Information Provision in Service Platforms: Optimizing for Supply

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While information design has gained significant attention in the recent literature as a tool for shaping consumers' purchase behavior, little is known about its use and implications in two-sided marketplaces, where both supply and demand consist of self-interested strategic agents. In this paper, we develop a dynamic game-theoretic model of a two-sided platform that allows for heterogeneity and endogenous behavior on both sides of the market. We focus on illustrating the potential benefits of optimal information provision in terms of managing supply-side decisions, including supplier entry/exit and pricing. Our analysis identifies three distinct mechanisms through which information design may increase platform revenues. First, when the outside options available to consumers and service providers are relatively unattractive, information design can be used to mimic the so-called "damaged goods" effect, allowing the platform to fine-tune its *composition of providers* and achieve a more revenue-efficient matching between supply and demand. Second, when consumers and/or providers have access to relatively attractive outside options, information design can help the platform increase its *transaction volume* significantly; interestingly, we find that in order to ramp up its throughput, the platform may need to understate the quality of its best providers. Third, when the platform uses commission subsidies to resolve the "cold-start" problem and incentivize the entry of new providers, information design can help achieve the same goal while extracting *higher commission revenues*; thus, we highlight the role of information design as a substitute for commission subsidies. Overall, our numerical experiments suggest that, by influencing the providers' decisions, optimal information provision can lead to a substantial increase in platform revenues.

Key words: platform operations, information provision, social learning, product line design, gig economy

1. Introduction

Information design has received significant attention in the recent literature as a powerful non-monetary lever that can be used by firms to induce desirable consumer behavior in a variety of settings, including queueing systems (Lingenbrink and Iyer 2019), retailing (Drakopoulos et al. 2020), transportation (e.g., Meigs et al. 2020), entertainment (Che and Hörner 2018), service platforms (Papanastasiou et al. 2018), and content promotion (Candogan and Drakopoulos 2020), among others (see Bimpikis and Papanastasiou (2019) for a comprehensive survey).[1] Work in this area of research has focused so far on the "traditional" model of a firm that markets a good or service to consumers. In such settings, it is natural to treat the features of the firm's good (e.g., quality, service rate) as being exogenous to the information design process.

However, an increasing number of high-profile online firms (also referred to as two-sided platforms or marketplaces) operate under an entirely different business model, one that relies on connecting independent self-interested service providers to consumers (e.g., Airbnb, TaskRabbit, Upwork). For these firms, managing the strategic decisions of the supply side of the market, such as entry and pricing, can be just as important as managing those of the demand side. In particular, the platform's overall supply characteristics (and hence the platform's ability to attract consumers) depend critically on the individual service providers the platform is able to attract and maintain. As previous literature on the topic tends to treat supply as exogenous and fixed over time, it is unclear whether and how information design can play a beneficial role in managing supply in a two-sided platform.

The goal of this paper is thus twofold. First, to provide a tractable modeling framework for studying information design in a two-sided marketplace. Second, to highlight the potential supply-side benefits of optimal information provision for a revenue-maximizing platform, along with qualitative insights on how these can be achieved.

To achieve these goals, we develop a dynamic game-theoretic model of an online platform connecting independent service providers (the "supply") to consumers (the "demand"). Unlike the existing literature on information design, our model allows for endogenous behavior on both sides of the market. In each period, service providers (who differ in their service quality) choose whether to seek employment inside or outside the platform, as well as what price to charge for their service, while consumers (who differ in how they value service quality) choose whether to seek service inside or outside of the platform, as well as which provider to transact with. Importantly, the quality of each service provider is ex ante unknown, and is revealed only through transactions with consumers inside the platform. The platform maximizes its revenues using two levers: (i) a commission charged per transaction, and (ii) an information-provision policy, which determines the disclosure of information pertaining to the providers' service quality.

Although the model described above involves rich and complex inter-dependencies between the platform, the providers, and the consumers, we nevertheless demonstrate that it is amenable to

[1] Information design refers to the process of designing an optimal policy for disclosing private information. In the standard paradigm, a principal possesses some private information which he wishes to disclose to an agent in a manner that achieves some desirable action from the agent. A policy consists of a mapping from the firm's private information to a set of "messages" received by the agent and interpreted according to the policy, which is assumed to be disclosed and committed to by the principal.

tractable analysis; in particular, we show that there exists a steady-state equilibrium, from which we are able to extract high-level managerial insights. In solving for equilibrium, we focus on highlighting the implications of the platform's information-provision policy on the providers' participation and pricing decisions, which play an crucial role in the platform's ability to match supply with demand in a revenue-efficient manner. The key observation from our analysis is that while the platform cannot exercise any direct form of control on the providers' decisions, information design can be used as an indirect way of inducing desirable behavior, and can have a (sometimes surprisingly) significant impact on platform revenues.

More specifically, our results are structured to illustrate three distinct supply-related mechanisms through which information design can improve the platform's revenue potential. These mechanisms are briefly described as follows:

(i) *Improving the composition of active providers on the platform.* We consider first an environment where the consumers' and the providers' outside options are relatively unattractive. In such cases, although the platform's transaction volume is high even without optimizing its information-provision policy, we demonstrate that doing so may nevertheless still be valuable for the firm, by helping to induce a more revenue-efficient match between supply and demand. The details of the mechanism point to a two-sided platform's version of the "damaged goods" idea first described in Deneckere and McAfee (1996): the platform uses information provision to deliberately "damage" a fraction of its high-quality providers (i.e., by labeling them as providers of lesser quality), solely for the purpose of price discriminating more effectively. Interestingly, the described mechanism highlights the use of information by two-sided platforms as an indirect way to mimic a traditional firm's direct choice of output quality.

(ii) *Increasing the volume of transactions.* We then consider an environment where the consumers' and/or the providers' outside options are relatively attractive, so that in the absence of an optimized information-provision policy, the platform struggles to generate a high volume of transactions. In such cases, we find that the impact of information design on platform revenues can be dramatic. In particular, our analysis highlights an intriguing chain reaction: by labeling some of its best providers as providers of unknown quality, the demand for providers of unknown quality (a category which includes new providers) increases; as a result, new providers become more willing to join the platform and are more likely to be hired, leading to higher levels of experimentation inside the platform. In turn, more experimentation leads to a higher rate of discovery of high-quality providers, who then (choose to) remain active on the platform. In the process, the platform enjoys a ramp-up of both new and high-quality supply, which leads to a significant increase in volume and revenues.

(iii) *Reducing commission subsidies.* Many two-sided platforms in practice struggle to gain traction because of the so-called "cold-start" problem faced by new providers. A direct approach to alleviate this is to offer subsidies in the form of reduced commission rates for new providers, in order to incentivize their entry. We first show that, in the absence of an optimized information-provision policy, such subsidies can indeed be beneficial for the platform; in fact, we find that in some cases it may even be optimal for the firm to incur a loss on transactions with new providers, so as to increase entry substantially and, in this way, accelerate the discovery of high-quality providers. However, we demonstrate that such costly measures need not be necessary. Instead, we find that information design can significantly reduce the need for commission subsidies, allowing the platform to achieve a higher level of entry of new providers without forgoing a significant share of its revenues in the process. Thus, this mechanism highlights the use of information as a cost-effective substitute for monetary subsidies.

The rest of this paper is organized as follows. In Section 2 we review the related literature. In Section 3 we present our model and in Section 4 we establish the existence of a steady-state equilibrium. In Section 5 we solve for the platform's optimal policy and demonstrate the three main mechanisms through which information design can improve platform revenues. Section 6 concludes.

2. Related Literature

This work contributes to the growing literature that considers the applications and implications of information design (see Rayo and Segal (2010) and Kamenica and Gentzkow (2011)) in settings where a principal seeks to optimally disclose private information to agents whose actions affect the principal's payoff. Lingenbrink and Iyer (2019) illustrate how information design can help a service provider modulate consumers' queue-joining behavior when the system state (queue length) is not directly observable to the consumers (for work on queue-joining behavior when the system state is observable, see Veeraraghavan and Debo (2009) and Veeraraghavan and Debo (2011)). Kostami (2019) compares the relative merits of static versus dynamic leadtime information provision in inventory systems. Drakopoulos et al. (2020) show that selective disclosure of inventory information can help a retailer control consumers' buy-now-or-wait decisions. Alizamir et al. (2020) investigate how a principal's optimal disclosure/warnings relating to harmful events depends on the perceived accuracy of his private information. Candogan and Drakopoulos (2020) consider information design in the context of promoting content in social networks. Anunrojwong et al. (2020) study the benefits of information design in the context of social services catering to individuals who may differ in their level of need.

Closer to our work is the paper by Papanastasiou et al. (2018), which focuses on information design in a two-sided service platform, but takes the supply side of the market as exogenous and fixed over time. The authors show that partial information structures can induce self-interested consumers to take actions that benefit their peers. In the same spirit, Kremer et al. (2014) and Che and Hörner (2018) also consider how information design may affect demand-side decisions while taking the supply as given. Mansour et al. (2015) take a more algorithmic approach and develop an algorithm with asymptotically optimal regret. For a more comprehensive survey of related work that abstracts away from supply-side considerations, see Bimpikis and Papanastasiou (2019). The current paper considers a two-sided platform setting, but our focus here is on the implications of information design for the supply side of the market. In investigating these implications, we model service providers as strategic agents who make their own entry/exit and pricing decisions, and we show that information design can help the platform increase its revenues by influencing the providers' short- and long-run equilibrium behavior. Furthermore, unlike the aforementioned work that focuses predominantly on total welfare as the objective to be maximized, this paper takes the perspective of revenue maximization, which renders the analysis significantly more complicated.

A central feature of the market we consider is that service providers (the supply side) are heterogeneous in quality, and that the quality of each provider can only be assessed "on the job" (i.e., when a provider engages in transactions with the platform's consumers). In work that addresses this market feature more directly, Terviö (2009) demonstrates that the potential inefficiencies associated with on-the-job discovery of quality may lead to a decrease in the average quality of workers in the market; Pallais (2014) show experimentally in an online labor marketplace that subsidizing inexperienced workers to generate information about their quality improves their subsequent employment outcomes; and Stanton and Thomas (2016) estimate that outsourcing agencies significantly increase workers' average earnings in online marketplaces by signaling the high quality of affiliated inexperienced workers. Collectively, these papers illustrate the potential for significant market level benefits of interventions that promote experimentation with new providers. Our work adds to this literature by highlighting the use of information design as a non-monetary intervention which may result in significant gains for market participants.

This work also complements the broader literature that studies the design and optimization of two-sided platforms. Belavina et al. (2020) explore the use of deferred payment mechanisms as a way to deter misconduct in crowdfunding platforms, while Du et al. (2017) consider the use of contingent stimulus policies to improve the success rate of crowdfunded projects. Tsoukalas and Hemenway Falk (2020) examine the effectiveness of token-based platforms to efficiently aggregate dispersed information from their users for applications ranging from curating content to on-chain governance. Feldman et al. (2019) study whether food-delivery platforms are beneficial to restaurants. Kanoria and Saban (2020) consider matching markets that may exhibit search inefficiencies and prescribe non-monetary interventions in the form of restrictions on agents' actions that improve welfare. Birge et al. (2020) provide a convex optimization formulation to calculate the revenue maximizing commission structure for a platform that facilitates trade between buyers and sellers of different types. Bimpikis et al. (2019) characterize optimal prices and commissions in the context of a ride-sharing platform that matches geographically dispersed demand for rides with a supply of drivers, who aim to maximize their expected earnings. Vellodi (2020) focuses on whether the availability of reviews about the quality of firms may adversely affect entry and derives design recommendations that lead to higher welfare. Finally, recent work by Johari et al. (2020) considers a problem of information disclosure in a two-sided market and derive conditions under which banning a fraction of the supply while not sharing any information about the rest is optimal for the platform.

3. Model Description

We consider a dynamic model of a two-sided platform that connects service providers with consumers. The model consists of three types of players, who interact with one another over an infinite discrete-time horizon: (i) the platform, which chooses a commission to be charged per transaction and an information-provision policy; (ii) the supply (i.e., a population of service providers), who choose whether to join the platform and, if so, what price to charge for providing service, and (iii) the demand (i.e., a population of consumers), who choose whether to seek service on the platform and, if so, with which service provider.

Demand. We assume that in each time period there is a short-lived population of consumers with total mass normalized to one, who enter the platform seeking service. Consumers are heterogeneous in their willingness to pay for service quality. We use θ to denote a consumer's type, and we assume that consumer types are uniformly distributed on the interval $[1,2]$. The net utility for a consumer with type θ from transacting with provider k is given by:

$$u = \theta q_k - p_k,$$

where q_k is the provider's service quality and p_k is the provider's service price. Upon entering the platform, each consumer observes the set of available service providers, the price set by each provider, and any information on the provider's service quality provided by the platform (the latter is determined by the platform's information-provision policy, as described below). Then, each consumer chooses among the available service providers with the goal of maximizing her expected utility. Apart from the providers available on the platform, consumers also have the option of seeking service outside of the platform; we assume that doing so results in expected service quality $q_0 \in (0,1)$ at price $p_0 \in (0,1)$. We further assume that $q_0 \geq p_0$, so that the outside option results in non-negative utility for all consumer types.

Supply. We assume that there is a large pool of potential service providers, a fraction $1-\beta$ of whom cease to exist in each time period and are replaced by new potential providers of equal mass. In every period, the potential providers may choose to enter the platform or pursue employment outside of it. Employment outside of the platform yields an expected revenue of w_0 per period. Inside the platform, expected revenues depend on the platform's commissions and information-provision policy and the resulting equilibrium behavior of the service providers and consumers. We assume that the service quality of each provider k can be either high or low, $q_k \in \{q_H, q_L\}$ and we normalize $q_H = 1$ and $q_L = 0$. The probability of a randomly chosen provider being of high quality is denoted by $\gamma \triangleq P(q_k = q_H)$. In our analysis, we will focus on the more interesting environments where the supply of high-quality providers is relatively scarce; accordingly, we assume that:

(i) $E[q_j] = \gamma q_H + (1-\gamma) q_L < q_0$ (i.e., although there are high-quality providers in the market, the expected quality of a randomly drawn provider is lower than that of the outside option); and

(ii) $\gamma\beta < 1-\beta$ (i.e., the volume of surviving high-quality providers in each period does not exceed the total volume of providers who cease to exist).

To capture the learning process that is characteristic of platforms with independent service providers, we assume that the service quality of a new provider is initially unknown and is revealed only after the provider engages in a transaction with a consumer inside the platform.[2] In every period, each provider, taking into account the platform's commissions and information-provision policy, chooses whether to join the platform and, if so, what price to charge for service; with respect to the latter, we assume that the net payment (i.e., the service price minus the commission fee), received for providing service must be at least b_0, for some $b_0 \in [0, p_0]$ (for instance, this may represent the provider's per-period cost of providing service).

Platform. The platform is long-lived and seeks to maximize its expected per-period revenue, by choosing a commission rate τ and an information-provision policy. The commission rate amounts to a percentage fee collected by the platform on any transaction that occurs between a consumer and a provider. The information-provision policy specifies a message or label attached to each provider, which is displayed to the consumers.[3] The label is meant to convey information on the provider's service quality based on the provider's past service outcomes, which we assume are observable to the platform (e.g., via consumer feedback or reviews). According to the platform's quality-learning process described above, the platform's information about provider k's quality

[2] Alternatively, one may consider a more gradual quality revelation process (i.e., each transaction generates a noisy signal about the provider's quality). However, this increases the problem's state-space and complicates the analysis considerably, without adding to the qualitative insights we aim to illustrate.

[3] Consistent with the rest of the information design literature, we assume that the platform's information-provision policy is announced and committed to (e.g., see Kamenica and Gentzkow 2011, Lingenbrink and Iyer 2019, Papanastasiou et al. 2018).

in any given period is described by state $j_k \in \{H, L, U\}$, corresponding to high, low, or unknown quality, respectively. The information-provision policy employed by the platform is then expressed as a (possibly stochastic) mapping from the platform's private information about provider k to a "label" which is assigned to the provider and published on the platform,

$$g(j_k) = \begin{cases} \mathcal{H} \text{ ("high quality")} & \text{w.p. } \rho_{\mathcal{H}}^{j_k} \\ \mathcal{L} \text{ ("low quality")} & \text{w.p. } \rho_{\mathcal{L}}^{j_k} \\ \mathcal{U} \text{ ("unknown quality")} & \text{w.p. } \rho_{\mathcal{U}}^{j_k}, \end{cases} \tag{1}$$

where $\rho_{\mathcal{H}}^{j_k} + \rho_{\mathcal{L}}^{j_k} + \rho_{\mathcal{U}}^{j_k} = 1$, for all $j_k \in \{H, L, U\}$. Thus, designing an information-provision policy consists of choosing the probability with which each label is assigned to each provider state. At one extreme, a policy such that $\rho_{\mathcal{H}}^{H} = \rho_{\mathcal{L}}^{L} = \rho_{\mathcal{U}}^{U} = 1$ corresponds to full information disclosure, since the platform's information can be perfectly inferred from the labels it assigns. At the other extreme, any policy with $\rho_{\mathcal{H}}^{j_k}, \rho_{\mathcal{L}}^{j_k}$, and $\rho_{\mathcal{U}}^{j_k}$ chosen independently of j_k corresponds to no information disclosure, since none of the platform's information can be inferred from the labels it assigns to the providers. Policies involving intermediate levels of information provision can be constructed by choosing the probabilities $\rho_{\mathcal{H}}^{j_k}, \rho_{\mathcal{L}}^{j_k}$, and $\rho_{\mathcal{U}}^{j_k}$ appropriately between the above two extremes.

With respect to the design of information provision policies, it is intuitive and straightforward to show that the platform cannot benefit from concealing information about providers whose quality is known to be low. However, it is less clear whether the platform can benefit from concealing information about providers known to be of high quality. Accordingly, in the analysis that follows we focus on the class of policies satisfying

$$\rho_{\mathcal{U}}^{U} = 1, \;\; \rho_{\mathcal{L}}^{L} = 1, \; \text{ and } \rho_{\mathcal{H}}^{U} = 1 - \rho_{\mathcal{H}}^{H} =: \alpha \in [0, 1]. \tag{2}$$

In words, under the class of policies defined in (2), the platform always assigns label $\mathcal{U}$ to providers of unknown quality (i.e., U-type providers) and label $\mathcal{L}$ to providers of low quality (i.e., known L-type providers). However, providers of high quality (i.e., known H-type providers) may be assigned label $\mathcal{U}$ with positive probability. At first glance, it may appear counter-intuitive for the platform to conceal the quality of its best providers, given that these providers are its highest earners; however, in Section 5 we demonstrate three distinct mechanisms through which such an approach to information disclosure can improve the platform's revenues.

From a practical point of view, we note that the class of policies described in (2) is particularly appealing in that it is operationally equivalent to a policy that simply delays disclosing information pertaining to the quality of high-quality service providers, an approach that is already observed in practice (for example, high-quality freelancers on the online labor marketplace Upwork can be labeled as "top-rated" only if they remain active on the marketplace for at least twelve months after their first transaction).[4,5] To emphasize this connection, we refer to a policy of the form (2) as an *"information-delay"* policy with delay parameter α.

4. Equilibrium

Given that the underlying supply and demand processes are time-invariant in our model, our analysis will focus on steady-state equilibria of the supply-demand game, for a fixed platform policy $\{\tau, \alpha\}$, where recall that τ denotes the commission fee that the platform extracts from every transaction and α represents the information-provision policy (i.e., the probability with which a high quality provider is assigned the label $\mathcal{U}$). To establish the existence of a steady-state equilibrium, the supply-demand game must simultaneously satisfy a number of conditions relating to supply-side participation and pricing, demand-side participation and provider choice, and supply-demand matching. We next describe these conditions in detail.

Consider first the platform participation decisions of individual providers. In a steady-state equilibrium, the expected lifetime earnings of a high-quality provider who is assigned label $\mathcal{H}$ in any period are given by

$$V_{\mathcal{H}}^{H} = \max\left\{\frac{(1-\tau)p_{\mathcal{H}}}{1-\beta}, \frac{w_0}{1-\beta}\right\}. \tag{3}$$

That is, such a provider will stay in the platform provided the price he can charge as an $\mathcal{H}$-labeled provider is sufficiently high and/or the platform's commission rate is sufficiently low, while he will seek employment outside the platform otherwise. The expected lifetime earnings of a high-quality provider who is assigned label $\mathcal{U}$ are given by

$$V_{\mathcal{U}}^{H} = \max\left\{\eta(1-\tau)p_{\mathcal{U}} + \beta\left(\alpha V_{\mathcal{U}}^{H} + (1-\alpha)V_{\mathcal{H}}^{H}\right), \frac{w_0}{1-\beta}\right\}, \tag{4}$$

where the (endogenous) parameter $\eta \in [0,1]$ here accounts for rationing that may occur if in equilibrium the demand for $\mathcal{U}$-labeled providers is lower than the availability of such providers.[6] Finally, for a provider of unknown quality (i.e., who has not yet transacted on the platform), the expected lifetime earnings are given by

$$V_{\mathcal{U}}^{U} = \max\left\{\eta\left((1-\tau_{\mathcal{U}})p_{\mathcal{U}} + \beta\left(\gamma\left(\alpha V_{\mathcal{U}}^{H} + (1-\alpha)V_{\mathcal{H}}^{H}\right) + (1-\gamma)\frac{w_0}{1-\beta}\right)\right) + (1-\eta)\beta V_{\mathcal{U}}^{U}, \frac{w_0}{1-\beta}\right\}. \tag{5}$$

[4] In particular, the platform here assigns label $\mathcal{H}$ to a high-quality provider only after a geometrically distributed number of transactions with parameter α, after which the label remains unchanged for the remainder of the provider's time on the platform.

[5] See https://support.upwork.com/hc/en-us/articles/211068468-Become-Top-Rated.

[6] Note that there can never be rationing among providers labeled $\mathcal{H}$ in equilbrium, since in such a case a provider could increase his earnings by unilaterally lowering his price slightly, which would guarantee being matched to a consumer.

We note that the expressions for $V_{\mathcal{H}}^{H}$, $V_{\mathcal{U}}^{H}$, and $V_{\mathcal{U}}^{U}$ involve prices $p_{\mathcal{U}}$ and $p_{\mathcal{H}}$ that are determined endogenously as a function of both the demand for each provider type as well as the competition between service providers (we provide additional details on this below). Moreover, we point out that our assumption that each provider must receive a minimum net payment $b_0 \geq 0$ for providing service implies that in any equilibrium with positive i-labeled provider participation, where $i \in \{\mathcal{U}, \mathcal{H}\}$, we must have $(1-\tau)p_i \geq b_0 \geq 0$. Moreover, free entry of providers implies that $V_{\mathcal{U}}^{U} = \frac{w_0}{1-\beta}$. Assuming that a steady-state equilibrium exists, we use δ_i^j to denote the mass of providers of quality $j \in \{U, H\}$ with label $i \in \{\mathcal{U}, \mathcal{H}\}$, who are active on the platform in any time period.

We next discuss the demand for different provider types (i.e., labels) in a steady-state equilibrium. Recall that each consumer chooses a provider to maximize her expected utility. In particular, a consumer with type θ chooses

$$\underset{i \in \{0, \mathcal{U}, \mathcal{H}\}}{\arg\max} \; \theta q_i - p_i,$$

where $\{0, \mathcal{U}, \mathcal{H}\}$ represents the set of available options to consumers, i.e., transacting with the outside option or with a provider on the platform with label $\mathcal{U}$ or $\mathcal{H}$. According to the information policy defined by (2), the expected quality of a provider with label $\mathcal{H}$ is equal to $q_{\mathcal{H}} = 1$, i.e., only high quality providers are assigned label $\mathcal{H}$. On the other hand, the expected quality of a provider with label $\mathcal{U}$ depends on α and is given by

$$q_{\mathcal{U}}(\alpha) \triangleq \frac{\delta_{\mathcal{U}}^{U} q_U + \eta \delta_{\mathcal{U}}^{H} q_H}{\delta_{\mathcal{U}}^{U} + \eta \delta_{\mathcal{U}}^{H}}. \tag{6}$$

The latter expression reflects the fact that, as a result of the platform's information-delay policy, the set of providers who get assigned label $\mathcal{U}$ may contain providers of high quality in addition to providers of unknown quality, so that $q_{\mathcal{U}}(\alpha) \in [q_U, q_H]$. Given expected qualities q_i and equilibrium prices p_i, let ζ_i denote the mass of consumers that engage in a transaction with a provider carrying label i, for $i \in \{\mathcal{U}, \mathcal{H}\}$. The following result describes how the quantities q_i, p_i and ζ_i are related, and provides the main structure of a steady-state equilibrium.

Lemma 1. *Consider a steady-state equilibrium under policy $\{\tau, \alpha\}$.*

Suppose $q_{\mathcal{U}}(\alpha) < q_0$. Then the equilibrium takes the form of Figure 1. In particular:

(i) If $1 < \frac{p_{\mathcal{U}} - p_0}{q_{\mathcal{U}}(\alpha) - q_0} < \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{1 - q_{\mathcal{U}}(\alpha)} < \frac{p_{\mathcal{H}} - p_0}{1 - q_0} < 2$, then

$$\zeta_{\mathcal{U}} = \frac{p_{\mathcal{U}} - p_0}{q_{\mathcal{U}}(\alpha) - q_0} - 1 \text{ and } \zeta_{\mathcal{H}} = 2 - \frac{p_{\mathcal{H}} - p_0}{1 - q_0}. \tag{7}$$

(ii) If $\max\left(\frac{p_{\mathcal{H}} - p_0}{1 - q_0}, 1\right) < \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{1 - q_{\mathcal{U}}(\alpha)} < \min\left(\frac{p_{\mathcal{U}} - p_0}{q_{\mathcal{U}}(\alpha) - q_0}, 2\right)$, then

$$\zeta_{\mathcal{U}} = \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{1 - q_{\mathcal{U}}(\alpha)} - 1 \text{ and } \zeta_{\mathcal{H}} = 2 - \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{1 - q_{\mathcal{U}}(\alpha)}. \tag{8}$$

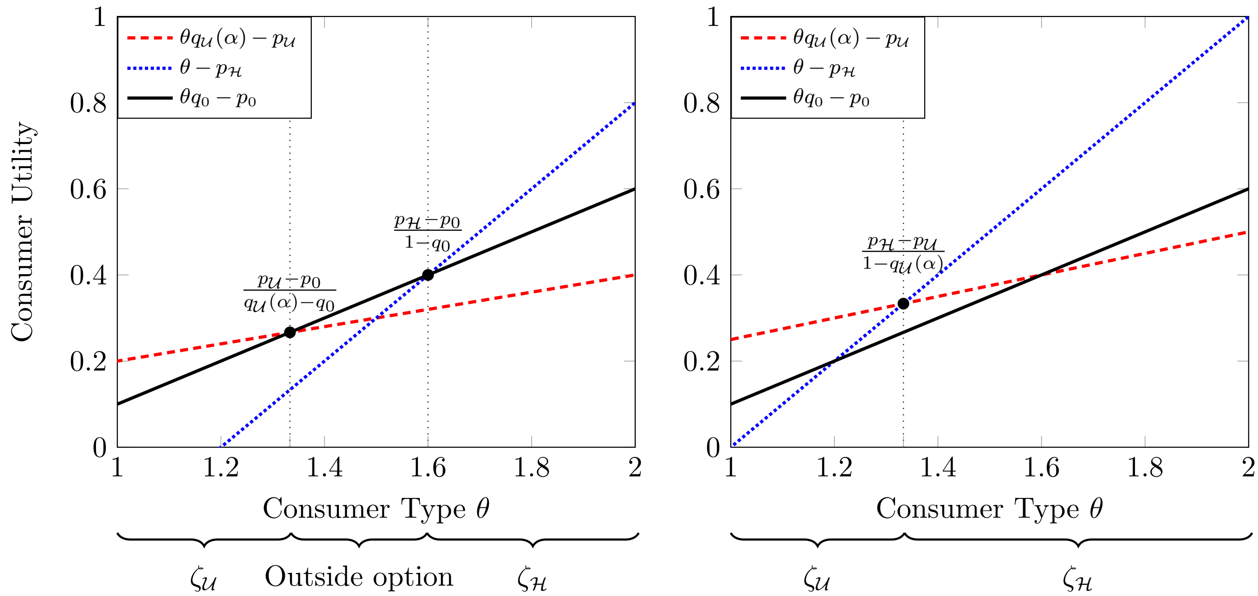

Figure 1 **Consumers' equilibrium choices of providers when $q_{\mathcal{U}}(\alpha) < q_0$. Left figure: If $1 < \frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}(\alpha)-q_0} < \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}(\alpha)} < \frac{p_{\mathcal{H}}-p_0}{1-q_0} < 2$ holds, then consumers with low (high) types transact inside the platform with providers labeled $\mathcal{U}$ ($\mathcal{H}$), whereas those with intermediate types take their outside option. Right figure: If $\max\left(\frac{p_{\mathcal{H}}-p_0}{1-q_0}, 1\right) < \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}(\alpha)} < \min\left(\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}(\alpha)-q_0}, 2\right)$, then the platform covers the entire consumer demand and low (high) types transact with providers labeled $\mathcal{U}$ ($\mathcal{H}$).**

(iii) Otherwise, $\zeta_{\mathcal{U}} = \zeta_{\mathcal{H}} = 0$.

Suppose $q_{\mathcal{U}}(\alpha) \geq q_0$. *Then the equilibrium takes the form of Figure 2. In particular:*

(i) If $\max\left(\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}(\alpha)-q_0}, 1\right) < \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}(\alpha)} < 2$, *then*

$$\zeta_{\mathcal{U}} = \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}(\alpha)} - \max\left(\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}(\alpha)-q_0}, 1\right) \text{ and } \zeta_{\mathcal{H}} = 2 - \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}(\alpha)}. \tag{9}$$

(ii) Otherwise, $\zeta_{\mathcal{U}} = \zeta_{\mathcal{H}} = 0$.

To conclude this section, we establish that a steady-state equilibrium as described above indeed exists for any given platform policy.

Proposition 1. *For any policy* $\{\tau, \alpha\}$, *a steady-state equilibrium exists.*

In summary, a steady-state equilibrium exists in our model for any platform policy $\{\tau, \alpha\}$and the supply-demand interactions induced by the platform's policy are fully described by the endogenous quantities δ_i^j, ζ_i, and p_i, for $j \in \{U, H\}$ and $i \in \{\mathcal{U}, \mathcal{H}\}$.

5. Value Drivers of Optimal Information Provision

The analysis of §4 establishes the existence and properties of a steady-state equilibrium for any *given* platform commissions-and-information policy $\{\tau, \alpha\}$. In this section, we solve for the platform's

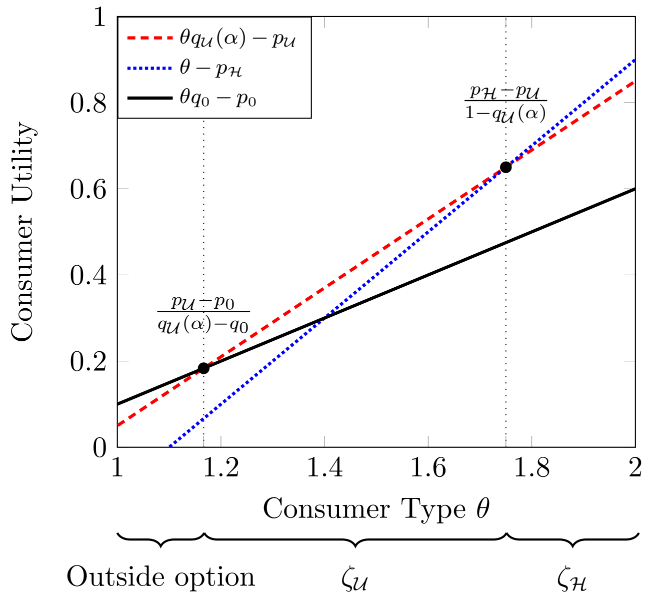

Figure 2 **Consumers' equilibrium choices of providers when $q_{\mathcal{U}}(\alpha) \geq q_0$ and $\max\left(\frac{p_{\mathcal{U}} - p_0}{q_{\mathcal{U}}(\alpha) - q_0}, 1\right) < \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{1 - q_{\mathcal{U}}(\alpha)} < 2$. Consumers with low types take their outside option, whereas the rest transact with providers inside the platform.**

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revenue-maximizing policy, with a focus on delivering the main qualitative insights associated with this policy. In particular, we identify and describe three distinct mechanisms through which an optimized information-provision policy can increase the platform's equilibrium revenues. The way in which information design can benefit the platform depends on the market conditions in which the platform operates, and in particular on the relative attractiveness for consumers and providers of transacting through the platform as opposed to taking their respective outside options.

To clearly illustrate each mechanism, our exposition proceeds in two steps. First, we characterize the equilibrium outcome assuming that the platform employs a full-information provision policy (i.e., in any period, the platform discloses all information it has in its possession regarding the quality of each provider active on the platform). Then, we show that employing an appropriately designed policy with information delay $\alpha > 0$ (i.e., of the form given in (2)) leads to higher revenues for the platform, highlighting the mechanics and drivers underlying the revenue improvement.

Taking into account the analysis of §4, before proceeding to the first mechanism we place two assumptions on our model primitives for the remainder of this section.

Assumption 1. *The following inequalities hold: (a) $q_0 - p_0 < E[q_j] - b_0$; (b) $q_0 < \frac{(2-\beta)\gamma}{2(1-\beta)+\beta\gamma}$.*

Assumption 1(a) ensures that the platform's revenue under a full-information provision policy is positive. In particular, when this assumption does not hold, no consumer would ever transact with a provider inside the platform, preferring instead the outside option. Assumption 1(b) is a technical condition we impose for tractability (i.e., the condition is in general sufficient but not necessary for our results), which essentially places an upper bound on the quality of the outside option.

5.1. Mechanism I: Improving the Provider Composition

The first setting we consider is one where the outside options available to consumers and providers are relatively unattractive compared to the utility they may derive by transacting inside the platform. In particular, in this section we consider a setting with the following characteristics: (i) q_0 is relatively low and/or p_0 is relatively high (i.e., the service quality of the outside option is relatively low and/or its price is relatively high); (ii) γ is relatively high (i.e., the fraction of high-quality providers in the population is sufficiently high); and (iii) b_0 is relatively low (i.e., the providers' per period cost of providing service is low).[7] In such a setting, it can be shown that when the platform employs a full-information provision policy, it is optimal to set the commission τ so as to induce an equilibrium where the platform satisfies all consumer demand in each period.

Proposition 2. *When the platform employs a full-information provision policy (i.e., $\alpha = 0$) and sets the commission optimally, all consumers choose a provider inside the platform.*

In particular, under a full-information provision policy, the platform's revenues are maximized at the highest commission rate such that all consumers prefer to seek service inside the platform. Increasing the commission rate further would increase the platform's revenue per transaction, but would result in a positive measure of consumers choosing the outside option instead of the platform. The associated loss in transaction volume from doing so outweighs the increase in revenue per transaction, leading to the result of the proposition.

Proposition 2 describes a situation where the platform already holds a strong position in the market relative to the outside options available to the consumers and the service providers. Nevertheless, even in such a case there may still be an opportunity for revenue growth if the platform can find a way to "soften" the tradeoff between revenue per transaction and overall transaction volume. Proposition 3 establishes that this can be achieved via an appropriately-designed policy for information provision.

Proposition 3. *The optimal platform policy features information provision with positive delay (i.e., $\alpha^* > 0$). In the equilibrium induced by the optimal policy, the total volume of transactions stays the same but the volume of transactions with $\mathcal{H}$-labeled ($\mathcal{U}$-labeled) providers is strictly lower (strictly higher), as compared to when the platform employs a full-information provision policy.*

[7] For example, the conditions $\gamma > q_0 - \frac{p_0}{2}$ and $b_0 = 0$ are sufficient.

Proposition 3 reveals that the mechanism underlying the benefits of an information policy with positive delay is one of optimizing the *composition* of transactions occurring on the platform, while maintaining the total volume of transactions constant. In other words, a policy with positive information delay allows the platform to match supply with demand in a more revenue-efficient manner. We next discuss how this is achieved.

Observe that relative to a full-information provision policy, a policy with positive delay (i.e., $\alpha > 0$) effectively amounts to assigning label $\mathcal{U}$ to a positive fraction of H-type providers in each period. This has two implications. The first implication is that the relative proportion of providers labeled $\mathcal{H}$ versus $\mathcal{U}$ changes; in particular, as α increases, the relative proportion of $\mathcal{H}$-labeled providers decreases. The second implication is that the consumers' quality expectation when hiring a $\mathcal{U}$-labeled provider changes, because this label now includes both providers of unknown quality (a proportion γ of whom are high-quality providers) as well as providers who are already known to be of high quality. In particular, the expected quality of a $\mathcal{U}$-labeled provider increases with the ratio of H-type providers included in this label. Conversely, the quality expectation when hiring an $\mathcal{H}$-labeled provider is unchanged.

The first implication tends to drive equilibrium prices for transactions involving $\mathcal{H}$-labeled (respectively, $\mathcal{U}$-labeled) providers up (down). In particular, $\mathcal{H}$-labeled providers now face less intense competition from their peers, which allows them to increase their price. On the other hand, $\mathcal{U}$-labeled providers now face more intense competition from their peers, which tends to drive their prices down; however, this tendency is counteracted by the second implication, namely, the consumers' increased expectation of service quality when hiring a $\mathcal{U}$-labeled provider, which increases their willingness-to-pay for $\mathcal{U}$-labeled service. As we show in the proof of Proposition 3, the latter effect dominates, causing equilibrium prices for $\mathcal{U}$-labeled transactions to increase.

As a result, we observe that as α increases, (i) the relative volume of $\mathcal{H}$-labeled ($\mathcal{U}$-labeled) transactions decreases (increases), and (ii) the equilibrium price of both $\mathcal{H}$- and $\mathcal{U}$-labeled transactions increases. Given that in any equilibrium $\mathcal{H}$-labeled transactions are more profitable for the platform, the optimal policy α^* then consists of identifying the best possible combination of commission revenues and relative transaction volumes. Figure 3 presents an example of the mechanism underlying the result of Proposition 3. Observe that the improvement in platform revenue from employing a policy with information delay is substantial, even though the platform's "starting position" (i.e., the steady-state transaction volume achieved under full information) in the setting considered here is already strong.

On an intuitive level, the mechanism described in Proposition 3 is the two-sided platform analogue of the well-known "damaged goods" effect, which applies under the traditional model of a firm: in order to price discriminate more effectively, the firm deliberately reduces (i.e., "damages")

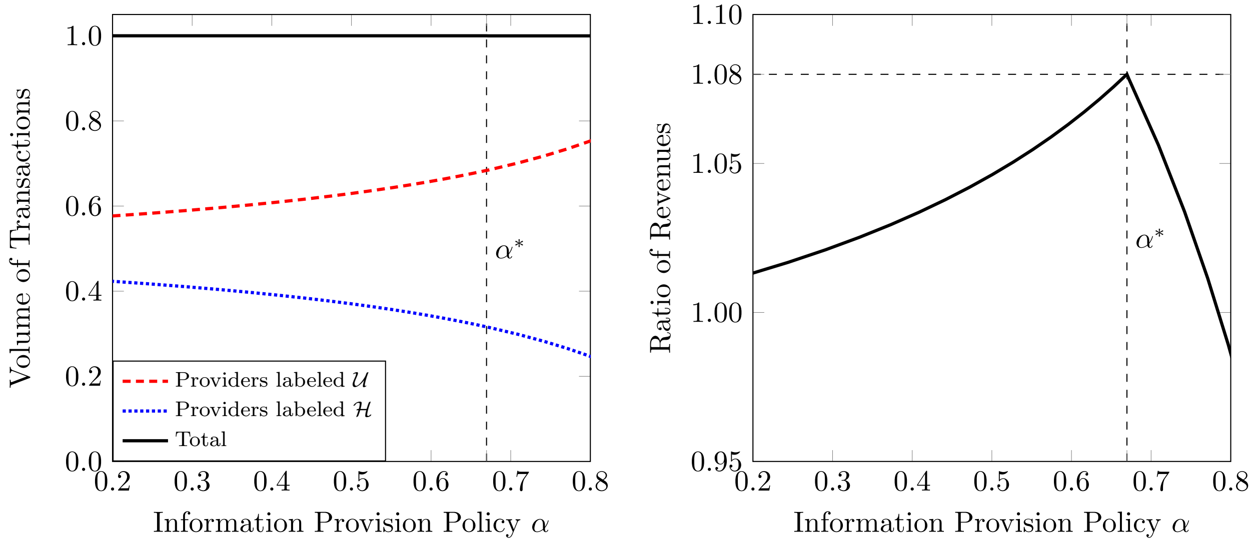

Figure 3 **Mechanism I: Improving the Provider Composition. The left plot depicts the volume of transactions with providers labeled $\mathcal{U}$ and $\mathcal{H}$ inside the platform as a function of the information provision policy α. The right plot depicts the ratio of revenues under information provision policy α over the revenues under full information disclosure (parameter values: $q_0 = 0.35$, $p_0 = 0.32$, $w_0 = 0.31$, $b_0 = 0$, $\gamma = 0.2$, and $\beta = 0.8$).**

the quality of a fraction of its output (see Deneckere and McAfee (1996)). In the two-sided platform model, the platform does not have direct control over the quality of its providers. However, by setting $\alpha^* > 0$ the platform essentially uses information to “damage” a fraction of its high-quality providers, by labeling them as providers of lesser quality. In turn, this leads to higher equilibrium prices set by the providers, as compared to when the platform employs a full-information provision policy. Thus, although the volume of transactions with the more-profitable $\mathcal{H}$-labeled providers is lower, the platform’s revenue per transaction is higher, leading to an overall increase in revenues.

5.2. Mechanism II: Increasing the Transaction Volume

The mechanism described in §5.1 applies to cases where the market conditions are such that the platform finds it optimal to cover the entire demand in the absence of an optimized information-provision policy. The second mechanism we describe is relevant when the consumers’ and/or the providers’ outside options are sufficiently attractive relative to transacting inside the platform, so that the platform does not find it optimal to satisfy the entire consumer demand under full information (i.e., a positive fraction of consumers choose to transact with the outside option).[8] This is formalized in the following proposition.

Proposition 4. *When the platform employs a full-information provision policy (i.e., $\alpha = 0$)*

[8] For example, the condition $\frac{p_0}{q_0 - \gamma} < 1 + \min\left(\frac{1}{4}, w_0\right)$ is sufficient.

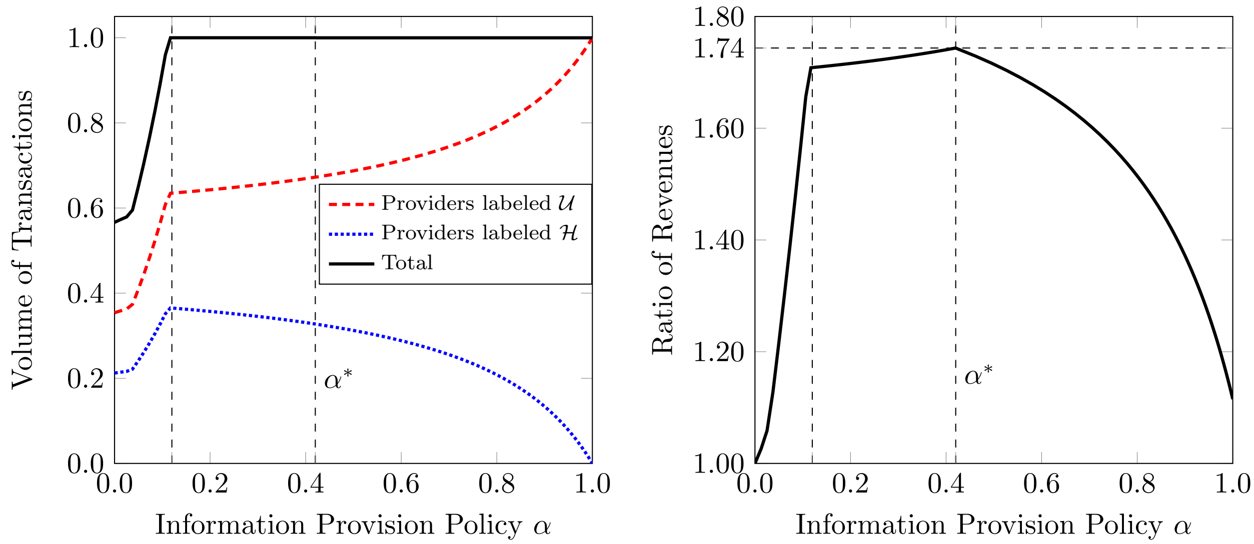

Figure 4 **Mechanism II: Increasing the Transaction Volume. The left plot depicts the volume of transactions with providers labeled $\mathcal{U}$ and $\mathcal{H}$ inside the platform as a function of the information provision policy α. The right plot depicts the ratio of revenues under information provision policy α over the revenues under full information disclosure (parameter values:** $q_0 = 0.21$, $p_0 = 0.085$, $w_0 = 0.035$, $b_0 = 0.005$, $\gamma = 0.15$, **and** $\beta = 0.8$**).**

and sets the commission optimally, the mass of consumers that choose the outside option in each period is strictly positive.

Proposition 4 describes an equilibrium where it is optimal for the platform to serve only a subset of the consumer demand, as a result of the difficulty in attracting providers and consumers to the platform in a profitable manner. While the platform in these cases finds itself in a difficult situation, Proposition 5 documents a powerful mechanism through which optimized information provision can help improve the platform's circumstance significantly.

PROPOSITION 5. *The optimal platform policy features information provision with positive delay (i.e., $\alpha^* > 0$). In the equilibrium induced by the optimal policy, both the overall volume of transactions as well as the volume of transaction with providers labeled $\mathcal{H}$ are strictly higher as compared to when the platform employs a full-information provision policy.*

In the mechanism described in Proposition 5, a policy with information delay increases the platform's revenue by increasing the total volume of transactions occurring inside the platform. Interestingly, the result also establishes that concealing (i.e., delaying the release of) information on the quality of some of its known high-quality providers in each period in fact allows the platform to maintain and reveal a higher volume of high-quality providers active inside the platform in steady state.

It is helpful to discuss the details of the mechanism underlying Proposition 5 in the context of the example of Figure 4, where we plot the volume of $\mathcal{U}$- and $\mathcal{H}$-labeled transactions occurring inside the platform in steady state, as a function of the delay parameter α. The plot can be explained on the basis of two qualitatively different regions depending on the value of α.

Consider first the region where $\alpha < 0.1$. Note that, in this region, the total volume of transactions occurring inside the platform is strictly increasing in α up until the entire market is captured by the platform. Moreover, observe that this occurs through a simultaneous increase in both the volume of transactions with providers labeled $\mathcal{U}$ but also with providers labeled $\mathcal{H}$. With regards to the first, recall that assigning label $\mathcal{U}$ to a positive measure of known H-type providers increases the expected quality of a transaction with a provider labeled $\mathcal{U}$. As a result, the demand for those providers increases, which in turn translates into an increase in the volume of $\mathcal{U}$-labeled transactions. It may appear counter-intuitive that the described manner in which the volume of transactions increases—which effectively involves a "re-labeling" of H-type providers from $\mathcal{H}$ to $\mathcal{U}$—is seen to result in a simultaneous *increase* of $\mathcal{H}$-labeled transactions in the platform. The key lies in the significant increase in participation of new (i.e., U-type) providers inside the platform (which occurs as a result of the increased demand for $\mathcal{U}$-labeled providers described above). The increase in new provider participation implies that the platform is able to discover H-type providers at a higher rate than before, through a higher rate of "experimentation" of consumers with new providers. Because the rate at which H-type providers are discovered and added to the platform's "inventory" is higher than the rate at which H-type providers are hidden from the consumers through the platform's information policy, the result is a net increase in the steady-state volume of $\mathcal{H}$-labeled transactions. Thus, somewhat paradoxically, by hiding high-quality providers from the consumers, the platform ultimately is able to make a higher volume of such providers available to consumers.

As for the region where $\alpha \geq 0.1$, here what we observe is the "composition" mechanism described in §5.1. In particular, once the platform successfully increases the total volume of transactions occurring on the platform through the mechanism described above, injecting further delay into its information policy allows the platform fine-tune the composition of transaction occurring on the platforms in order to achieve the most revenue-efficient set of transactions possible. As demonstrated in Figure 4, in this example the optimal policy increases the delay parameter α to a value higher than 0.1 so as to increase the relative volume of $\mathcal{U}$-labeled transactions while maintaining the same absolute volume of transactions; however, note that most of the revenue increase for the platform is attributed to the increase in the overall volume of transactions and the corresponding higher rate of discovery of high-quality providers (described above) which occurs over the region $\alpha < 0.1$.

5.3. Mechanism III: Reducing Commission Subsidies

Given the analysis in the previous sections, it is reasonable to ask whether the same effect (i.e., increasing the platform's revenues through a higher volume of transactions and/or a better composition of providers) can be achieved through an optimized menu of commissions, and without the need for redesigning the platform's information-provision policy. To answer this question, we now consider the case where the platform's commission policy can be label-dependent. Let the platform's *actual* commission from a $\mathcal{U}$- and $\mathcal{H}$-labeled transaction under the information-provision policy α be denoted by $\tau^c_{\mathcal{U}}(\alpha)$ and $\tau^c_{\mathcal{H}}(\alpha)$, respectively.[9] The following result establishes that the optimal commission menu when the platform employs a full-information provision policy "subsidizes" providers for which the platform does not hold any information.[10]

Proposition 6. *When the platform employs a full-information provision policy, the optimal commission menu satisfies $\tau^{c,*}_{\mathcal{U}}(0) < \tau^{c,*}_{\mathcal{H}}(0)$. Moreover, under the optimal commission menu, all consumers choose to transact on the platform.*

Note that the optimal differentiated-commissions policy described in Proposition 6 is such that the platform captures the entire consumer demand. In this way, the result underscores a key objective for the platform, namely, to increase the entry of new providers into the platform. Doing so has two main benefits: (i) the capacity to serve consumers demanding a $\mathcal{U}$-labeled service provider increases, and (ii) the capacity to serve consumers demanding $\mathcal{H}$-labeled providers also increases (indirectly, given the higher rate of discovery of H-type providers that occurs as a result of more experimentation with new providers). In the proof of Proposition 6, we further show that under the optimal policy, the platform subsidizes the entry of new providers, by either offering a reduced commission rate for $\mathcal{U}$-labeled transactions, or (in more extreme cases) offering to pay $\mathcal{U}$-labeled providers a premium over and above a zero commission rate.

Despite the fact that a differentiated-commissions policy allows the platform to maximize the volume of transactions occurring inside the platform, optimizing its information-provision policy still adds value, as we establish in the proposition that follows.

Proposition 7. *When the platform employs a differentiated-commissions policy, the optimal information-provision policy features positive delay (i.e., $\alpha^* > 0$). Moreover, under the optimal platform policy, the commissions on all transactions are strictly higher as compared to when the platform employs a full-information provision policy, (i.e., $\tau^{c,*}_{\mathcal{U}}(\alpha^*) > \tau^{c,*}_{\mathcal{U}}(0)$ and $\tau^{c,*}_{\mathcal{H}}(\alpha^*) > \tau^{c,*}_{\mathcal{H}}(0)$).*

[9] To ease the exposition, it is convenient to present the results in this section in terms of the actual commissions that the platform imposes on transactions, rather than in terms of percentages of the equilibrium prices; it is straightforward to show that when the platform's commissions can be label-dependent, the two ways of expressing results are equivalent.

[10] Note that the menu of commissions we consider is quite general in that it allows for the possibility that the platform reimburses a fraction of providers for them being active on the platform.

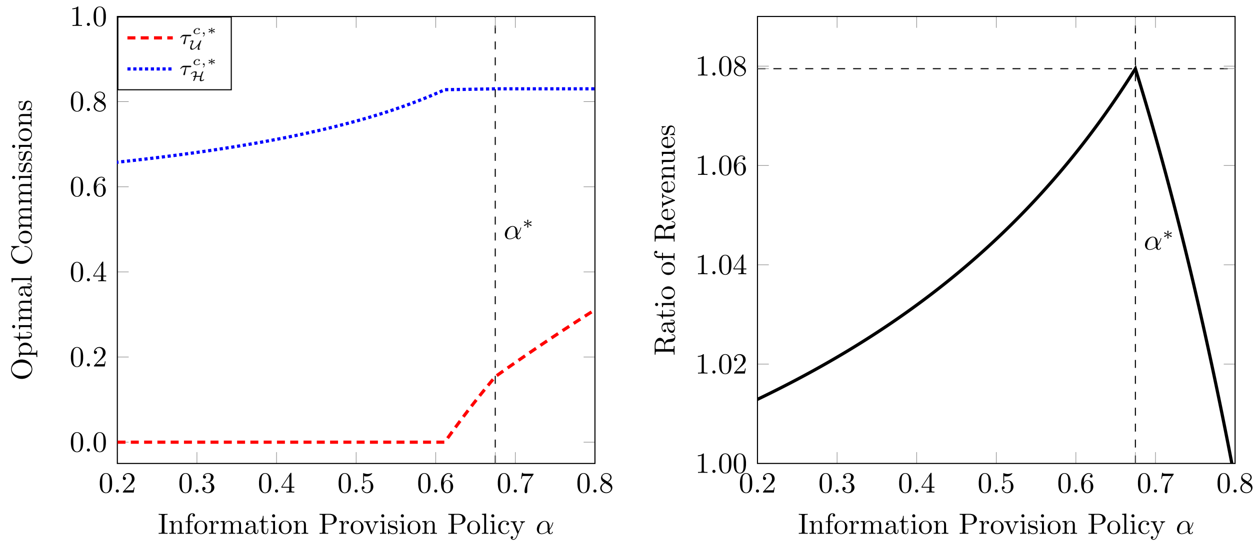

Figure 5 **Mechanism III: Reducing Commission Subsidies. The left plot depicts the optimal commission fees for transactions with providers labeled $\mathcal{U}$ and $\mathcal{H}$ inside the platform as a function of the information provision policy α. The right plot depicts the ratio of revenues under information provision policy α over the revenues under full information disclosure (parameter values: $q_0 = 0.38$, $p_0 = 0.26$, $w_0 = 0.22$, $b_0 = 0$, $\gamma = 0.22$, and $\beta = 0.8$).**

The key driver underlying the increase in revenue under the optimal information-provision policy is the platform's ability to charge higher commissions—and, in particular, a lower commission "subsidy" for new providers—without this resulting in a decrease in the entry of new providers or the overall volume of transactions.

To describe this mechanism in more detail, it is useful to consider the example of Figure 5. As the platform starts injecting delay into its information-provision policy (i.e. at low values of α), $\mathcal{H}$-labeled providers face less intense competition and are therefore able to increase their prices. This price increase is appropriated by the platform through a higher commission. At the same time, consumer demand for $\mathcal{U}$-labeled providers increases (owing to a higher expected service quality) but the commission for $\mathcal{U}$-labeled transactions remains low, as the platform still finds it optimal to subsidize new providers. As the information delay increases, consumer demand for $\mathcal{U}$-labeled providers eventually increases enough to drive a higher level of entry of new providers; at this point, the platform is able to increase the commission for $\mathcal{U}$-labeled providers without jeopardizing entry. The optimal policy strikes a balance between the composition of transactions occurring on the platform and the resulting revenue per transaction.

6. Concluding Remarks

This paper explores the benefits of information design for a two-sided platform in the presence of heterogeneity and endogenous behavior on both sides of the market. Service providers (the supply side) differ in their quality whereas consumers (the demand side) differ in how much they value service quality. The platform acts as an intermediary between the two sides and aims to maximize its revenues by appropriately designing two levers: (i) a commission charged per transaction, and (ii) an information-provision policy, which determines the disclosure of information pertaining to the providers' service quality inside the platform.

Depending on the market conditions in which the platform operates, we illustrate three main mechanisms through which information design can lead to higher revenues. When transacting outside the platform is relatively unattractive for providers and consumers, we demonstrate that the platform may find it optimal to delay disclosing the quality of a fraction of its best providers. This mechanism can be thought of as a two-sided platform analogue of the "damaged goods" effect (see Deneckere and McAfee (1996)) and its main purpose is to allow for more effective price discrimination. On the other hand, when consumers and/or service providers have access to relatively attractive outside options, we show that the platform may still benefit from delaying labeling a fraction of its high quality providers as such. However, the mechanism driving the increase in revenues in this case is entirely different qualitatively: bundling high-quality and new providers increases the overall volume of transactions on the platform and subsequently the rate at which consumers generate information about the quality of providers. As a result, the platform ends up featuring more providers labeled as high quality (although each high quality provider is labeled as such with a delay). Finally, we find that the benefits of information design persist even when the platform may set different commissions for providers of different labels. Here, we show that information can effectively act as a substitute for commission subsidies.

Throughout the paper, we focus on revenue maximization as the platform's primary objective and we show that information design can lead to a substantial increase in revenues (e.g., see Figures 3 and 4). At the same time, the platform's information-provision policy affects consumer surplus as well.[11] In Figure 6, we plot the consumer surplus as a function of the information delay parameter α, in the two numerical examples presented in §5.1 and §5.2 (see Figures 3 and 4, respectively). In the left plot (which corresponds to the "composition" mechanism of §5.1), the platform uses information design to alter the composition of providers who are active inside the platform, which essentially results in more effective price discrimination (albeit indirectly, through the providers' equilibrium pricing decisions). Accordingly, consumer surplus at the platform's optimal policy α^* is lower than under the full-information provision policy (i.e., $\alpha = 0$). In contrast, in the right plot of Figure 6 (which corresponds to the "volume" mechanism of §5.2), we observe that both the platform and consumers benefit substantially from the higher rate of experimentation and

[11] We note that free entry on the supply side implies that the providers' expected earnings inside the platform are equal to their outside option; that is, information design does not affect the providers' surplus.

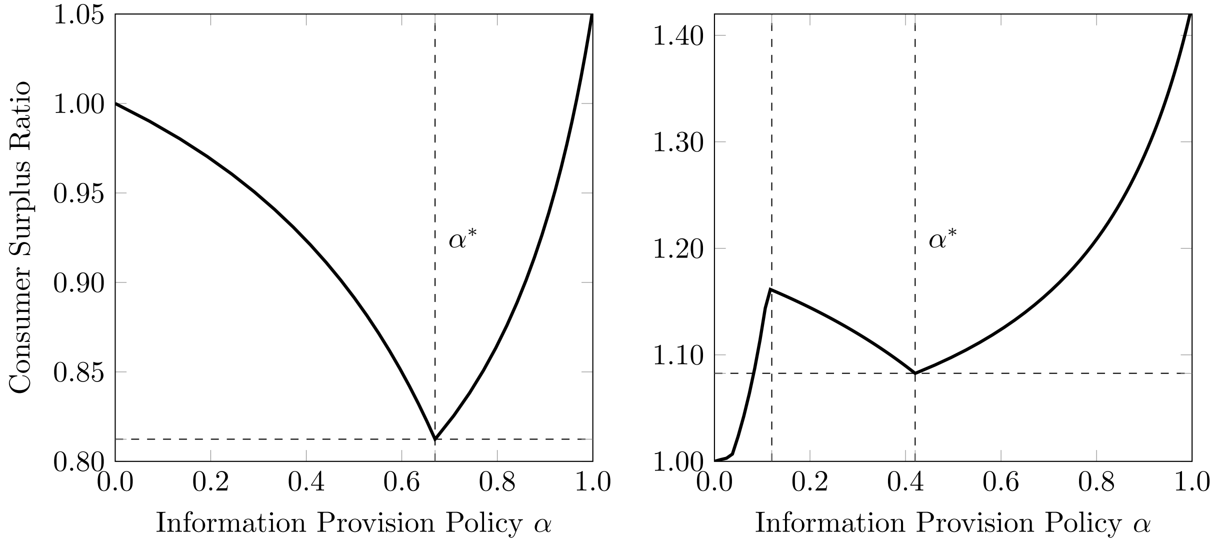

Figure 6 **Consumer Surplus under Mechanisms I and II (see §5.1 and §5.2, respectively). The left plot depicts the ratio of the consumer surplus under information provision policy α over the consumer surplus under full information, for the parameter values used in Figure 3 (Mechanism I). The right plot depicts the same ratio for the parameter values used in Figure 4 (Mechanism II). The optimal information provision policy for the platform α^* results in lower consumer surplus under Mechanism I (due to price discrimination) but higher consumer surplus under Mechanism II (due to a higher volume of transactions and rate of discovery of high-quality providers).**

the resulting discovery of high-quality providers induced under the optimal information-provision policy. Interestingly, here we find that the delayed release of information that occurs under the platform's optimal policy in fact leads to a Pareto improvement.

Our findings complement the growing literature on the design and operations of two-sided platforms. Unlike much of the prior work that treats supply as exogenous and fixed over time, we mainly explore the impact of information design on the supply side of the marketplace and illustrate the significant benefits that this may entail. To best illustrate how jointly optimizing commissions and information provision leads to benefits for the platform, we make a number of assumptions, including that demand is time-invariant; that providers' quality is revealed to the platform after a single transaction; and that providers are only vertically differentiated. Relaxing these assumptions constitute fruitful, albeit likely challenging, directions for future research. More generally, in light of the growing prominence of the gig economy and its increasing role in the market for labor, work that explores the interplay between the platform's design levers and the (equilibrium) behavior they induce from market participants represents an avenue for future research that is both theoretically interesting and practically relevant.

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Appendix

A. Preliminaries

Throughout the Appendix, we use the following notation to simplify the exposition of our analysis and results:

(i) $\lambda \triangleq \frac{\alpha}{1-\beta\alpha}$. Note that λ is increasing in α, i.e., the delay parameter in the platform's information provision policy and $\lambda = 0$ when $\alpha = 0$.

(ii) $\underline{\eta} \triangleq \frac{1-\beta}{\beta\gamma} \cdot \frac{q_0-\gamma}{1-q_0}$. Note that $\underline{\eta} > 0$ and Assumption 1(b) implies that $\underline{\eta} < 1/2$.

A.1. Formal Description of the Equilibrium Concept

This subsection provides a formal description of the equilibrium concept we employ under a general policy $(\tau_{\mathcal{U}}, \tau_{\mathcal{H}}, \alpha)$ by the platform. In particular, an equilibrium under information provision policy $\{\alpha\}$ and commission structure $\{\tau_{\mathcal{U}}, \tau_{\mathcal{H}}\}$ is a tuple of prices $\{p_{\mathcal{U}}, p_{\mathcal{H}}\}$, a mass of providers who engage with the platform $\{\delta_{\mathcal{U}}^U, \delta_{\mathcal{U}}^H, \delta_{\mathcal{H}}^H\}$, and a mass of consumers who transact inside the platform $\{\zeta_{\mathcal{U}}, \zeta_{\mathcal{H}}\}$ such that

(i) *Expected earnings:* If high-quality providers labeled $\mathcal{U}$ and $\mathcal{H}$ choose to remain active on the platform, then their expected future earnings are given as

$$V_{\mathcal{U}}^H = \eta(1-\tau_{\mathcal{U}})p_{\mathcal{U}} + \beta\alpha \max\left\{V_{\mathcal{U}}^H, \frac{w_0}{1-\beta}\right\} + \beta(1-\alpha)\max\left\{V_{\mathcal{H}}^H, \frac{w_0}{1-\beta}\right\}, \tag{10}$$

$$\text{and } V_{\mathcal{H}}^H = \frac{(1-\tau_{\mathcal{H}})p_{\mathcal{H}}}{1-\beta}, \tag{11}$$

respectively. Here η denotes the rationing rate among $\mathcal{U}$-labeled providers. If new providers choose to join the platform, then their expected lifetime earnings are given as

$$V_{\mathcal{U}}^U = \eta\left((1-\tau_{\mathcal{U}})p_{\mathcal{U}} + \beta\alpha\gamma \max\left\{V_{\mathcal{U}}^H, \frac{w_0}{1-\beta}\right\} + \beta(1-\alpha)\gamma\max\left\{V_{\mathcal{H}}^H, \frac{w_0}{1-\beta}\right\} + \beta(1-\gamma)\frac{w_0}{1-\beta}\right) + (1-\eta)\beta V_{\mathcal{U}}^U. \tag{12}$$

When there is entry to the platform and subsequently transaction, the expected earnings for new providers should be at least as high as their outside options. This together with our assumption that the supply of potential providers is infinite result in the following *free-entry condition*

$$V_{\mathcal{U}}^U = \frac{w_0}{1-\beta}.$$

(ii) *Providers' retention decisions:* High-quality providers (i.e., H-type providers) with label $\mathcal{U}$ and $\mathcal{H}$ decide whether or not to remain active on the platform to maximize their expected earnings. In particular, the decisions of high-quality providers with label $\mathcal{U}$ and high-quality providers with label $\mathcal{H}$ are characterized by

$$s_{\mathcal{U}} = \arg\max_{s\in[0,1]} sV_{\mathcal{U}}^H + (1-s)\frac{w_0}{1-\beta} \text{ and } s_{\mathcal{H}} = \arg\max_{s\in[0,1]} sV_{\mathcal{H}}^H + (1-s)\frac{w_0}{1-\beta},$$

where $s_{\mathcal{U}}$ and $s_{\mathcal{H}}$ represent the fractions of high-quality providers with label $\mathcal{U}$ and label $\mathcal{H}$, respectively, who remain active on the platform. Finally, the mass of providers is time-invariant and satisfies the following condition

$$\delta_{\mathcal{H}}^H = \beta\Big(s_{\mathcal{H}}\delta_{\mathcal{H}}^H + (1-\alpha)(s_{\mathcal{U}}\delta_{\mathcal{U}}^H + \gamma\delta_{\mathcal{U}}^U)\Big) \text{ and } \delta_{\mathcal{U}}^H = \beta\alpha\Big(s_{\mathcal{U}}\delta_{\mathcal{U}}^H + \gamma\delta_{\mathcal{U}}^U\Big). \tag{13}$$

(iii) *Customers' choice of providers:* Given that the mass of providers on the platform is positive, customers choose providers with different labels so as to maximize their expected utility. Specifically, the choice of a customer with type θ is given by

$$\arg\max_{i\in\{\varnothing,\mathcal{U},\mathcal{H}\}} \theta q_i - p_i,$$

where $\{\varnothing,\mathcal{U},\mathcal{H}\}$ represents the set of available options for customers, i.e., transacting with the outside option or with a provider with labels $\mathcal{U}$ and $\mathcal{H}$ inside the platform, respectively. The mass of customers who find it optimal to transact with providers with labels $\mathcal{U}$ and $\mathcal{H}$ is given by $\zeta_{\mathcal{U}}$ and $\zeta_{\mathcal{H}}$, respectively. Finally, the induced demand clears the available supply of providers. That is,

$$\zeta_{\mathcal{U}} = \delta^{U}_{\mathcal{U}} + s_{\mathcal{U}}\eta\delta^{H}_{\mathcal{U}} \text{ and } \zeta_{\mathcal{H}} = s_{\mathcal{H}}\delta^{H}_{\mathcal{H}}. \tag{14}$$

(iv) *Minimum payment:* The payment received by each provider should be no less than b_0. That is,

$$(1-\tau_{\mathcal{U}})p_{\mathcal{U}} \geq b_0 \text{ and } (1-\tau_{\mathcal{H}})p_{\mathcal{H}} \geq b_0.$$

Remarks: The existence of an equilibrium as defined above is established by Proposition A.1. When $\tau_{\mathcal{U}} = \tau_{\mathcal{H}} = \tau$, this equilibrium coincides with the equilibrium defined in Section 4.

A.2. Sufficient Conditions for Mechanisms I and II

This subsection formalizes the conditions underlying Mechanism I and II discussed in Section 5. In particular, Assumption 2 below provides *sufficient* conditions under which Propositions 2 and 3 hold.

ASSUMPTION 2. *The following conditions on the modeling primitives are sufficient for Mechanism I to hold at equilibrium, i.e.,*

$$\begin{aligned}
&\frac{p_0}{q_0-\gamma} > 1 + \frac{1-\beta}{1-\beta+\beta\gamma}, \text{ and} \\
&b_0 < \min\left\{ \frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}, \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} \right\} w_0.
\end{aligned} \tag{15}$$

Next, we provide *sufficient* conditions on the modeling primitives for Mechanism II to hold at equilibrium, i.e., the following conditions are sufficient for Propositions 4 and 5 to hold.

ASSUMPTION 3. *The following condition on the modeling primitives is sufficient for Mechanism II to hold at equilibrium*

$$\frac{p_0}{q_0-\gamma} < 1 + \min\left(\frac{1}{4}, w_0\right).$$

A.3. Auxiliary Technical Results

This subsection provides a number of technical results, which are required to establish our main findings. We point out that the following results hold under the general case when $\tau_{\mathcal{U}}$ and $\tau_{\mathcal{H}}$ may be different.

Proposition A.1 *An equilibrium as defined in Appendix A.1 exists under any* $\{\tau_{\mathcal{U}}, \tau_{\mathcal{H}}, \alpha\}$.

Proof. We proceed in three steps to establish the existence of an equilibrium. First, we construct an auxiliary normal form game with a finite number of players and convex and compact strategy spaces. Second, we establish the existence of a pure strategy Nash equilibrium of the auxiliary game using the results in Dasgupta and Maskin (1986). Third, we show that the equilibrium of the auxiliary game corresponds to an equilibrium of the original game, i.e., it satisfies the equilibrium conditions presented in Appendix A.1.zzzz

Step 1: In the first step, we construct an auxiliary game that involves 15 agents. In what follows, we characterize each agent's strategy space and payoff function, denoted by u_i, where $i \in \{1,2,...,15\}$. Given agent i, we further establish that u_i is upper semi-continuous (u.s.c.) in the actions of all agents and quasi-concave (q.c.) in agent i's action, and that $\max u_i$ is lower semi-continuous (l.s.c.) in the actions of all agents other than agent i. In terms of notation, we let $\mathbf{a}$ denote the action vector of all agent and $\mathbf{a}_{-i}$ denote the action vector of all agents except agent i. We then let $\{M_i\}$ denote a set of constants such that $M_j \gg M_i \gg 0$ if $i < j$.

1. Agent 1: We denote the agent's action space by $\tilde{\delta}_{\mathcal{U}}^{U} \in [0, M_1]$ and her payoff function by

$$u_1(\tilde{\delta}_{\mathcal{U}}^{U}, \mathbf{a}_{-1}) = -\eta\tilde{\delta}_{\mathcal{U}}^{U}\left|V_{\mathcal{U}}^{U} - \frac{w_0}{1-\beta}\right|,$$

   where η is the action of agent 13 with action space $[0, M_1]$, and $V_{\mathcal{U}}^{U}$ is the action of agent 6 with action space $[0, M_4]$. Therefore, u_1 is u.s.c. in $\mathbf{a}$ and q.c. in $\tilde{\delta}_{\mathcal{U}}^{U}$. In addition, $\max_{\tilde{\delta}_{\mathcal{U}}^{U}} u_1 = 0$, which is l.s.c. in $\mathbf{a}_{-1}$.

2. Agent 2: We denote the agent's action space by $\delta_{\mathcal{U}}^{H} \in [0, M_2]$ and her payoff function by

$$u_2(\delta_{\mathcal{U}}^{H}, \mathbf{a}_{-2}) = -\left|\delta_{\mathcal{U}}^{H} - \frac{\beta\gamma\alpha}{1-\beta\alpha s_{\mathcal{U}}}\eta\tilde{\delta}_{\mathcal{U}}^{U}\right|,$$

   where $s_{\mathcal{U}}$ is the action of agent 4 with action space $[0,1]$. Therefore, u_2 is u.s.c. in $\mathbf{a}$ and q.c. in $\delta_{\mathcal{U}}^{H}$. In addition, $\max_{\delta_{\mathcal{U}}^{H}} u_2 = 0$, which is l.s.c. in $\mathbf{a}_{-2}$.

3. Agent 3: We denote the agent's action space by $\delta_{\mathcal{H}}^{H} \in [0, M_3]$ and her payoff function by

$$u_3(\delta_{\mathcal{H}}^{H}, \mathbf{a}_{-3}) = -\left|\delta_{\mathcal{H}}^{H} - \frac{\beta(1-\alpha)}{1-\beta s_{\mathcal{H}}}\left(s_{\mathcal{U}}\delta_{\mathcal{U}}^{H} + \gamma\eta\tilde{\delta}_{\mathcal{U}}^{U}\right)\right|.$$

   Therefore, u_3 is u.s.c. in $\mathbf{a}$ and q.c. in $\delta_{\mathcal{H}}^{H}$. In addition, $\max_{\delta_{\mathcal{H}}^{H}} u_3 = 0$, which is l.s.c. in $\mathbf{a}_{-3}$.

4. Agent 4: We denote the agent's action space by $s_{\mathcal{U}} \in [0,1]$ and her payoff function by

$$u_4(s_{\mathcal{U}}, \mathbf{a}_{-4}) = -s_{\mathcal{U}}\max\left(\frac{w_0}{1-\beta} - V_{\mathcal{U}}^{H}, 0\right) - (1-s_{\mathcal{U}})\max\left(V_{\mathcal{U}}^{H} - \frac{w_0}{1-\beta}, 0\right),$$

   where $V_{\mathcal{U}}^{H}$ is the action of agent 7 with action space $[0, M_3]$. Therefore, u_4 is u.s.c. in $\mathbf{a}$ and q.c. in $s_{\mathcal{U}}$. In addition, $\max_{s_{\mathcal{U}}} u_4 = 0$, which is l.s.c. in $\mathbf{a}_{-4}$.

5. Agent 5: We denote the agent's action space by $s_{\mathcal{H}} \in [0,1]$ and her payoff function by

$$u_5(s_{\mathcal{H}}, \mathbf{a}_{-5}) = -s_{\mathcal{H}}\max\left(\frac{w_0}{1-\beta} - V_{\mathcal{H}}^{H}, 0\right) - (1-s_{\mathcal{H}})\max\left(V_{\mathcal{H}}^{H} - \frac{w_0}{1-\beta}, 0\right),$$

   where $V_{\mathcal{H}}^{H}$ is the action of agent 9 with action space $[0, M_2]$. Therefore, u_5 is u.s.c. in $\mathbf{a}$ and q.c. in $s_{\mathcal{H}}$. In addition, $\max_{s_{\mathcal{H}}} u_5 = 0$, which is l.s.c. in $\mathbf{a}_{-5}$.

6. Agent 6: We denote the agent's action space by $V_{\mathcal{U}}^{U} \in [0, M_4]$ and her payoff function by

$$u_6(V_{\mathcal{U}}^{U}, \mathbf{a}_{-6}) = -\Bigg| V_{\mathcal{U}}^{U} - \frac{\eta}{1-(1-\eta)\beta}\Big((1-\tau_{\mathcal{U}})p_{\mathcal{U}} + \beta\gamma\alpha s_{\mathcal{H}} V_{\mathcal{U}}^{H} + \beta\gamma(1-\alpha)s_{\mathcal{H}} V_{\mathcal{H}}^{H} + \beta\Big(\gamma\alpha(1-s_{\mathcal{U}}) + \gamma(1-\alpha)(1-s_{\mathcal{H}}) + 1 - \gamma\Big)\frac{w_0}{1-\beta}\Big)\Bigg|,$$

where $p_{\mathcal{U}}$ is the action of agent 14 with action space $[\frac{b_0}{1-\tau_{\mathcal{U}}}, M_1]$. Therefore, u_6 is u.s.c. in $\mathbf{a}$ and q.c. in $V_{\mathcal{U}}^{U}$. In addition, $\max_{V_{\mathcal{U}}^{U}} u_6 = 0$, which is l.s.c. in $\mathbf{a}_{-6}$.

7. Agent 7: We denote the agent's action space by $V_{\mathcal{U}}^{H} \in [0, M_3]$ and her payoff function by

$$u_7(V_{\mathcal{U}}^{H}, \mathbf{a}_{-7}) = -\Bigg| V_{\mathcal{U}}^{H} - \frac{1}{1-\beta\alpha s_{\mathcal{U}}}\Big(\eta(1-\tau)p_{\mathcal{U}} + \beta(1-\alpha)s_{\mathcal{H}} V_{\mathcal{H}}^{H} + \beta\Big(\alpha(1-s_{\mathcal{U}}) + (1-\alpha)(1-s_{\mathcal{H}})\Big)\frac{w_0}{1-\beta}\Big)\Bigg|.$$

Therefore, u_7 is u.s.c. in $\mathbf{a}$ and q.c. in $V_{\mathcal{U}}^{H}$. In addition, $\max_{V_{\mathcal{U}}^{H}} u_7 = 0$, which is l.s.c. in $\mathbf{a}_{-7}$.

8. Agent 8: We denote the agent's action space by $V_{\mathcal{H}}^{H} \in [0, M_2]$ and her payoff function by

$$u_8(V_{\mathcal{H}}^{H}, \mathbf{a}_{-8}) = -\Big| V_{\mathcal{H}}^{H} - \frac{(1-\tau_{\mathcal{H}})p_{\mathcal{H}}}{1-\beta}\Big|,$$

where $p_{\mathcal{H}}$ is the action of agent 12 with action space $[\frac{b_0}{1-\tau_{\mathcal{H}}}, M_1]$. Therefore, u_8 is u.s.c. in $\mathbf{a}$ and q.c. in $V_{\mathcal{H}}^{H}$. In addition, $\max_{V_{\mathcal{H}}^{H}} u_9 = 0$, which is l.s.c. in $\mathbf{a}_{-8}$.

9. Agent 9: We denote the agent's action space by $\zeta_{\mathcal{H}} \in [0,1]$ and denote her payoff function by

$$u_9(\zeta_{\mathcal{H}}, \mathbf{a}_{-9}) = -\left| \zeta_{\mathcal{H}} - \left(2 - \max\left\{\frac{p_{\mathcal{H}} - p_0}{q_{\mathcal{H}} - q_0}, \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_{\mathcal{H}} - q_{\mathcal{U}}}, 1\right\}\right)\right|,$$

where $q_{\mathcal{H}} = 1$. Therefore, u_9 is u.s.c. in $\mathbf{a}$ and q.c. in $\zeta_{\mathcal{H}}$. In addition,

$$\max_{\zeta_{\mathcal{H}}} u_9 = \begin{cases} 0, & \text{if } \max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}, 1\right\} \le 2, \\ -\left|2 - \max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}, 1\right\}\right|, & \text{otherwise.} \end{cases}$$

Therefore, $\max_{\zeta_{\mathcal{H}}} u_9$ is l.s.c. in $\mathbf{a}_{-9}$.

10. Agent 10: We denote the agent's action space by $\tilde{\zeta}_{\mathcal{U}} \in [0,1]$ and her payoff function by

$$u_{10}(\tilde{\zeta}_{\mathcal{U}}, \mathbf{a}_{-10}) = -\left|\tilde{\zeta}_{\mathcal{U}} - D_{\mathcal{U}}(\mathbf{a}_{-10})\right|\Big(\mathbb{1}_{\{p_{\mathcal{U}}<p_0\}}(\mathbf{a}_{-10}) + \mathbb{1}_{\{p_{\mathcal{U}}>p_0\}}(\mathbf{a}_{-10}) + \mathbb{1}_{\{q_{\mathcal{U}}<q_0\}}(\mathbf{a}_{-10}) + \mathbb{1}_{\{q_{\mathcal{U}}>q_0\}}(\mathbf{a}_{-10})\Big),$$

where $D_{\mathcal{U}}(\mathbf{a}_{-10}) =$

$$\begin{cases} \max\left\{\min\left\{\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}, 2\right\}, 1\right\} - 1, & \text{if } q_{\mathcal{U}} < q_0, \\ \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}, 2\right\}, 1\right\} - \min\left\{\max\left\{\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0}, 1\right\}, \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}, 2\right\}, 1\right\}\right\}, & \text{if } q_{\mathcal{U}} > q_0, \\ \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}, 2\right\}, 1\right\} - 1, & \text{if } q_{\mathcal{U}} = q_0 \text{ and } p_{\mathcal{U}} < p_0, \\ 0, & \text{if } q_{\mathcal{U}} = q_0 \text{ and } p_{\mathcal{U}} \ge p_0. \end{cases}$$

In the characterization of $D_{\mathcal{U}}(\mathbf{a}_{-10})$, $q_{\mathcal{H}} = 1$, and $q_{\mathcal{U}}$ is the action of agent 15 with action space $[\gamma, 1]$.

Next, we show that $D_{\mathcal{U}}(\mathbf{a}_{-10})$ is continuous in $\{\mathbf{a}_{-10}|q_{\mathcal{U}} \neq q_0\} \cup \{\mathbf{a}_{-10}|p_{\mathcal{U}} \neq p_0\}$. First, it follows that $D_{\mathcal{U}}(\mathbf{a}_{-10})$ is continuous in $\{\mathbf{a}_{-10}|q_{\mathcal{U}} < q_0\} \cup \{\mathbf{a}_{-10}|q_{\mathcal{U}} > q_0\}$ by its definition. Second, we show that $D_{\mathcal{U}}(\mathbf{a}_{-10})$ is continuous at any point within $\{\mathbf{a}_{-10}|q_{\mathcal{U}} = q_0 \text{ and } p_{\mathcal{U}} < p_0\}$. In particular, it suffices to show that the following holds given $\underline{p} < p_0$:

$$\lim_{(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) \to (q_0^-, \underline{p}^-)} D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) = \lim_{(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) \to (q_0^-, \underline{p}^+)} D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) = D_{\mathcal{U}}(q_0, \underline{p}), \tag{16}$$

$$\text{and} \quad \lim_{(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) \to (q_0^+, \underline{p}^-)} D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) = \lim_{(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) \to (q_0^+, \underline{p}^+)} D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) = D_{\mathcal{U}}(q_0, \underline{p}). \tag{17}$$

The first equalities in Expression (16) and Expression (17), respectively, hold because $D_{\mathcal{U}}(\mathbf{a}_{-10})$ is continuous in $\{\mathbf{a}_{-10}|q_{\mathcal{U}} < q_0\}$ and $\{\mathbf{a}_{-10}|q_{\mathcal{U}} > q_0\}$, respectively. Then, we establish the second equality in Expression (16). Note that when $q^i_{\mathcal{U}} \to q_0^-$ and $p^i_{\mathcal{U}} \to \underline{p}^+$,

$$D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) = \max\left\{\min\left\{\frac{p^i_{\mathcal{U}} - p_0}{q^i_{\mathcal{U}} - q_0}, \frac{p_{\mathcal{H}} - p^i_{\mathcal{U}}}{q_{\mathcal{H}} - q^i_{\mathcal{U}}}, 2\right\}, 1\right\} - 1$$

converges to

$$D_{\mathcal{U}}(q_0, \underline{p}) = \max\left\{\min\left\{\frac{p_{\mathcal{H}} - \underline{p}}{q_{\mathcal{H}} - q_0}, 2\right\}, 1\right\} - 1,$$

as $\frac{p^i_{\mathcal{U}} - p_0}{q^i_{\mathcal{U}} - q_0} \to +\infty$ and $\frac{p_{\mathcal{H}} - p^i_{\mathcal{U}}}{q_{\mathcal{H}} - q^i_{\mathcal{U}}} \to \frac{p_{\mathcal{H}} - \underline{p}}{q_{\mathcal{H}} - q_0}$. Therefore, the second equality in Expression (16) holds. Similarly, we show that the second equality in Expression (17) holds. In particular, note that when $q^i_{\mathcal{U}} \to q_0^+$ and $p^i_{\mathcal{U}} \to \underline{p}^+$,

$$D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) = \max\left\{\min\left\{\frac{p_{\mathcal{H}} - p^i_{\mathcal{U}}}{q_{\mathcal{H}} - q^i_{\mathcal{U}}}, 2\right\}, 1\right\} - \min\left\{\max\left\{\frac{p^i_{\mathcal{U}} - p_0}{q^i_{\mathcal{U}} - q_0}, 1\right\}, \max\left\{\min\left\{\frac{p_{\mathcal{H}} - p^i_{\mathcal{U}}}{q_{\mathcal{H}} - q^i_{\mathcal{U}}}, 2\right\}, 1\right\}\right\}$$

converges to

$$D_{\mathcal{U}}(q_0, \underline{p}) = \max\left\{\min\left\{\frac{p_{\mathcal{H}} - \underline{p}}{q_{\mathcal{H}} - q_0}, 2\right\}, 1\right\} - 1,$$

as $\frac{p^i_{\mathcal{U}} - p_0}{q^i_{\mathcal{U}} - q_0} \to -\infty$ and $\frac{p_{\mathcal{H}} - p^i_{\mathcal{U}}}{q_{\mathcal{H}} - q^i_{\mathcal{U}}} \to \frac{p_{\mathcal{H}} - \underline{p}}{q_{\mathcal{H}} - q_0}$. Therefore, $D_{\mathcal{U}}(\mathbf{a}_{-10})$ is continuous at any point with $q_{\mathcal{U}} = q_0$ and $p_{\mathcal{U}} = \underline{p} < p_0$. Third, we show that $D_{\mathcal{U}}(\mathbf{a}_{-10})$ is continuous at any point within $\{\mathbf{a}_{-10}|q_{\mathcal{U}} = q_0 \text{ and } p_{\mathcal{U}} > p_0\}$. Given $\bar{p} > p_0$,

$$\lim_{(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) \to (q_0^-, \bar{p}^-)} D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) = \lim_{(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) \to (q_0^-, \bar{p}^+)} D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) = D_{\mathcal{U}}(q_0, \bar{p}), \tag{18}$$

$$\text{and} \quad \lim_{(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) \to (q_0^+, \bar{p}^-)} D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) = \lim_{(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) \to (q_0^+, \bar{p}^+)} D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) = D_{\mathcal{U}}(q_0, \bar{p}). \tag{19}$$

The first equalities in Expression (18) and Expression (19) hold because $D_{\mathcal{U}}(\mathbf{a}_{-10})$ is continuous in $\{\mathbf{a}_{-10}|q_{\mathcal{U}} < q_0\}$ and $\{\mathbf{a}_{-10}|q_{\mathcal{U}} > q_0\}$, respectively. To show that the second equality in Expression (18) holds, we note that when $q^i_{\mathcal{U}} \to q_0^-$ and $p^i_{\mathcal{U}} \to \bar{p}^+$,

$$D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) = \max\left\{\min\left\{\frac{p^i_{\mathcal{U}} - p_0}{q^i_{\mathcal{U}} - q_0}, \frac{p_{\mathcal{H}} - p^i_{\mathcal{U}}}{q_{\mathcal{H}} - q^i_{\mathcal{U}}}, 2\right\}, 1\right\} - 1$$

converges to $D_{\mathcal{U}}(q_0,\bar{p})=0$, as $\frac{p^i_{\mathcal{U}}-p_0}{q^i_{\mathcal{U}}-q_0}\to-\infty$ and $\frac{p_{\mathcal{H}}-p^i_{\mathcal{U}}}{q_{\mathcal{H}}-q^i_{\mathcal{U}}}\to\frac{p_{\mathcal{H}}-\bar{p}}{q_{\mathcal{H}}-q_0}$. Similarly, to show the second equality in Expression (19), we note that when $q^i_{\mathcal{U}}\to q_0^+$ and $p^i_{\mathcal{U}}\to\bar{p}^+$,

$$D_{\mathcal{U}}(q^i_{\mathcal{U}},p^i_{\mathcal{U}})=\max\left\{\min\left\{\frac{p_{\mathcal{H}}-p^i_{\mathcal{U}}}{q_{\mathcal{H}}-q^i_{\mathcal{U}}},2\right\},1\right\}-\min\left\{\max\left\{\frac{p^i_{\mathcal{U}}-p_0}{q^i_{\mathcal{U}}-q_0},1\right\},\max\left\{\min\left\{\frac{p_{\mathcal{H}}-p^i_{\mathcal{U}}}{q_{\mathcal{H}}-q^i_{\mathcal{U}}},2\right\},1\right\}\right\}$$

converges to $D_{\mathcal{U}}(q_0,\bar{p})=0$, as $\frac{p^i_{\mathcal{U}}-p_0}{q^i_{\mathcal{U}}-q_0}\to+\infty$ and $\frac{p_{\mathcal{H}}-p^i_{\mathcal{U}}}{q_{\mathcal{H}}-q^i_{\mathcal{U}}}\to\frac{p_{\mathcal{H}}-\bar{p}}{q_{\mathcal{H}}-q_0}$. Therefore, $D_{\mathcal{U}}(\mathbf{a}_{-10})$ is continuous at any point with $q_{\mathcal{U}}=q_0$ and $p_{\mathcal{U}}=\bar{p}>p_0$. In sum, we have established the continuity of $D_{\mathcal{U}}(\mathbf{a}_{-10})$ in $\{\mathbf{a}_{-10}|q_{\mathcal{U}}\neq q_0\}\cup\{\mathbf{a}_{-10}|p_{\mathcal{U}}\neq p_0\}$.

Then, based on the continuity of $D_{\mathcal{U}}(\mathbf{a}_{-10})$, we rewrite $u_{10}(\tilde{\zeta}_{\mathcal{U}},\mathbf{a}_{-10})$ as follows:

$$u_{10}(\tilde{\zeta}_{\mathcal{U}},\mathbf{a}_{-10})=\begin{cases}0, & \text{if } q_{\mathcal{U}}=q_0 \text{ and } p_{\mathcal{U}}=p_0\\ -\left|\tilde{\zeta}_{\mathcal{U}}-D_{\mathcal{U}}(\mathbf{a}_{-10})\right|, & \text{if } q_{\mathcal{U}}>q_0 \text{ and } p_{\mathcal{U}}=p_0\\ & \text{or } q_{\mathcal{U}}<q_0 \text{ and } p_{\mathcal{U}}=p_0\\ & \text{or } q_{\mathcal{U}}=q_0 \text{ and } p_{\mathcal{U}}>p_0\\ & \text{or } q_{\mathcal{U}}=q_0 \text{ and } p_{\mathcal{U}}<p_0\\ -2\left|\tilde{\zeta}_{\mathcal{U}}-D_{\mathcal{U}}(\mathbf{a}_{-10})\right|, & \text{if } q_{\mathcal{U}}>q_0 \text{ and } p_{\mathcal{U}}>p_0\\ & \text{or } q_{\mathcal{U}}<q_0 \text{ and } p_{\mathcal{U}}>q_0\\ & \text{or } q_{\mathcal{U}}>q_0 \text{ and } p_{\mathcal{U}}<q_0\\ & \text{or } q_{\mathcal{U}}<q_0 \text{ and } p_{\mathcal{U}}<q_0\end{cases}.$$

Based on the above characterization, it immediately follows that u_{10} is u.s.c. in $\mathbf{a}$ and q.c. in $\tilde{\zeta}_{\mathcal{U}}$. In addition, $\max_{\tilde{\zeta}_{\mathcal{U}}}u_{10}=0$, which is l.s.c.

Moreover, we observe that when $q_{\mathcal{U}}\neq q_0$ or $p_{\mathcal{U}}\neq p_0$, we have

$$\zeta^*_{\mathcal{H}}+D_{\mathcal{U}}(\mathbf{a}_{-10})\le 1, \tag{20}$$

where

$$\zeta^*_{\mathcal{H}}=\arg\max_{\zeta_{\mathcal{H}}}u_9=\max\left\{2-\max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0},\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},1\right\},0\right\}.$$

In what follows, we show that inequality (20) holds in all four possible cases, i.e., (1) $q_{\mathcal{U}}<q_0$, (2) $q_{\mathcal{U}}>q_0$, (3) $q_{\mathcal{U}}=q_0$ and $p_{\mathcal{U}}<p_0$, and (4) $q_{\mathcal{U}}=q_0$ and $p_{\mathcal{U}}>p_0$, separately.

(1) In this case, we note that

$$D_{\mathcal{U}}(\mathbf{a}_{-10})=\max\left\{\min\left\{\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0},\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},2\right\},1\right\}\le 1.$$

Therefore,

$$\zeta^*_{\mathcal{H}}+D_{\mathcal{U}}(\mathbf{a}_{-10})=\left\{D_{\mathcal{U}}(\mathbf{a}_{-10}),1+\min\left\{\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0},\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},2\right\}-\max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0},\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},1\right\}\right\}.$$

Then, to show that the above expression is no greater than 1, it is equivalent to show

$$\min\left\{\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0},\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},2\right\}\le\max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0},\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},1\right\}. \tag{21}$$

It is straightforward to verify that under $q_{\mathcal{U}}<q_0$, either

$$\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}\le\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}\le\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0}\ \text{or}\ \frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0}\le\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}\le\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}$$

holds. In the first case, inequality (21) is equivalent to $\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},2\right\}\le\max\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},1\right\}$, which holds, and in the second case, inequality (21) is equivalent to $\min\left\{\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0},2\right\}\le\max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0},1\right\}$, which also holds.

(2) In this case, we note that

$$D_{\mathcal{U}}(\mathbf{a}_{-10}) = \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},2\right\},1\right\} - \min\left\{\max\left\{\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0},1\right\},\max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},2\right\},1\right\}\right\} \leq 1.$$

Then, to show that inequality (20) holds, it is equivalent to show

$$\max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0},\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},1\right\} - \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},2\right\},1\right\} + \min\left\{\max\left\{\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0},1\right\},\max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},2\right\},1\right\}\right\} \geq 1.$$

The above inequality holds as

$$\max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0},\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},1\right\} - \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},2\right\},1\right\} \geq 0, \tag{22}$$

and its third term is no less than 1.

(3) In this case, we note that

$$D_{\mathcal{U}}(\mathbf{a}_{-10}) = \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},2\right\},1\right\} - 1.$$

Then, we have

$$\zeta^*_{\mathcal{H}} + D_{\mathcal{U}}(\mathbf{a}_{-10}) = \max\left\{1 - \max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0},\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},1\right\} + \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}},2\right\},1\right\}, D_{\mathcal{U}}(\mathbf{a}_{-10})\right\} \leq 1,$$

where the inequality holds because $D_{\mathcal{U}}(\mathbf{a}_{-10}) \leq 1$, and inequality (22) holds.

(4) Note that $D_{\mathcal{U}}(\mathbf{a}_{-10}) = 0$ in this case. Since $\zeta^*_{\mathcal{H}} \leq 1$, inequality (20) follows.

Therefore, we conclude that inequality (20) holds when $q_{\mathcal{U}} \neq q_0$ or $p_{\mathcal{U}} \neq p_0$.

11. Agent 11: We denote the agent's action space by $\zeta_{\mathcal{U}} \in [0,1]$ and her payoff function by

$$u_{11}(\zeta_{\mathcal{U}}, \mathbf{a}_{-11}) = -\left|\zeta_{\mathcal{U}} - \min(\tilde{\zeta}_{\mathcal{U}}, 1-\zeta_{\mathcal{H}})\right|.$$

Therefore, u_{11} is u.s.c. in $\mathbf{a}$ and q.c. in $\zeta_{\mathcal{U}}$. In addition, $\max_{\zeta_{\mathcal{U}}} u_{11} = 0$, which is l.s.c. in $\mathbf{a}_{-11}$.

12. Agent 12: We denote the agent's action space by $p_{\mathcal{H}} \in [\frac{b_0}{1-\tau_{\mathcal{H}}}, M_1]$ and her payoff function by

$$u_{12}(p_{\mathcal{H}}, \mathbf{a}_{-12}) = \Big(p_{\mathcal{H}} - \frac{b_0}{1-\tau_{\mathcal{H}}}\Big)(\zeta_{\mathcal{H}} - s_{\mathcal{H}}\delta^H_{\mathcal{H}}).$$

Therefore, u_{12} is u.s.c. in $\mathbf{a}$ and q.c. in $p_{\mathcal{H}}$. In addition, $\max_{p_{\mathcal{H}}} u_{12}(p_{\mathcal{H}}, \mathbf{a}_{-12}) = \max\Big\{\Big(M_1 - \frac{b_0}{1-\tau_{\mathcal{H}}}\Big)(\zeta_{\mathcal{H}} - s_{\mathcal{H}}\delta^H_{\mathcal{H}}), 0\Big\}$, which is l.s.c. in $\mathbf{a}_{-12}$.

13. Agent 13: We denote the agent's action space by $\eta \in [0, M_1]$ and her payoff function by

$$u_{13}(\eta, \mathbf{a}_{-13}) = -\left|\zeta_{\mathcal{U}} - \eta(\tilde{\delta}^U_{\mathcal{U}} + s_{\mathcal{U}}\delta^H_{\mathcal{U}})\right|.$$

Therefore, u_{13} is u.s.c. in $\mathbf{a}$ and q.c. in η. In addition,

$$\max_{\eta} u_{13}(\eta, \mathbf{a}_{-13}) = \min\Big\{-\zeta_{\mathcal{U}} + M_1(\tilde{\delta}^U_{\mathcal{U}} + s_{\mathcal{U}}\delta^H_{\mathcal{U}}), 0\Big\},$$

which is l.s.c. in $\mathbf{a}_{-13}$.

14. Agent 14: We denote the agent's action space by $p_{\mathcal{U}} \in [\frac{b_0}{1-\tau_{\mathcal{U}}}, M_1]$ and her payoff function by

$$u_{14}(p_{\mathcal{U}}, \mathbf{a}_{-14}) = \Big(p_{\mathcal{U}} - \frac{b_0}{1-\tau_{\mathcal{U}}}\Big)(\eta - 1).$$

Therefore, u_{14} is u.s.c in $\mathbf{a}$ and q.c. in $p_{\mathcal{U}}$. In addition,

$$\max_{p_{\mathcal{U}}} u_{14}(p_{\mathcal{U}}, \mathbf{a}_{-14}) = \max\Big\{\Big(M_1 - \frac{b_0}{1-\tau_{\mathcal{U}}}\Big)(\eta-1), 0\Big\},$$

which is l.s.c. in $\mathbf{a}_{-14}$.

15. Agent 15: We denote the agent's action space by $q_{\mathcal{U}} \in [\gamma, 1]$ and her payoff function by

$$u_{15}(q_{\mathcal{U}}, \mathbf{a}_{-15}) = -\Big|q_{\mathcal{U}} - \frac{\eta\gamma(\tilde{\delta}^U_{\mathcal{U}} + s_{\mathcal{U}}\delta^H_{\mathcal{U}}) + s_{\mathcal{H}}\delta^H_{\mathcal{H}}}{\eta(\tilde{\delta}^U_{\mathcal{U}} + s_{\mathcal{U}}\delta^H_{\mathcal{U}}) + s_{\mathcal{H}}\delta^H_{\mathcal{H}}}\Big|\eta\tilde{\delta}^U_{\mathcal{U}}.$$

Therefore, u_{15} is u.s.c. in $\mathbf{a}$ and q.c. in $q_{\mathcal{U}}$. In addition, $\max_{q_{\mathcal{U}}} u_{15} = 0$, which is l.s.c. in $\mathbf{a}_{-15}$.

This concludes the description of the auxiliary game that we will employ in the proof of equilibrium existence.

Step 2: Next, we show that the auxiliary game has a pure strategy Nash equilibrium, which follows directly by the Corollary of Theorem 2 in Dasgupta and Maskin (1986). In addition, we note that the following conditions hold in any given equilibrium:

1. By the equilibrium actions of agents 2 and 3, we have

$$\delta^H_{\mathcal{U}} = \frac{\beta\gamma\alpha}{1-\beta\alpha s_{\mathcal{U}}}\eta\tilde{\delta}^U_{\mathcal{U}} \text{ and } \delta^H_{\mathcal{H}} = \frac{\beta(1-\alpha)}{1-\beta s_{\mathcal{H}}}\Big(s_{\mathcal{U}}\delta^H_{\mathcal{U}} + \gamma\eta\tilde{\delta}^U_{\mathcal{U}}\Big). \tag{23}$$

2. By the equilibrium actions of agents 6, 7, and 8, we have

$$\begin{aligned} V^U_{\mathcal{U}} = \frac{\eta}{1-(1-\eta)\beta}\Big(&(1-\tau_{\mathcal{U}})p_{\mathcal{U}} + \beta\gamma\alpha s_{\mathcal{H}}V^H_{\mathcal{U}} + \beta\gamma(1-\alpha)s_{\mathcal{H}}V^H_{\mathcal{H}} \\ &+ \beta\Big(\gamma\alpha(1-s_{\mathcal{U}}) + \gamma(1-\alpha)(1-s_{\mathcal{H}}) + 1-\gamma\Big)\frac{w_0}{1-\beta}\Big), \end{aligned} \tag{24}$$

$$V^H_{\mathcal{U}} = \frac{1}{1-\beta\alpha s_{\mathcal{U}}}\Big(\eta(1-\tau)p_{\mathcal{U}} + \beta(1-\alpha)s_{\mathcal{H}}V^H_{\mathcal{H}} + \beta\Big(\alpha(1-s_{\mathcal{U}}) + (1-\alpha)(1-s_{\mathcal{H}})\Big)\frac{w_0}{1-\beta}\Big), \tag{25}$$

$$\text{and } V^H_{\mathcal{H}} = \frac{(1-\tau_{\mathcal{H}})p_{\mathcal{H}}}{1-\beta}. \tag{26}$$

Furthermore, if $\eta\tilde{\delta}^U_{\mathcal{U}} > 0$ holds in equilibrium, the following should also hold

1. By the equilibrium action of agent 1, we have

$$V^U_{\mathcal{U}} = \frac{w_0}{1-\beta}. \tag{27}$$

2. By the equilibrium action of agent 15, we have

$$q_{\mathcal{U}} = \frac{\eta\gamma(\tilde{\delta}^U_{\mathcal{U}} + s_{\mathcal{U}}\delta^H_{\mathcal{U}}) + s_{\mathcal{H}}\delta^H_{\mathcal{H}}}{\eta(\tilde{\delta}^U_{\mathcal{U}} + s_{\mathcal{U}}\delta^H_{\mathcal{U}}) + s_{\mathcal{H}}\delta^H_{\mathcal{H}}}. \tag{28}$$

3. By the equilibrium action of 14, we have

$$0 < \eta \le 1. \tag{29}$$

We show Expression (29) by way of a contradiction. Suppose $\eta > 1$, which implies that $p_{\mathcal{U}} = M_1 \to +\infty$ by the equilibrium action of agent 14. As a result, $\tilde{\zeta}_{\mathcal{U}} = 0$ by the equilibrium action of agent 10, and hence $\zeta_{\mathcal{U}} = 0$ by the equilibrium action of agent 11. Thus, we have $\eta = 0$ by the equilibrium action of agent 13 (note that $\tilde{\delta}^U_{\mathcal{U}} + s_{\mathcal{U}}\delta^H_{\mathcal{U}} \ge \tilde{\delta}^U_{\mathcal{U}} > 0$), which yields a contradiction.

4. By the equilibrium action of agent 13, we have

$$\zeta_{\mathcal{U}} = \eta(\tilde{\delta}_{\mathcal{U}}^{U} + s_{\mathcal{U}}\delta_{\mathcal{U}}^{H}). \tag{30}$$

5. By the payoff function of agent 12, we have

$$\zeta_{\mathcal{H}} = s_{\mathcal{H}}\delta_{\mathcal{H}}^{H}. \tag{31}$$

We first show $\zeta_{\mathcal{H}} \le s_{\mathcal{H}}\delta_{\mathcal{H}}^{H}$ by way of another contradiction. Suppose $\zeta_{\mathcal{H}} > s_{\mathcal{H}}\delta_{\mathcal{H}}^{H}$. Then, agent 12's equilibrium action is $p_{\mathcal{H}} = M_1 \to +\infty$. Then, we have $\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0} \to +\infty$ and $\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}} \to +\infty$. By the equilibrium action of agent 9, we have $\zeta_{\mathcal{H}} = 0$, which yields a contradiction. Then, we show $\zeta_{\mathcal{H}} \ge s_{\mathcal{H}}\delta_{\mathcal{H}}^{H}$ by way of yet another contradiction. Suppose $\zeta_{\mathcal{H}} < s_{\mathcal{H}}\delta_{\mathcal{H}}^{H}$. Then, agent 12's action is $p_{\mathcal{H}} = \frac{b_0}{1-\tau_{\mathcal{H}}} < \frac{w_0}{1-\tau_{\mathcal{H}}}$, which then results in $V_{\mathcal{H}}^{H} < \frac{w_0}{1-\beta}$. By the equilibrium action of agent 5, we have $s_{\mathcal{H}} = 0$, which yields a contradiction.

Step 3: Finally, we show that the equilibrium of the auxiliary game coincides with the equilibrium we defined in Appendix A.1. It is straightforward to verify that (i) the free-entry condition holds because of Expression (27); (ii) providers' lifetime earnings as characterized in Equation (12), Equation (10), and Equation (11) coincide with Expression (24), Expression (25), and Expression (26), respectively; (iii) providers' retention decisions, i.e., $s_{\mathcal{U}}$ and $s_{\mathcal{H}}$, coincide with the equilibrium actions of agents 4 and 5; (iv) the mass of providers given in Equation (13) coincides with Expression (23); (v) condition Equation (14) holds because of Expression (30) and Expression (31); and (vi) the minimum payment constraints hold because of the action spaces of agents 12 and 14.

Next, we verify that $\zeta_{\mathcal{U}}$ and $\zeta_{\mathcal{H}}$ are equal to the mass of customers who choose providers with labels $\mathcal{U}$ and label $\mathcal{H}$, respectively, as their *optimal* choice. In terms of notation, we let $\Theta_{\mathcal{U}}$ and $\Theta_{\mathcal{H}}$ denote the set of customers within $[1,2]$, whose optimal choice are providers with label $\mathcal{U}$ and label $\mathcal{H}$, respectively. We use $\mu(\cdot)$ to denote the measure of a given customer set.

We first verify that $\zeta_{\mathcal{H}} = \mu(\Theta_{\mathcal{H}})$, where $\zeta_{\mathcal{H}}$ is given by the equilibrium action of agent 9. Note that given $\theta \in \Theta_{\mathcal{H}}$, we have $\theta q_{\mathcal{H}} - p_{\mathcal{H}} \ge \theta q_0 - p_0$ and $\theta q_{\mathcal{H}} - p_{\mathcal{H}} \ge \theta q_{\mathcal{U}} - p_{\mathcal{U}}$, from which we obtain $\theta \ge \max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}\right\}$. Therefore,

$$\Theta_{\mathcal{H}} = \left[\max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}\right\}, +\infty\right) \cap [1,2],$$

and $\mu(\Theta_{\mathcal{H}}) = \max\left\{2 - \max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}, 1\right\}, 0\right\} = \zeta_{\mathcal{H}}$.

Then, we verify $\zeta_{\mathcal{U}} = \mu(\Theta_{\mathcal{U}})$, where $\zeta_{\mathcal{U}}$ is given by the equilibrium action of agent 11. By the definition of $\Theta_{\mathcal{U}}$, we have

$$\theta q_{\mathcal{U}} - p_{\mathcal{U}} \ge \theta q_0 - p_0 \text{ and } \theta q_{\mathcal{U}} - p_{\mathcal{U}} \ge \theta q_{\mathcal{H}} - p_{\mathcal{H}}. \tag{32}$$

We then show that $\zeta_{\mathcal{U}} = \mu(\Theta_{\mathcal{U}})$ in the following 5 possible cases:

1. Suppose $q_{\mathcal{U}} < q_0$ holds in equilibrium. Then inequality (32) results in $\theta \le \min\left\{\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}\right\}$. Therefore,

$$\Theta_{\mathcal{U}} = \left(-\infty, \min\left\{\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}\right\}\right] \cap [1,2],$$

and $\mu(\Theta_{\mathcal{U}}) = \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}, 2\right\}, 1\right\} - 1 = \tilde{\zeta}_{\mathcal{U}} = \zeta_{\mathcal{U}}$, where the first equality holds because of agent 10's equilibrium actionand thesecond equality holds because of inequality (20) and the equilibrium action of agent 11.

2. Suppose $q_{\mathcal{U}} > q_0$ holds in equilibrium. Then, inequality (32) results in $\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0} \leq \theta \leq \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}$. Therefore,
$$\Theta_{\mathcal{U}} = \left[\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}\right] \cap [1,2],$$
and $\mu(\Theta_{\mathcal{U}}) = \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}, 2\right\}, 1\right\} - \min\left\{\max\left\{\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0}, 1\right\}, \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}, 2\right\}, 1\right\}\right\} = \tilde{\zeta}_{\mathcal{U}} = \zeta_{\mathcal{U}}$, where the first equality holds because of agent 10's equilibrium actionand thesecond equality holds because of inequality (20) and the equilibrium action of agent 11.

3. Suppose $q_{\mathcal{U}} = q_0$ and $p_{\mathcal{U}} < p_0$ hold in equilibrium. Then inequality (32) results in $\theta \leq \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}$. Therefore,
$$\Theta_{\mathcal{U}} = \left(-\infty, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}\right] \cap [1,2],$$
and $\mu(\Theta_{\mathcal{U}}) = \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}, 2\right\}, 1\right\} - 1 = \tilde{\zeta}_{\mathcal{H}} = \zeta_{\mathcal{U}}$, where the first equality holds because of agent 10's equilibrium actionand thesecond equality holds because of inequality (20) and the equilibrium action of agent 11.

4. Suppose $q_{\mathcal{U}} = q_0$ and $p_{\mathcal{U}} > p_0$ hold in equilibrium. Note that the first inequality of inequality (32) cannot hold for any θ. Therefore, $\Theta_{\mathcal{U}} = \emptyset$, and $\mu(\Theta_{\mathcal{U}}) = 0 = \tilde{\zeta}_{\mathcal{U}} = \zeta_{\mathcal{U}}$, where the first equality holds because of agent 10's equilibrium actionand thesecond equality holds because of inequality (20) and the equilibrium action of agent 11.

5. Suppose $q_{\mathcal{U}} = q_0$ and $p_{\mathcal{U}} = p_0$ hold in equilibrium. Then inequality (32) results in $\theta \leq \frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}$. In addition, since providers $\mathcal{U}$ and customers' outside options are identical, customers are indifferent when choosing between them, and any
$$\Theta_{\mathcal{U}} \subseteq \left(-\infty, \frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}\right] \cap [1,2]$$
is valid. Therefore, $\mu(\Theta_{\mathcal{U}}) \leq \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}, 2\right\}, 1\right\} - 1$. On the other hand, we note that $\zeta_{\mathcal{U}} \leq 1 - \zeta_{\mathcal{H}}$, since $\tilde{\zeta}_{\mathcal{U}}$ can take any value within $[0,1]$. To establish that given $\zeta_{\mathcal{U}}$ there exists a $\Theta_{\mathcal{U}}$ such that $\mu(\Theta_{\mathcal{U}}) = \zeta_{\mathcal{U}}$, it is equivalent to show that $\max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}, 2\right\}, 1\right\} - 1 = 1 - \zeta_{\mathcal{H}}$. By the characterization of $\zeta_{\mathcal{H}}$ from the equilibrium action of agent 9, it is equivalent to show
$$\begin{aligned} 2 - \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}, 2\right\}, 1\right\} &= \max\left\{2 - \max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}}, 1\right\}, 0\right\} \\ &= \max\left\{2 - \max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}, 1\right\}, 0\right\} \\ &= 2 + \max\left\{-\max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}, 1\right\}, -2\right\} \\ &= 2 - \min\left\{\max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}, 1\right\}, 2\right\}. \end{aligned}$$
The second equality holds as $\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_{\mathcal{H}}-q_{\mathcal{U}}} = \frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}$ in this case. Then, it is straightforward to verify that
$$\max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}, 2\right\}, 1\right\} = \min\left\{\max\left\{\frac{p_{\mathcal{H}}-p_0}{q_{\mathcal{H}}-q_0}, 1\right\}, 2\right\}.$$
Therefore, we show that there exists a $\Theta_{\mathcal{U}}$ such that $\mu(\Theta_{\mathcal{U}}) = \zeta_{\mathcal{U}}$ given $\zeta_{\mathcal{U}}$.

This concludes the proof of existence of an equilibrium as formalized in Appendix A.1. $\square$

Lemma A.1 *Suppose providers with label $\mathcal{H}$ remain active on the platform in equilibrium. Then, the mass of high-quality providers with labels $\mathcal{U}$ and label $\mathcal{H}$, respectively, are:*

$$\delta_{\mathcal{U}}^{H} = \beta\gamma\lambda\delta_{\mathcal{U}}^{U} \text{ and } \delta_{\mathcal{H}}^{H} = \frac{\beta\gamma}{1-\beta}\Big(1-(1-\beta)\lambda\Big)\delta_{\mathcal{U}}^{U}. \tag{33}$$

Proof. Equation (33) follows directly from Equation (13). $\square$

Lemma A.2 *Suppose providers with label $\mathcal{H}$ remain active on the platform in equilibrium. Then, the free-entry condition for new providers (i.e., $V_{\mathcal{U}}^{U} = \frac{w_0}{1-\beta}$) is equivalent to*

$$\frac{\beta\gamma}{1-\beta}\cdot\frac{1-(1-\beta)\lambda}{1+\beta\gamma\lambda\eta}\Big((1-\tau_{\mathcal{H}})p_{\mathcal{H}} - w_0\Big) = \frac{w_0}{\eta} - (1-\tau_{\mathcal{U}})p_{\mathcal{U}}. \tag{34}$$

Proof. First, by Equation (12) and the fact that $V_{\mathcal{U}}^{H} \geq \frac{w_0}{1-\beta}$ and $V_{\mathcal{H}}^{H} \geq \frac{w_0}{1-\beta}$, we can rewrite $V_{\mathcal{U}}^{U} = \frac{w_0}{1-\beta}$ as:

$$(1-\tau_{\mathcal{U}})p_{\mathcal{U}} + \beta\alpha\gamma V_{\mathcal{U}}^{H} + \beta(1-\alpha)\gamma V_{\mathcal{H}}^{H} - \beta\gamma\frac{w_0}{1-\beta} = \frac{w_0}{\eta}.$$

Second, we substitute $V_{\mathcal{U}}^{H}$ and $V_{\mathcal{H}}^{H}$ in the equality above using Equation (10) and Equation (11), respectively, and obtain:

$$(\delta_{\mathcal{U}}^{U} + \eta\delta_{\mathcal{U}}^{H})\Big((1-\tau_{\mathcal{U}})p_{\mathcal{U}} - \frac{w_0}{\eta}\Big) + \delta_{\mathcal{H}}^{H}\Big((1-\tau_{\mathcal{H}})p_{\mathcal{H}} - w_0\Big) = 0.$$

Lastly, by substituting $\delta_{\mathcal{U}}^{H}$ and $\delta_{\mathcal{H}}^{H}$ in the equality above using Equation (33), we obtain Equation (34). $\square$

Lemma A.3 *Suppose $\tau_{\mathcal{U}} = \tau_{\mathcal{H}} = \tau$. Then, L-type providers, i.e., low-quality providers that have completed a transaction inside the platform, take their outside option under any information provision policy given by Equation* (2)*.*

Proof. We prove the lemma by showing that it holds for L-type providers with label $\mathcal{U}$ and label $\mathcal{L}$, respectively.

1. For a L-type provider with label $\mathcal{L}$, we claim that she takes the outside option at each period as her lifetime earnings inside the platform are lower than the outside option. That is,

$$\frac{(1-\tau)p_{\mathcal{L}}}{1-\beta} < \frac{w_0}{1-\beta}, \tag{35}$$

where $p_{\mathcal{L}}$ is the price for her service. Note that $p_{\mathcal{L}} < p_{\mathcal{U}}$ since $q_{\mathcal{L}} = 0 < q_{\mathcal{U}}$ (otherwise, no customer would hire providers with label $\mathcal{L}$ given that they are dominated by providers with label $\mathcal{U}$, whose supply is infinite). Therefore, to show inequality (35), it suffices to show that $(1-\tau)p_{\mathcal{U}} \leq w_0$. We verify the last inequality by way of a contradiction. Suppose $(1-\tau)p_{\mathcal{U}} > w_0$. Then, the lifetime earnings of a provider that is assigned label $\mathcal{U}$ satisfy the following

$$V_{\mathcal{U}}^{U} \geq (1-\tau)p_{\mathcal{U}} + \beta\frac{w_0}{1-\beta} > \frac{w_0}{1-\beta}.$$

This holds given that the provider can take her outside option from the subsequent period onward. In turn, this contradicts the free-entry condition, i.e., $V_{\mathcal{U}}^{U} = \frac{w_0}{1-\beta}$. Therefore, we have $(1-\tau)p_{\mathcal{L}} < (1-\tau)p_{\mathcal{U}} \leq w_0$, and L-type providers with label $\mathcal{L}$ will take the outside option.

2. For a L-type provider with label $\mathcal{U}$, it follows that remaining active on the platform is a suboptimal option using a similar argument as under the case above.

In sum, L-type providers choose the outside option in equilibrium under any given information provision policy. □

Lemma A.4 *Suppose $\tau_{\mathcal{U}} = \tau_{\mathcal{H}} = \tau$. Then, in equilibrium, the mass and the expected quality of providers with label $\mathcal{U}$ are*

$$\delta_{\mathcal{U}} = (1+\beta\gamma\lambda\eta)\delta^U_{\mathcal{U}} \text{ and } q_{\mathcal{U}} = \frac{\gamma+\beta\gamma\lambda\eta}{1+\beta\gamma\lambda\eta},$$

respectively. Moreover, if $q_{\mathcal{U}} \le q_0$, then the structure of the equilibrium is described by one of the following cases:

Case 1 (Eq1): *There is no rationing among $\mathcal{U}$-labeled providers (i.e., $\eta = 1$), and all customers transact with providers inside the platform (i.e., $\delta_{\mathcal{U}} + \delta^H_{\mathcal{H}} = 1$). In addition,*

(i) The mass of new providers who are active on the platform satisfy $\delta^U_{\mathcal{U}} = \frac{1-\beta}{1-\beta+\beta\gamma}$.

(ii) Prices of providers with label $\mathcal{U}$ and label $\mathcal{H}$ are given by

(a) $p_{\mathcal{U}} = \frac{w_0}{1-\tau} - \frac{\beta\gamma}{1-\beta+\beta\gamma}\Big(1-(1-\beta)\lambda\Big)\Big(\frac{1}{1+\beta\gamma\lambda} + \frac{1-\beta}{1-\beta+\beta\gamma}\Big)(1-\gamma)$, *and*

(b) $p_{\mathcal{H}} = p_{\mathcal{U}} + \Big(\frac{1}{1+\beta\gamma\lambda} + \frac{1-\beta}{1-\beta+\beta\gamma}\Big)(1-\gamma)$.

(iii) Platform revenues are given by $\frac{\tau}{1-\tau}w_0$.

(iv) Finally, the equilibrium satisfies the following conditions:

- *Providers with label $\mathcal{U}$ are not financially constrained, i.e., $(1-\tau)p_{\mathcal{U}} \ge b_0$.*
- *The customer with type $1+\delta_{\mathcal{U}}$ prefers providers with label $\mathcal{U}$ to the outside option, i.e.,*

$$p_{\mathcal{U}} \le p_0 - \left(\frac{1}{1+\beta\gamma\lambda} + \frac{1-\beta}{1-\beta+\beta\gamma}\right)\Big(q_0 - \gamma - \beta\gamma\lambda(1-q_0)\Big).$$

Case 2 (Eq2): *There is rationing among $\mathcal{U}$-labeled providers (i.e., $\eta < 1$), and all customers transact with providers inside the platform (i.e., $\delta_{\mathcal{U}} + \delta^H_{\mathcal{H}} = 1$). In addition,*

(i) The mass of new providers who are active on the platform satisfy

$$\delta^U_{\mathcal{U}} = \frac{1}{\frac{1-\beta+\beta\gamma}{1-\beta} - \beta\gamma\lambda(1-\eta)}.$$

(ii) Prices of providers with label $\mathcal{U}$ and label $\mathcal{H}$ are given by

(a) $p_{\mathcal{U}} = \frac{b_0}{1-\tau}$.

(b) $p_{\mathcal{H}} = p_{\mathcal{U}} + \Big(\frac{1}{1+\beta\gamma\lambda\eta} + \delta^U_{\mathcal{U}}\Big)(1-\gamma)$.

(iii) Platform revenues are given by $\tau\Big(p_{\mathcal{U}} + \Big(\frac{1}{1+\beta\gamma\lambda\eta} + \delta^U_{\mathcal{U}}\Big)\delta^H_{\mathcal{H}}(1-\gamma)\Big)$.

(iv) Finally, the equilibrium satisfies the following conditions:

- *There is rationing among providers with label $\mathcal{U}$, and η is given as the solution to Equation* (34).
- *The customer with type $1+\delta_{\mathcal{U}}$ prefers providers with label $\mathcal{U}$ to the outside option, i.e.,*

$$1+\delta_{\mathcal{U}} \le \frac{p_0 - p_{\mathcal{U}}}{q_0 - q_{\mathcal{U}}}.$$

Case 3 (Eq3): *There is rationing among $\mathcal{U}$-labeled providers (i.e., $\eta < 1$), and there is a positive mass of customers who choose the outside option (i.e., $\delta_{\mathcal{U}} + \delta_{\mathcal{H}}^H < 1$). In addition,*

(i) The mass of new providers who are active on the platform satisfy

$$\delta_{\mathcal{U}}^U = \frac{p_0 - \frac{b_0}{1-\tau}}{q_0 - \gamma - \beta\gamma\lambda\eta(1-q_0)} - \frac{1}{1+\beta\gamma\lambda\eta}.$$

(ii) Prices of providers with label $\mathcal{U}$ and label $\mathcal{H}$ are given by

(a) $p_{\mathcal{U}} = \frac{b_0}{1-\tau}$.

(b) $p_{\mathcal{H}} = p_0 + \left(2 - \frac{\beta\gamma}{1-\beta}\left(1-(1-\beta)\lambda\right)\delta_{\mathcal{U}}^U\right)(1-q_0)$.

(iii) Platform revenues are given by

$$\pi_{r,3}(\tau,\lambda) \triangleq \frac{\tau}{1-\tau}\left(\frac{\beta\gamma}{1-\beta} + \frac{1}{\eta}\right) w_0 \delta_{\mathcal{U}}^U.$$

(iv) Finally, the equilibrium satisfies the following conditions:

- *There is rationing among providers with label $\mathcal{U}$, and η is given as the solution to Equation* (34).
- *The volume of transactions inside the platform with providers labeled $\mathcal{U}$ is strictly positive.*
- *The volume of transactions inside the platform is strictly positive. In addition, a strictly positive fraction of consumers take their outside option.*

Case 4 (Eq4): *There is no rationing among $\mathcal{U}$-labeled providers (i.e., $\eta = 1$), and there is a positive mass of customers choosing the outside option (i.e., $\delta_{\mathcal{U}} + \delta_{\mathcal{H}}^H < 1$). In addition,*

(i) The mass of new providers who are active on the platform satisfy

$$\delta_{\mathcal{U}}^U = \frac{\frac{1-\beta+\beta\gamma}{1-\beta}(p_0 - \frac{w_0}{1-\tau}) + \frac{2\beta\gamma}{1-\beta}(1-q_0) - (q_0-\gamma) - (1-q_0)\beta\gamma\lambda}{(\frac{\beta\gamma}{1-\beta})^2(1-q_0) + (q_0-\gamma) - \left(\frac{1-\beta+2\beta\gamma}{1-\beta}(1-q_0) - (q_0-\gamma)\right)\beta\gamma\lambda}.$$

(ii) Prices of providers with label $\mathcal{U}$ and label $\mathcal{H}$ are given by

(a) $p_{\mathcal{U}} = p_0 - (\frac{1}{1+\beta\gamma\lambda} + \delta_{\mathcal{U}}^U)\left(q_0 - \gamma - \beta\gamma\lambda(1-q_0)\right)$.

(b) $p_{\mathcal{H}} = p_0 + \left(2 - \frac{\beta\gamma}{1-\beta}\left(1-(1-\beta)\lambda\right)\delta_{\mathcal{U}}^U\right)(1-q_0)$.

(iii) Platform revenues are given by

$$\pi_{r,4}(\tau,\lambda) \triangleq \frac{\tau}{1-\tau} \cdot \frac{1-\beta+\beta\gamma}{1-\beta} w_0 \delta_{\mathcal{U}}^U.$$

(iv) Finally, the equilibrium satisfies the following conditions:

- *Providers with label $\mathcal{U}$ are not financially constrained, i.e., $(1-\tau)p_{\mathcal{U}} \geq b_0$.*
- *The volume of transactions inside the platform is strictly positive. In addition, a strictly positive fraction of consumers take their outside option.*

Proof. First, note that the mass of providers with label $\mathcal{U}$ who are active on the platform is given by

$$\delta_{\mathcal{U}} = \zeta_{\mathcal{U}} = \delta_{\mathcal{U}}^U + \eta\delta_{\mathcal{U}}^H = (1+\beta\gamma\lambda\eta)\delta_{\mathcal{U}}^U,$$

where the last equality is obtained from Equation (33). Then, their expected quality is given by

$$q_{\mathcal{U}} = \frac{\delta_{\mathcal{U}}^U\gamma + \eta\delta_{\mathcal{U}}^H}{\delta_{\mathcal{U}}^U + \eta\delta_{\mathcal{U}}^H} = \frac{\gamma + \beta\gamma\lambda\eta}{1+\beta\gamma\lambda\eta},$$

where the second equality follows from Equation (33).

Next, we provide a characterization of the equilibrium quantities corresponding to each of the four cases described above.

Eq1: By the definition of this case, there is no rationing among providers with label $\mathcal{U}$ and all customers transact with providers inside the platform. Therefore, $\eta = 1$ and $\delta_{\mathcal{U}} + \delta_{\mathcal{H}}^H = 1$. From these two equalities and Equation (33), we further obtain that

$$\delta_{\mathcal{U}}^U = \frac{1-\beta}{1-\beta+\beta\gamma}.$$

Next, we obtain the equilibrium prices by Expression (8) as

$$p_{\mathcal{H}} = p_{\mathcal{U}} + (1+\zeta_{\mathcal{U}})(1-q_{\mathcal{U}}) = p_{\mathcal{U}} + \Big(\frac{1}{1+\beta\gamma\lambda} + \frac{1-\beta}{1-\beta+\beta\gamma}\Big)(1-\gamma).$$

The expression for $p_{\mathcal{U}}$ is obtained directly from Equation (34). Finally, from Equation (34) we also obtain that $(1-\tau)(\delta_{\mathcal{H}}^H p_{\mathcal{H}} + \delta_{\mathcal{U}} p_{\mathcal{U}}) = w_0$. In turn, this implies that revenues for the platform are given by $\frac{\tau}{1-\tau} w_0$.

Eq2: By the definition of this case, providers with label $\mathcal{U}$ are financially constrained given that there is rationing among them, i.e., $\eta < 1$. Furthermore, all customers transact with providers inside the platform. Therefore,

$$p_{\mathcal{U}} = \frac{b_0}{1-\tau} \text{ and } \delta_{\mathcal{U}} + \delta_{\mathcal{H}}^H = 1.$$

First, we solve for $\delta_{\mathcal{U}}^U$ from the last equality above and Equation (33). Then, we characterize $p_{\mathcal{H}}$ based on Expression (8) and we obtain

$$p_{\mathcal{H}} = p_{\mathcal{U}} + (1+\zeta_{\mathcal{U}})(1-q_{\mathcal{U}}) = \frac{b_0}{1-\tau} + \Big(\frac{1}{1+\beta\gamma\lambda\eta} + \frac{1}{\frac{1-\beta+\beta\gamma}{1-\beta} - \beta\gamma\lambda(1-\eta)}\Big)(1-\gamma).$$

Given the characterization of $p_{\mathcal{U}}$ and $p_{\mathcal{H}}$, we determine η from Equation (34). Finally, for the platform's revenues, we have

$$\pi_{r,2} = \tau(p_{\mathcal{U}}\delta_{\mathcal{U}} + p_{\mathcal{H}}\delta_{\mathcal{H}}^H) = \tau\Big(\frac{b_0}{1-\tau} + (\frac{1}{1+\beta\gamma\lambda\eta} + \delta_{\mathcal{U}}^U)\delta_{\mathcal{H}}^H(1-\gamma)\Big),$$

where the second equality is obtained by substituting $p_{\mathcal{H}}$ from above.

The characterization of the equilibrium quantities corresponding to the remaining cases follows by similar arguments and straightforward algebra, which we omit for brevity. □

Proposition A.2 *Suppose $\tau_{\mathcal{U}} = \tau_{\mathcal{H}} = \tau$ and $\alpha = 0$. Then, the structure of the equilibrium as a function of τ and the modeling primitives can be described as follows.*

(0) If $p_0 \le q_0 - \gamma$, then there is no provider on the platform.

(1) If $q_0 - \gamma < p_0 \le \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)$ and $0 < b_0 \le \frac{p_0-(q_0-\gamma)}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} w_0$, then

(1-a) The structure of the equilibrium follows case Eq3 from Lemma A.4 if

$$0 \le \tau < 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma} w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma}) b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}} - 1)p_0}. \tag{36}$$

(1-b) The structure of the equilibrium follows case Eq4 from Lemma A.4 if

$$1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma} w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma}) b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}} - 1)p_0} \le \tau < 1 - \frac{w_0}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}. \tag{37}$$

(1-c) Otherwise, there is no provider on the platform.

(2) If $q_0-\gamma<p_0\le\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)$ *and* $\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0<b_0\le w_0$*, then*

(2-a) The structure of the equilibrium follows case Eq3 from Lemma A.4 if

$$0<\tau<1-\frac{b_0}{p_0-(q_0-\gamma)}. \tag{38}$$

(2-b) Otherwise, there is no provider on the platform.

(3) If $\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)<p_0\le\frac{2(1-\beta)+\beta\gamma}{1-\beta}\cdot\frac{1-q_0}{1/\underline{\eta}-1}$ *and*

$$0<b_0\le\frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\big(2(1-\beta)+\beta\gamma\big)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0,$$

then

(3-a) The structure of the equilibrium follows case Eq2 from Lemma A.4 if

$$0<\tau<1-\frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\big(2(1-\beta)+\beta\gamma\big)}\frac{w_0-b_0}{1-\gamma}. \tag{39}$$

(3-b) The structure of the equilibrium follows case Eq1 from Lemma A.4 if

$$1-\frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\big(2(1-\beta)+\beta\gamma\big)}\frac{w_0-b_0}{1-\gamma}\le\tau\le1-\frac{w_0}{p_0+\frac{\big(2(1-\beta)+\beta\gamma\big)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}. \tag{40}$$

(3-c) The structure of the equilibrium follows case Eq4 from Lemma A.4 if

$$1-\frac{w_0}{p_0+\frac{\big(2(1-\beta)+\beta\gamma\big)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}<\tau<1-\frac{w_0}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}. \tag{41}$$

(3-d) Otherwise, there is no provider on the platform.

(4) If $\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)<p_0\le\frac{2(1-\beta)+\beta\gamma}{1-\beta}\frac{1-q_0}{1/\underline{\eta}-1}$ *and*

$$\frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\big(2(1-\beta)+\beta\gamma\big)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0<b_0<\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0,$$

then

(4-a) The structure of the equilibrium follows case Eq2 from Lemma A.4 if

$$0<\tau\le1-\frac{b_0}{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}. \tag{42}$$

(4-b) The structure of the equilibrium follows case Eq3 from Lemma A.4 if

$$1-\frac{b_0}{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}<\tau<1-\frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0-(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)-(\frac{1}{\underline{\eta}}-1)p_0}.$$

(4-c) The structure of the equilibrium follows case Eq4 from Lemma A.4 if Expression (37) *holds.*

(4-d) Otherwise, there is no provider on the platform.

(5) If $\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)<p_0\le\frac{2(1-\beta)+\beta\gamma}{1-\beta}\cdot\frac{1-q_0}{1/\underline{\eta}-1}$ *and* $\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0\le b_0\le w_0$*, then*

(5-a) The structure of the equilibrium follows case Eq2 from Lemma A.4 if Expression (42) *holds.*

(5-b) The structure of the equilibrium follows case Eq3 from Lemma A.4 if

$$1-\frac{b_0}{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}<\tau<1-\frac{b_0}{p_0-(q_0-\gamma)}. \tag{43}$$

(5-c) Otherwise, there is no provider on the platform.

(6) If $p_0>\frac{2(1-\beta)+\beta\gamma}{1-\beta}\cdot\frac{1-q_0}{1/\underline{\eta}-1}$ *and* $0<b_0<\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0$*, then*

(6-a) The structure of the equilibrium follows case Eq2 from Lemma A.4 if Expression (39) *holds.*

(6-b) The structure of the equilibrium follows case Eq1 from Lemma A.4 if Expression (40) *holds.*

(6-c) The structure of the equilibrium follows case Eq4 from Lemma A.4 if Expression (41) *holds.*

(6-d) Otherwise, there is no provider on the platform.

(7) If $p_0>\frac{2(1-\beta)+\beta\gamma}{1-\beta}\cdot\frac{1-q_0}{1/\underline{\eta}-1}$ *and* $\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0\le b_0\le\frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0$*, then*

(7-a) The structure of the equilibrium follows case Eq2 from Lemma A.4 if Expression (39) *holds.*

(7-b) The structure of the equilibrium follows case Eq1 from Lemma A.4 if Expression (40) *holds.*

(7-c) The structure of the equilibrium follows case Eq4 from Lemma A.4 if

$$1-\frac{w_0}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}<\tau\le1-\frac{(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0-\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0}{(\frac{1}{\underline{\eta}}-1)p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)}.$$

(7-d) The structure of the equilibrium follows case Eq3 from Lemma A.4 if

$$1-\frac{(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0-\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0}{(\frac{1}{\underline{\eta}}-1)p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)}<\tau<1-\frac{b_0}{p_0-q_0+\gamma}.$$

(7-e) Otherwise, there is no provider on the platform.

(8) If $p_0>\frac{2(1-\beta)+\beta\gamma}{1-\beta}\cdot\frac{1-q_0}{1/\underline{\eta}-1}$ *and* $\frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0<b_0\le w_0$*, then*

(8-a) The structure of the equilibrium follows case Eq2 from Lemma A.4 if Expression (42) *holds.*

(8-b) The structure of the equilibrium follows case Eq3 from Lemma A.4 if Expression (43) *holds.*

(8-c) Otherwise, there is no provider on the platform.

Proof. To establish the proposition, we employ Lemma A.4 and verify that the equilibrium conditions are satisfied for each of the cases.

(0) When $p_0\le q_0-\gamma$, no providers choose to remain active on the platform, as the platform cannot induce an equilibrium that features a positive volume of transactions and leads to non-negative revenues.

(1) Under the assumptions of case (1), we note that

$$\frac{w_0}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}\le\frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0-(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)-(\frac{1}{\underline{\eta}}-1)p_0}.$$

Thus, the partition of the interval for τ in cases (1-a) and (1-b) is valid.

(1-a) To show that the structure of the equilibrium follows case Eq3 under (1-a), it suffices to verify conditions $\delta_{\mathcal{U}}^U+\delta_{\mathcal{H}}^H<1$, $0<\eta<1$, and $\delta_{\mathcal{U}}^U>0$. Note that $\delta_{\mathcal{U}}^H=0$ and $\delta_{\mathcal{H}}^H=\frac{\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^U$. Condition $\delta_{\mathcal{U}}^U+\delta_{\mathcal{H}}^H<1$ is equivalent to $\delta_{\mathcal{U}}^U<\frac{1-\beta}{1-\beta+\beta\gamma}$, which in turn can be rewritten as

$$p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)<\frac{b_0}{1-\tau}.$$

The last inequality holds since its left-hand side is non-positive while the right-hand side is positive. Then, we solve for η as

$$\eta = \frac{w_0}{\frac{\beta\gamma}{1-\beta}(1-\tau)p_{\mathcal{H}} - \frac{\beta\gamma}{1-\beta}w_0 + b_0}.$$

Note that $\eta > 0$ is equivalent to

$$\tau < 1 - \frac{w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}} - 1)p_0}.$$

The inequality above holds from Expression (36). Moreover, again from from Expression (36), we observe that $\eta < 1$. Finally, condition $\delta_{\mathcal{U}}^{U} > 0$ follows from Expression (38).

(1-b) To show that the structure of the equilibrium follows case Eq4 under (1-b), it suffices to verify conditions $\delta_{\mathcal{U}}^{U} + \delta_{\mathcal{H}}^{H} < 1$, $(1-\tau)p_{\mathcal{U}} \geq b_0$, and $\delta_{\mathcal{U}}^{U} > 0$. The first condition is equivalent to $\delta_{\mathcal{U}}^{U} < \frac{1-\beta}{1-\beta+\beta\gamma}$, which holds because $\tau < 1$, $b_0 > 0$, and $p_0 \leq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0 - \gamma)$. Next, we solve for $p_{\mathcal{U}}$ as

$$p_{\mathcal{U}} = \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}\frac{w_0}{1-\tau} + (\frac{1}{\underline{\eta}} - 1)p_0 - \left(2 + \frac{\beta\gamma}{1-\beta}\right)(1-q_0)}{\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma}}.$$

Note that condition $(1-\tau)p_{\mathcal{U}} \geq b_0$ is equivalent to the first inequality of Expression (37). Then, condition $\delta_{\mathcal{U}}^{U} > 0$ holds as it is equivalent to the second inequality of Expression (37).

(1-c) Under case (1-c), none of the equilibrium structures Eq1, Eq2, Eq3, and Eq4 can arise and no providers join the platform.

(2) Under the assumptions of case (2), there are two cases to consider:

(2-a) To show that the structure of the equilibrium follows case Eq3 under (2-a), it suffices to show that conditions $0 < \delta_{\mathcal{U}}^{U} < \frac{1-\beta}{1-\beta+\beta\gamma}$ and $0 < \eta < 1$ hold. Following the same argument as in (1-a), we verify that these conditions hold.

(2-b) Under case (2-b), none of the equilibrium structures Eq1, Eq2, Eq3, and Eq4 can arise and no providers join the platform.

The remaining cases, i.e., cases (3)-(8), can be shown using similar arguments. We omit the details for the sake of brevity. □

Proposition A.3 *Suppose $\alpha = 0$ and let*

$$\bar{\tau}^* \triangleq 1 - \frac{2w_0}{p_0 + w_0 + \frac{2\beta\gamma}{1-\beta+\beta\gamma}(1 - \frac{1}{2}\underline{\eta})(1-q_0)}.$$

Then, the optimal commission under the full-information provision policy, $\tau^*(0)$ and thestructure of the equilibrium under $(\tau^*(0), 0)$ can be described as follows:

(1) If $q_0 - \gamma < p_0 < \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)$ and $0 < b_0 \leq \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0$, then we have the following:

(a) *If*

$$b_0 > \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma/\underline{\eta}}\left(1 - \frac{(2+\frac{\beta\gamma}{1-\beta})(1-q_0) - (\frac{1}{\underline{\eta}}-1)p_0}{\frac{1-\beta+\beta\gamma}{2\beta\gamma}(p_0+w_0) + (1-\frac{1}{2}\underline{\eta})(1-q_0)}\right)w_0, \tag{44}$$

then

$$\tau^*(0) = \arg\max \pi_{r,3}(\tau, 0)\mathbb{1}\left\{0 < \tau < 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}}-1)p_0}\right\} \tag{45}$$

and the structure of the equilibrium under $(\tau^*(0), 0)$ follows case Eq3.

(b) *Otherwise, if inequality (44) does not hold, then* $\tau^*(0) = \bar{\tau}^*$ *and the structure of the equilibrium under* $(\tau^*(0), 0)$ *follows case Eq4.*

(2) If $q_0 - \gamma < p_0 < \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)$ *and* $\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0 < b_0 \le w_0$*, then*

$$\tau^*(0) = \arg\max \pi_{r,3}(\tau,0)\mathbb{1}\left\{0 < \tau < 1 - \frac{b_0}{p_0 - (q_0-\gamma)}\right\}$$

and the structure of the equilibrium under $(\tau^*(0), 0)$ *follows case Eq3.*

(3) If $\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma) < p_0 < \frac{2(1-\beta)+\beta\gamma}{1-\beta}\cdot\frac{1-q_0}{1/\underline{\eta}-1}$ *and* $0 < b_0 \le \frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0$*, then*

$$\tau^*(0) = \max\left\{\bar{\tau}^*, 1 - \frac{w_0}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}\right\}. \tag{46}$$

Moreover, if

$$p_0 \ge w_0 + \frac{2(1-\beta)}{1-\beta+\beta\gamma}\left(\frac{3(1-\beta)+\beta\gamma}{2(1-\beta+\beta\gamma)} - \frac{1-\beta}{1-\beta+\beta\gamma}\frac{1}{\underline{\eta}}\right)(q_0-\gamma), \tag{47}$$

then the structure of the equilibrium under $(\tau^*(0), 0)$ *follows case Eq1; otherwise, the structure of the equilibrium under* $(\tau^*(0), 0)$ *follows case Eq4.*

(4) If $\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma) < p_0 < \frac{2(1-\beta)+\beta\gamma}{1-\beta}\cdot\frac{1-q_0}{1/\underline{\eta}-1}$ *and*

$$\frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0 < b_0 < \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0,$$

then:

(a) *If inequality (44) holds, then*

$$\tau^*(0) = \arg\max \pi_{r,3}(\tau,0)\mathbb{1}\left\{1 - \frac{b_0}{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)} \le \tau < 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}}-1)p_0}\right\} \tag{48}$$

and the structure of equilibrium under $(\tau^*(0), 0)$ *follows case Eq2 or Eq3.*

(b) *Otherwise, if inequality (44) does not hold, then* $\tau^*(0) = \bar{\tau}^*$ *and the structure of the equilibrium under* $(\tau^*(0), 0)$ *follows case Eq4.*

(5) If $\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma) < p_0 < \frac{2(1-\beta)+\beta\gamma}{1-\beta}\cdot\frac{1-q_0}{1/\underline{\eta}-1}$ *and* $\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0 \le b_0 \le w_0$*, then*

$$\tau^*(0) = \arg\max \pi_{r,3}(\tau,0)\mathbb{1}\left\{1 - \frac{b_0}{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)} \le \tau < 1 - \frac{b_0}{p_0-(q_0-\gamma)}\right\} \tag{49}$$

and the structure of equilibrium under $(\tau^*(0), 0)$ *follows case Eq2 or Eq3.*

(6) If $p_0 > \frac{2(1-\beta)+\beta\gamma}{1-\beta}\cdot\frac{1-q_0}{1/\underline{\eta}-1}$ *and* $0 < b_0 < \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0$*, then* $\tau^*(0)$ *is given by Expression (46). Moreover, if inequality (47) holds, then the structure of the equilibrium under* $(\tau^*(0), 0)$ *follows case Eq1; otherwise, the structure of the equilibrium under* $(\tau^*(0), 0)$ *follows case Eq4.*

(7) If $p_0 > \frac{2(1-\beta)+\beta\gamma}{1-\beta}\frac{1-q_0}{1/\underline{\eta}-1}$ *and*

$$\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0 \le b_0 \le \frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0,$$

then we have the following:

(a) *If inequality* (44) *holds, then*

$$\tau^*(0) = \arg\max \pi_{r,3}(\tau,0)\mathbb{1}\left\{1-\frac{(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0-\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0}{(\frac{1}{\underline{\eta}}-1)p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)} < \tau < 1-\frac{b_0}{p_0-q_0+\gamma}\right\} \tag{50}$$

and the structure of the equilibrium under $(\tau^*(0),0)$ *follows case Eq3.*

(b) *Otherwise, if inequality* (44) *does not hold, then* $\tau^*(0)$ *is given by Expression* (46)*. Moreover, if inequality* (47) *also holds the structure of equilibrium under* $(\tau^*(0),0)$ *follows case Eq1; otherwise, if* (47) *does not hold, the structure of the equilibrium under* $(\tau^*(0),0)$ *follows case Eq4.*

(8) If $p_0 > \frac{2(1-\beta)+\beta\gamma}{1-\beta}\cdot\frac{1-q_0}{1/\underline{\eta}-1}$ *and* $\frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0 < b_0 \le w_0$*, then* $\tau^*(0)$ *is given by Expression* (49) *and the structure of equilibrium under* $(\tau^*(0),0)$ *follows case Eq2 or Eq3.*

Proof. The proof of the proposition is based on the following five observations.

Observation 1. By Lemma A.4, it is straightforward to show that revenues for the platform increase in τ when the structure of the equilibrium follows cases Eq1 and Eq2. Therefore, the optimal commission $\tau^*(0)$ can only be at the boundary of the intervals specified under cases (3-a), (3-b), (4-a), (5-a), (6-a), (6-b), (7-a), (7-b), and (8-a) of Proposition A.2.

Observation 2. If $\tau^*(0)$ is in the interior of the interval when the structure of the equilibrium follows case Eq4, we show that $\tau^*(0) = \bar{\tau}^*$. That is, $\frac{\partial \pi_{r,4}}{\partial\tau}(\bar{\tau}^*,0)=0$, where $\pi_{r,4}(\tau,\lambda)$ is given by Lemma A.4. This follows from Lemma A.4 given that maximizing revenues turns out to be equivalent to maximizing the following quadratic function:

$$(u-1)\left(\frac{1}{w_0}\left(p_0+\frac{2\beta\gamma}{1-\beta+\beta\gamma}\left(1-\frac{1}{2}\underline{\eta}\right)(1-q_0)\right)-u\right),$$

where $u=\frac{1}{1-\tau}$. The expression above is maximized at $\tau^*(0)=\bar{\tau}^*$.

Observation 3. Consider the expressions for thr platform's revenues when the structure of the equilibrium follows cases Eq3 and Eq4, i.e., $\pi_{r,3}(\tau,0)$ and $\pi_{r,4}(\tau,0)$, respectively, given by Lemma A.4. There exist unique $\hat{\tau}_3$ and $\hat{\tau}_4$, such that

$$\frac{\partial \pi_{r,3}}{\partial\tau}(\hat{\tau}_3,0)=0, \frac{\partial \pi_{r,4}}{\partial\tau}(\hat{\tau}_4,0)=0, \frac{\partial^2 \pi_{r,3}}{\partial\tau^2}(\hat{\tau}_3,0)<0, \text{ and } \frac{\partial^2 \pi_{r,4}}{\partial\tau^2}(\hat{\tau}_4,0)<0. \tag{51}$$

In addition, $\pi_{r,i}(\tau,0)$ is increasing in τ within $[0,\hat{\tau}_i]$ and decreasing in τ within $[\hat{\tau}_i,1]$, where $i\in\{3,4\}$.

For $\pi_{r,4}(\tau,0)$, the observation follows from $\pi_{r,4}(0,0)=0$, $\pi_{r,4}(1^-,0)=-\infty$, $\max_{\tau\in[0,1]}\pi_{r,4}(\tau,0)>0$and thefact that that $\pi_{r,4}(\tau,0)$ is quadratic in $\frac{1}{1-\tau}$.

For $\pi_{r,3}(\tau,0)$, the observation follows from

$$\pi_{r,3}(0,0)=0, \pi_{r,4}(1^-,0)=-\infty, \frac{\partial \pi_{r,3}}{\partial\tau}(0,0)>0, \frac{\partial \pi_{r,3}}{\partial\tau}(1^-,0)=-\infty,$$

and the fact that
$$\frac{\partial^2 \pi_{r,3}}{\partial \tau^2}(\tau,0) > 0 \text{ only if } \tau < 1 - \frac{3b_0}{b_0 + p_0 + \frac{\left(\frac{1}{\underline{\eta}}-1\right)p_0-(1-q_0)}{\frac{1-\beta}{\beta\gamma}+\frac{1}{\underline{\eta}}}}.$$

Observation 4. Consider cases (1) and (4) of Proposition A.2. Then
$$\hat{\tau}_3 > \hat{\tau}_4, \tag{52}$$
where $\hat{\tau}_3$ and $\hat{\tau}_4$ are given by Expression (51).

Given inequality (52), the optimal commission $\tau^*(0)$ in cases (1) and (4) can be determined by checking whether the structure of the equilibrium follows case Eq4 under $\hat{\tau}_4$. In particular, if the structure of the equilibrium follows case Eq4 under $\hat{\tau}_4$, then $\tau^*(0) = \hat{\tau}_4$. Otherwise, $\tau^*(0)$ can be determined by Expressions (45) and (48).

Before verifying inequality (52), we rewrite the expression for the platform's revenues under cases Eq3 and Eq4 as $\pi(\tau) = \tau h(p_{\mathcal{U}})$, where $h(p_{\mathcal{U}})$ is a function of $p_{\mathcal{U}}$. In particular,
$$\begin{aligned} h(p_{\mathcal{U}}) &\triangleq \delta^U_{\mathcal{U}} p_{\mathcal{U}} + \delta^H_{\mathcal{H}} p_{\mathcal{H}} = \frac{\beta\gamma}{1-\beta}\delta^U_{\mathcal{U}}\left(\frac{1-\beta}{\beta\gamma}p_{\mathcal{U}} + p_0 + \left(2 - \frac{\beta\gamma}{1-\beta}\delta^U_{\mathcal{U}}\right)(1-q_0)\right) \\ &= \frac{1+\frac{\beta\gamma}{1-\beta}\frac{1}{\underline{\eta}}}{q_0-\gamma}\Big(p_0 - (q_0-\gamma) - p_{\mathcal{U}}\Big)\left(p_{\mathcal{U}} + \frac{\left(2+\frac{\beta\gamma}{1-\beta}\right)(1-q_0) - \left(\frac{1}{\underline{\eta}}-1\right)p_0}{\frac{1-\beta}{\beta\gamma}+\frac{1}{\underline{\eta}}}\right). \end{aligned}$$
Note that $\pi'(\tau) = 0$ is equivalent to $\tau p'_{\mathcal{U}} = -\frac{h(p_{\mathcal{U}})}{h'(p_{\mathcal{U}})}$, where $p'_{\mathcal{U}}$ is the derivative of $p_{\mathcal{U}}$ with respect to τ. Furthermore,
$$\frac{h'(p_{\mathcal{U}})}{h(p_{\mathcal{U}})} = -\frac{1}{p_0-(q_0-\gamma)-p_{\mathcal{U}}} + \frac{1}{p_{\mathcal{U}} + \frac{\left(2+\frac{\beta\gamma}{1-\beta}\right)(1-q_0)-\left(\frac{1}{\underline{\eta}}-1\right)p_0}{\frac{1-\beta}{\beta\gamma}+\frac{1}{\underline{\eta}}}},$$
which is decreasing in $p_{\mathcal{U}}$.

Then, we establish inequality (52) by way of a contradiction. Suppose that $\hat{\tau}_3 \le \hat{\tau}_4$. Then, we obtain
$$\hat{\tau}_3 p'_{\mathcal{U}}(\hat{\tau}_3) = -\frac{h\Big(p_{\mathcal{U}}(\hat{\tau}_3)\Big)}{h'\Big(p_{\mathcal{U}}(\hat{\tau}_3)\Big)}, \text{ and } \hat{\tau}_4 p'_{\mathcal{U}}(\hat{\tau}_4) = -\frac{h\Big(p_{\mathcal{U}}(\hat{\tau}_4)\Big)}{h'\Big(p_{\mathcal{U}}(\hat{\tau}_4)\Big)},$$
by Expression (51). Next, we claim that $p'_{\mathcal{U}}(\hat{\tau}_3) < p'_{\mathcal{U}}(\hat{\tau}_4)$ holds. Note that this can be rewritten as
$$\frac{b_0}{(1-\hat{\tau}_3)^2} < \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma 1/\underline{\eta}} \cdot \frac{w_0}{(1-\hat{\tau}_4)^2}$$
by Lemma A.4. To show that this inequality holds, it suffices to show that
$$b_0 \le \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma\frac{1}{\underline{\eta}}} w_0. \tag{53}$$
We also note that
$$b_0 \le \frac{p_0-(q_0-\gamma)}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} w_0$$
holds under cases (1) and (4) of Proposition A.2. Then, to verify inequality (53), it suffices to show that
$$\frac{p_0-(q_0-\gamma)}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} w_0 < \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma\frac{1}{\underline{\eta}}} w_0.$$

The inequality above is equivalent to $p_0 < \frac{2(1-\beta)+\beta\gamma}{1-\beta} \cdot \frac{1-q_0}{1/\underline{\eta}-1}$, which holds under cases (1) and (4). Therefore, we have shown that $p'_{\mathcal{U}}(\hat{\tau}_3) < p'_{\mathcal{U}}(\hat{\tau}_4)$. Furthermore, we obtain

$$\hat{\tau}_3 p'_{\mathcal{U}}(\hat{\tau}_3) < \hat{\tau}_4 p'_{\mathcal{U}}(\hat{\tau}_4) \text{ and } -\frac{h'(p_{\mathcal{U}}(\hat{\tau}_3))}{h(p_{\mathcal{U}}(\hat{\tau}_3))} > -\frac{h'(p_{\mathcal{U}}(\hat{\tau}_4))}{h(p_{\mathcal{U}}(\hat{\tau}_4))},$$

which results in a contradiction. Therefore, $\hat{\tau}_3 \leq \hat{\tau}_4$ cannot hold.

Observation 5. Consider case (7) of Proposition A.2. Then

$$\hat{\tau}_3 < \hat{\tau}_4 \tag{54}$$

where $\hat{\tau}_3$ and $\hat{\tau}_4$ are given by Expression (51).

Given inequality (54), the optimal commission $\tau^*(0)$ in case (7) can be determined by checking whether the structure of the equilibrium follows case Eq3 under $\hat{\tau}_4$. In particular, if the structure of the equilibrium follows case Eq2, Eq1, or Eq4 under $\hat{\tau}_4$, then $\tau^*(0)$ is given by Expression (46). Otherwise, if the structure of the equilibrium follows case Eq3 under $\hat{\tau}_4$, then $\tau^*(0)$ is given by Expression (50).

The proof of inequality (54) follows similar arguments as those in Observation 4 above.

Given the five observations above, we can characterize $\tau^*(0)$ and the structure of the equilibrium under $(\tau^*(0), 0)$ for each of the eight cases stated in the proposition.

(1) Under case (1), it is straightforward to show that Expression (44) is equivalent to

$$\bar{\tau}^* < 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma} w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma}) b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}} - 1)p_0}. \tag{55}$$

If Expression (44) holds, then by Observations 2, 3, and 4 we obtain that $\tau^*(0)$ is given by Expression (45) and the structure of the equilibrium follows case Eq3 under $\tau^*(0)$. Based on the same observations, if Expression (44) does not hold, we obtain $\tau^*(0) = \bar{\tau}^*$ and the structure of the equilibrium follows case Eq4.

(2) The characterization of case (2) is straightforward. We omit the details for brevity.

(3) Under case (3), Observation 1 implies that $\tau^*(0)$ is either at the left boundary or the interior of the interval where the structure of the equilibrium follows case Eq4. It is straightforward to show that inequality (47) is equivalent to

$$\bar{\tau}^* \leq 1 - \frac{w_0}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}.$$

By Observation 2, if inequality (47) holds, revenues are maximized at

$$1 - \frac{w_0}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}.$$

In addition, the structure of the equilibrium under $(\tau^*(0), 0)$ follows case Eq1. Otherwise, revenues are maximized at $\bar{\tau}^*$ and the structure of the equilibrium follows case Eq4. In sum, $\tau^*(0)$ is given by (46).

(4) Under case (4), we note that Expression (44) is equivalent to Expression (55). By Observations 1-4, if Expression (44) holds, then $\tau^*(0)$ is obtained by Expression (48)and thestructure of the equilibrium under $(\tau^*(0), 0)$ follows case Eq2 or Eq3. By the same observations, if Expression (55) does not hold, we have $\tau^*(0) = \bar{\tau}^*$and thestructure of the equilibrium under $(\tau^*(0), 0)$ follows case Eq4.

(5) Case (5) follows from Observation 1 in a straightforward manner.

(6) Case (6) follows from Observations 1 and 2 using similar arguments as in case (3).

(7) Under case (7), we note that Expression (44) is equivalent to

$$\bar{\tau}^* > 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma} w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma}) b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}} - 1) p_0}.$$

On the one hand, by Observations 1-3, and 5, it follows that $\tau^*(0)$ is given by Expression (50) if Expression (44) holds. Moreover, the structure of the equilibrium follows case Eq3 under $(\tau^*(0), 0)$ by case (7-d) in Proposition A.2.

On the other hand, by the same observations, the optimal commission $\tau^*(0)$ is either equal to the left boundary or lies in the interior of the interval where the structure of the equilibrium follows case Eq4. Note that

$$\bar{\tau}^* \le 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma} w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma}) b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}} - 1) p_0},$$

is equivalent to

$$b_0 \le \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma/\underline{\eta}} \Big(1 - \frac{(2 + \frac{\beta\gamma}{1-\beta})(1-q_0) - (\frac{1}{\underline{\eta}} - 1) p_0}{\frac{1-\beta+\beta\gamma}{2\beta\gamma}(p_0 + w_0) + (1 - \frac{1}{2}\underline{\eta})(1-q_0)} \Big) w_0.$$

Following the same arguments as in case (3), we show that if Expression (44) does not hold, $\tau^*(0)$ is given by Expression (46). In addition, we can determine whether the structure of the equilibrium follows case Eq1 or Eq4 under $\tau^*(0)$ based on inequality (47).

(8) Case (8) follows by Observation 1 and we omit the proof for brevity.

We have completed the characterization of $\tau^*(0)$ as well as the corresponding structure of the equilibrium under $(\tau^*(0), 0)$. □

A.4. Differentiated Commissions

The remainder of this appendix provides a number of results that apply to the case when the platform may set different commissions depending on a provider's label, i.e., $\tau_{\mathcal{U}}$ and $\tau_{\mathcal{H}}$ may be different. We consider the general case when $\tau_{\mathcal{U}}, \tau_{\mathcal{H}} \in \mathbb{R}$. In particular, the platform may find it optimal to compensate customers to transact with providers on the platform, i.e., $p_{\mathcal{U}}, p_{\mathcal{H}}$ may be negative, and subsidize providers to join the platform by choosing $\tau_{\mathcal{U}}, \tau_{\mathcal{H}} > 1$ or $\tau_{\mathcal{U}}, \tau_{\mathcal{H}} < 0$ (so that $(1-\tau_{\mathcal{U}})p_{\mathcal{U}} \ge b_0$ and $(1-\tau_{\mathcal{H}})p_{\mathcal{H}} \ge b_0$ hold).

Lemma A.5 *The platform's revenue maximization problem can be formulated as*

$$\max_{\tau_{\mathcal{U}} \in \mathbb{R}, \tau_{\mathcal{H}} \in \mathbb{R}, \lambda \ge 0} \tau_{\mathcal{U}} p_{\mathcal{U}} \delta_{\mathcal{U}} + \tau_{\mathcal{H}} p_{\mathcal{H}} \delta_{\mathcal{H}}^H$$

$$s.t.$$

$$\textit{(Steady state condition)}\ \delta_{\mathcal{H}}^H = \frac{\beta\gamma}{1-\beta}\Big(1-(1-\beta)\lambda\Big)\delta_{\mathcal{U}}^U,$$

$$\delta_{\mathcal{U}}^H = \beta\gamma\lambda\delta_{\mathcal{U}}^U,\ \textit{and}\ \delta_{\mathcal{U}} + \delta_{\mathcal{H}}^H \le 1$$

$$\textit{(Mass of } \mathcal{U}\textit{-labeled providers)}\ \delta_{\mathcal{U}} = (1+\beta\gamma\lambda\eta)\delta_{\mathcal{U}}^U$$

$$
\begin{aligned}
&\text{(Mass of } \mathcal{H}\text{-labeled providers)}\ \delta_{\mathcal{H}}^{H} = \frac{\beta\gamma}{1-\beta}\Big(1-(1-\beta)\lambda\Big)\delta_{\mathcal{U}}^{U} \\
&\text{(Expected quality of } \mathcal{U}\text{-labeled providers)}\ q_{\mathcal{U}} = \frac{\gamma+\beta\gamma\lambda\eta}{1+\beta\gamma\lambda\eta} \\
&\text{(Price of } \mathcal{U}\text{-labeled providers)}\ p_{\mathcal{U}} = \begin{cases} p_0-(1+\delta_{\mathcal{U}})(q_0-q_{\mathcal{U}}), & \text{if } q_{\mathcal{U}}\le q_0 \\ p_0+(2-\delta_{\mathcal{U}}-\delta_{\mathcal{H}}^{H})(q_{\mathcal{U}}-q_0), & \text{if } q_{\mathcal{U}}> q_0 \end{cases} \\
&\text{(Price of } \mathcal{H}\text{-labeled providers)}\ p_{\mathcal{H}} = \begin{cases} p_0+(2-\delta_{\mathcal{H}}^{H})(1-q_0), & \text{if } q_{\mathcal{U}}\le q_0 \\ p_{\mathcal{U}}+(2-\delta_{\mathcal{H}}^{H})(1-q_{\mathcal{U}}), & \text{if } q_{\mathcal{U}}> q_0 \end{cases} \\
&\text{(Free-entry condition)}\ \frac{\beta\gamma}{1-\beta}\cdot\frac{1-(1-\beta)\lambda}{1+\beta\gamma\lambda\eta}\Big((1-\tau_{\mathcal{H}})p_{\mathcal{H}}-w_0\Big) = \frac{w_0}{\eta}-(1-\tau_{\mathcal{U}})p_{\mathcal{U}}. \\
&\text{(Participation constraint for } \mathcal{H}\text{-labeled providers)}\ (1-\tau_{\mathcal{H}})p_{\mathcal{H}}\ge w_0 \\
&\text{(Financial constraint)}\ (1-\tau_{\mathcal{U}})p_{\mathcal{U}}\ge b_0
\end{aligned}
\tag{56}
$$

Proof. This proof focuses on characterizing the prices of $\mathcal{U}$-labeled and $\mathcal{H}$-labeled providers, as the remaining constraints are straightforward. First of all, we note that $\zeta_{\mathcal{U}} = \delta_{\mathcal{U}}^{U}+\eta\delta_{\mathcal{U}}^{H}$ and $\zeta_{\mathcal{H}}=\delta_{\mathcal{H}}^{H}$, which follow from market clearing. Then, we characterize $p_{\mathcal{U}}$ and $p_{\mathcal{H}}$ based on the following possible equilibrium structures.

1. When $q_{\mathcal{U}}\le q_0$ and a positive mass of customers choosing the outside option, we have

$$\zeta_{\mathcal{U}} = \frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0}-1 \text{ and } \zeta_{\mathcal{H}} = 2-\frac{p_{\mathcal{H}}-p_0}{1-q_0},$$

which follows by Lemma 1. We then obtain

$$p_{\mathcal{U}} = p_0-(1+\delta_{\mathcal{U}}^{U}+\eta\delta_{\mathcal{U}}^{H})(q_0-q_{\mathcal{U}}) \text{ and } p_{\mathcal{H}} = p_0+(2-\delta_{\mathcal{H}}^{H})(q_{\mathcal{H}}-q_0)$$

by the market-clearing conditions.

2. When $q_{\mathcal{U}}\le q_0$ and there are no customers choosing the outside option, we obtain

$$\zeta_{\mathcal{U}} = \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}}-1 \text{ and } \zeta_{\mathcal{H}} = 2-\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}}$$

by Lemma 1. Then, by the market-clearing conditions, we obtain

$$p_{\mathcal{H}} = p_{\mathcal{U}}+(2-\delta_{\mathcal{H}}^{H})(q_{\mathcal{H}}-q_{\mathcal{U}}).$$

In addition, a customer with type $1+\zeta_{\mathcal{U}}$ prefers providers with label $\mathcal{U}$ to the outside option (i.e., $p_{\mathcal{U}}\le p_0-(\zeta_{\mathcal{U}}+1)(q_0-q_{\mathcal{U}})$). Next, we claim that any policy with $p_{\mathcal{U}}< p_0-(\zeta_{\mathcal{U}}+1)(q_0-q_{\mathcal{U}})$ is suboptimal. By Equation (34), it is straightforward to verify that increasing $\tau_{\mathcal{H}}$ and/or $\tau_{\mathcal{U}}$ can increase $p_{\mathcal{U}}$ given that all other equilibrium quantities are fixed. Therefore, if $p_{\mathcal{U}}< p_0-(\zeta_{\mathcal{U}}+1)(q_0-q_{\mathcal{U}})$ holds, the platform can always increase its revenues by increasing either commission. In other words, if the optimal policy leads to $q_{\mathcal{U}}\le q_0$ and all customers transact with providers inside the platform, it must be that $p_{\mathcal{U}}= p_0-(\zeta_{\mathcal{U}}+1)(q_0-q_{\mathcal{U}})$ holds. Therefore, we obtain

$$p_{\mathcal{U}} = p_0-(1+\delta_{\mathcal{U}}^{U}+\eta\delta_{\mathcal{U}}^{H})(q_0-q_{\mathcal{U}}) \text{ and } p_{\mathcal{H}} = p_0+(2-\delta_{\mathcal{H}}^{H})(1-q_0).$$

3. When $q_{\mathcal{U}}\ge q_0$ and there are no customers choosing the outside option, we obtain

$$\zeta_{\mathcal{U}} = \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}}-1 \text{ and } \zeta_{\mathcal{H}} = 2-\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}}$$

by Lemma 1. Then, by the market-clearing conditions, we obtain $p_{\mathcal{H}} = p_{\mathcal{U}} + (2-\delta_{\mathcal{H}}^H)(q_{\mathcal{H}} - q_{\mathcal{U}})$. In addition, a customer with type 1 prefers providers with label $\mathcal{U}$ to the outside option, i.e., $p_{\mathcal{U}} \leq p_0 + q_{\mathcal{U}} - q_0$. Next, we claim that any policy with $p_{\mathcal{U}} < p_0 + q_{\mathcal{U}} - q_0$ is suboptimal. By Equation (34), it is straightforward to verify that increasing $\tau_{\mathcal{H}}$ and/or $\tau_{\mathcal{U}}$ can increase $p_{\mathcal{U}}$ given that all other equilibrium outcomes are fixed. Therefore, if $p_{\mathcal{U}} < p_0 + q_{\mathcal{U}} - q_0$ holds, the platform can always increase its revenues by increasing either commission. In other words, if the optimal policy leads to $q_{\mathcal{U}} \geq q_0$ and all customers transact with providers inside the platform, it must be that $p_{\mathcal{U}} = p_0 + q_{\mathcal{U}} - q_0$ holds. Therefore,

$$p_{\mathcal{H}} = p_{\mathcal{U}} + (2-\delta_{\mathcal{H}}^H)(q_{\mathcal{H}} - q_{\mathcal{U}}) \text{ and } p_{\mathcal{U}} = p_0 + (2-\delta_{\mathcal{U}}^U - \eta\delta_{\mathcal{U}}^H - \delta_{\mathcal{H}}^H)(q_{\mathcal{U}} - q_0),$$

where $\delta_{\mathcal{U}}^U + \eta\delta_{\mathcal{U}}^H + \delta_{\mathcal{H}}^H = 1$.

4. When $q_{\mathcal{U}} \geq q_0$ and a positive mass of customers choose outside option, we obtain

$$\zeta_{\mathcal{U}} = \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{1-q_{\mathcal{U}}} - \frac{p_{\mathcal{U}} - p_0}{q_{\mathcal{U}} - q_0} \text{ and } \zeta_{\mathcal{H}} = 2 - \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{1-q_{\mathcal{U}}}$$

by Lemma 1. By the market-clearing conditions, we obtain

$$p_{\mathcal{H}} = p_{\mathcal{U}} + (2-\delta_{\mathcal{H}}^H)(q_{\mathcal{H}} - q_{\mathcal{U}}) \text{ and } p_{\mathcal{U}} = p_0 + (2-\delta_{\mathcal{U}}^U - \eta\delta_{\mathcal{U}}^H - \delta_{\mathcal{H}}^H)(q_{\mathcal{U}} - q_0).$$

Therefore, we have established that the platform's revenue maximization problem is given by formulation (56). □

Lemma A.6 *Suppose $\gamma < \frac{1-\beta}{\beta}$ holds. Then, the optimal information provision policy with differentiated commissions results in no rationing (i.e., $\eta = 1$) in equilibrium. Moreover, the optimal differentiated commissions under the full-information provision policy (i.e., $\alpha = 0$) also results in no rationing in equilibrium.*

Proof. It suffices to show that if policy $(\tau_{\mathcal{U}}, \tau_{\mathcal{H}}, \lambda)$ induces an equilibrium with $\eta < 1$, we can always propose another policy, denoted by $(\tilde{\tau}_{\mathcal{U}}, \tilde{\tau}_{\mathcal{H}}, \tilde{\lambda})$, which leads to strictly higher revenues.

By Equation (34) and the objective function of Formulation (56), platform revenues can be rewritten as

$$\pi_r \triangleq \delta_{\mathcal{U}}^U \left(\frac{\beta\gamma}{1-\beta}\Big(1-(1-\beta)\lambda\Big)p_{\mathcal{H}} + (1+\beta\gamma\lambda\eta)p_{\mathcal{U}} - \Big(\frac{\beta\gamma}{1-\beta} + \frac{1}{\eta}\Big)w_0 \right). \tag{57}$$

Note that Equation (57) is independent of the commissions. Next, we consider the following two cases, $q_{\mathcal{U}} \leq q_0$ and $q_{\mathcal{U}} > q_0$, separately.

1. Suppose $q_{\mathcal{U}} \leq q_0$ and $\eta < 1$ hold under policy $(\tau_{\mathcal{U}}, \tau_{\mathcal{H}}, \lambda)$. Substituting the characterizations of $p_{\mathcal{U}}$ and $p_{\mathcal{H}}$ from Lemma A.5 in Equation (57), we obtain

$$\begin{aligned}\pi_r = \delta_{\mathcal{U}}^U \Bigg(&\Big(\frac{1-\beta+\beta\gamma}{1-\beta} - \beta\gamma\lambda(1-\eta)\Big)p_0 - \Big(q_0 - \gamma - \beta\gamma\lambda\eta(1-q_0)\Big) \\ &+ \frac{2\beta\gamma}{1-\beta}\Big(1-(1-\beta)\lambda\Big)(1-q_0) - \Big(\frac{\beta\gamma}{1-\beta} + \frac{1}{\eta}\Big)w_0 \\ &- \Big(\frac{\beta\gamma}{1-\beta}\Big(1-(1-\beta)\lambda\Big)\Big)^2(1-q_0)\delta_{\mathcal{U}}^U - (1+\beta\gamma\lambda\eta)\Big(q_0 - \gamma - \beta\gamma\lambda\eta(1-q_0)\Big)\delta_{\mathcal{U}}^U \Bigg).\end{aligned} \tag{58}$$

By Equation (58), it immediately follows that π_r is fully determined by λ, η, and $\delta_{\mathcal{U}}^U$ (i.e., it is independent of $\tau_{\mathcal{U}}$ and $\tau_{\mathcal{H}}$). Next, we show that $\frac{\partial \pi_r}{\partial \eta} > 0$. In particular, $\frac{\partial \pi_r}{\partial \eta}$ has the same sign as

$$\beta\gamma\lambda p_0 + \beta\gamma\lambda(1-q_0) + 2(\beta\gamma\lambda)^2(1-q_0)\eta\delta_{\mathcal{U}}^U + \beta\gamma\lambda(1+\gamma-2q_0)\delta_{\mathcal{U}}^U + \frac{w_0}{\eta^2} > 0. \tag{59}$$

The inequality above holds as $1+\gamma-2q_0 > 0$ (note that $\underline{\eta} < \frac{1}{2}$ and $\gamma < \frac{1-\beta}{\beta}$).

We use $\delta_{\mathcal{U}}^U$, η, and $p_{\mathcal{U}}$ to denote the equilibrium quantities under policy $(\tau_{\mathcal{U}}, \tau_{\mathcal{H}}, \lambda)$. To construct another policy that achieves higher revenues, we let $\tilde{\lambda} = \lambda$, and we choose $\tilde{\tau}_{\mathcal{U}}$ and $\tilde{\tau}_{\mathcal{H}}$ such that the following holds

$$\frac{\beta\gamma}{1-\beta}\frac{1-(1-\beta)\lambda}{1+\beta\gamma\lambda}\Big((1-\tilde{\tau}_{\mathcal{H}})\tilde{p}_{\mathcal{H}} - w_0\Big) = w_0 - (1-\tilde{\tau}_{\mathcal{U}})\tilde{p}_{\mathcal{U}}, \text{ and } (1-\tilde{\tau}_{\mathcal{U}})\tilde{p}_{\mathcal{U}} = (1-\tau_{\mathcal{U}})p_{\mathcal{U}},$$

where

$$\tilde{p}_{\mathcal{H}} = p_0 + \Big(2 - \frac{\beta\gamma}{1-\beta}\Big(1-(1-\beta)\lambda\Big)\delta_{\mathcal{U}}^U\Big)(1-q_0) \text{ and } \tilde{p}_{\mathcal{U}} = p_0 - \delta_{\mathcal{U}}^U\Big(q_0 - \gamma - \beta\gamma\lambda(1-q_0)\Big).$$

Therefore, under policy $(\tilde{\tau}_{\mathcal{U}}, \tilde{\tau}_{\mathcal{H}}, \tilde{\lambda})$, we observe $\tilde{\delta}_{\mathcal{U}}^U = \delta_{\mathcal{U}}^U$, yet $\tilde{\eta} = 1$. In addition, it is straightforward to verify that $(1-\tilde{\tau}_{\mathcal{H}})\tilde{p}_{\mathcal{H}} \geq w_0$ and $w_0 \geq (1-\tilde{\tau}_{\mathcal{U}})\tilde{p}_{\mathcal{U}} = (1-\tau_{\mathcal{U}})p_{\mathcal{U}} \geq b_0$. Lastly, by inequality (59), policy $(\tilde{\tau}_{\mathcal{U}}, \tilde{\tau}_{\mathcal{H}}, \tilde{\lambda})$ strictly outperforms policy $(\tau_{\mathcal{U}}, \tau_{\mathcal{H}}, \lambda)$ with regards to revenues.

2. Suppose $q_{\mathcal{U}} > q_0$ and $\eta < 1$ hold under policy $(\tau_{\mathcal{U}}, \tau_{\mathcal{H}}, \lambda)$. We let $\delta_{\mathcal{U}}^U$ and η denote the equilibrium quantities under the original policy. Then, we construct a policy, $(\tilde{\tau}_{\mathcal{U}}, \tilde{\tau}_{\mathcal{H}}, \tilde{\lambda})$, such that its equilibrium outcomes satisfy $\tilde{\delta}_{\mathcal{U}}^U = \delta_{\mathcal{U}}^U$, $\tilde{\eta} = 1$, $\tilde{q}_{\mathcal{U}} = q_{\mathcal{U}}$, and $\tilde{\lambda} = \lambda\eta$. In addition, we choose $\tilde{\tau}_{\mathcal{U}}$ and $\tilde{\tau}_{\mathcal{H}}$ such that

$$\frac{\beta\gamma}{1-\beta} \cdot \frac{1-(1-\beta)\tilde{\lambda}}{1+\beta\gamma\tilde{\lambda}}\Big((1-\tilde{\tau}_{\mathcal{H}})\tilde{p}_{\mathcal{H}} - w_0\Big) = w_0 - (1-\tilde{\tau}_{\mathcal{U}})\tilde{p}_{\mathcal{U}}, \text{ and } (1-\tilde{\tau}_{\mathcal{U}})\tilde{p}_{\mathcal{U}} = (1-\tau_{\mathcal{U}})p_{\mathcal{U}},$$

where

$$\tilde{p}_{\mathcal{U}} = p_0 + \Big(2 - \frac{1-\beta+\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^U\Big)(q_{\mathcal{U}} - q_0) \text{ and } \tilde{p}_{\mathcal{H}} = \tilde{p}_{\mathcal{U}} + \Big(2 - \frac{\beta\gamma}{1-\beta}\Big(1-(1-\beta)\tilde{\lambda}\Big)\delta_{\mathcal{U}}^U\Big)(1-q_{\mathcal{U}}).$$

Next, we claim that the proposed policy leads to higher revenues than the original policy. To simplify the notation, we let $u \triangleq \lambda\eta = \tilde{\lambda}\tilde{\eta} = \tilde{\lambda}$, which is the same under both policies. Then, given u, we rewrite the expression for revenues as follows

$$\begin{aligned}\pi_r(\lambda) =& \delta_{\mathcal{U}}^U\Bigg(\Big(\frac{1-\beta+\beta\gamma}{1-\beta} - \beta\gamma\lambda + \beta\gamma u\Big)\Big(p_0 + \Big(2 - \Big(\frac{1-\beta+\beta\gamma}{1-\beta} - \beta\gamma\lambda + \beta\gamma u\Big)\delta_{\mathcal{U}}^U\Big)(q_{\mathcal{U}} - q_0)\Big) \\ &+ \frac{2\beta\gamma}{1-\beta}\Big(1-(1-\beta)\lambda\Big)(1-q_{\mathcal{U}}) - \Big(\frac{\beta\gamma}{1-\beta}\Big(1-(1-\beta)\lambda\Big)\Big)^2(1-q_{\mathcal{U}})\delta_{\mathcal{U}}^U - \Big(\frac{\beta\gamma}{1-\beta} + \frac{\lambda}{u}\Big)w_0\Bigg).\end{aligned} \tag{60}$$

To establish that the proposed policy leads to higher revenues than the original policy, it suffices to show that $\pi_r(\lambda)$ is decreasing in λ given u and $\delta_{\mathcal{U}}^U$, which follows from straightforward algebra. Since $\tilde{\lambda} < \lambda$, the proposed policy without rationing (i.e., $\tilde{\eta} = 1$) results in higher revenues than the original policy with rationing ($\eta < 1$). Therefore, any policy with $q_{\mathcal{U}} \geq q_0$ and $\eta < 1$ is suboptimal.

In sum, any policy that induces an equilibrium with $\eta < 1$ is suboptimal, which conclude the proof.[12] □

[12] Noticeably, the proof applies to the special case of finding the optimal full-information provision policy. We simply let $\lambda = 0$ and follow the argument under the case of $q_{\mathcal{U}} < q_0$. As a result, we again conclude that the optimal full-information provision policy results in no rationing either.

Lemma A.7 *Suppose $\gamma < \frac{1-\beta}{\beta}$, $\underline{\eta} < 1$, and α is such that $0 \le \lambda \le \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$. Then, the mass of new providers under the optimal differentiated commissions given α can be characterized as*

$$\delta_{\mathcal{U}}^{U,*}(\lambda) = \min\left(\frac{1}{2}\cdot\frac{\frac{1-\beta+\beta\gamma}{1-\beta}(p_0-w_0)-(q_0-\gamma)+\frac{2\beta\gamma}{1-\beta}(1-q_0)-\beta\gamma\lambda(1-q_0)}{q_0-\gamma+\left(\frac{\beta\gamma}{1-\beta}\right)^2(1-q_0)-\beta\gamma\lambda\left(1+\gamma-2q_0+\frac{2\beta\gamma}{1-\beta}(1-q_0)\right)}, \frac{1-\beta}{1-\beta+\beta\gamma}\right). \tag{61}$$

In addition, $\delta_{\mathcal{U}}^{U,*}(\lambda)$ is increasing in λ.[13]

Proof. When $\gamma < \frac{1-\beta}{\beta}$, by Lemma A.6, we only need to consider the policy with no rationing among providers with label $\mathcal{U}$ (i.e., $\eta = 1$). For $\lambda \in [0, \frac{q_0-\gamma}{\beta\gamma(1-q_0)}]$, we have $q_{\mathcal{U}} \le q_0$. Then, by Equation (58), the platform's revenues can be simplified as

$$\begin{aligned}
\pi_r(\delta_{\mathcal{U}}^U, \lambda) &= \delta_{\mathcal{U}}^U\Bigg(\frac{1-\beta+\beta\gamma}{1-\beta}(p_0-w_0) - \Big(q_0-\gamma-\beta\gamma\lambda(1-q_0)\Big) + \frac{2\beta\gamma}{1-\beta}\Big(1-(1-\beta)\lambda\Big)(1-q_0) \\
&\quad - \left((1+\beta\gamma\lambda)\Big(q_0-\gamma-\beta\gamma\lambda(1-q_0)\Big) + \left(\frac{\beta\gamma}{1-\beta}-\beta\gamma\lambda\right)^2(1-q_0)\right)\delta_{\mathcal{U}}^U\Bigg) \\
&= \delta_{\mathcal{U}}^U\Bigg(\frac{1-\beta+\beta\gamma}{1-\beta}(p_0-w_0) - (q_0-\gamma) + \frac{2\beta\gamma}{1-\beta}(1-q_0) - \beta\gamma\lambda(1-q_0) \\
&\quad - \left(q_0-\gamma+\left(\frac{\beta\gamma}{1-\beta}\right)^2(1-q_0) - \beta\gamma\lambda\left(1+\gamma-2q_0+\frac{2\beta\gamma}{1-\beta}(1-q_0)\right)\right)\delta_{\mathcal{U}}^U\Bigg).
\end{aligned} \tag{62}$$

Note that given λ, Equation (62) is quadratic in $\delta_{\mathcal{U}}^U$; hence, it is maximized by $\delta_{\mathcal{U}}^{U,*}(\lambda)$, given in Expression (61).

To show that $\delta_{\mathcal{U}}^{U,*}(\lambda)$ is increasing in λ, it suffices to show that it holds when $\delta_{\mathcal{U}}^{U,*}(\lambda) < \frac{1-\beta}{1-\beta+\beta\gamma}$. In this case, we note that $\frac{d\delta_{\mathcal{U}}^{U,*}}{d\lambda}(\lambda)$ has the same sign as the following expression

$$\begin{aligned}
&\left(1+\gamma-2q_0+\frac{2\beta\gamma}{1-\beta}(1-q_0)\right)\left(\frac{1-\beta+\beta\gamma}{1-\beta}(p_0-w_0)-(q_0-\gamma)+\frac{2\beta\gamma}{1-\beta}(1-q_0)\right) \\
&\quad -(1-q_0)\left(q_0-\gamma+\left(\frac{\beta\gamma}{1-\beta}\right)^2(1-q_0)\right).
\end{aligned}$$

Straightforward algebra implies that when $\underline{\eta} < 1$, the quantity above is positive. □

Lemma A.8 *Suppose $\gamma < \frac{1-\beta}{\beta}$ and $\underline{\eta} < 1$. Then, all customers transact with providers inside the platform under the optimal differentiated commissions when $\lambda = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$. That is,*

$$\delta_{\mathcal{U}}^{U,*}\left(\frac{q_0-\gamma}{\beta\gamma(1-q_0)}\right) = \frac{1-\beta}{1-\beta+\beta\gamma},$$

where $\delta_{\mathcal{U}}^{U,*}(\lambda)$ is given by Expression (61).

Proof. Given the expression for $\delta_{\mathcal{U}}^{U,*}(\lambda)$ given in Expression (61), it is equivalent to show that

$$\frac{1}{2}\cdot\frac{\frac{1-\beta+\beta\gamma}{1-\beta}(p_0-w_0)-(q_0-\gamma)+\frac{2\beta\gamma}{1-\beta}(1-q_0)-(q_0-\gamma)}{q_0-\gamma+\left(\frac{\beta\gamma}{1-\beta}\right)^2(1-q_0)-\frac{q_0-\gamma}{1-q_0}\left(1+\gamma-2q_0+\frac{2\beta\gamma}{1-\beta}(1-q_0)\right)} \ge \frac{1-\beta}{1-\beta+\beta\gamma}.$$

[13] The superscript * represents that the corresponding quantity is obtained under the optimal differentiated commissions given α.

The inequality above can be rewritten as

$$\begin{aligned}
&\frac{1-\beta+\beta\gamma}{1-\beta}(p_0-w_0)+\frac{2\beta\gamma}{1-\beta}(1-q_0)-2(q_0-\gamma)\\
&\geq \frac{2(1-\beta)}{1-\beta+\beta\gamma}\left(q_0-\gamma+\left(\frac{\beta\gamma}{1-\beta}\right)^2(1-q_0)-\frac{q_0-\gamma}{1-q_0}\left(\left(1+\frac{2\beta\gamma}{1-\beta}\right)(1-q_0)-(q_0-\gamma)\right)\right)\\
&= \frac{2(1-\beta)}{1-\beta+\beta\gamma}\left(\left(\frac{\beta\gamma}{1-\beta}\right)^2(1-q_0)-\frac{2\beta\gamma}{1-\beta}(q_0-\gamma)+\frac{(q_0-\gamma)^2}{1-q_0}\right)\\
&= \frac{2(1-\beta)}{1-\beta+\beta\gamma}\cdot\frac{1}{1-q_0}\left(\frac{\beta\gamma}{1-\beta}(1-q_0)-(q_0-\gamma)\right)^2.
\end{aligned}$$

Note that $p_0-w_0>0$. To show that the preceding inequality holds, it suffices to show that

$$2>\frac{2(1-\beta)}{1-\beta+\beta\gamma}\cdot\frac{1}{1-q_0}\left(\frac{\beta\gamma}{1-\beta}(1-q_0)-(q_0-\gamma)\right)=\frac{2\beta\gamma}{1-\beta+\beta\gamma}\left(1-\frac{1-\beta}{\beta\gamma}\frac{q_0-\gamma}{1-q_0}\right),$$

which follows, since $\gamma<\frac{1-\beta}{\beta}$ and $q_0>\gamma$. □

Lemma A.9 *Suppose $\gamma<\frac{1-\beta}{\beta}$, $\underline{\eta}<1$, and α is such that $0\leq\lambda\leq\frac{q_0-\gamma}{\beta\gamma(1-q_0)}$. Then, if $\delta_{\mathcal{U}}^{U,*}(\lambda)<\frac{1-\beta}{1-\beta+\beta\gamma}$, where $\delta_{\mathcal{U}}^{U,*}(\lambda)$ is given by Expression (61), the platform's revenues under the optimal differentiated commissions given λ are convex in λ. Otherwise, the platform's revenues under the optimal differentiated commissions given λ are linearly increasing in λ.*

Proof. The proof of the lemma follows from straightforward algebra using Expressions (61) and (62). □

Proposition A.4 *Suppose $\gamma<\frac{1-\beta}{\beta}$ and $\underline{\eta}<1$. Then, the optimal policy satisfies*

$$\lambda^*=\begin{cases}\frac{q_0-\gamma}{\beta\gamma(1-q_0}, & \text{if } \delta_{\mathcal{U}}^{U,*}(0)=\frac{1-\beta}{1-\beta+\beta\gamma}\\ \frac{q_0-\gamma}{\beta\gamma(1-q_0}, & \text{if } \delta_{\mathcal{U}}^{U,*}(0)<\frac{1-\beta}{1-\beta+\beta\gamma} \text{ and } \frac{q_0-\gamma}{\beta\gamma(1-q_0}\geq\bar{\lambda}\ ,\\ 0, & \text{otherwise}\end{cases} \tag{63}$$

where $\delta_{\mathcal{U}}^{U,*}(\lambda)$ is given by Expression (61), and

$$\bar{\lambda}=\frac{\left(\left(\frac{1-\beta+\beta\gamma}{1-\beta}(p_0-w_0)-(q_0-\gamma)+\frac{2\beta\gamma}{1-\beta}(1-q_0)\right)-2\left(\frac{1-\beta}{1-\beta+\beta\gamma}\right)\left(q_0-\gamma+\left(\frac{\beta\gamma}{1-\beta}\right)^2(1-q_0)\right)\right)^2}{4\left(q_0-\gamma+\left(\frac{\beta\gamma}{1-\beta}\right)^2(1-q_0)\right)\left(\frac{1-\beta}{1-\beta+\beta\gamma}\right)\left(\frac{1-\beta}{1-\beta+\beta\gamma}\left(\beta\gamma\left(1+\gamma-2q_0+\frac{2\beta\gamma}{1-\beta}(1-q_0)\right)\right)-\beta\gamma(1-q_0)\right)}.$$

Moreover, the optimal differentiated commissions are

$$\tau_{\mathcal{U}}^*(\lambda^*)=Median\left\{1-\frac{w_0}{p_{\mathcal{U}}^*(\lambda^*)},1-\frac{b_0}{p_{\mathcal{U}}^*(\lambda^*)},0\right\}. \tag{64}$$

$$\text{and } \tau_{\mathcal{H}}^*(\lambda^*)=1-\frac{\frac{1-\beta+\beta\gamma}{1-\beta}w_0-(1+\beta\gamma\lambda^*)\left(1-\tau_{\mathcal{U}}^*(\lambda^*)\right)p_{\mathcal{U}}^*(\lambda^*)}{\frac{\beta\gamma}{1-\beta}\left(1-(1-\beta)\lambda^*\right)p_{\mathcal{H}}^*(\lambda^*)}, \tag{65}$$

where

$$\begin{aligned}
p_{\mathcal{U}}^*(\lambda)&=p_0-\left(1+(1+\beta\gamma\lambda)\delta_{\mathcal{U}}^{U,*}(\lambda)\right)\left(q_0-\frac{\gamma+\beta\gamma\lambda}{1+\beta\gamma\lambda}\right),\\
\text{and } p_{\mathcal{H}}^*(\lambda)&=p_0+\left(2-\frac{\beta\gamma}{1-\beta}\left(1-(1-\beta)\lambda\right)\delta_{\mathcal{U}}^{U,*}(\lambda)\right)(1-q_0).
\end{aligned}$$

Proof. First, the optimality of λ^* as stated above follows directly from Lemmas A.10 and A.11, which are specified as follows.

Lemma A.10 *Suppose $\gamma < \frac{1-\beta}{\beta}$, $\underline{\eta} < 1$, and α is such that $0 \le \lambda \le \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$. Then, the informational delay of the optimal policy, λ^*, is given by* (63).

Proof. Note that $q_{\mathcal{U}} \le q_0$ when $0 \le \lambda \le \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$, and we only need to consider the policies such that $\eta = 1$ by Lemma A.6 (under $\gamma < \frac{1-\beta}{\beta}$).

First, note that the optimal informational delay satisfies $\lambda^* = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$ if $\delta_{\mathcal{U}}^{U,*}(0) = \frac{1-\beta}{1-\beta+\beta\gamma}$. This holds because the platform's revenues are linearly increasing in λ under $\gamma < \frac{1-\beta}{\beta}$ (Lemma A.9).

Next, we consider the converse case, i.e., $\delta_{\mathcal{U}}^{U,*}(0) < \frac{1-\beta}{1-\beta+\beta\gamma}$. Platform's revenues at $\lambda = 0$ (i.e., $\alpha = 0$) are given by

$$\pi_r(0) = \frac{1}{4} \cdot \frac{\left(\frac{1-\beta+\beta\gamma}{1-\beta}(p_0 - w_0) - (q_0 - \gamma) + \frac{2\beta\gamma}{1-\beta}(1-q_0)\right)^2}{q_0 - \gamma + \left(\frac{\beta\gamma}{1-\beta}\right)^2 (1-q_0)}.$$

Similarly, platform revenues at $\lambda = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$ are

$$\begin{aligned}\pi_r\Big(\frac{q_0-\gamma}{\beta\gamma(1-q_0)}\Big) = \Bigg[&\frac{1-\beta+\beta\gamma}{1-\beta}(p_0 - w_0) - (q_0-\gamma) + \frac{2\beta\gamma}{1-\beta}(1-q_0) - \left(q_0 - \gamma + \Big(\frac{\beta\gamma}{1-\beta}\Big)^2(1-q_0)\right)\frac{1-\beta}{1-\beta+\beta\gamma} \\ &+ \left(\Big(\beta\gamma\big(1+\gamma-2q_0 + \frac{2\beta\gamma}{1-\beta}(1-q_0)\big)\Big)\frac{1-\beta}{1-\beta+\beta\gamma} - \beta\gamma(1-q_0)\right)\frac{q_0-\gamma}{\beta\gamma(1-q_0)}\Bigg]\frac{1-\beta}{1-\beta+\beta\gamma}.\end{aligned}$$

It is straightforward to verify that $\pi_r\big(\frac{q_0-\gamma}{\beta\gamma(1-q_0)}\big) \ge \pi_r(0)$ if and only if $\frac{q_0-\gamma}{\beta\gamma(1-\beta)} \ge \bar{\lambda}$. Given the convexity of $\pi_r(\lambda)$ in λ (Lemma A.9), it follows that that the optimal informational delay within $\lambda \in [0, \frac{q_0-\gamma}{\beta\gamma(1-q_0)}]$ is $\lambda = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$. Likewise, if $\frac{q_0-\gamma}{\beta\gamma(1-q_0)} < \bar{\lambda}$, then $\pi_r\big(\frac{q_0-\gamma}{\beta\gamma(1-q_0)}\big) < \pi_r(0)$. Thus, the optimal informational delay within $0 \le \lambda \le \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$ is $\lambda = 0$. ☐

Lemma A.11 *Suppose $\gamma < \frac{1-\beta}{\beta}$, $\underline{\eta} < 1$, and α is such $\lambda \ge \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$. Then, the optimal policy satisfies $\lambda^* = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$.*

Proof. First, we show that given a policy, $(\tau_{\mathcal{U}}, \tau_{\mathcal{H}}, \lambda)$, with $\lambda > \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$, we can find an alternative policy $(\tilde{\tau}_{\mathcal{U}}, \tilde{\tau}_{\mathcal{H}}, \tilde{\lambda})$ with $\frac{q_0-\gamma}{\beta\gamma(1-q_0)} \le \tilde{\lambda} < \lambda$. We use $\delta_{\mathcal{U}}^U$ and η to denote the equilibrium quantities under the original policy, and we use $\tilde{\delta}_{\mathcal{U}}^U$ and $\tilde{\eta}$ to denote the equilibrium quantities under the alternative policy. In particular, we select $\tilde{\tau}_{\mathcal{U}}$ and $\tilde{\tau}_{\mathcal{H}}$ such that $\tilde{\delta}_{\mathcal{U}}^U = \delta_{\mathcal{U}}^U$, $\tilde{\eta} = 1$,

$$\frac{\beta\gamma}{1-\beta} \cdot \frac{1-(1-\beta)\tilde{\lambda}}{1+\beta\gamma\tilde{\lambda}}\Big((1-\tilde{\tau}_{\mathcal{H}})\tilde{p}_{\mathcal{H}} - w_0\Big) = w_0 - (1-\tilde{\tau}_{\mathcal{U}})\tilde{p}_{\mathcal{U}}, \text{ and } (1-\tilde{\tau}_{\mathcal{U}})\tilde{p}_{\mathcal{U}} = (1-\tau_{\mathcal{U}})p_{\mathcal{U}},$$

where

$$\tilde{p}_{\mathcal{U}} = p_0 + \Big(2 - \frac{1-\beta+\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^U\Big)(\tilde{q}_{\mathcal{U}} - q_0) \text{ and } \tilde{p}_{\mathcal{H}} = \tilde{p}_{\mathcal{U}} + \Big(2 - \frac{\beta\gamma}{1-\beta}\Big(1-(1-\beta)\tilde{\lambda}\Big)\delta_{\mathcal{U}}^U\Big)(1-\tilde{q}_{\mathcal{U}}).$$

It is straightforward to show that such $\tilde{\tau}_{\mathcal{U}}$ and $\tilde{\tau}_{\mathcal{H}}$ exist.

Then, we show that the alternative policy results in higher platform revenues compared to the original policy. It suffices to show that given $\eta = 1$ and $\delta_{\mathcal{U}}^U \in [0, \frac{1-\beta}{1-\beta+\beta\gamma}]$, we have $\frac{\partial \pi_r}{\partial \lambda}(\delta_{\mathcal{U}}^U, \lambda) < 0$, where $\pi_r(\delta_{\mathcal{U}}^U, \lambda)$ is given by Equation (60). In particular, we have

$$\pi_r(\delta_{\mathcal{U}}^U, \lambda) = \delta_{\mathcal{U}}^U \Bigg(\frac{1-\beta+\beta\gamma}{1-\beta}(p_0 - w_0) + \frac{1-\beta+\beta\gamma}{1-\beta}\Big(2 - \frac{1-\beta+\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^U\Big)\frac{\beta\gamma\lambda(1-q_0)-(q_0-\gamma)}{1+\beta\gamma\lambda}$$
$$+ \frac{\beta\gamma}{1-\beta}\Big(1-(1-\beta)\lambda\Big)\Big(2 - \frac{\beta\gamma}{1-\beta}\Big(1-(1-\beta)\lambda\Big)\delta_{\mathcal{U}}^U\Big)\frac{1-\gamma}{1+\beta\gamma\lambda}\Bigg).$$

We obtain that derivative $\frac{\partial \pi_r}{\partial \lambda}(\delta_{\mathcal{U}}^U, \lambda)$ has the same sign as

$$-\frac{1-\beta+\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^U + \frac{\beta\gamma}{1-\beta}\Big(1-(1-\beta)\lambda\Big)\delta_{\mathcal{U}}^U,$$

which is negative as $\frac{1-\beta+\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^U$ is the mass of all providers on the platform and $\frac{\beta\gamma}{1-\beta}\Big(1-(1-\beta)\lambda\Big)\delta_{\mathcal{U}}^U$ is mass of providers with label $\mathcal{H}$. Note that the proposed policy is such that $\tilde{\lambda} < \lambda$, so it results in higher platform revenues than the original one.

Therefore, when α satisfies $\lambda \geq \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$, platform revenues are decreasing in λ, and, hence, the optimal policy satisfies $\lambda^* = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$. □

Next, we characterize the optimal commissions (i.e., $\tau_{\mathcal{U}}^*(\lambda^*)$ and $\tau_{\mathcal{H}}^*(\lambda^*)$). Note that $\tau_{\mathcal{U}}^*(\lambda^*)$ and $\tau_{\mathcal{H}}(\lambda^*)$ may not be unique. An optimal pair of commissions can be determined as follows: First, $\tau_{\mathcal{U}}^*(\lambda)$ is determined by Expression (64). It follows that under such $\tau_{\mathcal{U}}^*(\lambda)$, the financial constraint of $\mathcal{U}$-labeled providers holds (i.e., $\big(1-\tau_{\mathcal{U}}^*(\lambda)\big)p_{\mathcal{U}}^*(\lambda) \geq b_0$). Second, given $\tau_{\mathcal{U}}^*(\lambda)$, we determine the corresponding $\tau_{\mathcal{H}}^*(\lambda)$ using Equation (34), where $p_{\mathcal{U}}^*(\lambda)$ and $p_{\mathcal{H}}^*(\lambda)$ are determined by Lemma A.5. □

Corollary A.1 *Suppose $\gamma < \frac{1-\beta}{\beta}$ and $\underline{\eta} < 1$. Then, the optimal differentiated commissions under the full-information provision policy (i.e., $\alpha = 0$), $\tau_{\mathcal{U}}^*(0)$ and $\tau_{\mathcal{H}}^*(0)$, can be characterized by Expression* (64) *and Expression* (65)*, where*

$$p_{\mathcal{U}}^*(0) = p_0 - \Big(1+\delta_{\mathcal{U}}^{U,*}(0)\Big)(q_0-\gamma),\ p_{\mathcal{H}}^*(0) = p_0 + \Big(2 - \frac{\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^{U,*}(0)\Big)(1-q_0),$$
$$\textit{and } \delta_{\mathcal{U}}^{U,*}(0) = \min\left(\frac{\frac{1-\beta+\beta\gamma}{1-\beta}(p_0-w_0) - (q_0-\gamma) + \frac{2\beta\gamma}{1-\beta}(1-q_0)}{2\Big(q_0-\gamma+\Big(\frac{\beta\gamma}{1-\beta}\Big)^2(1-q_0)\Big)}, \frac{1-\beta}{1-\beta+\beta\gamma}\right). \tag{66}$$

In addition,

$$\tau_{\mathcal{U}}^*(0)p_{\mathcal{U}}^*(0) < \tau_{\mathcal{H}}^*(0)p_{\mathcal{H}}^*(0). \tag{67}$$

Proof. First, by Lemma A.7 (under $\gamma < \frac{1-\beta}{\beta}$) and $\alpha = 0$, it is straightforward to verify that $\delta_{\mathcal{U}}^{U,*}(0)$ is given by Expression (66). Next, the optimal commissions are given by Proposition A.4. In addition, the expressions for $p_{\mathcal{U}}^*(0)$ and $p_{\mathcal{H}}^*(0)$ follow from Lemma A.5.

Next, we show that inequality (67) holds. By Expressions (64) and (65), we obtain

$$\tau_{\mathcal{U}}^*(0)p_{\mathcal{U}}^*(0) = p_{\mathcal{U}}^*(0) - w_0 + \text{Median}\Big\{0, w_0 - b_0, w_0 - p_{\mathcal{U}}^*(0)\Big\},$$
$$\text{and } \tau_{\mathcal{H}}^*(0)p_{\mathcal{H}}^*(0) = p_{\mathcal{H}}^*(0) - \frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 + \frac{1-\beta}{\beta\gamma}p_{\mathcal{U}}^*(0) - \frac{1-\beta}{\beta\gamma}\tau_{\mathcal{U}}^*(0)p_{\mathcal{U}}^*(0).$$

Given the expressions above, inequality (67) becomes

$$\frac{1-\beta+\beta\gamma}{\beta\gamma}\text{Median}\Big\{0, w_0-b_0, w_0-p_{\mathcal{U}}^*(0)\Big\} < p_{\mathcal{H}}^*(0)-p_{\mathcal{U}}^*(0). \tag{68}$$

If $p_{\mathcal{U}}^*(0)\geq w_0$, then we claim that inequality (68) holds as its left-hand side is zero and its right-hand side is positive. If $p_{\mathcal{U}}^*(0)<w_0$, then the left-hand side of inequality (68) is no more than $\frac{1-\beta+\beta\gamma}{\beta\gamma}\Big(w_0-p_{\mathcal{U}}^*(0)\Big)$. So, it suffices to show that

$$p_{\mathcal{H}}^*(0)-p_{\mathcal{U}}^*(0) > \frac{1-\beta+\beta\gamma}{\beta\gamma}\Big(w_0-p_{\mathcal{U}}^*(0)\Big) = \frac{1-\beta+\beta\gamma}{\beta\gamma}\Big(w_0-p_0+\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)\Big).$$

Note that $w_0\leq p_0$. To verify the inequality above, it suffices to show that

$$p_{\mathcal{H}}^*(0)-p_{\mathcal{U}}^*(0) = \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(1-\gamma) > \frac{2(1-\beta)+\beta\gamma}{\beta\gamma}(q_0-\gamma).$$

This holds since it is equivalent to $\underline{\eta}<1$. □

Lemma A.12 *Suppose $\gamma<\frac{1-\beta}{\beta}$ and $\underline{\eta}<\frac{1}{2}$. Then, all customers transact with providers inside the platform under the optimal commissions and a full-information provision policy. That is, $\delta_{\mathcal{U}}^{U,*}(0)=\frac{1-\beta}{1-\beta+\beta\gamma}$, where $\delta_{\mathcal{U}}^{U,*}(0)$ is given by Expression (66).*

Proof. By Expression (66), note that $\delta_{\mathcal{U}}^{U,*}(0)=\frac{1-\beta}{1-\beta+\beta\gamma}$ is equivalent to

$$\frac{\frac{1-\beta+\beta\gamma}{1-\beta}(p_0-w_0)-(q_0-\gamma)+\frac{2\beta\gamma}{1-\beta}(1-q_0)}{2\Big(q_0-\gamma+\Big(\frac{\beta\gamma}{1-\beta}\Big)^2(1-q_0)\Big)} \geq \frac{1-\beta}{1-\beta+\beta\gamma}.$$

It is straightforward to verify that the inequality above is equivalent to the following inequality

$$\frac{1-\beta+\beta\gamma}{1-\beta}\frac{p_0-w_0}{q_0-\gamma}+\frac{2(1-\beta)}{1-\beta+\beta\gamma}\frac{1}{\underline{\eta}} \geq 1+\frac{2(1-\beta)}{1-\beta+\beta\gamma}.$$

Note that $p_0\geq w_0$. Therefore, to show that the inequality above holds, it suffices to show that $\underline{\eta}\leq\frac{2(1-\beta)}{3(1-\beta)+\beta\gamma}$. The preceding inequality follows by the assumptions that $\gamma<\frac{1-\beta}{\beta}$ and $\underline{\eta}<\frac{1}{2}$. □

Corollary A.2 *Suppose $\gamma<\frac{1-\beta}{\beta}$, $\underline{\eta}<\frac{1}{2}$. Then, the platform generates more revenues from each $\mathcal{U}$-labeled provider and from each $\mathcal{H}$-labeled provider under the optimal policy than those generated under the optimal commissions and a full-information provision policy. In particular,*

$$\tau_{\mathcal{U}}^*(0)p_{\mathcal{U}}^*(0) < \tau_{\mathcal{U}}^*(\lambda^*)p_{\mathcal{U}}^*(\lambda^*), \text{ and} \tag{69}$$

$$\tau_{\mathcal{H}}^*(0)p_{\mathcal{H}}^*(0) < \tau_{\mathcal{H}}^*(\lambda^*)p_{\mathcal{H}}^*(\lambda^*), \tag{70}$$

where $\tau_{\mathcal{U}}^*(\lambda)$, $p_{\mathcal{U}}^*(\lambda)$, $\tau_{\mathcal{H}}^*(\lambda)$, and $p_{\mathcal{H}}^*(\lambda)$ are given by Proposition A.4.

Proof. Under the conditions of the corollary, we first note that the optimal informational delay α^* satisfies $\lambda^*=\frac{q_0-\gamma}{\beta\gamma(1-\beta)}$ by Proposition A.4 and Lemma A.12. In addition, we note that $p_{\mathcal{U}}^*(\lambda^*)=p_0$ by Lemma A.5.

First, we establish inequality (69). Note that $p_{\mathcal{U}}^*(0)<p_0=p_{\mathcal{U}}^*(\lambda^*)$ by Corollary A.1 and Lemma A.5. Then, by Expression (64), we obtain

$$\tau_{\mathcal{U}}^*(\lambda)p_{\mathcal{U}}^*(\lambda)=\text{Median}\Big\{p_{\mathcal{U}}^*(\lambda^*)-w_0, p_{\mathcal{U}}^*(\lambda^*)-b_0, 0\Big\},$$

which implies that $\tau^*_{\mathcal{U}}(0)p^*_{\mathcal{U}}(0) < p_0 - w_0 = \tau^*_{\mathcal{U}}(\lambda^*)p^*_{\mathcal{U}}(\lambda^*)$.

Next, we establish inequality (70). Since $\tau^*_{\mathcal{H}}(\lambda)p^*_{\mathcal{H}}(\lambda^*) = p^*_{\mathcal{H}}(\lambda^*) - w_0$, it is equivalent to show that

$$\tau^*_{\mathcal{H}}(0)p^*_{\mathcal{H}}(0) = p^*_{\mathcal{H}}(0) - \frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 + \frac{1-\beta}{\beta\gamma}\Big(1-\tau^*_{\mathcal{U}}(0)\Big)p^*_{\mathcal{U}}(0) \le p^*_{\mathcal{H}}(\lambda^*) - w_0.$$

Note that $p^*_{\mathcal{H}}(0) < p^*_{\mathcal{H}}(\lambda^*)$ because of $\delta^{H*}_{\mathcal{H}}(0) > \delta^{H*}_{\mathcal{H}}(\lambda^*)$ and Lemma A.5. So it suffices to show that

$$w_0 \le \frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 - \frac{1-\beta}{\beta\gamma}\Big(1-\tau^*_{\mathcal{U}}(0)\Big)p^*_{\mathcal{U}}(0).$$

Note that the inequality above is equivalent to $\big(1-\tau^*_{\mathcal{U}}(0)\big)p^*_{\mathcal{U}}(0) \le w_0$, which holds because of Expression (64) and $b_0 \le w_0$. □

B. Proofs for Sections 4 and 5

Proof of Lemma 1

First, we note that the equilibrium involves three types of *threshold* customers who are indifferent between two options. Specifically,

(i) Let $\theta_{\mathcal{U}0}$ denote the customer, who is indifferent between providers with label $\mathcal{U}$ and the outside option. By definition,

$$\theta_{\mathcal{U}0}q_{\mathcal{U}}(\alpha) - p_{\mathcal{U}} = \theta_{\mathcal{U}0}q_0 - p_0,$$

which implies

$$\theta_{\mathcal{U}0} = \frac{p_{\mathcal{U}} - p_0}{q_{\mathcal{U}}(\alpha) - q_0}.$$

Next, note that if $q_{\mathcal{U}}(\alpha) < q_0$, then customers with $\theta < \theta_{\mathcal{U}0}$ prefer $\mathcal{U}$-labeled providers to the outside option, whereas those with $\theta > \theta_{\mathcal{U}0}$ prefer the outside option to providers with label $\mathcal{U}$. On the other hand, if $q_{\mathcal{U}}(\alpha) \ge q_0$, customers with $\theta > \theta_{\mathcal{U}0}$ prefer $\mathcal{U}$-labeled providers to the outside option, whereas those with $\theta < \theta_{\mathcal{U}0}$ prefer the outside option to providers with label $\mathcal{U}$.

(ii) Let $\theta_{\mathcal{H}0}$ denote the customer, who is indifferent between providers with label $\mathcal{H}$ and the outside option. That is,

$$\theta_{\mathcal{H}0} - p_{\mathcal{H}} = \theta_{\mathcal{H}0}q_0 - p_0,$$

which implies

$$\theta_{\mathcal{H}0} = \frac{p_{\mathcal{H}} - p_0}{1 - q_0}.$$

Then, customers with $\theta < \theta_{\mathcal{H}0}$ prefer the outside option to providers with label $\mathcal{H}$, whereas those with $\theta > \theta_{\mathcal{H}0}$ prefer $\mathcal{H}$-labeled providers to the outside option.

(iii) Let $\theta_{\mathcal{H}\mathcal{U}}$ denote the customer, who is indifferent between providers with label $\mathcal{U}$ and providers with label $\mathcal{H}$. That is,

$$\theta_{\mathcal{H}\mathcal{U}} - p_{\mathcal{H}} = \theta_{\mathcal{H}\mathcal{U}}q_{\mathcal{U}} - p_{\mathcal{U}},$$

which implies

$$\theta_{\mathcal{H}\mathcal{U}} = \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{1 - q_{\mathcal{U}}(\alpha)}.$$

Then, customers with $\theta < \theta_{\mathcal{H}\mathcal{U}}$ prefer $\mathcal{U}$-labeled providers to $\mathcal{H}$-labeled providers, whereas customers with $\theta > \theta_{\mathcal{H}\mathcal{U}}$ prefer providers with label $\mathcal{U}$ to providers with label $\mathcal{H}$.

Based on this characterization of customers who are indifferent between different providers, we proceed to characterize the structure of the equilibrium under $q_{\mathcal{U}}(\alpha) < q_0$ and $q_{\mathcal{U}}(\alpha) \geq q_0$, separately. First, we characterize the equilibrium under $q_{\mathcal{U}}(\alpha) < q_0$. It is straightforward to verify that either $\theta_{\mathcal{U}0} < \theta_{\mathcal{H}\mathcal{U}} < \theta_{\mathcal{H}0}$ or $\theta_{\mathcal{H}0} \leq \theta_{\mathcal{H}\mathcal{U}} \leq \theta_{\mathcal{U}0}$.

(a) Suppose $\theta_{\mathcal{U}0} < \theta_{\mathcal{H}\mathcal{U}} < \theta_{\mathcal{H}0}$ holds in equilibrium. Then, customers with $\theta < \theta_{\mathcal{U}0}$ choose $\mathcal{U}$-labeled providers given that $\theta < \theta_{\mathcal{U}0}$ and $\theta < \theta_{\mathcal{H}\mathcal{U}}$. Using a similar argument, it follows that customers with $\theta \geq \theta_{\mathcal{H}0}$ choose providers with label $\mathcal{H}$. Finally, customers with types in interval $[\theta_{\mathcal{U}0}, \theta_{\mathcal{H}0}]$ take the outside option. Thus, $\zeta_{\mathcal{U}}$ and $\zeta_{\mathcal{H}}$ are given by

$$\zeta_{\mathcal{U}} = \ell\Big((-\infty, \theta_{\mathcal{U}0}] \cap [1,2]\Big) \text{ and } \zeta_{\mathcal{H}} = \ell\Big([\theta_{\mathcal{H}0}, +\infty) \cap [1,2]\Big),$$

where $\ell(\cdot)$ measures the length of a given interval. Therefore, $\zeta_{\mathcal{U}} > 0$ if and only if $\theta_{\mathcal{U}0} > 1$, and $\zeta_{\mathcal{H}} > 0$ if and only if $\theta_{\mathcal{H}0} < 2$. By Equation (13), i.e., the time-invariant condition, and Equation (14), i.e., the market-clearing condition, we conclude that $\zeta_{\mathcal{U}}$ and $\zeta_{\mathcal{H}}$ are either both positive or both zero. Therefore, if $\theta_{\mathcal{U}0} > 1$ and $\theta_{\mathcal{H}0} < 2$, then Expression (7) holds; otherwise, we have $\zeta_{\mathcal{U}} = \zeta_{\mathcal{H}} = 0$ in equilibrium, , i.e., the equilibrium features no transactions inside the platform.

(b) Suppose $\theta_{\mathcal{H}0} \leq \theta_{\mathcal{H}\mathcal{U}} \leq \theta_{\mathcal{U}0}$ holds in equilibrium. Then, customers with $\theta < \theta_{\mathcal{H}\mathcal{U}}$ choose $\mathcal{U}$-labeled providers given that $\theta < \theta_{\mathcal{U}0}$ and $\theta < \theta_{\mathcal{H}\mathcal{U}}$. Using a similar argument, we obtain that customers with $\theta \geq \theta_{\mathcal{H}\mathcal{U}}$ choose $\mathcal{H}$-labeled providers and no customers take the outside option. Thus, $\zeta_{\mathcal{U}}$ and $\zeta_{\mathcal{H}}$ are given by

$$\zeta_{\mathcal{U}} = \ell\Big((-\infty, \theta_{\mathcal{H}\mathcal{U}}] \cap [1,2]\Big) \text{ and } \zeta_{\mathcal{H}} = \ell\Big([\theta_{\mathcal{H}\mathcal{U}}, +\infty) \cap [1,2]\Big).$$

Therefore, $\zeta_{\mathcal{U}} > 0$ if and only if $\theta_{\mathcal{H}\mathcal{U}} > 1$ and $\zeta_{\mathcal{H}} > 0$ if and only if $\theta_{\mathcal{H}\mathcal{U}} < 2$. By Equation (13) and Equation (14), we conclude that $\zeta_{\mathcal{U}}$ and $\zeta_{\mathcal{H}}$ are either both positive or both zero. Therefore, if $1 < \theta_{\mathcal{H}\mathcal{U}} < 2$, then Expression (8) holds; otherwise, we have $\zeta_{\mathcal{U}} = \zeta_{\mathcal{H}} = 0$ in equilibrium.

The equilibrium structure when $q_{\mathcal{U}}(\alpha) \geq q_0$ can be established by similar arguments. □

Proof of Proposition 1

Equilibrium existence follows directly by Proposition A.1. □

Proof of Proposition 2

We establish Proposition 2 under Assumption 2. To simplify notation, we let $\underline{\eta} \triangleq \frac{1-\beta}{\beta\gamma}\frac{q_0-\gamma}{1-\beta}$. We also point out that $\underline{\eta} < 1/2$ by Assumption 1 (b).

First, note that Assumption 2 implies that

$$p_0 > \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma).$$

So, to characterize $\tau^*(0)$ and determine the equilibrium structure, we only need to consider cases (3) to (8) in Proposition A.3. In addition, condition Expression (15) implies that only cases (3) and (6) are possible when b_0 is sufficiently small. Lastly, we show when $\underline{\eta} < 1/2$ holds, inequality (47) also holds. In particular, the right-hand side of inequality (47) is less than w_0 given that

$$\frac{3(1-\beta)+\beta\gamma}{2(1-\beta+\beta\gamma)} - \frac{1-\beta}{1-\beta+\beta\gamma}\frac{1}{\underline{\eta}} < 0$$

when $\underline{\eta} < 1/2$ and $\gamma < \frac{1-\beta}{\beta}$. Therefore, Proposition A.3 implies that the equilibrium takes form Eq1. In particular, the all customers transact inside the platform at equilibrium.

Proof of Proposition 3

We proceed in three steps to establish the proposition under Assumption 2. In step 1, we argue that the following policy results in a type Eq1 equilibrium

$$\lambda^* = \frac{q_0 - \gamma}{\beta\gamma(1-q_0)} > 0, \tag{71}$$

$$\text{and } \tau^* = 1 - \frac{w_0}{p_0 + \frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1 - q_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big)}, \tag{72}$$

where recall that we use $\lambda = \frac{\alpha}{1-\beta\alpha}$ to simplify notation.

In step 2, we show that the proposed policy is optimal by establishing the equivalence between the equilibrium it induces and the equilibrium under the optimal policy with differentiated commissions, as characterized by Lemma A.12 and Proposition A.4. Finally, on step 3, we show that the mass of $\mathcal{H}$-labeled ($\mathcal{U}$-labeled) providers under the proposed policy is lower (higher) compared to the optimal full-information provision information.

Step 1: We show that a type Eq1 equilibrium arises under the λ^* and τ^* given above. It is straightforward to verify that $q_{\mathcal{U}} = q_0$ and $p_{\mathcal{U}} = p_0$ follow under λ^* and τ^*. By Lemma A.4, it suffices to verify that the customer with type $1+\delta_{\mathcal{U}}$ prefers $\mathcal{U}$-labeled providers to the outside option, and that $\mathcal{U}$-labeled providers are not financially constrained. That is, $p_{\mathcal{U}} \le p_0$ and $(1-\tau^*)p_{\mathcal{U}} \ge b_0$. In addition, we need to verify that the free-entry condition (i.e., Equation (34)) holds. First, we note that $p_{\mathcal{U}} \le p_0$ holds trivially since $p_{\mathcal{U}} = p_0$. Then, $(1-\tau^*)p_{\mathcal{U}} \ge b_0$ is equivalent to

$$\tau^* \ge 1 - \frac{w_0 - b_0}{\frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1 - q_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big)}.$$

The inequality above is equivalent to

$$\frac{w_0 - b_0}{\frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1 - q_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big)} \ge \frac{w_0}{p_0 + \frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1 - q_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big)},$$

which is further equivalent to

$$b_0 \le \frac{p_0}{p_0 + \frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1 - q_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big)} w_0. \tag{73}$$

Then, we claim that Expression (73) holds under Assumption 2. In particular, note that $b_0 < \frac{p_0 - (q_0-\gamma)}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)\left(\frac{2}{\underline{\eta}}-1\right)} w_0$ by Expression (15). Thus, to show Expression (73), it suffices to show the following

$$\frac{p_0 - (q_0-\gamma)}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)\left(\frac{2}{\underline{\eta}}-1\right)} w_0 < \frac{p_0}{p_0 + \frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1 - q_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big)} w_0,$$

which follows from simple algebra given that $0 < \underline{\eta} < 1$. Finally, we need to verify that the free-entry condition holds. This is equivalent to showing that the following equality holds (note that $\eta = 1$ and $\delta_{\mathcal{U}}^U = \frac{1-\beta}{1-\beta+\beta\gamma}$ under a type Eq1 equilibrium)

$$\frac{\beta\gamma}{1-\beta} \cdot \frac{1-\underline{\eta}}{1 + \frac{q_0-\gamma}{1-q_0}} \Big((1-\tau^*)p_{\mathcal{H}} - w_0\Big) = w_0 - (1-\tau^*)p_{\mathcal{U}},$$

where

$$(1-\tau^*)p_{\mathcal{H}} = w_0 + (1-\tau^*)\Big(\frac{1-\beta}{1-\beta+\beta\gamma} + \frac{\beta\gamma}{1-\beta+\beta\gamma}\underline{\eta}\Big)\Big(1-q_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)\Big), \text{ and}$$

$$(1-\tau^*)p_{\mathcal{U}} = w_0 - (1-\tau^*)\frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1-q_0+\frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big).$$

Then, the free-entry condition becomes

$$\begin{aligned}&(1-\tau^*)\frac{\beta\gamma}{1-\beta}\cdot\frac{1-\underline{\eta}}{1+\frac{q_0-\gamma}{1-q_0}}\Big(\frac{1-\beta}{1-\beta+\beta\gamma}+\frac{\beta\gamma}{1-\beta+\beta\gamma}\underline{\eta}\Big)\Big(1-q_0+\frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big)\\ =&(1-\tau^*)\frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1-q_0+\frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big),\end{aligned}$$

which can be shown to hold from simple algebra.

Step 2: Next, we show that the equilibrium under $(\tau^*,\, \lambda^*)$ is the same as the equilibrium under the optimal joint policy with differentiated commissions. In particular, the informational delay of the optimal joint policy with differentiated commissions satisfies $\lambda = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$, which is the same as λ^*, since $\underline{\eta} < 1/2$ (Proposition A.4 and Lemma A.12). In addition, it is straightforward to verify that under the optimal joint policy with differentiated commissions, the induced revenues for the platform are equal to those under $(\tau^*,\, \lambda^*)$ (Lemma A.5 and Equation (57)). This establishes that (τ^*, λ^*) is indeed the optimal policy for the platform.

Step 3: Finally, we show that under (τ^*, λ^*), the mass of providers with label $\mathcal{H}$ who are active on the platform, i.e., $\delta^H_{\mathcal{H}}$, is lower than that under the optimal full-information provision policy. In particular, we have that

$$\delta^{H,*}_{\mathcal{H}} = (1-\underline{\eta})\frac{\beta\gamma}{1-\beta+\beta\gamma},$$

under (τ^*, λ^*). On the other hand, under the full-information provision policy, we have by the proof of Proposition 2) that

$$\delta^{H,0}_{\mathcal{H}} = \frac{\beta\gamma}{1-\beta+\beta\gamma}.$$

Then, it follows that $\delta^{H,*}_{\mathcal{H}} < \delta^{H,0}_{\mathcal{H}}$, and $\delta^*_{\mathcal{U}} = 1-\delta^{H,*}_{\mathcal{H}} > \delta^0_{\mathcal{U}} = 1-\delta^{H,0}_{\mathcal{H}}$.

In sum, we have shown that the optimal policy for the platform features positive delay (i.e., $\alpha^* > 0$) in this setting. In addition, we established that the mass of providers with label $\mathcal{H}$ who are active on the platform is lower than that under the optimal full-information provision policy. □

Proof of Proposition 4

We establish Proposition 4 under Assumption 3. Since

$$\frac{p_0}{q_0-\gamma} < 1+\frac{1}{4} < 1+\frac{1-\beta}{1-\beta+\beta\gamma},$$

only cases (1) and (2) of Proposition A.3 can arise. So, the equilibrium can take either form Eq3 or Eq4. Both feature customers who take their outside option, which concludes the proof of the proposition. □

Proof of Proposition 5

The proof follows arguments similar to those in the proof of Proposition 3. In particular, we proceed in three steps to establish that policy (τ^*, λ^*) with λ^* and τ^* given by Expression (71) and Expression (72), respectively, is optimal under Assumption 3. We also establish that (τ^*, λ^*) results in a higher volume of transaction with $\mathcal{H}$-labeled providers compared to the optimal full-information provision policy. We omit the details for brevity. □

Proof of Proposition 6

First, by the definition of $\tau_{\mathcal{U}}^{c,*}(0)$ and $\tau_{\mathcal{H}}^{c,*}(0)$, we have

$$\tau_{\mathcal{U}}^{c,*}(0) = \max\left(\tau_{\mathcal{U}}^{*}(0)p_{\mathcal{U}}^{*}(0), 0\right) \text{ and } \tau_{\mathcal{H}}^{c,*}(0) = \max\left(\tau_{\mathcal{H}}^{*}(0)p_{\mathcal{H}}^{*}(0), 0\right),$$

where $\tau_{\mathcal{U}}^{*}(0)$, $\tau_{\mathcal{H}}^{*}(0)$, $p_{\mathcal{U}}^{*}(0)$, and $p_{\mathcal{H}}^{*}(0)$ are given by Corollary A.1. In addition, we note that $\tau_{\mathcal{H}}^{*}(0)p_{\mathcal{H}}^{*}(0) > 0$; otherwise, the platform's revenues will become negative. Therefore, it follows that $\tau_{\mathcal{U}}^{c,*}(0) < \tau_{\mathcal{H}}^{c,*}(0)$ by Corollary A.1. □

Proof of Proposition 7

First, by the definition of $\tau_{\mathcal{U}}^{c,*}(\alpha^*)$ and $\tau_{\mathcal{H}}^{c,*}(\alpha^*)$, we have

$$\tau_{\mathcal{U}}^{c,*}(\alpha^*) = \max\left(\tau_{\mathcal{U}}^{*}(\lambda^*)p_{\mathcal{U}}^{*}(\lambda^*), 0\right) \text{ and } \tau_{\mathcal{H}}^{c,*}(\alpha^*) = \max\left(\tau_{\mathcal{H}}^{*}(\lambda^*)p_{\mathcal{H}}^{*}(\lambda^*), 0\right),$$

where $\tau_{\mathcal{U}}^{*}(\alpha^*)$, $\tau_{\mathcal{H}}^{*}(\alpha^*)$, $p_{\mathcal{U}}^{*}(\alpha^*)$, and $p_{\mathcal{H}}^{*}(\alpha^*)$ are given by Corollary A.2. The proof of the corollary also implies that

$$\tau_{\mathcal{U}}^{*}(\lambda^*)p_{\mathcal{U}}^{*}(\lambda^*) = p_0 - w_0 > 0.$$

Therefore, it follows that $\tau_{\mathcal{U}}^{c,*}(0) < \tau_{\mathcal{U}}^{c,*}(\alpha^*)$ and $\tau_{\mathcal{H}}^{c,*}(0) < \tau_{\mathcal{H}}^{c,*}(\alpha^*)$ by Corollary A.2. □