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Efficient entry

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Abstract

We present a dynamic entry game, in which entry costs become sunk gradually. In equilibrium the most profitable firms enter, as they commit faster not to exit. This rationalizes an equilibrium selection assumption often employed in the empirical entry literature.

Keywords: Entry; Commitment

JEL classification: C7; C73

1. Introduction

We study a simple entry situation in which N potential entrants decide whether to enter into a new market. Post-entry competition is assumed to be symmetric, but potential entrants vary in the sunk costs they must incur in order to enter. This framework has proved quite attractive for empirical work (Bresnahan and Reiss, 1990; Berry, 1992; Scott Morton, 1999). Such entry models introduce the possibility of multiple equilibria, which raises several econometric problems. Bresnahan and Reiss (1990) and Berry (1992) address this multiplicity problem by conditioning only on the number of entrants, n, which is uniquely determined. While this solution is agnostic with respect to equilibrium selection, it has two important drawbacks. First, it is econometrically inefficient as it does not use the

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information about the identities of entrants provided by the data. Second, it does not provide sharp predictions as to the identities of the entrants; this puts limitations on welfare analysis and counterfactual exercises.

An alternative approach is to assume a particular equilibrium selection criterion, which uniquely determines the identities of the n entrants and provides a sharper prediction. Scott Morton (1999) assumes that the selected equilibrium is the socially efficient one, namely the one in which the entrants are the n firms with the lowest entry costs. The sequential entry model of Berry (1992) makes a similar assumption, by assuming that entry decisions are made sequentially with lower cost entrants moving first. While this equilibrium selection assumption is convenient, it is reasonable to ask what underlying mechanism may rationalize it. This is the goal of this note. Our model introduces a simple dynamic element into the entry game. We require potential entrants to sink their entry costs gradually over time, rather than instantly and simultaneously. Beyond being perhaps more realistic, this extension allows lower cost entrants to commit faster to entry, thereby selecting the efficient entry outcome as the unique subgame perfect equilibrium of the dynamic game.

In our model firms play an off-equilibrium war of attrition, which is won by the lower cost firms. In that sense, the idea behind the model is similar to that of Judd (1985), who models an off-equilibrium post-entry war of attrition between a multiproduct firm and a new entrant. In contrast, our war of attrition takes place before entry, not after. We also extend the model to more players and more periods. Similar to us, Bulow and Klemperer (1999) model a multi-player war of attrition. In their model the value of entry is exogenous because the number of eventual entrants is fixed in advance. In contrast, we allow the number of entrants, and therefore the value of entry, to be endogenously determined. Finally, our approach is closely related to the dynamic framework proposed by Caruana and Einav (2005), which uses increasing switching costs as a way to model gradual commitment opportunities. The gradual sinking of entry costs in the current context serves a similar role.

2. Model and results

We first introduce the static game used in the literature, and later extend it to our dynamic setting. There are N potential entrants. Each has to decide whether to enter a new market. Post-entry competition is symmetric. This competition is modeled by a reduced-form per-firm profit function $v(n)$, which is positive and strictly decreasing in n, the number of actual entrants. As competition is symmetric, $v(n)$ does not vary across players. A player choosing to enter incurs a sunk cost of entry, c_i. Formally, let $a_i=1(a_i=0)$ if player i enters (stays out), so payoffs are given by

$$\pi_i(a_i, a_{-i}) = \begin{cases} v\left(\sum_j a_j\right) - c_i & \text{if} \quad a_i = 1 \\ 0 & \text{if} \quad a_i = 0 \end{cases} \tag{1}$$

We assume that firms are heterogenous in their entry costs. Without loss of generality, let $c_1<c_2<\ldots<c_N$. Our focus is on generic cases, so we ignore the non-generic cases where two firms have identical entry costs. To make the analysis meaningful, we assume also that $c_1<v(1)$ (so at least one firm enters) and $c_N>v(N)$ (so at least one potential entrant stays out).

As already mentioned, all pure-strategy Nash equilibria of the static game have the same number of entrants (see Berry (1992) for a proof). The identities of entrants, however, may not be uniquely determined. Consider the following example. Let $N=5$, $v(n)=10/n$, and entry costs be 1, 2, 3, 3.1, and 5. It is easy to check that in any pure-strategy Nash equilibrium, firms 1, 2, and a third firm enter; the third entrant may be either firm 3 or firm 4. The socially efficient equilibrium would have the lower cost firm, firm 3, in the market.

Let us now introduce simple dynamics. Suppose that all entry occurs by some exogenously specified time T. As in Scott Morton (1999), one can think of players as potential producers of a generic drug, and of T as the date at which the incumbent's patent expires. Prior to this date, from time 0 to time T, potential entrants sink entry costs. We normalize $T=1$ and assume that entry costs are sunk linearly over the time interval [0,1]. As we discuss later, this linearity assumption is not particularly important. At any time $t\in[0,1]$ firms can observe all previous actions by other firms, and can act (drop out) at any time.[1,2] The game can be thought of as a war of attrition; all firms which have not dropped out by $t=1$ enter the market, while all other firms lose the costs that they sank prior to dropping out. Formally, let $t_i<1$ be the time at which firm i drops out, $t_i=1$ if firm i does not drop out, and let $n=|\{i:t_i=1\}|$ be the number of firms which enter. Payoffs are given by

$$\pi_i(t_i,t_{-i}) = \begin{cases} v(n)-c_i & \text{if} \quad t_i=1 \\ -c_it_i & \text{if} \quad t_i<1 \end{cases} \tag{2}$$

Theorem 1 presents our result: in any subgame perfect equilibrium (spe) of the dynamic game, the n highest profit (lowest cost) firms stay in the market, while the rest drop out immediately.

Theorem 1. *Let n uniquely satisfy $c_n<v(n)$ and $c_{n+1}>v(n+1)$.*[3] *In any spe, firms 1,2,…,n enter, while firms $n+1$, $n+2$,…,N drop out at $t=0$.*

Proof. See Appendix. □

To gain intuition for the way the dynamic game excludes the higher-cost firms, let us use the five-firm example presented above. The key insight comes from considering a subgame in which firms 1–4 have yet to drop out and $t\in(0.5/3, 0.6/3.1)$. In any such subgame, firms 1, 2, and 3 are fully committed to stay in, while firm 4 is not. To see this, note that continuation payoffs (net of the entry costs already sunk) for firm 3 are at least $v(4)-(1-t)c_3=10/4-3(1-t)=3t-0.5>0$. This implies that at this point firm 3 is fully committed to never drop out, regardless of the actions of the other firms. The same holds for firms 1 and 2. Firm 4 understands this, and may still gain from dropping out since $v(4)-(1-t)c_4=10/4-3.1(1-t)=3.1t-0.6<0$. Therefore, in equilibrium firms 1–3 do not drop out, while firm 4 foresees this and drops out immediately at time zero, to save on the sunk entry costs.

The proof generalizes this example by showing that it is credible for each of the actual entrants (the n lowest cost firms) to never drop out. This is so because at some point in the game, the other firms will find it profitable to drop out, guaranteeing the entrants positive payoffs. Since never dropping out is credible for the entrants, each of the other firms cannot make positive payoffs, and therefore prefers to drop out immediately to save on sunk entry costs. More generally, the multiplicity problem in the static game is driven by the very discrete action space: firms can be either in or out. The dynamic framework

[1] The assumption that dropping out is once and for all is not important. The results carry through if firms which drop out could reverse decisions, as long as by doing so they would still have to sink all their remaining entry costs.

[2] We assume that players can move in continuous time. All the results will carry through if time is discrete and sufficiently fine, or if players move sequentially rather than simultaneously at each point in time.

[3] As already mentioned, we focus on generic games, so we assume that both inequalities are strict.

smoothes out the entry process and allows actions to be more continuous; this creates a more gradual commitment process, which generates efficient sorting. Even if two firms are similar, the gradual entry process guarantees that there would be some point in time where the asymmetry bites and allows the more efficient firm to gain an advantage.

3. Discussion

We extend the simple simultaneous entry model by allowing entry costs to become sunk only gradually. This dynamic element selects the efficient entry as the unique equilibrium outcome, as it allows firms with lower entry costs to commit faster not to exit. This setup provides economic foundations for the assumption regarding equilibrium selection in entry models, which has been often employed in the empirical entry literature.

In our model, we assumed that entry costs are sunk linearly. This assumption is only made for convenience. The model extends naturally to any pattern of expenditure, provided that it is smooth and that all firms follow the same pattern. In fact, a weaker assumption is possible: all we require is a monotonicity property, under which the ranking of firms according to their level of remaining unsunk entry costs does not change over time.

Since firms derive commitment power from their ability to sink their entry costs, it might be appealing to endogenize the rate at which these costs are sunk. To prevent the model from degenerating into a static game where firms sink all costs at $t=0$, we would need to consider technological constraints (or added costs) on the firms' ability to sink entry costs quickly. However these constraints are modeled, as long as firms are similar in their ability to speed up sinking entry costs, it seems natural to conjecture that our baseline results remain the same. In equilibrium, firms may sink entry costs faster than is optimal in order to gain a commitment advantage, but ultimately the lowest cost firms will enter.

Two other extensions to the baseline model could introduce either symmetric uncertainty or private information into the model. For the former, a simple assumption that might be analytically tractable is to model post-entry profits by $\epsilon v(\cdot)$, where $v(\cdot)$ is known and ϵ is learned over time. For the latter, a natural assumption would be for each firm to know its own level of entry costs but not those of its competitors. In either case, it seems likely that one or more marginal firms might enter initially, and then drop out later as more information arrives or as other firms fail to drop out; such "late exit" does not occur on the equilibrium path of the complete-information game.

Finally, one can extend the dynamic model to include more complicated entry situations, in which post-entry competition is not symmetric and player identities matter (Ciliberto and Tamer, 2004; Einav, 2004). Under such assumptions the equilibrium number of firms is no longer uniquely determined in the static model. The dynamic model, however, will again select one particular equilibrium outcome, and therefore may be even more useful.[4,5] In such situations, however, characterizing the number and identities of the entrants will depend on the parameters of the model in a complicated and less intuitive way.

[4] We should note that in such cases, uniqueness of the equilibrium in the dynamic model would rely on a slight modification to the model to prevent firms from acting simultaneously.

[5] This prediction can be seen as a selection of a particular equilibrium of the static game, and could, in principle, be tested using the approach proposed by Ciliberto and Tamer (2004).

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Appendix. Proof of Theorem 1

Lemma 1. *Choose $k>n$. Suppose that at least k firms remain at time t, and relabel them $1, 2,\ldots,m(m\geq k)$. If $c_1<\ldots<c_{k-1}<v(k)/1-t<c_k<\ldots<c_m$ then in the spe of the continuation game starting at time t, firms $k,\ldots,m$ all drop out immediately.*

Proof. We prove the lemma by induction on k (from N to $n+1$). In the base case $k=N$, so by assumption $c_1<\ldots<c_{N-1}<v(N)/1-t<c_N$ .Firms $1,\ldots,N-1$ all have continuation value of at least $v(N)-(1-t)c_{N-1}>0$ from entering, so they all enter regardless of other firms' actions. Given this, firm N faces continuation value $v(N)-(1-t)c_N<0$ from entering, so firm N drops out immediately, proving the base case.

Now, assume that the lemma is proved for $k+1$, and that the condition is satisfied for k, that is $c_1<\ldots<c_{k-1}<v(k)/1-t<c_k<\ldots<c_m$. We claim that firms $1,\ldots,k-1$ know that if they stay in, they are guaranteed at most k actual entrants, so they all choose to stay in, forcing the other firms out. Suppose, towards contradiction, that this is not true. Choose firm $i\in\{1,\ldots,k-1\}$, assume that firm i stays in and all other firms play spe strategies, and suppose that more than k firms enter. Let $l(\tau)$ $(\tau\epsilon[t,1])$ be the number of firms still active at time τ which satisfy $(1-\tau)\ c_j<v(k+1)$. By construction, $c_k>v(k)/1-t>v(k+1)/1-t$, so at most $k-1$ firms have costs below $v(k+1)/1-t$, so $l(t)<k$. At $\tau=1$ all firms still active satisfy $c_j\ (1-\tau)<v(k+1)$; since we assumed that more than k firms enter, $l(1)>k$. Finally, we argue that at any time, $l(\cdot)$ can only increase by one at a time. That is, for any τ there is some $\epsilon>0$ sufficiently small such that $l(\tau+\epsilon)-l(\tau-\epsilon)=1$. This is because $l(\cdot)$ increases whenever $v(k+1)/1-\tau$ crosses one firm's cost level; since the firms have distinct costs, this generically never happens to two firms at the same time.

Given that $l(t)<k$, $l(1)>k$, and $l(\cdot)$ never increases by more than one at a time, there must be some time τ where $l(\tau)=k$. But then the conditions for the lemma at $k+1$ would have been satisfied, so all firms at $k+1,\ldots, a_m$ drop out in equilibrium at time τ. Since firm i is one of the k most efficient firms and all the other firms are playing equilibrium strategies, all but k firms should have dropped out at time τ, contradicting our assumption that more than k enter.

Thus, if i enters, at most k firms enter. Knowing this, firm i faces continuation value of at least $v(k)-(1-t)c_i>0$ from entering, and so firm i enters in equilibrium. Repeating this argument for all $i<k$, all of these firms enter, so at least $k-1$ firms will enter for sure. Knowing this, any firm j with $j\geq k$ has a continuation value of at most $v(k)-(1-t)c_j<0$ from entering, so all these firms drop out immediately. □

The remainder of the proof of Theorem 1 is analogous to the proof of the inductive step of the lemma. Fix $i\leq n$. We will show that if firm i enters and the rest play spe strategies, there cannot be more than n entrants. As before, let $l(\tau)$ be the number of firms remaining at time τ with $c_j(1-\tau)<v(n+1)$ along the "quasi-equilibrium-path" where firm i stays in and the rest of the firms play spe strategies. Note that

$l(0) \leq n$ (since $c_{n+1} > v(n+1)$ by the definition of n). Suppose that more than n firms stayed in until the end. Then $l(1) \geq n+1$. Since (as argued above) $l(\cdot)$ can only increase by one at a time, there must be some time τ where $l(\tau) = n+1$, at which point the conditions for the lemma are satisfied for $k = n+1$ and all but the most efficient n firms surviving must drop out immediately in equilibrium. Since these n firms include firm i, at most n firms survive, including firm i, proving our assertion by contradiction.

So since firm $i \leq n$ is guaranteed at most n entrants if it enters, it is guaranteed positive profit from entering (recall $c_i < v(n)$), so firms $1, \ldots, n$ all enter. Knowing this, firms $n+1, \ldots, N$ all face negative profits if they enter, so they all drop out immediately. □

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