AP304 Laser Laboratory Course, Lab 2B

THE HELIUM-NEON LASER &
LASER RESONATORS

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Abstract

In this lab, we built a Helium-Neon laser and investigated the spatial mode properties of various resonator types as well as their regimes of angular stability. We also estimated the small signal gain and the saturation intensity of the gain medium by placing a variable loss element inside the cavity. Comparison with analytical expressions for these values provided sufficient agreement between theoretical and experimental results.

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1 \ A short introduction to Helium-Neon lasers

The invention of the Helium-Neon laser (HeNe) by Javan [1] followed Ted Maiman’s success in operating the first laser even within just a few months. While Maiman’s experiments with the ruby laser were based on Schawlow and Townes classical paper [2], Javan spectroscopically measured the energy levels of the helium-neon gas mixture and predicted the possibility for population inversion and thus lasing. Unlike Maiman, Javan had more success in publishing his results in the Physical Review Letters (Maiman’s paper on the ruby laser was rejected by the editors of Phys. Rev. Lett. and he only published a short article in “Nature” in the UK.). Short after succeeding in operating the HeNe at 1.15 \mu \text{m}, other transitions were observed, such as the ubiquitous 632.8 nm one [3].

The population inversion of the neon atoms is achieved by an electrical glow discharge which creates a plasma with high energy electrons while not affecting the kinetic energy of the remaining atoms\(^1\). The electrons ionize the gas mixture and excite the helium ions into a metastable state. The energy transfer between this metastable state and the neon atoms turns out to be very efficient and is therefore responsible for most of the neon’s population inversion process, even though some of the population inversion is facilitated by direct electron impact as well. The two strongest laser transitions of this gas mixture are in the infrared at 3.39 \mu \text{m} and 1.15 \mu \text{m}. These transitions experience more gain than the visible 632.8 nm transition. To achieve lasing on this line, one has to introduce loss mechanisms for the infrared transitions, such as wavelength selective elements inside the cavity.

For a detailed discussion on the dependence of the gain and output power as a function of discharge conditions see [4].

2 \ Laser Resonators

Every type of oscillator, be it mechanical, electrical, or optical, is based on positive feedback. The most common way to achieve the necessary feedback in lasers is to use some sort of mirrors in an open cavity configuration. In the case of low gain media (such as the helium-neon gas), these mirrors have to be highly reflective to minimize losses.

\(^1\)For more information on plasmas see Francis F. Chen’s book "Introduction to Plasma Physics and Controlled Fusion".

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There is a plethora of different mirror configurations. We will start with the most obvious choice, namely two plan-parallel mirrors forming a Fabry-Perot resonator. For information about the theoretical treatment of the Fabry-Perot resonator see [5, 6, 7]. It is easy to see that this simple plan-parallel cavity has some major drawbacks. To confine the oscillating optical power inside the cavity, the parallelism of the two mirrors has to be controlled to within a few arc seconds. To reduce scatter losses, the surface quality of the mirrors has to be very high, which becomes more and more difficult as the wavelength of the oscillating light decreases.

As a quick fix one could think of placing a lens inside the cavity to collimate the light and therefore reduce the sensitivity with respect to tilt and tip of the mirrors. In this case, reflection losses on the lens’ surface will make this a sub-optimal setup. Yet, the idea to use a spherical element inside the laser resonator to confine the back and forth sloshing photons leads us to an interesting alternative to a lens, namely spherical mirrors.

We will now take a brief look at geometrical optics to find a convenient way to describe spherical resonators.

2.1 Geometrical Optics and Ray Matrices

The propagation of electromagnetic radiation can be described by geometrical optics as long as the dimensions and radii of curvature of the implemented optical elements is much larger than the radiation’s wavelength. Furthermore, one can express the radiation’s propagation by ray matrices, as defined below and explained in much more detail in [8], under the following conditions:

- paraxial ray propagation
- lossless optical elements
- no diffraction
- no interference

Without further ado, we introduce the ray vector

\[ \vec{r}(z) = \begin{pmatrix} r(z) \\ r'(z) \end{pmatrix} \]
where \( r(z) \) is the ray’s distance from the optical axis and \( r'(z) \) its angle with respect to the optical axis. In the paraxial approximation, we assume small angles and therefore take the sine to be equal to its argument and cosine to be equal to one.

Many widely encountered optical elements can then be represented by \( 2 \times 2 \)-Matrices. The matrices that we will use in our analysis of the spherical resonator are the following ones:

\[
M_h = \begin{pmatrix}
1 & d \\
0 & 1
\end{pmatrix}
\]

spherical mirror \( M_s = \begin{pmatrix}
1 & 0 \\
\frac{1}{2} & 1
\end{pmatrix}\)

where \( d \) stands for the dimension of the medium and \( R \) for the curvature of the mirror.

### 2.2 Stable Resonators

Now that we found a means of describing the propagation of optical radiation by using simple ray matrices, let us think about the stability of resonators. We will consider a resonator as being ”stable”, if the paraxial ray will stay inside the cavity after any given number of roundtrips. The matrix for a single roundtrip can be written as

\[
M_u = M_s \cdot M_h \cdot M_s \cdot M_h
\]

\[
= \begin{pmatrix}
1 & 0 \\
\frac{2}{R_1^2} & 1
\end{pmatrix}
\begin{pmatrix}
1 & d \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
\frac{2}{R_2^2} & 1
\end{pmatrix}
\begin{pmatrix}
1 & d \\
0 & 1
\end{pmatrix}
\]

\[
= \begin{pmatrix}
1 - \frac{2d}{R_1^2} & \frac{2d^2}{R_1^2} \\
\frac{4d}{R_1 R_2^2} - \frac{2}{R_1^2} & \frac{4d}{R_1 R_2^2} - \frac{2d^2}{R_1^2} - \frac{4d^2}{R_1 R_2^2} + \frac{4d^2}{R_1 R_2}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
A & B \\
C & D
\end{pmatrix}
\]

After \( N \) roundtrips the ray vector will be of the following form:

\[
\begin{pmatrix}
{r_N} \\
{r'_N}
\end{pmatrix} = \begin{pmatrix}
A & B \\
C & D
\end{pmatrix}^N \begin{pmatrix}
{r_1} \\
{r'_1}
\end{pmatrix}
\]

Figure 1. Resonator stability diagram. The shaded area denotes the regime of stability.
With $AD - BC = 1$ and $\cos \theta := \frac{1}{2}(A + D)$ we arrive at Sylvester’s theorem;

$$
\begin{vmatrix}
A & B \\
C & D
\end{vmatrix}^N = \frac{1}{\sin \theta} \begin{vmatrix}
A \sin N\theta - \sin(N-1)\theta & B \sin N\theta \\
C \sin N\theta & D \sin N\theta - \sin(N-1)\theta
\end{vmatrix}
$$

For $r_N$ to remain finite (i.e. the ray stays inside the resonator) $\theta$ has to be real. Using this constraint and the definition of $\cos \theta$, we arrive at the following expression;

$$
-1 \leq 1 - \frac{2d}{R_1} - \frac{2d}{R_2} + \frac{2d^2}{R_1R_2} \leq 1
$$

$$
0 \leq 1 - \frac{d}{R_1} - \frac{d}{R_2} + \frac{d^2}{R_1R_2} \leq 1
$$

If we introduce the resonator parameters $g_i := 1 - d/R_i$, we can further simplify this stability condition to

$$
0 \leq g_1g_2 \leq 1
$$

Fig. 1 shows a plot of this stability criterion. The shaded area marks the region in which the resonator is considered stable.

### 2.3 The stability criterion for gaussian beams ($\text{TEM}_{00}$)

Since the description of radiation propagation by geometrical optics is only an approximation to reality to first order, one might ask whether the derived stability criterion holds in praxis. Without proof, we will state the beam parameters for the fundamental gaussian beam inside spherical resonators;

- Rayleigh length $z_R = \frac{L \cdot \sqrt{g_1g_2(1 - g_1g_2)}}{g_1 + g_2 - 2g_1g_2}$
- Beam waist $w_0 = \sqrt{\frac{\lambda \cdot z_R}{\pi}}$
- Spot size $w(z) = w_0 \cdot \sqrt{1 + \left(\frac{z}{z_R}\right)^2}$
- Divergence angle $\Theta = \lim_{z \to \infty} \frac{w}{z} = \frac{\lambda}{\pi w_o}$

Since the beam waist has to be real valued, we have the following constraint;

$$
g_1g_2(1 - g_1g_2) \geq 0
$$

This leads to the same stability criterion as derived in the limit of geometrical optics.
3. Experimental Setup

Figure 2. Spatial mode profiles of the Helium-Neon laser. The following resonator configuration was used: $R_1 = 60 \text{ cm}, R_2 = 60 \text{ cm}, d = 40 \text{ cm}$.

3 Experimental Setup

After all this dry theory, let’s finally come to the experiment. We were asked to build a working HeNe laser by setting up an optical resonator around a commercially available helium-neon plasma tube and to characterize the spatial mode properties of this laser in different configurations as well as its stability with respect to angular misalignment of the mirrors. Furthermore, the single pass gain of the gain medium, the small signal gain $g_0$, and the threshold intensity $I_{\text{sat}}$ of the laser had to be determined.

Recording the spatial mode profiles of the cavities using a CCD camera, we altered the configuration of the cavity by changing the distance between the mirrors. We tried to take pictures of “interesting” modes by tweaking the angular alignment of the mirrors in each configuration. We followed this procedure for two different sets of mirrors, namely for two spherical mirrors with 60 cm radius of curvature and with a combination of a flat mirror and one of the spherical mirrors. Furthermore, we investigated the effect of an aperture inside the laser cavity.

To measure the small signal gain $g_0$ and the threshold intensity $I_{\text{sat}}$, a variable loss element (with loss $a_{\text{var}}$) had to be placed inside the cavity. The output power was then measured as a function of the introduced loss. According to the theory, the output power of the laser should depend on these parameters as follows;

$$P_{\text{out}} = \frac{1}{2} A I_{\text{sat}} T \left( \frac{g_0}{a + T} - 1 \right)$$

(1)

with

- resonator losses \( a = a_{\text{var}} + a_{\text{fixed}} \)
- mirror transmittance \( T \)
- area of the beam \( A \)
Figure 3. Spatial mode profiles of the Helium-Neon laser. The following resonator configuration
was used: $R_1 = 60 \text{ cm}, R_2 = 60 \text{ cm}, d = 80 \text{ cm}$. This series of pictures shows the effect of an
intracavity aperture; 1st picture: no aperture, 2nd picture: small aperture, 3rd picture: increased
aperture.

The small signal gain can therefore be measured by increasing the losses until the
laser stops lasing. Estimating the amount of fixed losses and knowing the mirror
transmittance, one can then calculate the saturation intensity by fitting the data to
the above equation. One could also solve the equation for each data point to find an
average value of $I_{\text{sat}}$ if a least square fit turns out to be too complicated.

To measure the single pass gain, the power of an external HeNe laser was measured
after passing through the plasma tube after removing the resonator mirrors. Using
a lock-in technique, the power was measured with the plasma tube’s glow discharge
turned on and off. This procedure should have been repeated for varying input powers
to be able to observe possible gain saturation.

4 Experimental Results and Analysis

Fig. 2 shows the first set of pictures that we took with the CCD camera. It is
interesting to see that we could excite high order modes that resembled Hermite-
Gaussian functions as well as radially symmetric Laguerre-Gaussian modes. In a
slightly different configuration, we tried to place a variable aperture (i.e. an iris)
inside the cavity to observe its effect on the transverse mode pattern. The sequence
of pictures in Fig. 3 shows nicely the change in spatial mode profiles going from
multimode without aperture to fundamental mode with small aperture. The last
picture was taken after increasing the aperture slightly; a TEM$_{01}$-like mode is clearly
visible.
Last but not least we changed the cavity configuration from spherical to hemispherical by exchanging one of the spherical mirrors for a flat mirror. In this configuration we could observe clearly radially symmetric higher order Laguerre-Gaussian modes as shown in Fig. 4. This was a little bit surprising, since the brewster windows attached to the plasma tube should break the radial symmetry of the resonator. This concludes the observation of the spatial cavity modes.

For each cavity setup, we also qualitatively measured the cavity’s sensitivity to angular misalignment of the resonator mirrors. It became obvious that the cavity became more and more unstable as we approached the limits of stability as shown in Fig. 1. After leaving the stable region, we could not get the laser to oscillate.

We will now come to a more quantitative measurement. The following data on the gain measurement was kindly provided by Sarah Katherine Braden. Taking a closer look at Eqn. 1, one sees that the bracketed quantity approaches zero as the intracavity losses becomes comparable to the small signal gain $g_0$. We can therefore determine $g_0$ by introducing a variable loss element and increasing the loss until the laser stops lasing. In our case, the variable loss element was a variable attenuator [9] using two counter rotating wedged optical plates controlled by a micrometer. Changing the angle of the plates, one changes the reflection of the interfaces according to Fresnel’s formula (assuming air being one of the interfaces)

$$\rho_p = \frac{\sqrt{n^2 - \sin^2 \theta_i - n^2 \cos \theta_i}}{\sqrt{n^2 - \sin^2 \theta_i + n^2 \cos \theta_i}}$$

Due to the symmetry of the device, the attenuation happens with negligible output beam deviation and dynamic range proportional to the fourth power of the transmission of a single wedge. A commercial version of this device is being sold by Newport.

**Figure 4.** Spatial mode profiles of the Helium-Neon laser. The following resonator configuration was used: $R_1 = 60\,\text{cm}, R_2 = \infty, d = 60\,\text{cm}$. 

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Figure 5. Variable loss element using two wedged optical plates. The inset shows an enlarged version of the marked interval which represents the measurement range during the experiment.

Fig. 5 shows the calculated roundtrip power loss as a function of plate angle assuming fused silica \((n = 1.46)\) for the optical wedge’s material.

Placing this attenuator inside the confocal cavity (i.e. \(R_1 = R_2 = d = 60\) cm, \(A = \pi \omega_0^2 \approx 1.90 \cdot 10^{-3}\) cm\(^2\)), the maximum output power was recorded at an angle of 55.35° which comes close to the theoretical value of 55.59° predicted for Brewster’s angle with \(n = 1.46\). The blue graph in Fig. 6 shows the measured output power as a function of power loss corresponding to a range of plate angles as shown in the inset in Fig. 5. To analyze the experimental data, we used two different procedures. Assuming no knowledge of any of the parameters in Eqn. 1, we tried a hyperbolic least square fit to the data with the following function:

\[
f(x) = y_0 + \frac{ab}{b + x}
\]

The result of this fit is represented by the red dashed line in Fig. 6. Identifying the terms in the fit function, we get the following values

\[
g_0 \approx 1.3\% \quad \text{and} \quad I_{\text{sat}} \approx 29.2\ \text{W/cm}^2
\]

In this calculation, we didn’t take advantage of the fact that the small signal gain had already been measured by increasing the losses until the laser stopped oscillating.

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4. Experimental Results and Analysis

In this measurement, $g_0$ was measured to be 2.1% assuming fixed losses of only 0.3%. This value for $g_0$ seems to be closer to the real value, since the mirror transmittance $T$ was 1% and the laser stopped lasing when the variable losses had been increased to about 0.8%. Using this information about $g_0$, $T$, and $a_{\text{fixed}}$, we solved Eqn. 1 for $I_{\text{sat}}$ for each experimental data point. Averaging over all these values, we arrive at $I_{\text{sat}} \approx 27 \text{W/cm}^2$. A plot of Eqn. 1 using these values is shown as the black line in Fig. 6. We can clearly see that this method doesn’t agree as closely with the experimental data as the hyperbolic fit, even though the results for the system parameters seem to make more sense from a physical point of view.

As the ultimate experiment, the single pass gain of the plasma tube was measured as described above. Unfortunately, the group that took the data didn’t measure the single pass gain as a function of input power. This means that we only have a single point in phase space, which makes this part of the measurement fairly unreliable. Nevertheless, we would like to quote the result of this measurement, since it nicely agrees with the results of the previous calculations;-) The single pass gain was measured to be 2.1%. It would be false, though, to conclude that this value represents either the small signal gain $g_0$ or the saturated gain of the gain medium, since we only have this single value and don’t know anything about the accuracy of the mea-
surement. We leave it up to the reader’s experience to judge whether or not to trust this result.

5 Conclusion

In this lab we have explored the properties of the Helium-Neon laser and experimented with various optical resonator configurations, such as the spherical and hemispherical resonator. The advantages and disadvantages of these cavities became clear by examining the stability properties of each configuration and comparing it to the theoretically predicted values according to the stability diagram (see Fig. 1). Furthermore, we learned how to measure the small signal gain and saturation intensity of laser gain media by introducing a variable loss element into the resonator. The experimental data turned out to be in good agreement with the theory and various methods to measure $g_0$ and $I_{\text{sat}}$ resulted in comparable values.

Having learned these measurement techniques using a fairly harmless low-power laser, we can now move on to apply this knowledge to more intricate high-power laser systems.

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References


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