

3 Hints

3.1 Divisor calculus

1.
 - (a)–(d) Use a cubic through all the points and J_1, J_2 (in (d), all but one of the eight points).
 - (e) Put the nine points in a 3×3 grid whose rows and columns are lines: the rows and the columns have the same total.
 - (f) Work in $(ABCD) \cup EF$.
 - (g) Work in $\omega \cup QR$, where Q, R are the diagonal points outside ω .
2. Pick a non-torsion point Q on an elliptic curve and consider its integer multiples.
3. Work in $\Gamma \cup \ell$.
4. Work in $A_1A_2 \cup A_3A_4 \cup A_5A_6 \cup A_7A_8$.
5. Let (ADX) meet AB again at Z , (BCY) meet CD again at W . Work in $(ABCD) \cup XY \cup WZ$.
6. Work in the union of the 8-point circle and a conic that passes through the four outer points and one inner point.
7. Work in the quartic made of the bisectors at A and C , the perpendicular bisector of AC , and BD .
8. Work in the quartic $\omega_1 \cup AC \cup BD$.

3.2 Cubic curves

3.2.1 Exercises/Folklore

1. Suffices to show that $(AM, AN; AJ_1, AJ_2) = -1$, where AM, AN are the angle bisectors of $\angle BAC$. Equivalently: $(\infty_{AM}, \infty_{AN}; J_1, J_2) = -1$.
2. M is the center of an involution swapping $A \leftrightarrow D, B \leftrightarrow E, C \leftrightarrow F$.
3. (a) Equivalently: the feet of the projections of P to AB, BC, CD, DA are concyclic.
 (b) Use complex numbers.
 (c) Normalise the involution swapping $A \leftrightarrow C, B \leftrightarrow D$ to $z \mapsto 1/z$, and use $\hat{z} = z + 1/z$.

3.2.2 Problems

1. If T is the target point, then A, T, P, D are concyclic. Work in a cubic through $B, C, D, E, F, I, T, J_1, J_2$.
2. Work in the isoptic cubic $\mathcal{C}(ABDC)$.
3. The points whose pedal circle passes through a fixed point form a non-pivotal isocubic.
4. The two conditions are the Darboux and McCay cubics.
5. Work in the pivotal isocubic whose pivot is the isogonal conjugate of D .
6. The circle with diameter I_AI_C (excenters of ABC) is part of the Neuberg cubic of ABC .
7. If AK meets Ω at R, S , then $\{P : PB \cdot PC = PR \cdot PS\}$ is the isoptic cubic of $WYXZ$.
8. Work in the circular pivotal isocubic of ABC through the common point of AD, BE, CF .