A Note on The Analogies between Empirical Models of Auctions and of Differentiated Product Markets.*

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PRELIMINARY AND INCOMPLETE
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Abstract

Under standard equilibrium assumptions, a bidder in an independent private value auction chooses her bid by trading off between markup and probability of winning. Under similar assumptions, a producer in a differentiated product market chooses its price by trading off between markup and residual demand. This similarity in the nature of the problem lands itself to a striking similarity in the first order conditions, which form the basis for estimation. Despite these similarities, the two literatures evolved in very different directions; much of the auction literature emphasizes nonparametric identification and nonparametric estimation techniques, while the demand literature concentrates on (parametric) ways to deal with endogeneity of prices. This note tries to provide a unified framework which nests the two literatures, thereby clarifying the differences in assumptions that made the literatures go in such distinct directions.

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1 Introduction

This note tries to create a conceptual link between two important strands of the recent empirical literature in Industrial Organization. The first focuses on estimation of demand and costs in differentiated product markets (Bresnahan, 1987; Berry, Levinsohn, and Pakes, 1995; Nevo, 2001; and many others). The second attempts to estimate the cost/valuation distribution in auction markets (Paarsch, 1992; Laffont, Ossard, and Vuong, 1995; Guerre, Perrigne, and Vuong, 2000; and many others).

A standard assumption in the differentiated-product literature is that prices are set as an outcome of a Nash Equilibrium behavior, typically with constant marginal costs. Under these assumptions, single-product firms choose prices to solve

$$\max_{p_i}(p_i - c_i)D_i(p_i, p_{-i}) \tag{1}$$

Consequently, in a Nash Equilibrium this choice satisfies the following first order condition:

$$c_i = p_i + \left(\frac{\partial D_i(p_i, p_{-i})}{\partial p_i}\right)^{-1} D_i(p_i, p_{-i}) \tag{2}$$

Similarly, in independent-private-value (IPV) (procurement) auction, the literature typically assumes that bids are set as an outcome of a Bayesian Nash Equilibrium behavior. Under this assumption, bidders place bids to solve

$$\max_{b_i}(b_i - c_i)\Pr(b_i < b_j \forall j \neq i) \tag{3}$$

Consequently, in equilibrium this choice satisfies the following first order condition:

$$c_i = b_i + \left(\frac{\partial \Pr(b_i < b_j \forall j \neq i)}{\partial b_i}\right)^{-1} \Pr(b_i < b_j \forall j \neq i) \tag{4}$$

The similarity is, of course, not incidental. After all, one way of thinking about IPV auctions is as an incomplete information version of a Bertrand price competition in homogeneous product markets. Despite this striking similarity in the nature of the problem, the two literatures evolved in very different directions. Much of the auction literature emphasizes nonparametric identification and nonparametric estimation techniques, while the demand literature concentrates on parametric ways to deal with endogeneity of prices. This note tries to provide a conceptual mapping between the two literatures, and to clarify the differences in assumption that made the literatures go in such distinct directions.

One common belief should be dismissed at the outset. Thinking about auctions as a homogeneous product Bertrand competition has led many economists to believe that auctions are simpler; unlike multi-dimensional demand systems for differentiated products, so the argument goes, the allocation rule in auctions is known and simple: the lowest bid wins. It turns out that thinking about the mapping helps to clarify why this argument does not hold. Taking the original Bertrand case as a benchmark, the move towards an incomplete information version of it complicates the problem just as much, if not more, as the move towards product differentiation. I argue in this note that the so-called simplicity of the auction literature, which allows nonparametric treatment, is achieved through a set of simplifying assumptions rather than through a fundamentally simpler problem. The key simplifying assumption is about the wedge between the information available to the econometrician and that available to agents. The demand literature assumes such a wedge exists, and takes the form of unobserved product quality. The auction literature assumes this wedge away, therefore faces no endogeneity problem, allowing it to be more flexible on other dimensions. Which set of assumptions is more reasonable depends on the particular question and particular industry analyzed. It should not depend, however, on whether it is an auction or a demand application.

This note has several goals. First, the unified framework should help in facilitating communication between the two literatures, which so far evolved quite independently of each other. Second, the unified framework may help in thinking and evaluating the implicit trade-offs made by making certain assumptions. Third, one may view certain markets as a combination of differentiated product markets and auctions. In such cases, using ideas from both literatures may be important. For example, one can think about differentiated product markets in which firms cannot change prices too often, and therefore prices are set without complete information about opponents' prices. Alternatively, one can think about auctions in which the allocation rule is somewhat vague, and depends not only on prices. In such cases, the allocation rule cannot be assumed, but has to be estimated, just as any other demand system.

The rest of the note continues as follows. Section 2 provides a unified framework. Section 3 maps each literature to the unified framework, and maps the assumptions made by each literature to an equivalent set of assumptions in the other. Section 4 discusses several related analogies, as well as differences in the nature of the problem, which may have made the literatures go in distinct directions. Section 5 concludes.

2 An Innocent Model

The Model

Consider a simultaneous-move game of the following structure. There are N players. Each player has to choose an action $x_i \in [0, \infty]$. The utility of player i from choosing x_i and a (potentially random) action of $\widetilde{x}_{-i}$ for the other players is given by $(x_i - \theta_i)E(\widetilde{S}_i(x_i, \widetilde{x}_{-i}))$. We assume that $\widetilde{S}_i(x_i, x_{-i})$ is non-negative, is weakly decreasing in x_i, and weakly increasing in x_j for $j \neq i$. Let $S_i(x_i) \equiv E(\widetilde{S}_i(x_i, \widetilde{x}_{-i}))$. We assume that $S_i(x_i)$ is continuous and twice continuously differentiable in all its arguments. It is easy to think about $(x_i - \theta_i)$ as markup, and about $S_i(x_i)$ as a residual demand function.

Under these assumptions, player i solves

$$\max_{x_i} (x_i - \theta_i) S_i(x_i) \tag{5}$$

and, in equilibrium, the following first order condition is satisfied

$$(x_i - \theta_i)\frac{\partial S_i(x_i)}{\partial x_i} + S_i(x_i) = 0 \tag{6}$$

Second order conditions are assumed to be satisfied. Namely, we assume that

$$(x_i - \theta_i)\frac{\partial^2 S_i(x_i)}{\partial x_i^2} + 2\frac{\partial S_i(x_i)}{\partial x_i} < 0 \tag{7}$$

or, in words, that $S_i(x_i)$ is not too convex. This also guarantees that the solution $x_i(\theta_i)$ is increasing in θ_i (keeping fixed the opponents' strategies, $\widetilde{x}_{-i}$).

We can rearrange the first order condition to obtain

$$\theta_i = x_i + \left(\frac{\partial S_i(x_i)}{\partial x_i}\right)^{-1} S_i(x_i) \tag{8}$$

Estimation

Suppose now the econometrician observes many outcomes of such a game, and that for each game the econometrician observes the action, x_i, for each player. If $S_i(x_i)$ was known or estimable, one could use equation (8) to back out θ_i. If $S_i(x_i)$ is unknown, however, this is not possible. A higher observed x_i can be driven either by a higher θ_i or by a "more favorable" $S_i(x_i)$.

To simplify the intuition, suppose all the information about a player can be characterized by a one-dimensional parameter. In particular, let $S_i(x_i) = S(x_i; \lambda_i, \lambda_{-i})$, with $S(x_i; \lambda_i, \lambda_{-i})$ increasing in λ_i and decreasing in λ_j for $j \neq i$. A player with a higher λ is "stronger": *Ceteris paribus*, she faces higher demand, and makes her opponents face lower demand.

Suppose also that $S(x_i; \lambda_i, \lambda_{-i})$ is known to the econometrician up to the λ's parameters, which are unobserved (to the econometrician only; the players know everything). A player is now defined by a pair (θ_i, λ_i). The econometric task is to infer costs, θ_i's, from observing x_i's. It is quite clear that there are "too many" degrees of freedom: without further assumptions, it is impossible to infer a two-dimensional error structure by observing only a one-dimensional set of variables. Thus, the system is not identified. The only way to identify the system would be to either make parametric restrictions on (θ_i, λ_i), or to use other sources of data which would provide independent information about (θ_i, λ_i).

The key conceptual distinction between the two error terms is the way they show up in the objective function. θ_i is a non-strategic error term, as it affects player i utility, but has no effect on the utility of the other players, other than through the indirect effect on player i's choice of x_i. In contrast, λ_i is a strategic error: it directly affects the objective function of both player i and her opponents.

3 Mapping to The Literature

Differentiated Product Demand

Consider first the case of differentiated product markets. It is easy to see that equation (1) maps itself directly to equation (5) of Section 2: x is the price, θ is the marginal costs, and $S(\cdot)$ is the residual demand function. To fix ideas, consider, as a benchmark example, a simple logit discrete-choice demand model (Berry, 1994). The utility for consumer h from product i is given by $u_{hi} = \delta_i - \alpha p_i + \varepsilon_{hi}$ where δ_i is the average quality of product i, p_i is its price, and ε_{hi} is an idiosyncratic taste preference, distributed according to a type I extreme value, and is i.i.d across consumers and products. The mean utility from the outside good (good 0) is normalized to zero. If the number of consumers in the market is M, this specification gives rise to the well-known logit demand function:

$$Q_i(p_i, p_{-i}) = \frac{\exp(\delta_i - \alpha p_i)}{1 + \sum_{j \in J} \exp(\delta_j - \alpha p_j)} \tag{9}$$

This demand function satisfies the restriction that δ_i is a sufficient statistic for player i, so all heterogeneity can be summarized by a one-dimensional parameter. In the language of the end of Section 2, we can write $S(x_i; \lambda_i, \lambda_{-i}) = \frac{\exp(\delta_i - \alpha x_i)}{1+\sum_{j \in J} \exp(\delta_j - \alpha x_j)}$, where λ is now δ. As already indicated, if we only observed prices, we will not be able to determine whether the price of a certain product is higher because of higher marginal costs or because of higher quality. Clearly, the distinction is of crucial importance for any counterfactual analysis. Loosely speaking, products of higher quality are good to have, and products of higher marginal costs are inefficiently manufactured. In that sense, prices are endogenous: they are correlated with the (potentially unobserved) quality of the product.

Luckily, in demand models we can solve the indeterminacy problem by exploiting an additional source of data. We typically observe quantities, as well as prices. Quantities can identify the λ_i's, and therefore prices can nonparametrically identify the marginal costs, θ_i. For example, in the logit case described above, with single-product firms, equation (8) becomes

$$c_i = p_i - \frac{1}{\alpha(1 - q_i/M)} \tag{10}$$

and marginal costs can be backed out from information about prices and quantities, and the parameters α and M. Without quantity data, however, the system is not identified, unless we know (or make assumptions about) the product qualities, δ_i's. One should note, however, the in order to identify the system, one has to make the parametric assumptions which make δ_i a one-dimensional sufficient statistic for player i. As we discuss below, such parametric assumptions may have strong implication in counterfactuals, so other alternatives may be worth exploring.

IPV Auctions

Consider now an IPV procurement auction. A quick comparison of equation (3) and equation (5) reveals that one can think of the probability of winning as the demand function, S. In addition, just as before, x is the bid, and θ is the costs (which are private information). In thinking about this mapping, it is important to emphasize what may make the probability of winning function different from bidder to bidder. In demand models, this variation is due to quality differences among products. In auction models, this is due to differences among bidders, which are *common knowledge*. Common knowledge differences are strategic: they make one bidder's expectation about her probability of winning, given a bid, be different than those expectation of other bidders. Cost variation which is *private information* is non-strategic: by construction, it does not enter the opponent's optimization problem.

In an auction setting, the corresponding quantity data (i.e. the probability of winning) is not observable. Thus, the literature has solved the indeterminacy problem by imposing restrictions on the structure of the strategic error term, or, in other words, on the shape of possible asymmetries across bidders within and across auctions. Guerre, Perrigne, and Vuong (2000), for example, assumes that all variation among bidders is due to private information. This implies that the demand function faced by each bidder varies only with the number of bidders, but not with their identities. Thus, in the language of Section 2, one can think of this symmetry assumption as assuming that $\lambda_i = \lambda_j \ \forall i, j$. This allows us to solve the indeterminacy problem. It "shuts down" one source of variation among bidders, and therefore allows us to map the one-dimensional data (bids, in this case) to the remaining one-dimensional error term (the θ_i's, which are private information to each bidder). It is now easy to see the equivalence with the differentiated product demand system. The corresponding assumption on demand would be that all products are symmetric. A logit system (as above) with all product qualities being the same is one example.[1] A Dixit-Stiglitz-Spence preferences is a different example (Spence, 1976; and Dixit and Stiglitz, 1977). These are parametric examples. With these assumptions and enough data, however, just as in auctions, one can relax all parametric assumptions on the demand structure. Under these assumptions prices vary only due to idiosyncratic shocks to marginal costs, and therefore are not endogenous in an econometric sense. Thus, with enough independent markets, one can observe prices and quantities for each (symmetric) product, and nonparametrically back out the demand function.

Several auction papers relax the symmetry assumption by allowing bidders to differ from each other. Campo, Perrigne, and Vuong (2002) and Kransutskaya (2002) rely on the fact that bidders can be categorized to a finite set of types (i.e., in the language of Section 2, the λ_i's can take a finite set of values), and that the econometrician knows which bidders are of the which type. With enough data, one can then nonparametrically estimate $S(x_i; \lambda_i, \lambda_{-i})$ for any finite combination of $(\lambda_i, \lambda_{-i})$ and continue as before. Two comments are in place. First, in the context of demand models, these assumptions are equivalent to an assumption that the mean quality of a product does not vary across markets. In such a case, within a logit framework, using product fixed effects would eliminate the endogeneity problem of prices. Moreover, the auction literature implies that and with enough markets and corresponding quantity data the demand function, as before, can be nonparametrically backed out. Second, one should note that the data requirements for nonparametric estimation increase exponentially with the number of values the λ_i's can take, forcing the econometrician, in practice, to either rely on a very small number of types (Hortacsu, 2002) or to use stylized environments, in which the same set of bidders play against each other repeatedly (Bajari and Hortacsu, 2003).

Finally, a different set of papers (Bajari, 1999; Bajari and Ye, 2003; and Pesendorfer and Jofre-Bonet, 2002) assume that bidders' types are drawn from a continuous set, but are known (exactly, or up to a small set of parameters) by the econometrician. Either way, this forces them to rely on parametric assumptions, making it somewhat more similar to the demand literature.

[1] A nested-logit demand model with all inside goods being identical, and the outside good in a different nest will also work. See an application of such a model in Berry and Waldfogel (1999).

Discussion

As the analogy makes clear, the probability of winning is the ex-ante demand function faced by a bidder in an auction. Unlike differentiated product markets, in which the realization of demand (quantity) is observed, the realization of demand in auction markets, namely the probability of winning, is not an observable variable. Thus, we do not have a quantity analog in auction models, which will help us identify the two-dimensional error structure. At the same time, the absence of quantity data makes it harder to falsify the symmetry assumptions. This is, I believe, one of the key reasons that the auction literature made these different set of assumptions. One should note, however, that one can identify the two-dimensional error structure by imposing parametric assumptions, which is the goal of an ongoing research project. This may lead to very different counterfactual results, and is somewhat analogous to the logit demand model described in the beginning of the section.

To summarize, both demand and IPV auction models have very similar structure, which can be unified by the unified model of Section 2. In order to identify this model with typically available data, one has to make certain assumptions. There are, in general, two sets of assumptions one may consider. The first, typically made in the demand literature, relies on parametric assumptions about the demand function, still allowing one-dimensional asymmetry across players. The second, which is popular in the auction literature, relies on stronger symmetry assumptions, but allows non-parametric estimation of the remaining heterogeneity.

More generally, however, the choice between these two sets of assumptions should not be linked with the demand/auction model, but rather with the application in hand. For example, as is well known, logit demand (or related techniques) restrict the relationship between demand and marginal revenue in a certain way. Therefore, when the focus of the analysis is on this relationship, one may want to consider relaxing the functional-form assumptions and put stronger restriction on the asymmetry among products (Hortacsu and Syverson (2003) is a recent example of such an approach). Analogously, in certain auction applications asymmetries and unobserved bidder-specific heterogeneity may be important, in which case one may consider imposing parametric distributional assumptions, while allowing a more flexible pattern of bidder asymmetries. As I already emphasized, it is the particular application and the particular research question that should guide us in making these modeling decisions.

4 Other Analogies, and Differences

Few other analogies between the two literatures should be pointed out. First, multi-unit auctions are very much like multi-product firms if winning more than one auction gives rise to some synergies (in costs or profits). Such synergies, in the form of substitutability between units, arise by the structure of the allocation rule if combination bids are allowed (Cantillon and Pesendorfer, 2003). In such a case, an increase in the probability of winning one unit alone reduces the probability of winning the bundle. This type of auctions would be a direct analog to demand models if multi-product firms could also price the bundle in a discount, which is something we rarely observe in applications.

Second, much of the demand literature has departed from the simple logit model presented in the previous section, in favor of extensions which allow for a less restrictive (but still parametric) substitution pattern (e.g. Berry, Levinsohn, and Pakes, 1995). This is somewhat analogous to private-value auctions with affiliated values. Typical symmetric models of auctions with affiliated values allow for a symmetric structure of affiliation. With heterogeneous bidders, however, the affiliation structure is likely to be asymmetric. Two bidders may be more similar to each other than they are to a third one. Thus, their costs may be more affiliated to each other, than they are with the third bidder's costs. Just as in demand models, allowing full flexibility for the affiliation pattern is not practical in typical data sets. Therefore, one could follow similar ideas to those in the demand literature, and parametrize the affiliation structure. Such parametrization would allow stronger affiliation between bidders who are more similar on observables.

Third, both literatures focus on testing the assumptions about the rules of the game. This amounts to testing between competition and collusion in the demand models, and to testing between competition and collusion and between private value and common value models in the auction literature. In demand models, these tests rely on exogenous rotation of the demand curve (Bresnahan, 1989), e.g. by (exogenously) having different sets of products offered in different markets. In auction models the tests rely on a similar idea of exogenous rotation, driven by exogenous change in the number of bidders (Athey and Haile, 2002; Haile, Hong, and Shum, 2003).

Despite these similarities, there are important differences between the two literatures. One has already been mentioned: data sets will be different. The analog to quantity data is not observable in an auction setting, thereby forcing us to rely more on structural assumptions. A second difference is computational: while in demand models we solve for a Nash Equilibrium, auction models require us to solve (or to avoid solving) for a Bayesian Nash Equilibrium. Thus, we need to solve for the full equilibrium strategy rather than for the equilibrium price. This makes things somewhat more computationally intensive (e.g. we need to solve a system of ODE's rather than a system of equations). Most important, the literatures differ in the object of interest. In demand models, the demand system is taken as given and drives many of the counterfactuals, thus allowing for a flexible demand system is crucial. In auctions, we may influence the structure of the game by changing the rules of the auction (i.e. changing the "demand function"), thus we may be interested in a different set of counterfactuals, and may need to be more flexible on other dimensions.

5 Concluding Remarks

This note lays out a conceptual mapping between empirical models of differentiated product demand systems and empirical models of IPV auctions. It highlights the fact that the striking differences in the emphasis of the two literatures is mainly driven by the nature of the assumptions employed. The auction literature can emphasize nonparametric estimation because it makes stronger assumption about the data generating process. Most importantly, it assumes that the agents do not possess any informational advantage about the environment compared to that known to the econometrician. The reasons, I believe, are twofold: the nature of the data (quantity data can both identify the demand shifters and, at the same time, falsify the restriction) and complexity of computations (solving for the asymmetric Bayesian Nash Equilibrium is harder than for the full-information Nash Equilibrium).

Is this difference in the set of assumptions driven by fundamental differences in the economics of these two types of markets? My belief is that in an abstract sense the answer is no, although certain cases may be "cleaner" and hence may satisfy this stronger set of restrictions. The goal of this note, however, is not to evaluate the different sets of assumption, but rather to put them within the same framework. My hope is that it will bring closer together the two literatures, and will shed some light on questions such as: what can we get in differentiated demand estimation if we assume away the endogeneity problem but relax the functional-form assumptions, or what would be the main issues in the auction literature if we relax the symmetry assumptions.

In addition, the note makes us think about application in which problems from both literatures are combined. This can be thought of in the context of incomplete information Bertrand competition, when products are differentiated. This may be the case when prices *cannot* be changed costlessly as a response to opponents' prices. Equivalently, one can think of applications in which the allocation rule of the auction is unknown to the econometrician, as it depends not only on price, but on other variables. This will force the econometrician to estimate the allocation rule at the same time as it estimates the cost distribution. These two problems are interesting avenues for future research.

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