Official Course Description: A rigorous introduction to PDE accessible to advanced undergraduates. Elliptic, parabolic, and hyperbolic equations in many space dimensions including basic properties of solutions such as maximum principles, causality, and conservation laws. Methods include the Fourier transform, as well as more classical methods.

Teaching Staff:

Instructor: Dr. Laura Fredrickson
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Office: 380-382L
Office Hours: Wednesdays 4:30-6pm

Course Assistant: Leila Sloman
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Office: 380-381M
Office Hours: TuTh 3-5pm

Lecture: MWF 9:30-10:20am | 380-380F

Exams: There will be one 70 minute midterm exam, taken in class, from 9:10-10:20am. The final is comprehensive.

Midterm: Friday, February 7 (in class, 9:10-10:20am) Note the early start time!
Final: Thursday, March 19 from 8:30-11:30am (380-380F)

Prerequisites: 171 or equivalent.

This course is similar to Math 220, but is designed for undergraduate math majors. It is the continuation of the honors analysis course 171, emphasizing rigorous proofs, in the spirit of 171. The knowledge of measure theory, as presented in 172 is not a prerequisite for the class, though $L^p$ spaces will be discussed as completions. The Fourier analysis part of 172 will be covered in this class.

Textbook: Due to the availability of Andras Vasy’s lecture notes, the following texts are all “recommended”:

- Strauss’ *Partial Differential Equations: An introduction* covers most of the topics, but at a lower level than in our course. This is especially true regarding first order PDEs, which is the first major topic covered, as well as distributions and the Fourier transform.

- Evans’ *Partial Differential Equations* is a more advanced text, and it covers course topics not dealt with in Strauss’ book.

- Pinchover and Rubenstein’s *An introduction to partial differential equations* is a fairly good match for the level of difficulty of the course. However, it only covers some of the topics; it offers a different and complementary perspective.
Course website: Course announcements, homework, solutions will be posted on Canvas. Additionally, the syllabus will be posted on my website http://web.stanford.edu/~ljfred4/.

Grading Policy: On all work, your grade will be computed as a percentage: the number of points you earned divided by the number of points possible. The weekly homework and exams are weighted as follows:

- Homework: 25% (lowest score dropped)
- Midterm: 30%
- Final: 45%

Your letter grade will be given based on your numerical average earned in the class, on a scale not stricter than the following: you are guaranteed a D for 60.0 or above, C- for 70.0 or above, C for 73.0 or above, C+ for 77.0 or above, B- for 80.0 or above, B for 83.0 or above, B+ for 87.0 or above, A- for 90.0 or above, and an A for 93.0 or above.

E-mail: If you send me an e-mail, you can expect 24-hour turn-around on school days.

Homework: Weekly homework assignments are to be submitted via Gradescope by 9:30am each Friday. The assignments will be posted on Canvas by the previous Friday. For ease of grading, each problem should be on its own sheet(s) of paper.

The lowest score will be dropped to accommodate exceptional situations such as a serious illness. Because the lowest score is dropped, you can miss one assignment without penalty. No late homework will be accepted, and no make-up homework will be given.

Your solutions should be readable and well-explained. E.g., you should try to use complete sentences, insert explanations, and err on the side of writing out “for all,” etc. rather than using the symbol. Professor Keith Conrad at the University of Connecticut has written a helpful guide to common errors in mathematical writing: http://www.math.uconn.edu/~kconrad/math216/mathwriting.pdf.

I encourage you to form study groups and work together. A good strategy is to try each problem yourself first, then get together with others to discuss your solutions and questions, and finally you should write up the solutions by yourself. (The Honor Code applies to this and all other written aspects of the course.)

Alternate Sitting for the Midterm Exam: In exceptional circumstances, and by prearrangement only, you may take the midterm exam at a fixed alternate time. The alternate sitting will always occur before the standard sitting for the exam. To arrange an alternate sitting you must e-mail me at least two weeks before the midterm.

Final Exam Policy: (See registrar.stanford.edu/students/final-exams.)

- Students must not register for classes with conflicting end-quarter exams.
- Alternative arrangements for the final may only be made for the following unforeseen circumstances: illness, personal emergency, or the student’s required participation in special events (for example, athletic championships) approved as exceptions by the Committee on Undergraduate Standards and Policy (C-USP).

Schedule: This course is structured with the expectation that you will attend every lecture. Of course, sometimes an absence is necessary. In such a situation, you should contact a classmate to get notes and other information for the class you missed.

We will have 28 lectures in total. Here is a tentative schedule, which may be adjusted as the quarter goes on.
• **Week 1** [Jan 6, 8, 10] Introduction, Classification of PDE (Vasy 1, Strauss 1.1, Evans 1.1-1.3); Where do PDE come from (Vasy 2); First order PDEs, Method of characteristics (Vasy 3, Strauss 1.2, Evans 2.1);

• **Week 2** [Jan 13, 15, 17] Quasilinear first order PDEs (Vasy, Evans 3.2); Distributions (Vasy 4, Strauss 12.1)

• **Week 3** [Jan 22, 24] Distributions, weak solutions, shocks (Vasy 5, Strauss 12.1 & 14.1, Evans 3.4); Classification of second order equations (Vasy 6, Strauss 1.5 & 2.1, Evans 2.4)

• **Week 4** [Jan 27, 29, 31] The wave equation on \( \mathbb{R} \), domain of dependence, propagation of singularities (Vasy 7, Strauss 2.1-2.2); Energy conservation for the wave equation, the maximum principle for Laplace’s equation (Vasy 7, Strauss 2.2-2.3)

• **Week 5** [February 3, 5] The maximum principle and energy decay for the heat equation; energy estimates for Laplace’s equation (Strauss 2.3, Evans 4.3.1) [February 7] **Midterm**

• **Week 6** [February 10, 12, 14] The Fourier transform and solutions of PDEs(Vasy 8, Strauss 12.3-4, Evans), Convolutions

• **Week 7** [February 19, 21] Tempered distributions, convolutions, solutions of Laplace’s equation and the wave equation in terms of convolutions (Vasy 9); Heat and wave equations in half space and on intervals (Vasy 10, Strauss 3.1-3.2)

• **Week 8** [February 24, 26, 28] Inhomogeneous PDE; Duhamel’s principle (Vasy 11, Strauss 3.3-3.4, Evans 2.4.2); Separation of variables, eigenvalue problems (Vasy 12, Strauss 4.1-4.3)

• **Week 9** [March 2, 4, 6] Inner produce spaces and symmetric boundary conditions; Fourier series (Vasy 13, Strauss 5.1-5.3); Convergence of Fourier series (Vasy 14, 5.4-5.5) Laplace’s equations on the disk (Vasy 14, Strauss 6.3);

• **Week 10** [March 9, 11, 13] Solvability of PDE by duality (Vasy); **Review**

**Students with Documented Disabilities:** Students who may need an academic accommodation based on the impact of a disability must initiate the request with the Office of Accessible Education (OAE). Professional staff will evaluate the request with required documentation, recommend reasonable accommodations, and prepare an Accommodation Letter for faculty dated in the current quarter in which the request is made. Students should contact the OAE by the end of the first week of the quarter, since timely notice is needed to coordinate accommodations. The OAE is located at 563 Salvatierra Walk (723-1066, studentaffairs.stanford.edu).

**Textbooks and Other Resources:** The lecture notes are of high quality, and you should read them. The recommended textbooks provide supplementary resources.

You are encouraged to attend the office hours provided by the instructor and course assistant. Another resource which may be of use is Counseling and Psychological Services. See vaden.stanford.edu/caps-and-wellness.

**Computers:** If you wish to use a computer in class, you must speak with me first.

**Academic Integrity:** The Honor Code articulates Stanford University’s expectations of students and faculty in establishing and maintaining the highest standards in academic work. Examples of
conduct that have been regarded as being in violation of the Honor Code (and are most relevant for this course) include copying from another’s examination paper or allowing another to copy from one’s own paper; plagiarism; revising and resubmitting an exam for regrading, without the instructor’s knowledge and consent; representing as one’s own work the work of another; and giving or receiving aid on an academic assignment under circumstances in which a reasonable person should have known that such aid was not permitted. See communitystandards.stanford.edu for more information on the Honor Code.

Important Dates:

- First Day of Classes .......................................................... January 6
- Martin Luther King, Jr., Day (no classes) ................................. January 20
- Add/Drop Deadline ............................................................ January 24
- Midterm Exam (in class) ..................................................... February 7
- Presidents’ Day (no classes) .................................................. February 17
- Course Withdrawal & Change of Grading Basis Deadlines ........... March 28
- Last Day of Classes, Last Day to Arrange an Incomplete ............ March 13
- Final Exam ................................................................. March 19 (8:30-11:30pm)