to monitor and enforce cross-contingent contracts. Our results thus provide a rationale other than information asymmetries (see Kayis, Erhun, and Plambeck (2013)) for why direct sourcing may be more attractive when the risk and/or costs associated with disruptive events is high.[11]

6. Concluding Remarks

This paper argues that in the presence of non-convexities, the sourcing preferences of firms at different tiers of a supply chain may not fully coincide, leading to the formation of inefficient chains. Our results highlight that strategies focusing on optimizing sourcing decisions in each tier in isolation may not be sufficient for mitigating risks at an aggregate level. As we argue below, properly assessing and securing against disruptive events affecting a firm's production process necessitates a concerted effort at collecting detailed information about the entire structure of the supply network, developing risk metrics that account for network propagation effects, and providing an appropriate set of incentives for the sourcing decisions of other firms further upstream.

Mapping the Supply Chain Network. As already argued, our results emphasize that the sourcing decisions of a firm's suppliers at various tiers can have first-order implications for the firm's exposure to disruption risk. As such, the first step in any effort to address such misalignment of incentives is to map out the supply chain's detailed network structure. This view is consistent with the practice of various large manufacturers over the past few years of building detailed databases on their

suppliers (including relatively small upstream companies) and gathering information on the types of products they manufacture, the locations of their plants, and how they source their inputs (Wall Street Journal (2016b), Simchi-Levi et al. (2014), and Choi et al. (2015)). Such efforts can also assist with identifying alternatives for further diversifying disruption risks throughout the chain.

Measuring Risk Exposure. Recently, efforts to improve supply chain resilience have generated new metrics to quantify risk exposure by taking into account the entire structure of a firm's supply network. For example, Simchi-Levi et al. (2014, 2015) develop a risk-exposure model that aims to assess the overall impact of a supplier's disruption on the chain. Our findings illustrate that, in addition to accounting for the supply network interactions and the market structure, accounting for the firms' operational environment (as captured by their production functions) can result in risk-exposure metrics that provide a more comprehensive picture of the risks firms are exposed to.

Forming More Resilient Chains. Many efforts, such as those summarized above, are aimed at identifying and potentially quantifying sources of risk in a firm's existing supply network. Mitigating a firm's risk exposure, however, may require a series of steps at the strategic level to ensure the formation of more resilient chains. Note that our findings illustrate that supply chains that form as the result of firms' individual sourcing decisions may exhibit too much overlap from the point of view of the downstream manufacturer. Our results thus provide an alternative rationale for resorting to *segmentation* and *regionalization* strategies. Chopra and Sodhi (2014) argue that adoption of such strategies can be effective in reducing supply chain fragility by containing the adverse effects of a disruptive event to a small subset of product lines or geographic regions. Our model suggests that, in addition to the benefits identified by Chopra and Sodhi (2014), adoption of segmentation and regionalization strategies may also eliminate the possibility of the formation of overly-diversified supply chains to begin with. More concretely in the context of our model, such strategies would ensure that Tier 1 suppliers have access to different sets of Tier 2 fabricators (e.g., because they source from different regions), thus ruling out the case identified in Proposition 2.

In addition, large manufacturers have begun to standardize parts used in multiple product lines in order to enable shifting production among their suppliers in the event of an emergency (Wall Street Journal (2016b) and Sáenz and Revilla (2014)). For example, Sáenz and Revilla (2014) document the initiatives that Cisco undertook to reconfigure its supply chain by adding flexible manufacturers capable of producing a large set of Cisco's products at short notice. Interpreted in the context of our model and results, note that one of the main drivers of the inefficiency we identify is the fact that disruptive events in Tier 2 affect the entirety of the suppliers in Tier 1 (due to the overlap in their procurement channels that results from equilibrium decision making). However, if parts were standardized and moving production from one supplier to another could be done smoothly and in short notice, then transshipping a Tier 1 supplier's (insufficient) input to another Tier 1 supplier would mitigate the adverse effects of this inefficiency.

Aligning Incentives via Better Contracts. In addition to the efforts summarized above, downstream manufacturers are increasingly realizing the need for providing better contract terms to their suppliers to decrease the likelihood of production disruptions (see also a related discussion in Section 4). For example, in a somewhat unusual move, General Motors pledged to offer to a subset of its suppliers, especially, those operating in regions of high economic uncertainty, the opportunity to renegotiate their contract terms periodically in order to reduce their risk of failure and bankruptcy (Wall Street Journal (2016a), Supply Chain Digest (2016)).

Furthermore, Section 5 argues that firms could sustain coordination (at least to some extent) in their supply chains by leveraging cross-contingent contracts. Although enforcing and monitoring such contracts could be prohibitively costly, spot market trading is a potential alternative (as the resulting price depends on the aggregate supply of the good and is not bilaterally determined) that is practically implementable and widely adopted in some industries, e.g., semiconductors. Thus, our findings provide another explanation for the proliferation of such markets in addition to asymmetric information among market participants (e.g., Mendelson and Tunca (2007)).

Finally, aside from efforts to contain the impact of disruptive events (e.g., by regionalizing the supply chain or enabling quick shifting of production among a firm's suppliers) or providing better financial terms to reduce the likelihood of supplier failure, firms are increasingly engaging in close collaborations with their supply chain partners and in some cases they directly manage relationships with subsets of their upstream suppliers. For example, as Choi and Linton (2011) argue, Honda and Toyota directly negotiate contracts with select upstream vendors and explicitly request that their direct suppliers use those vendors exclusively and execute the negotiated terms (thus effectively implementing some form of direct sourcing).

To conclude, we emphasize that, in order to illustrate the role of non-convexities in a parsimonious fashion, our model abstracted away from other potentially relevant issues, such as competition, supplier asymmetries, more intricate contracts, etc. Although incorporating such important features into a unified model would result in a more comprehensive framework for examining the formation of multi-tier supply chains and is a natural next step for future research, we chose to focus on the interplay between non-convexities and the firms' risk considerations as the only source of any misalignment between their sourcing preferences. This enables us to unambiguously attribute the formation of suboptimal chains to how firms at different tiers in the supply chain decide to deal with the disruption risk they face.

Appendix : Proofs

Proof of Proposition 1

If the production function h is concave, then $\Delta_{zz'} < 0$ for all $z, z' \in [0, 1]$. It is thus immediate that inequality (1) is satisfied, implying that the suppliers choose a dual-sourcing strategy. On the other hand, in light of Assumption 1, we have for all $z, z' \in [0, 1]$:

$$\psi(h(z) + h(z')) - \psi(h(z) + h(z') - 2\Delta_{zz'}) < 0,$$

thus guaranteeing that inequality (2) is also satisfied. That is, the manufacturer's net profit is higher when the suppliers dual-source, proving that the incentives of all parties are fully aligned. The proof for the case that h is convex is similar, and is hence omitted.   Q.E.D.

Proof of Proposition 2

Recall that

$$\Pi_s - \Pi_d = p \int_0^1 \int_0^1 \Delta_{zz'} f(z) f(z') dz dz'.$$

Assumption (a) immediately implies that tier 1 suppliers strictly prefer to dual-source. It is thus sufficient to show that the profits of the manufacturer are maximized if both tier 1 suppliers single-source. Recall that

$$\Phi_s - \Phi_d = \int_0^1 \int_0^1 \left[\psi(h(z) + h(z')) - \psi(h(z) + h(z') - 2\Delta_{zz'}) \right] f(z) f(z') dz dz'.$$

Given that $\psi(y) = \phi(y) - py$ is concave, the first-order condition for concavity implies that:

$$\Phi_s - \Phi_d \geq 2 \int_0^1 \int_0^1 \Delta_{zz'} \psi'(h(z) + h(z')) f(z) f(z') dz dz'.$$

On the other hand, by assumption (b), there exists $\bar{x}$ such that $\Delta_{zz'} > 0$ for all $z, z' < \bar{x}$. Pick a small enough $\underline{x} < \bar{x}$ such that if $\max\{z, z'\} \geq \bar{x}$, then

$$2h(\underline{x}) \leq h(z) + h(z'). \tag{7}$$

Note that such a $\underline{x}$ exists as long as h is strictly increasing. Thus,

$$\Phi_s - \Phi_d \geq 2 \int_0^{\underline{x}} \int_0^{\underline{x}} \Delta_{zz'} \psi'(h(z) + h(z')) f(z) f(z') dz dz' + 2 \int_{\max\{z,z'\} \geq \bar{x}} \Delta_{zz'} \psi'(h(z) + h(z')) f(z) f(z') dz dz',$$

where we use the fact that $\Delta_{zz'} > 0$ whenever $\max\{z, z'\} \in [\underline{x}, \bar{x}]$ and that $\psi' > 0$. Consequently,

$$\Phi_s - \Phi_d \geq 2 \int_0^{\underline{x}} \int_0^{\underline{x}} \Delta_{zz'} \psi'(h(z) + h(z')) f(z) f(z') dz dz' - 2 \int_{\max\{z,z'\} \geq \bar{x}} |\Delta_{zz'}| \psi'(h(z) + h(z')) f(z) f(z') dz dz'$$

$$\geq 2 \int_0^{\underline{x}} \int_0^{\underline{x}} \Delta_{zz'} \psi'(h(z) + h(z')) f(z) f(z') dz dz' - 2\psi'(2h(\underline{x})) \int_{\max\{z,z'\} \geq \bar{x}} |\Delta_{zz'}| f(z) f(z') dz dz',$$

where the second inequality is a consequence of inequality (7) and concavity of ψ. On the other hand, by the mean value theorem, there exists $x_1^*, x_2^* \in (0, \bar{x})$, possibly dependent on ψ, such that

$$\int_0^{\bar{x}} \int_0^{\bar{x}} \Delta_{zz'} \psi'\left(h(z) + h(z')\right) f(z) f(z') dz dz' = \psi'\left(h(x_1^*) + h(x_2^*)\right) \int_0^{\bar{x}} \int_0^{\bar{x}} \Delta_{zz'} f(z) f(z') dz dz'.$$

As a result,

$$\Phi_s - \Phi_d \geq 2\psi'\left(h(x_1^*) + h(x_2^*)\right) \int_0^{\bar{x}} \int_0^{\bar{x}} \Delta_{zz'} f(z) f(z') dz dz' - 2\psi'\left(2h(\underline{x})\right) \int_{\max\{z,z'\} \geq \bar{x}} |\Delta_{zz'}| f(z) f(z') dz dz'. \tag{8}$$

The proof is complete once we show that there exists a concave function ϕ such that the first term is larger than the second. This, however, is immediate once one notes that the domains over which the two integrals are taken are non-overlapping and f is a continuous distribution.   Q.E.D.

Proof of Proposition 5

Throughout the proof, we focus on a supplier who obtains a fraction $\lambda \geq 1/2$ of its inputs from fabricator 1. The case of $\lambda \leq 1/2$ can be handled symmetrically. Before presenting the proof of the proposition, we state and prove two lemmas.

LEMMA 1. *For $\beta \leq 1 - \lambda$, let*

$$Q(\beta) = \{(z_1, z_2) \in [0,1]^2 : \lambda z_1 + (1-\lambda)z_2 \leq \beta\}. \tag{9}$$

If $\lambda \geq 1/2$, then $G(\beta) = \int_{Q(\beta)} (z_1 - z_2) dz_1 dz_2 \leq 0$.

Proof: First, note that $Q(\beta) = S_1 \cup S_2 \cup S_3$, where

$$S_1 = \{(z_1, z_2) : \lambda z_1 + (1-\lambda)z_2 \leq \beta, z_2 \leq z_1\},$$

$$S_2 = \{(z_1, z_2) : \lambda z_1 + (1-\lambda)z_2 \leq \beta, \lambda z_2 + (1-\lambda)z_1 \leq \beta, z_2 \geq z_1\},$$

$$S_3 = \{(z_1, z_2) : \lambda z_1 + (1-\lambda)z_2 \leq \beta, \lambda z_2 + (1-\lambda)z_1 \geq \beta, z_2 \geq z_1\}.$$

On the other hand, given that $\lambda \geq 1/2$, it is immediate that if $z_2 \geq z_1$ and $\lambda z_2 + (1-\lambda)z_1 \leq \beta$, then $\lambda z_1 + (1-\lambda)z_2 \leq \beta$. Thus, $S_2 = \{(z_1, z_2) : \lambda z_2 + (1-\lambda)z_1 \leq \beta, z_2 \geq z_1\}$, which implies that

$$\int_{S_1} (z_1 - z_2) dz_1 dz_2 + \int_{S_2} (z_1 - z_2) dz_1 dz_2 = 0,$$

and therefore, $\int_{Q(\beta)} (z_1 - z_2) dz_1 dz_2 = \int_{S_3} (z_1 - z_2) dz_1 dz_2$. Finally, since $z_1 \leq z_2$ over S_3, it is immediate that the integral on the right-hand side above is non-positive, completing the proof.

LEMMA 2. *If $\lambda \geq 1/2$, then $G(\beta)$ is non-increasing in β.*

Proof: We prove the lemma by showing that the derivative of $G(\beta)$ with respect to β is non-positive as long as $\lambda \geq 1/2$. By the Leibniz integral rule, we directly obtain

$$dG/d\beta = \frac{1}{\lambda^2} \int_0^{\beta/(1-\lambda)} (\beta - z_2) dz_2 = \frac{\beta^2(1/2 - \lambda)}{\lambda^2(1-\lambda)^2} \leq 0, \qquad \text{for } \lambda \geq 1/2.$$

Proof of Proposition 5. Consider a tier 1 supplier that orders n units in total from the fabricators and sources a fraction $\lambda \geq \frac{1}{2}$ of its order from fabricator 1. We prove the theorem by showing that the derivative of the supplier's profit function $\Pi(n, \lambda)$ is decreasing for $\lambda \geq 1/2$, which implies that dual-sourcing maximizes its profits. Recall that the latter are given by (5). Thus,

$$
\begin{aligned}
\Pi(n, \lambda) = & -qn\mathbb{E}[z] + \int_0^1 \int_0^1 pn\mu_m z \left[1 - H\left(z - \frac{x_m}{n}\right)\right] dz_1 dz_2 \\
& + \int_0^1 \int_0^1 [pn\mu_a z - v_a] \left[H\left(z - \frac{x_m}{n}\right) - H\left(z - \frac{x_a}{n}\right)\right] dz_1 dz_2 \\
& + \int_0^1 \int_0^1 \left[pn\left(\mu_m(z + (\mu_a - \mu_m)\frac{x_a}{n}\right) - v_a\right] H\left(z - \frac{x_a}{n}\right) dz_1 dz_2,
\end{aligned}
\tag{10}
$$

where $H(\cdot)$ denotes the Heaviside step function, and $z = \lambda z_1 + (1 - \lambda)z_2$ is the total number of units obtained by the supplier. Differentiating $\Pi(n, \lambda)$ with respect to its sourcing decision λ implies that

$$
d\Pi(n, \lambda)/d\lambda = pn(\mu_a - \mu_m) \int_{\frac{x_m}{n} \leq z \leq \frac{x_a}{n}} (z_1 - z_2) dz_1 dz_2 + pn\mu_m \int (z_1 - z_2) dz_1 dz_2.
\tag{11}
$$

The proof is complete once we show that the integral on the right-hand size is negative for $\lambda \geq 1/2$. To this end, we consider five different cases depending on the value of λ relative to x_m/n and x_a/n. For notational simplicity, we denote the set $\{(z_1, z_2) : x_m/n \leq z \leq x_a/n\}$ with Δ.

(i) First, suppose that $x_m, x_a \leq (1 - \lambda)n$. It is immediate to verify that

$$
\int_\Delta (z_1 - z_2) dz_1 dz_2 = \int_{Q(x_a/n)} (z_1 - z_2) dz_1 dz_2 - \int_{Q(x_m/n)} (z_1 - z_2) dz_1 dz_2,
$$

where $Q(\beta)$ is defined in (9). Given that $x_a \geq x_m$, Lemma 2 implies that the above is non-positive.

(ii) Suppose that $x_m \leq n(1 - \lambda) \leq n\lambda \leq x_a$. In this case, we have

$$
\int_\Delta (z_1 - z_2) dz_1 dz_2 = -\int_{Q(x_m/n) \cup Q'} (z_1 - z_2) dz_1 dz_2,
$$

where $Q' = [0, 1]^2 \setminus Q(x_a/n)$. On the other hand, a simple change of variables implies that

$$
\int_{Q'} (z_1 - z_2) dz_1 dz_2 = -\int_{Q(1 - x_a/n)} (z_1 - z_2) dz_1 dz_2,
$$

and therefore, $\int_\Delta (z_1 - z_2) dz_1 dz_2 = \int_{Q(1 - x_a/n)} (z_1 - z_2) dz_1 dz_2 - \int_{Q(x_m/n)} (z_1 - z_2) dz_1 dz_2$. Furthermore, it must be the case that $1 - x_a/n > x_m/n$. Consequently, once again by Lemma 2, it is immediate that the the right-hand side of the above expression is non-positive.

(iii) Next suppose that $n\lambda \leq x_a$ and $n(1 - \lambda) \leq x_m$. This case, however, cannot arise as $x_a + x_m \geq n$ contradicts the assumption that $x_m/n + x_a/n < 1$.

(iv) Next, suppose that $n(1 - \lambda) \leq x_m, x_a \leq n\lambda$. In this case, we have

$$
\int_\Delta (z_1 - z_2) dz_1 dz_2 = \int_0^1 \int_{x_m/n\lambda - (1-\lambda)z_2/\lambda}^{x_a/n\lambda - (1-\lambda)z_2/\lambda} (z_1 - z_2) dz_1 dz_2 = \frac{x_a^2 - x_m^2}{2n^2\lambda^2} - \left(\frac{x_a - x_m}{n\lambda^2}\right) \int_0^1 z_2 dz_2
$$

$$= \frac{x_a - x_m}{2n^2\lambda^2}(x_a + x_m - n) \le 0,$$

where the last inequality follows from $m - x_a > x_m$, which implies that $x_m + x_a < n$.

(v) Finally, suppose that $x_m \le (1 - \lambda)n \le x_a \le \lambda n$. In this case,

$$\int_\Delta (z_1 - z_2)dz_1dz_2 = \int_{Q''}(z_1 - z_2)dz_1dz_2 + \int_{Q'''}(z_1 - z_2)dz_1dz_2, \qquad \text{where ,} \qquad (12)$$

$Q'' = \{(z_1, z_2) : x_m/n \le \lambda z_1 + (1 - \lambda)z_2 \le 1 - \lambda\}$ and $Q''' = \{(z_1, z_2) : 1 - \lambda \le \lambda z_1 + (1 - \lambda)z_2 \le x_a/n\}$.

From case (iv) above, we know that the second integral in (12) is non-positive. Furthermore, case (i) implies that the first term in (12) is also negative.

To summarize, the derivative of the expected profit of each supplier with respect to λ expressed in (11) is negative for $\lambda \ge 1/2$, implying that it's optimal for the suppliers to dual-source.   Q.E.D.

Proof of Proposition 6

Throughout the proof, for simplicity, we let $z_A = n(\lambda_A z_1 + (1 - \lambda_A)z_2)$ and $z_B = n(\lambda_B z_1 + (1 - \lambda_B)z_2)$ denote the input parts delivered to suppliers A and B, respectively. We first state a simple lemma, the proof of which is a straightforward consequence of Leibnitz' integral rule and is thus omitted.

LEMMA 3. *Suppose that $g(z_1, z_2)$ is bounded in both arguments with a support restricted to $[0, 1]^2$. Then, for any $\beta \ge 0$,*

$$\int_0^1 \int_0^1 g(z_1, z_2)\delta(z_A - \beta)dz_1dz_2 = \int_0^1 g\left(z, \frac{\beta/n - \lambda_A z}{1 - \lambda_A}\right)dz. \qquad (13)$$

Proof of Proposition 6. The first part of the proposition follows from extending Proposition 1. Specifically, note that when $v_a = 0$, the suppliers' production function is concave, which in turn implies that $2h(\bar{z}) \ge h(z_A) + h(z_B)$, where $\bar{z} = n(z_1 + z_2)/2$. Thus,

$$\Phi(n, 1/2, 1/2) - \Phi(n, \lambda_A, \lambda_B) = \int_0^1 \int_0^1 \psi(2h(\bar{z})) - \psi(h(z_A) + h(z_B))I(0 \le h(\bar{z}) < 1)dz_1dz_2 \quad (14)$$

$$+ \int_0^1 \int_0^1 \psi(2h(\bar{z})) - \psi(h(z_A) + h(z_B))I(1 \le h(\bar{z}))dz_1dz_2, \qquad (15)$$

where $\Phi(n, \lambda_A, \lambda_B)$ is the expected profit of the manufacturer given in (6) and recall that the manufacturer's optimal total order size to tier 1 is normalized to 2 in the absence of disruptions (the manufacturer though may over-order, i.e., set $2n > 2$). This implies that term (14) is positive since function ψ is increasing for $\bar{z} < 1$. The claim follows by noting that when $h(\bar{z}) > 1$, it has to be the case that $z_A, z_B \ge x_a$, which in turn implies that $2h(\bar{z}) = h(z_A) + h(z_B)$ and, thus, term (15) is equal to zero. To see this, assume for the sake of contraction that $z_A \le x_A$. Then,

$$h(\bar{z}) \le h(\mu_m((n + x_a)/2 - x_a) + \mu_a x_a) \le h(\mu_m(1 - x_a) + \mu_a x_a) = 1,$$

where the first inequality follows from $z_A \leq x_A$ and $z_B \leq n$, the second inequality follows from the assumption that $n \leq 2 - x_a$, and the last equality follows from the fact that we normalized the manufacturer's optimal order size in the absence of disruptions to one.

In order to prove the second part, we show that the manufacturer's profit can increase if one of the suppliers does not fully dual-source. In particular, we show that

$$\lim_{\lambda_A \downarrow 1/2} d\Phi(n, \lambda_A, 1/2)/d\lambda_A > 0,$$

To simplify notation, let $I(\cdot)$ denote the indicator function and define $\Delta_{am} = (\mu_a - \mu_m)x_a$. Then,

$$\Phi(n, \lambda_A, \lambda_B) = \int_0^1 \int_0^1 \psi(\mu_m z_A + h(z_B))I(0 \leq z_A < x_m)dz_1 dz_2$$

$$+ \int_0^1 \int_0^1 \psi(\mu_a z_A + h(z_B))I(x_m \leq z_A < x_a)dz_1 dz_2$$

$$+ \int_0^1 \int_0^1 \psi(\mu_m z_A + \Delta_{am} + h(z_B))I(x_a \leq z_A \leq n)dz_1 dz_2.$$

where recall that $\bar{z} = n(z_1 + z_2)/2$. The derivative of $\Phi(\cdot)$ with respect to λ_A satisfies

$$\frac{d}{d\lambda_A}\Phi(n, \lambda_A, 1/2) = \int_0^1 \int_0^1 \mu_m n(z_1 - z_2)\psi'(\mu_m z_A + h(\bar{z}))I(0 \leq z_A < x_m)dz_1 dz_2 \tag{16}$$

$$+ \int_0^1 \int_0^1 \mu_a n(z_1 - z_2)\psi'(\mu_a z_A + h(\bar{z}))I(x_m \leq z_A < x_a)dz_1 dz_2 \tag{17}$$

$$+ \int_0^1 \int_0^1 \mu_m n(z_1 - z_2)\psi'(\mu_m z_A + \Delta_{am} + h(\bar{z}))I(x_a \leq z_A \leq n)dz_1 dz_2 \tag{18}$$

$$+ \int_0^1 \int_0^1 n(z_1 - z_2)\psi(\mu_m z_A + h(\bar{z}))(\delta(z_A) - \delta(z_A - x_m))dz_1 dz_2 \tag{19}$$

$$+ \int_0^1 \int_0^1 n(z_1 - z_2)\psi(\mu_a z_A + h(\bar{z}))(\delta(z_A - x_m) - \delta(z_A - x_a))dz_1 dz_2 \tag{20}$$

$$+ \int_0^1 \int_0^1 n(z_1 - z_2)\psi(\mu_m z_A + \Delta_{am} + h(\bar{z}))(\delta(z_A - x_a) - \delta(z_A - n))dz_1 dz_2. \tag{21}$$

Next, we show that the limit of the first three terms (16)–(18) as $\lambda_A \downarrow 1/2$ is equal to zero. Note that the integrands in all three terms are continuous and bounded functions of λ_A. Hence, by the dominated convergence theorem, the limit as $\lambda_A \downarrow 1/2$ is simply equal to evaluating each term at $\lambda_A = 1/2$. This immediately implies that the first three terms are indeed all equal to zero at $\lambda_A = 1/2$. This is a consequence of the fact that the integrands in (16)–(18) are antisymmetric in z_1 and z_2, and hence, integrating each over the unit square leads to a value of zero. As for the remaining terms, we employ Lemma 3 and obtain the following

$$\lim_{\lambda_A \downarrow 1/2} d\Phi(n, \lambda_A, 1/2)/d\lambda_A = -2\int_0^{x_m/n}(nz - x_m)\psi(\mu_m x_m + \mu_a x_m)f(z)f(2x_m/n - z)dz \tag{22}$$

$$-2\int_{x_m/n}^1(nz - x_m)\psi(2\mu_m x_m)f(z)f(2x_m/n - z)dz \tag{23}$$

$$+2 \int_0^{x_m/n} (nz - x_m)\, \psi\,(2\mu_a x_m)\, f(z) f\,(2x_m/n - z)\, dz \tag{24}$$

$$+2 \int_{x_m/n}^1 (nz - x_m)\, \psi\,(\mu_a x_m + \mu_m x_m)\, f(z) f\,(2x_m/n - z)\, dz \tag{25}$$

$$-2 \int_0^1 (nz - x_a)\, \psi\,(2\mu_a x_a)\, f(z) f\,(2x_a/n - z)\, dz \tag{26}$$

$$+2 \int_0^1 (nz - x_a)\, \psi\,(\mu_m x_a + \mu_a x_a + \Delta_{am})\, f(z) f\,(2x_a/n - z)\, dz. \tag{27}$$

The fact that $\Delta_{am} = (\mu_a - \mu_m)x_a$ implies that (26) and (27) also cancel out. Consequently, a simple change of variables in (23) and (25) and using the facts that the support of $f(\cdot)$ is restricted to the unit interval and that $2x_m/n < 1$ lead to

$$\lim_{\lambda_A \downarrow 1/2} d\Phi(n, \lambda_A, 1/2)/d\lambda_A = 2 \int_0^{x_m/n} (nz - x_m) \Big[\psi(2\mu_m x_m) + \psi(2\mu_a x_m) - 2\psi(\mu_a x_m + \mu_m x_m) \Big] dz.$$

Finally, the fact that $\psi(\cdot)$ is strictly concave implies that the above integral is strictly negative. Thus, the expected profits of the monopolist would be strictly higher of supplier A deviates from complete dual-sourcing, completing the proof.   Q.E.D.

Endnotes

1. For a recent example, see Carvalho, Nirei, Saito, and Tahbaz-Salehi (2016).

2. Other examples include, Anupindi and Akella (1993), Federgruen and Yang (2008), Yang, Aydın, Babich, and Beil (2009, 2012) and Hu and Kostamis (2015). For a comprehensive survey of this literature, see Tomlin and Wang (2011) and Aydın, Babich, Beil, and Yang (2011).

3. For the case of single-sourcing, we constrain the suppliers to source from distinct fabricators (i.e., $\lambda_A \in \{0,1\}$ and $\lambda_B = 1 - \lambda_A$). This ensures that the systematic losses we study do not result from the concentration of suppliers' orders at a single fabricator. Furthermore, note that as long as ϕ is concave, the outcome in which the two suppliers single-source from the same fabricator is Pareto-dominated by the case in which they single-source from distinct fabricators.

4. It may be worthwhile to note that qualitatively the insight we illustrate in Proposition 2 would remain the same if tier 1 suppliers had different production functions: non-convexities in the chain, e.g., in the production function of one of the tier 1 suppliers, tend to create misalignments in the supply network configurations that firms operating in different tiers view as optimal.

5. Recall that if h has no inflection points, then Proposition 1 guarantees that all parties' incentives are fully aligned. Furthermore, note that if $x^* \neq 1/2$, symmetry around the inflection point means that h is symmetric over the interval $[\max\{0, 2x^* - 1\}, \min\{2x^*, 1\}]$.

6. For a more thorough treatment of the role of bankruptcies in supply chains, see Swinney and Netessine (2009), and Yang, Birge, and Parker (2015).

7. Equivalently, R can be interpreted as the firm's bankruptcy cost in the case of a default; a cost that is avoided if the firm is capable of paying the fixed cost v.

8. The above example highlights that the types of non-convexities that we studied in the previous section may not necessarily arise as a consequence of returns to scale in production. Rather, they can also arise in response to financial or operational constraints. In particular, for our results to hold, h does not have to be interpreted as a physical production function *per se*. In the context of the bankruptcy example above, even though the suppliers' production technology enables them to turn each unit of input to a unit of the intermediate good, the presence of the fixed cost v ensures that the number of units eventually delivered to the manufacturer follows equation (3).

9. For example, the following cross-contingent contract guarantees that the firms' endogenous sourcing decisions will result in a chain that is optimal for the downstream manufacturer for the setting we study in Subsection 4.2. The price for supplier A is given as a function of the quantities that the manufacturer receives from suppliers A, B, which we denote by y_A and y_B, by:

$$p(y_A, y_B) = \begin{cases} qy_A/\mu_m + 1/2\beta\phi(y_A, y_B) & \text{if } y_A \leq \mu_m x_m \\ qy_A/\mu_a + 1/2\beta\phi(y_A, y_B) & \text{if } \mu_m x_m < y_A \leq \mu_a x_a \\ q[y_A/\mu_m - \frac{x_a}{\mu_m}(\mu_a - \mu_m)] + 1/2\beta\phi(y_A, y_B) & \text{if } \mu_a x_a < y_A \leq \mu_a x_a + \mu_m(\bar{x} - x_a) \end{cases}$$

where β is some constant.

10. This argument also shows that even though, for the sake of simplicity, we restricted our attention to fixed-price contracts between the manufacturer and its suppliers, our results are not driven by this assumption. Rather, similar moral hazard problems would arise as long as the suppliers' sourcing decisions are not contractable and they are restricted to writing (linear or non-linear) bilateral contracts with their suppliers.

11. In a very different context, Shin and Tunca (2010) show that bilateral *pricing schemes* lead to over-investment in demand forecasting when firms compete in quantities. Also, they show that a cross-contingent contracting scheme can achieve coordination in the demand forecasting process.

Acknowledgments

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Electronic Companion

EC.1. Additional Proofs

Proof of Proposition 3

As in the proof of Proposition 2, recall that

$$\Pi_s - \Pi_d = p \int_0^1 \int_0^1 \Delta_{zz'} f(z) f(z') dz dz'.$$

Thus, by condition (a), suppliers employ a single-sourcing strategy. On the other hand, we have

$$\Phi_s - \Phi_d = \int_0^1 \int_0^1 \left[\psi(h(z) + h(z')) - \psi(h(z) + h(z') - 2\Delta_{zz'}) \right] f(z) f(z') dz dz'.$$

Once again, using the fact that ϕ, and hence ψ, are concave, we have

$$\Phi_s - \Phi_d \le 2 \int_0^1 \int_0^1 \Delta_{zz'} \psi'\left(h(z) + h(z') - 2\Delta_{zz'}\right) f(z) f(z') dz dz'$$

$$= 2 \int_0^1 \int_0^1 \Delta_{zz'} \psi'\left(2h\left(\frac{z+z'}{2}\right)\right) f(z) f(z') dz dz'.$$

On the other hand, by condition (b), there exists $\bar{x}$ such that $\Delta_{zz'} < 0$ for all $z, z' < \bar{x}$. Therefore,

$$\Phi_s - \Phi_d \le 2 \int_0^{\bar{x}/2} \int_0^{\bar{x}/2} \Delta_{zz'} \psi'\left(2h\left((z+z')/2\right)\right) f(z) f(z') dz dz' + 2 \int_{\max\{z,z'\} \ge \bar{x}} \Delta_{zz'} \psi'\left(2h\left((z+z')/2\right)\right) f(z) f(z') dz dz', \tag{EC.1}$$

where we are using the fact that $\Delta_{zz'} \psi'(2h((z+z')/2)) < 0$ whenever $\max\{z, z'\} \in [\bar{x}/2, \bar{x}]$, which in turn is a consequence of the fact that ψ is increasing. On the other hand, the fact that h is increasing implies that

$$h(\bar{x}/2) \le h\left(\frac{z+z'}{2}\right),$$

for all $(z, z') \in [0,1]^2$ such that $\max\{z, z'\} \ge \bar{x}$. Consequently, if ψ' is large enough over $[0, \bar{x}/2]$, the right-hand side of inequality (EC.1) can be made negative. In other words, for a given function h that satisfies conditions (a) and (b), one can always choose a concave enough function ϕ such that the right-hand side of (EC.1) is negative, implying that the manufacturer is better off when the suppliers choose a dual-sourcing strategy.   Q.E.D.

Proof of Corollary 1

We start by stating three auxiliary lemmas.

LEMMA EC.1. *Suppose that $x^* < 1/2$, and let*

$$M_1 = \{(z_1, z_2) : \Delta_{z_1 z_2} > 0, \max\{z_1, z_2\} \le 2x^*\}$$
$$M_2 = \{(z_1, z_2) : \Delta_{z_1 z_2} > 0, \max\{z_1, z_2\} > 2x^*\}$$
$$M_1' = \{(z_1, z_2) : (2x^* - z_1, 2x^* - z_2) \in M_1\}$$
$$M_2' = \{(z_1, z_2) : (z_1, 2x^* - z_2) \in M_2\}.$$

Then,

$$\Delta_{z_1 z_2} + \Delta_{2x^* - z_1, 2x^* - z_2} = 0 \qquad \forall (z_1, z_2) \in M_1' \qquad\qquad \text{(EC.2)}$$

$$\Delta_{z_1 z_2} + \Delta_{z_1, 2x^* - z_2} < 0 \qquad \forall (z_1, z_2) \in M_2'. \qquad\qquad \text{(EC.3)}$$

Proof: To prove the first statement, note that for any arbitrary $(z_1, z_2) \in M_1'$, we have

$$\Delta_{z_1 z_2} + \Delta_{2x^* - z_1, 2x^* - z_2} = \frac{1}{2}\left[h(z_1) + h(2x^* - z_1)\right] + \frac{1}{2}\left[h(z_2) + h(2x^* - z_2)\right]$$
$$- \left[h\left(\frac{z_1 + z_2}{2}\right) + h\left(2x^* - \frac{z_1 + z_2}{2}\right)\right].$$

The symmetry of h around x^* implies that $h(x) + h(2x^* - x) = 2h(x^*)$ for any $\max\{0, 2x^* - 1\} \le x \le \min\{2x^*, 1\}$. Therefore, it is immediate that the right-hand side of the above equation is equal to zero.

To prove the second statement, note that for any arbitrary $(z_1, z_2) \in M_2'$, we have $z_1 > 2x^*$. Furthermore,

$$\Delta_{z_1 z_2} + \Delta_{z_1, 2x^* - z_2} = \left[h(z_1) - h\left(\frac{z_1 + z_2}{2}\right)\right] - \left[h\left(x^* + \frac{z_1 - z_2}{2}\right) - h(x^*)\right],$$

where once again we are using the assumption that h is symmetric around x^*. Now the fact that $z_1 > x^* + (z_1 - z_2)/2$ alongside the assumption that h is concave for $x > x^*$ implies that the right-hand side of the above equality is strictly negative.

An identical argument leads to the following lemma.

LEMMA EC.2. *Suppose that $x^* > 1/2$, and let*

$$N_1 = \{(z_1, z_2) : \Delta_{z_1 z_2} < 0, \min\{z_1, z_2\} \ge 2x^* - 1\}$$
$$N_2 = \{(z_1, z_2) : \Delta_{z_1 z_2} < 0, \min\{z_1, z_2\} < 2x^* - 1\}$$
$$N_1' = \{(z_1, z_2) : (2x^* - z_1, 2x^* - z_2) \in N_1\}$$
$$N_2' = \{(z_1, z_2) : (z_1, 2x^* - z_2) \in N_2\}.$$

Then,

$$\Delta_{z_1 z_2} + \Delta_{2x^* - z_1, 2x^* - z_2} = 0 \qquad \forall (z_1, z_2) \in N_1' \qquad\qquad \text{(EC.4)}$$

$$\Delta_{z_1 z_2} + \Delta_{z_1, 2x^* - z_2} > 0 \qquad \forall (z_1, z_2) \in N_2'. \qquad\qquad \text{(EC.5)}$$

LEMMA EC.3. *For any $(z_1, z_2) \in N_1$,*

$$h(z_1) + h(z_2) > h(2x^* - z_1) + h(2x^* - z_2). \qquad (\text{EC.6})$$

Proof: Note that for any $(z_1, z_2) \in N_1$, we have $\max\{z_1, z_2\} > x^*$, as otherwise the convexity of h for values smaller than x^* would imply that $\Delta_{z_1 z_2} > 0$. Given this observation, three distinct cases may arise.

First, suppose that $z_1, z_2 > x^*$, which immediately implies that $2x^* - z_1 < z_1$ and $2x^* - z_2 < z_2$, thus guaranteeing that (EC.6) is satisfied.

Next, suppose that $z_1 > x^* > z_2$, while $z_1 + z_2 > 2x^*$. Consequently, $2x^* - z_1 < z_2$ and $2x^* - z_2 < z_1$, thus immediately leading to (EC.6).

Finally, suppose that $z_1 > x^* > z_2$ and $z_1 + z_2 < 2x^*$. We show that these two inequalities cannot hold simultaneously. Note that

$$
\begin{aligned}
\Delta_{z_1 z_2} &= \frac{1}{2}\left[h(x^*) + h(z_1 + z_2 - x^*) - 2h\left(\frac{z_1 + z_2}{2}\right)\right] + \frac{1}{2}\left[h(z_1) - h(x^*) + h(z_2) - h(z_1 + z_2 - x^*)\right] \\
&= \frac{1}{2}\left[h(x^*) + h(z_1 + z_2 - x^*) - 2h\left(\frac{z_1 + z_2}{2}\right)\right] + \frac{1}{2}\left[h(x^*) - h(2x^* - z_1) + h(z_2) - h(z_1 + z_2 - x^*)\right],
\end{aligned}
$$

where the second equality is a consequence of h's symmetry around x^*. Convexity of h below x^* simultaneously implies that the first term on the right-hand side above is non-negative, and that

$$h(x^*) - h(2x^* - z_1) \geq h(z_1 + z_2 - x^*) - h(z_2),$$

and hence, guaranteeing that $\Delta_{z_1 z_2} > 0$. This, however, is in contradiction with the assumption that $(z_1, z_2) \in N_1$, completing the proof.

Using the above lemmas, we can now present the proof of Corollary 1.

Proof of part (i) Suppose that $x^* < 1/2$. Lemma EC.1 implies $\Delta_{z_1 z_2} = -\Delta_{2x^* - z_1, 2x^* - z_2}$ for all $(z_1, z_2) \in M_1'$. Given that $(2x^* - z_1, 2x^* - z_2) \in M_1$, it is then immediate that $M_1 \cap M_1' = \varnothing$. Similarly, (EC.3) implies that $M_2 \cap M_2' = \varnothing$. Therefore, the pairwise intersections of sets M_1, M_2, M_1' and M_2' are empty. Consequently,

$$\int_0^1 \int_0^1 \Delta_{z_1 z_2} dz_1 dz_2 \leq \int_{M_1} \Delta_{zz'} dz dz' + \int_{M_1'} \Delta_{zz'} dz dz' + \int_{M_2} \Delta_{zz'} dz dz' + \int_{M_2'} \Delta_{zz'} dz dz', \quad (\text{EC.7})$$

where we are using the fact that $\Delta_{z_1 z_2} \leq 0$ for all $(z_1, z_2) \notin M_1 \cup M_2$. Furthermore, equation (EC.2) implies that the first two terms on the right-hand side of (EC.7) sum up to zero, whereas inequality (EC.3) guarantees that the sum of the last two terms is negative. This implies that the right-hand side of (EC.7) is strictly negative, a statement which coincides with condition (a) of Proposition 2. Thus, the suppliers choose a single-sourcing strategy, whereas for a concave enough revenue function ϕ, the manufacturer would be better off if the suppliers dual-source.   Q.E.D.

Proof of part (ii) Now suppose that $x^* > 1/2$. An argument similar to that of part (i) shows that under such an assumption,

$$\int_0^1 \int_0^1 \Delta_{z_1 z_2} dz_1 dz_2 > 0,$$

guaranteeing that the suppliers choose a single-sourcing strategy. Thus, it is sufficient to show that the manufacturer's net profit is higher when the suppliers single-source. Recall that

$$\Phi_s - \Phi_d = \int_0^1 \int_0^1 \left[\psi(h(z_1) + h(z_2)) - \psi(h(z_1) + h(z_2) - 2\Delta_{z_1 z_2}) \right] dz_1 dz_2.$$

Therefore,

$$
\begin{aligned}
\Phi_s - \Phi_d \geq & \int_{N_1 \cup N_1'} \left[\psi(h(z_1) + h(z_2)) - \psi(h(z_1) + h(z_2) - 2\Delta_{z_1 z_2}) \right] dz_1 dz_2 \\
& + \int_{N_2 \cup N_2'} \left[\psi(h(z_1) + h(z_2)) - \psi(h(z_1) + h(z_2) - 2\Delta_{z_1 z_2}) \right] dz_1 dz_2,
\end{aligned}
$$

where we are using the fact that $\Delta_{z_1 z_2} \geq 0$ for all $(z_1, z_2) \notin N_1 \cup N_2$ and that the pairwise intersections of N_1, N_2, N_1' and N_2' are empty. The proof is complete once we show that the two integrals on the right-hand side above are positive.

Consider an arbitrary $(z_1, z_2) \in N_1$. Given that $\Delta_{z_1 z_2} < 0$, Lemma EC.3 and concavity of ψ imply that

$$
\begin{aligned}
\psi(h(z_1) + h(z_2)) & - \psi\left(h(z_1) + h(z_2) - 2\Delta_{z_1 z_2}\right) \\
& > \psi(h(2x^* - z_1) + h(2x^* - z_2)) - \psi(h(2x^* - z_1) + h(2x^* - z_2) - 2\Delta_{z_1 z_2}).
\end{aligned}
$$

Thus, by equation (EC.4) in Lemma EC.2, we have

$$
\begin{aligned}
\psi(h(z_1) + h(z_2)) & - \psi\left(h(z_1) + h(z_2) - 2\Delta_{z_1 z_2}\right) \\
& > \psi(h(2x^* - z_1) + h(2x^* - z_2)) - \psi(h(2x^* - z_1) + h(2x^* - z_2) + 2\Delta_{2x^* - z_1, 2x^* - z_2}). \quad \text{(EC.8)}
\end{aligned}
$$

Integrating both sides of the above equation over N_1 and a change of variables imply

$$
\begin{aligned}
\int_{N_1} & \left[\psi(h(z_1) + h(z_2)) - \psi\left(h(z_1) + h(z_2) - 2\Delta_{z_1 z_2}\right) \right] dz_1 dz_1 \\
& > \int_{N_1'} \left[\psi(h(z_1) + h(z_2)) - \psi\left(h(z_1) + h(z_2) + 2\Delta_{z_1 z_2}\right) \right] dz_1 dz_1,
\end{aligned}
$$

and hence,

$$
\begin{aligned}
\int_{N_1 \cup N_1'} & \left[\psi(h(z_1) + h(z_2)) - \psi\left(h(z_1) + h(z_2) - 2\Delta_{z_1 z_2}\right) \right] dz_1 dz_1 \\
& > \int_{N_1'} \left[2\psi(h(z_1) + h(z_2)) - \psi\left(h(z_1) + h(z_2) + 2\Delta_{z_1 z_2}\right) - \psi\left(h(z_1) + h(z_2) - 2\Delta_{z_1 z_2}\right) \right] dz_1 dz_1.
\end{aligned}
$$

Now concavity of ψ guarantees that the right-hand side of the above inequality is positive, and as a result:

$$\int_{N_1 \cup N_1'} \left[\psi(h(z_1) + h(z_2)) - \psi(h(z_1) + h(z_2) - 2\Delta_{z_1 z_2}) \right] dz_1 dz_2 > 0.$$

A similar argument shows that

$$\int_{N_2 \cup N_2'} \left[\psi(h(z_1) + h(z_2)) - \psi(h(z_1) + h(z_2) - 2\Delta_{z_1 z_2}) \right] dz_1 dz_2 > 0,$$

completing the proof.   Q.E.D.

Proof of Proposition 4

First, we show that the suppliers find it optimal to choose $\lambda = 1/2$, even when they can choose any $\lambda \in [0, 1]$. Although the arguments below hold irrespective of the manufacturer's order, to simplify exposition we let $n = 1$.

Consider a tier 1 supplier that sources a fraction $\lambda \geq 1/2$ of its order from fabricator 1. The supplier's expected profit is equal to

$$\Pi(\lambda) = -q/2 + (R - v)\mathbb{P}(z_\lambda > x_d) + p\mathbb{E}\left(z_\lambda \mathbb{I}_{\{x_d \leq z_\lambda\}}\right),$$

where $z_\lambda = \lambda z_1 + (1 - \lambda)z_2$ is the size of the order delivered by the two fabricators to the supplier. The above equation can be rewritten as

$$\Pi(\lambda) = -q/2 + (R - v) - \int_{z_\lambda < x_d} (R - v + p z_\lambda) dz_1 dz_2 - p \int_{z_\lambda < 1} (1 - z_\lambda) dz_1 dz_2.$$

Therefore,

$$\frac{\partial \Pi}{\partial \lambda} = p \int_{x_d < z_\lambda < 1} (z_1 - z_2) dz_1 dz_2 + \frac{R - v + p x_d}{(1 - \lambda)^2} \int_{\max\{1 - (1 - x_d)/\lambda, 0\}}^{x_d/\lambda} (z_1 - x_d) dz_1, \quad \text{(EC.9)}$$

where we are using the assumption that $x_d < 1/2$ and $\lambda > 1/2$. It is easy to verify that the right-hand side above is equal to zero at $\lambda = 1/2$. Next, we show that the second term on the right-hand side above is negative for $\lambda > 1/2$. To this end, we consider two separate cases:

First suppose that $1 - (1 - x_d)/\lambda < 0$. Under this assumption, it is immediate that the integral on the right-hand side of (EC.9), which we denote by I, is equal to

$$I = \int_0^{x_d/\lambda} (z_1 - x_d) dz_1 = x_d^2 (1 - 2\lambda)/2\lambda^2$$

which is clearly negative for $\lambda > 1/2$. Now suppose that $1 - (1 - x_d)/\lambda > 0$. This implies that

$$I = \int_{1 - (1 - x_d)/\lambda}^{x_d/\lambda} (z_1 - x_d) dz_1 = (1 - 1/\lambda)^2 (x_d - 1/2)$$

which is again strictly negative.

Thus, regardless of the value of λ, the second term on the right-hand side of (EC.9) is negative. Therefore, as long as

$$R > \bar{R} = v - px_d - \inf_{\lambda \in (1/2,1]} \left\{ \frac{p(1-\lambda)^2}{I} \int_{x_d < z_\lambda < 1} (z_1 - z_2) dz_1 dz_2 \right\},$$

then $\partial \Pi / \partial \lambda < 0$ for all $\lambda \in (1/2, 1]$. Thus, for any $R > \bar{R}$, the expected profit of the suppliers is maximized when $\lambda = 1/2$. The claim is thus complete once we show that $\bar{R}$ is finite, which is satisfied as long as the term within the braces is uniformly bounded above for all $\lambda > 1/2$. Using L'Hospital's rule implies that the limit of this term as $\lambda \to 1/2$ is indeed finite.

Finally, one can show that sourcing equally from both fabricators, i.e., $\lambda = 1/2$, is never optimal from the point of view of the manufacturer for any order size n using arguments similar to those in the proof of Proposition 6; thus, we chose to omit the details here.   Q.E.D.

EC.2. Quantifying the Profit Loss

In this section, we present a simple example that illustrates the extent of the potential profit loss for the downstream manufacturer when its suppliers decide to dual source (in order to mitigate their own disruption risk). In particular, we assume that $\phi(y) = 1.5y - 0.5y^2$ for the manufacturer, $p = q = 0.5$, which ensures that there is no surplus loss in the chain due to the suppliers' producing in excess of the manufacturer's order quantity, and the suppliers' production function takes the same form as in the example of Subsection 4.1. The manufacturer chooses order quantities so as to maximize its expected profit taking the supply uncertainty into account. In particular, the manufacturer can (and typically does) order an aggregate quantity which is higher than one, i.e., the optimal order quantity in the absence of any disruption risk. The following table summarizes the relation between the probability that a tier 1 supplier is not able to procure enough inputs to initiate production at equilibrium and the profit loss for the manufacturer when tier 1 suppliers choose to dual source — which is optimal from their point of view — as opposed to adopting a single sourcing strategy, which would have been optimal for the manufacturer.

In addition, we provide the following discussion to illustrate the potential shortcomings of bilateral contracts. Suppose the suppliers' production function h takes the following form (depicted also in Figure EC.1):

$$h(x) = \begin{cases} x & \text{if } x \le 1/4 \\ 1/4 + 2(x - 1/4) & \text{if } x \in (1/4, 1/2] \\ 3/4 + 1/2(x - 1/2) & \text{if } x > 1/2, \end{cases}$$

i.e., it is a piecewise linear function with slopes 1, 2, and 1/2 for $x < 1/4$, $x \in [1/4, 1/2]$, and $x > 1/2$ respectively. Further, suppose that the manufacturer can write contracts that involve payments of the general form

$$p(x) = a_0 + a_1 x + a_2 x^2.$$

Disruption probability for at a tier 1 supplier	Percentage of profit loss for the manufacturer
2%	0.9%
4 %	3.9%
6%	9.9%
8%	17.8%
10%	27.8%

Table EC.1   The percentage of profit loss for the manufacturer, i.e., $(\Phi_s - \Phi_d)/\Phi_d$, as a function of the disruption risk at a tier 1 supplier at equilibrium. For this example, tier 1 suppliers always find dual-sourcing optimal in contrast to the manufacturer that would have preferred them to single-source.

This functional form subsumes wholesale price contracts, quantity discounts, linear penalty contracts, etc. Finally, assume that the manufacturer's order quantity is normalized to 1. It is easy to show that in general,

$$\Pi_s - \Pi_d = -\frac{2a_1}{48} - \frac{5a_2}{768}. \tag{EC.10}$$

This observation along with the fact that $a_1 h(x) + a_2(h(x))^2 \geq qx$, for all x (so that suppliers find it optimal to produce given input x) imply that there is no feasible combination of a_0, a_1, and a_2 that would induce suppliers to adopt single-sourcing instead of dual-sourcing, and thus illustrating that coordination through bilateral contracts may be quite challenging.
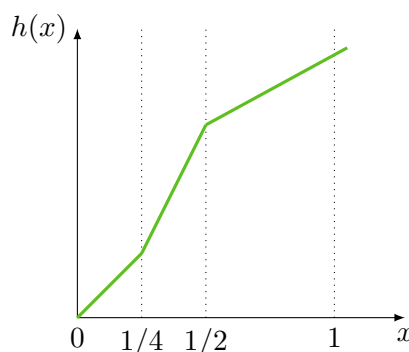

Figure EC.1   The production function h.

EC.3. Yield Shocks

As a way of further illustrating the generality of the insights we present in the paper, this section considers a variant of our original modeling framework in which we impose mild assumptions on the form of the supply uncertainty. In particular, we let ξ denote the probability that a fabricator does not experience any disruptive event and delivers the total order quantity requested by the suppliers in tier 1. In the event that fabricator i experiences a disruption (which occurs with

probability $1 - \xi$), its production output is significantly reduced, i.e., the fraction of the order quantity delivered by the fabricator is given by random variable γ_i that has probability density function f and support over the interval $[0, \Gamma]$, with $\Gamma < 1$. Furthermore, we focus exclusively on the case when the manufacturer's revenue function ϕ is strictly concave and, similar to Section 2, we normalize its optimal aggregate order quantity from tier 1 in the absence of any disruption risk to 2, i.e.,

$$\phi'(y) - p = 0, \text{ for } y = 2.$$

Finally, for simplicity we normalize the suppliers' production function h such that $h(0) = 0$ and $h(1) = 1$. We let m_s, m_d and n_s, n_d denote the order quantities from the manufacturer to each of the suppliers in tier 1 and from each of the suppliers to the fabricators in tier 2 under single-sourcing and dual-sourcing respectively. Importantly, we allow over-ordering from the manufacturer, i.e., $m_s \geq 1$ and/or $m_d \geq 1$. Similar to Section 3 we derive conditions on the production function h under which tier 1 suppliers prefer to dual-source. The expected profit of a component supplier under single-sourcing is equal to

$$\Pi_s = \xi[pm_s - qn_s] + (1 - \xi) \int_0^\Gamma [p\min\{h(n_s x), m_s\} - qn_s x] f(x) dx.$$

On the other hand, if the supplier sources equally from both fabricators, its expected profit is

$$\Pi_d = \xi^2 [pm_d - qn_d] + 2\xi(1 - \xi) \int_0^\Gamma [p\min\{h(n_d/2 + n_d x/2), m_d\} - q(n_d/2(1 + x))] f(x) dx$$
$$+ (1 - \xi)^2 \int_0^\Gamma \int_0^\Gamma [p\min\{h(n_d/2(x_1 + x_2)), m_d\} - q(n_d/2(x_1 + x_2))] f(x_1) f(x_2) dx_1 dx_2.$$

Similarly, the expected profit for the manufacturer when the suppliers single source is given as

$$\Phi_s = \xi^2 [\phi(2m_s) - 2pm_s] + 2\xi(1 - \xi) \int_0^\Gamma [\phi(m_s + \min\{h(n_s x), m_s\}) - p(m_s + \min\{h(n_s x), m_s\})] f(x) dx$$
$$+ (1 - \xi)^2 \int_0^\Gamma \int_0^\Gamma \phi(\min\{h(n_s x_1), m_s\} + \min\{h(n_s x_2), m_s\}) f(x_1) f(x_2) dx_1 dx_2,$$
$$- (1 - \xi)^2 \int_0^\Gamma \int_0^\Gamma p\Big(\min\{h(n_s x_1), m_s\} + \min\{h(n_s x_2), m_s\}\Big) f(x_1) f(x_2) dx_1 dx_2,$$

whereas when they dual source it takes the following form

$$\Phi_d = \xi^2 [\phi(2m_d) - 2pm_d]$$
$$+ 2\xi(1 - \xi) \int_0^\Gamma [\phi(2\min\{h(n_d/2(1 + x)), m_d\}) - 2p(\min\{h(n_d/2(1 + x)), m_d\})] f(x) dx$$
$$+ (1 - \xi)^2 \int_0^\Gamma \int_0^\Gamma \phi(2\min\{h((n_d/2(x_1 + x_2)))\}) - 2p\min\{h((n_d/2(x_1 + x_2)))\} f(x_1) f(x_2) dx.$$

We make Assumption EC.1 that guarantees that the suppliers' aggregate order quantity is equal to the order they receive from the manufacturer, i.e., $n_s = m_s$ and $n_d = m_d$ (the assumption

guarantees that the suppliers do not have an incentive to over-order irrespective of how they source). Note that the assumption is not too restrictive as it allows the manufacturer to strategically over-order (set m_s or m_d greater than one) or order different quantities depending on its equilibrium expectations about the suppliers' sourcing decisions (set $m_s \neq m_d$).

ASSUMPTION EC.1. *Parameters ξ, p, q, and Γ are such that*

$$-\xi q + (1-\xi) \int_0^\Gamma [ph'(1/2 + x/2) - q]\left(\frac{1+x}{2}\right)f(x)dx < 0.$$

Assumption EC.1 essentially implies that disruptive events are not too frequent, i.e., ξ is large enough, which is quite appropriate for models of catastrophic risk. The following two propositions that are analogous to Propositions 1 and 2 establish conditions under which the optimal degree of diversification for firms that operate in different tiers of the supply chain may coincide or be significantly different respectively. In agreement with our benchmark model, the results illustrate that the (mis)alignment in the firms' incentives is largely dependent on the suppliers' production function h.

PROPOSITION EC.1. *If the production function h is concave, then both the suppliers and the manufacturer are better off when tier 1 firms dual-source, as long as $m_d \leq 2/(1+\Gamma)$ and $m_s \leq 2/(1+\Gamma)$.*

Proof: First, note that $\Gamma < 1$ implies that $n_s = h^{-1}(m_s)$, i.e., when suppliers source from a single fabricator they have no incentive to strategically inflate their order. We show that when h is strictly concave then $\Pi_s(n_s) = \Pi_s(h^{-1}(m_s)) < \Pi_d(h^{-1}(m_s))$ which consequently implies that $\Pi_s(n_s) < \Pi_d(n_d)$, i.e., suppliers prefer dual-sourcing. We have

$$\Pi_d(n_s) - \Pi_s(n_s) \geq \xi(1-\xi) \int_0^\Gamma p[2h(n_s/2(1+x)) - h(n_s)] - q[2(n_s/2(1+x)) - n_s]f(x)dx$$
$$+ (1-\xi)^2 \int_0^\Gamma [ph(n_s x) - qn_s x]f(x)dx - (1-\xi) \int_0^\Gamma [ph(n_s x) - qn_s x]f(x)dx,$$
$$\text{(EC.11)}$$

where the inequality follows since h is concave and

$$\int_0^\Gamma \int_0^\Gamma [ph(n_s(x_1/2 + x_2/2)) - q(n_s(x_1/2 + x_2/2))]f(x_1)f(x_2)dx_1dx_2 \geq \int_0^\Gamma [ph(n_s x) - qn_s x]f(x)dx.$$

Next, note that since h is concave we have

$$h(n_s/2 + n_s x/2) - h(n_s x) \geq h(n_s) - h(n_s/2(1+x)),$$

which in combination with (EC.11) implies that suppliers prefer dual-sourcing. The fact that the manufacturer prefers dual sourcing follows in a straightforward manner from that fact that function h is concave and the manufacturer's order quantity does not exceed $2/(1+\Gamma)$.   Q.E.D.

When production in tier 1 is subject to diseconomies of scale, i.e., h is concave, suppliers and the downstream manufacturer find dual-sourcing optimal. This is no longer the case when h has a more general functional form.

PROPOSITION EC.2. *Suppose that the following conditions are satisfied:*

(a) The production function h is strictly concave on average in $[0, \Gamma]$, that is,

$$\int_0^\Gamma \int_0^\Gamma [h(n_s x_1) + h(n_s x_2) - 2h(n_s(x_1 + x_2)/2)] f(x_1) f(x_2) dx < 0.$$

(b) There exists $0 < \bar{x} < \Gamma$ such that h is strictly convex for all values below $\bar{x}$.

(c) The revenue function of the manufacturer, ϕ, is sufficiently concave.

Then, tier 1 suppliers find it optimal to dual-source, whereas the manufacturer would be better off if they employ a single-sourcing strategy.

Proof: The first part of the proposition, i.e., that tier 1 suppliers strictly prefer to dual-source, follows immediately from the assumption on h. It is thus sufficient to show that the profits of the manufacturer are maximized if both tier 1 suppliers single-source. Given the assumptions we made in the section, the suppliers' order quantities satisfy $n_s = h^{-1}(m_s)$ and $n_d = h^{-1}(m_d)$. Then, we have that $\Phi_s(n_s) - \Phi_d(n_d) \geq \Phi_s(n_d) - \Phi_d(n_d)$ and

$$\Phi_s(n_d) - \Phi_d(n_d) = \xi(1-\xi)\psi(2h(n_d)) + 2\xi(1-\xi) \int_0^\Gamma [\psi(h(n_d) + h(n_d x)) - \psi(2h(n_d/2(1+x)))] f(x) dx$$

$$+ (1-\xi)^2 \int_0^\Gamma \int_0^\Gamma [\psi(h(n_d x_1) + h(n_d x_2)) - \psi(2h(n_d(x_1/2 + x_2/2)))] f(x_1) f(x_2) dx_1 dx_2.$$

Given that $\psi(y) = \phi(y) - py$ is concave we have

$$\Phi_s(n_d) - \Phi_d(n_d) \geq \xi(1-\xi)\left[\psi(2h(n_d)) + 2\int_0^\Gamma \psi'(h(n_d) + h(n_d x))[h(n_d) + h(n_d x) - 2h(n_d/2(1+x))] f(x) dx\right]$$

$$+ (1-\xi)^2 \int_0^\Gamma \int_0^\Gamma \psi'(h(n_d x_1) + h(n_d x_2))[h(n_d x_1) + h(n_d x_2) - 2h(n_d/2(x_1 + x_2))] f(x_1) f(x_2) dx_1 dx_2.$$

Assumption (b) states that there exists $\bar{x} < \Gamma$ such that function h is convex for all $x < \bar{x}$. Pick a small enough $\underline{x} < \bar{x}$ such that if $\max\{x_1, x_2\} \geq \bar{x}$, then

$$2h(n_d \underline{x}) \leq h(n_d x_1) + h(n_d x_2). \tag{EC.12}$$

Note that

$$\Phi_s(n_d) - \Phi_d(n_d) \geq \xi(1-\xi)\left[\psi(2h(n_d)) + 2\int_0^\Gamma \psi'(h(n_d) + h(n_d x))[h(n_d) + h(n_d x) - 2h(n_d/2(1+x))] f(x) dx\right]$$

$$+ (1-\xi)^2 \int_0^x \int_0^x \psi'(h(n_d x_1) + h(n_d x_2))[h(n_d x_1) + h(n_d x_2) - 2h(n_d(x_1/2 + x_2/2))] f(x_1) f(x_2) dx_1 dx_2$$

$$+ (1-\xi)^2 \int_{\max\{x_1,x_2\}\geq \bar{x}} \psi'(h(n_d x_1) + h(n_d x_2))[h(n_d x_1) + h(n_d x_2) - 2h(n_d(x_1/2 + x_2/2))]f(x_1)f(x_2)dx_1 dx_2.$$

The proof is complete once we show that there exists a concave function ϕ such that the right-hand side of the above inequality becomes positive. This, however, is immediate as long as

$$h(n_d x_1) + h(n_d x_2) \leq h(n_d) + h(n_d x_2) \leq h(n_d) + h(\Gamma n_d) \leq 2, \qquad \text{for all } x_1, x_2 \leq \Gamma,$$

which guarantees that $\psi'(h(n_d) + h(n_d x)) > 0$ and $\psi'(h(n_d x_1) + h(n_d x_2)) > 0$ for $x, x_1, x_2 \leq \Gamma$ in the inequality above. In other words, the claim follows as long as the manufacturer's order quantity m_d is such that in the event that a disruption occurs (probability $(1-\xi^2)$), the manufacturer will not end up with excess inventory, i.e., the quantity of the (intermediate) good it will receive from the supplies will be less than 2 (this is a quite reasonable assumption in light of the interpretation of the yield shocks as *severe*).   Q.E.D.