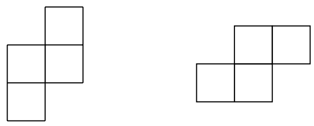
Singapore International Mathematical Olympiad 2018
Camp Quiz
Day 1

1. Sheldon and Bella play a game on an infinite grid of cells. On each of his turns, Sheldon puts one of the following tetrominoes (reflections and rotations aren't permitted)



somewhere on the grid without overlap. Then, Bella colors that tetromino such that it has a different color from any other tetromino that shares a side with it. After 2631 such moves by each player, the game ends, and Sheldon's score is the number of colors used by Bella.

What's the maximum N such that Sheldon can guarantee that his score will be at least N?

2. Given $\triangle ABC$, let I, O, Γ denote its incenter, circumcenter and circumcircle respecitvely. Let AI intersect Γ at $M(\neq A)$. Circle ω is tangent to AB, AC and Γ internally at T (i.e. the mixtilinear incircle opposite A). Let the tangents at A and T to Γ meet at P, and let PI and TM intersect at Q. Prove that QA and MO intersect at a point on Γ.

3. Suppose $f : \mathbb{N} \to \mathbb{N}$ is a function such that

$$f^n(n) = 2n$$

for all $n \in \mathbb{N}$. Must $f(n) = n + 1$ for all n?

B. Anana has an ordered n-tuple $(a_1, a_2, ..., a_n)$ if integers. Banana may make a guess on Anana's ordered integer n-tuple $(x_1, x_2, ..., x_n)$, upon which Anana will reveal the product of differences $(a_1 - x_1)(a_2 - x_2)...(a_n - x_n)$. How many guesses does Banana need to figure out Anana's n-tuple for certain?

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Day 2

4. Find all functions $f : \mathbb{N} \setminus \{1\} \to \mathbb{N}$ such that for all distinct $x, y \in \mathbb{N}$ with $y \geq 2018$,
$$\gcd(f(x), y) \cdot \operatorname{lcm}(x, f(y)) = f(x)f(y)$$

5. Let $x_1, x_2, x_3, y_1, y_2, y_3$ be real numbers in $[-1, 1]$. Find the maximum value of
$$(x_1y_2 - x_2y_1)(x_2y_3 - x_3y_2)(x_3y_1 - x_1y_3)$$

6. In $\triangle ABC$, let O, H, and N be its circumcenter, orthocenter, and nine-point center respectively. Let AN meet the circumcircle of $\triangle ABC$ at S. Let the tangents to the circumcircle of ABC at B and C meet at D. Show that $\angle DSH = \angle DOA$.

B. Simon plays a game on an $n \times n$ grid of cells. Initially, each cell is filled with an integer. Every minute, Simon picks a cell satisfying the following:

   (a) The magnitude of the integer in the chosen cell is less than n^{n^n}

   (b) The sum of all the integers in the neighboring cells (sharing one side with the chosen cell) is non-zero

   Simon then adds each integer in a neighboring cell to the chosen cell.

   Show that Simon will eventually not be able to make any valid moves.