STAT331 HW1, Due Friday, March/14

• Show that the generalized gamma distribution family is closed under the power transformation, i.e., if \( T \) follows a generalized gamma distribution, then \( T^s, s > 0 \) also follows a generalized gamma distribution. Show furthermore that any generalized gamma distribution can be written as a power transformation of a Gamma distribution.

• Assume that \( T \) and \( C \) are possibly correlated failure and censoring times, respectively. Furthermore, assume that

\[
\text{pr}(T > t, C > s) = S(t, s), t, s > 0
\]

is the continuously differentiable survival function for the bivariate random variable \((T, C)\). Let \( U = \min(T, C), \delta = I(T \leq C) \).

1. Show that

\[
\text{pr}(U > u, \delta = 1) = \int_u^\infty \left\{ -\frac{\partial S(t, s)}{\partial t} \right\}_{t=s=x} dx
\]

and

\[
\text{pr}(U > u, \delta = 0) = \int_u^\infty \left\{ -\frac{\partial S(t, s)}{\partial s} \right\}_{t=s=x} dx
\]

2. Show that

\[
S(t, s) = e^{-t^2-s^2-2ts} \quad \text{and} \quad S(t, s) = e^{-2(t^2+s^2)}
\]

induce the same joint distribution of \((U, \delta)\). (This is a toy example on the relationship between identifiability and informative censoring.)

• Suppose \((T, C)\) denote a survival time and potential censoring time, respectively, and that \( Z \) is a covariate (such as a binary indicator of treatment group or gender). Suppose that \( T \perp C \mid Z \). Does it follow that \( T \perp C \)? Suppose on the other hand that \( T \perp C \). Does it follow that \( T \perp C \mid Z \)? Prove the statements that are true and give specific examples if not.