Abstract—We consider an uncoded noncoherent uplink and downlink with a large antenna array at the base station. A ray tracing propagation model is assumed with knowledge at the base station of only the ray arrival angles and amplitudes. We identify the sources of performance degradation, quantify notions of diversity and multiplexing gains and present numerical results to demonstrate these gains. Our results indicate that in this noncoherent system, increasing the number of antenna elements increases multiplexing gain and reduces error rate. This contrasts with the fact that uncoded noncoherent systems yield no multiplexing gains in rich scattering.

I. INTRODUCTION

Large antenna arrays allow fine angular resolution in the beamspace, so that radio signals can be resolved spatially. This property is currently exploited in several wireless systems including phased array radar and astronomical imaging [1], [2], [3]. In this work, we explore the implications of this fine angular resolution for the design of a noncoherent uncoded wireless communication system.

Rich scattering [4] for MIMO systems, which results in i.i.d. fading across antenna elements, is widely used in system analysis due to its simplicity. This model has been used to develop theoretical performance limits, insights into practical designs, and channel models for performance analysis, especially for communication systems that use the instantaneous channel state information at the receiver, i.e., coherent communication systems. There has been significant interest recently in massive MIMO systems when phase information is available at the base station, even simple architectures yield large multiplexing gains in rich scattering.

On the contrary, under rich scattering, in a system that does not have information about the instantaneous channel phase at the transmitter and the receiver, the multiplexing gain is much less, possible none. However, the rich scattering model is a mathematical idealization that becomes less accurate as the number of users and/or antennas in the system grows. For example, in [5], the authors invoke the laws of physics and show that there is a certain limit to the number of independent fading/scattering elements in any environment. Moreover, it has been empirically observed [6] that there are correlation structures that exist once the number of antennas is large enough. This suggests that when the number of spatial dimensions in a communication node (either transmitter or receiver) is large enough, many of the underlying assumptions about the propagation environment and their implications need to be rethought. This motivates us to propose and analyze the performance of noncoherent massive MIMO systems outside the idealized rich scattering model. Propagation models with correlated fading in MIMO systems have been previously studied in the context of coherent systems [7], [8], [9], [10]. In this work, we assume a ray tracing model with a finite number of multipath components. Note that similar models have been defined in 3GPP for LTE. Under a ray tracing model, we show how the nature of fading with large antenna arrays can actually provide multiplexing gains even without instantaneous phase information.

In particular, we consider a base station with a large uniform linear antenna (ULA) array serving several single antenna users. The base station and users do not have knowledge of the instantaneous channel phases associated with the propagation environment, hence have to use noncoherent schemes. To quantify the gains in the limit of a large number of antennas, we define notions of outage, diversity and multiplexing as metrics to determine the number of users that may be supported under certain constraint on the uncoded bit error rate. We show that the finer angular resolution afforded by large antenna arrays not only makes it possible to drive the error probability down to zero, but also to support multiple data streams.

We obtain these desirable properties via a simple energy-detection based noncoherent transmission and detection scheme. We find that with this scheme, in both the uplink and the downlink, we can simultaneously support all users with a vanishing probability of significant inter-user interference (which we will later define in terms of an outage probability), as long as the number of users is smaller (in a sense made precise later) than the square root of the number of antennas at the base station. Defining diversity gain as the dominating exponent in the non-outage error probability, we derive a diversity-multiplexing tradeoff associated with non-outage transmissions. Since our analysis does not use the instantaneous phase of the channel gains, we describe our system as being noncoherent.

The rest of the paper is organized as follows. In Section II we describe our system model and our assumptions about the propagation model, transmission, and decoding schemes. We describe our performance metrics in Section III followed by
detailed results in Section IV. We finally present numerical results in Section V and our conclusions in Section VI.

II. SYSTEM MODEL

We now describe the propagation models considered in this paper, followed by a description of the transmission and detection schemes.

A. Notation

We use boldface fonts to refer to vectors. We use $o(g(N))$ to refer to any function $f(x)$ such that $\lim_{N \to \infty} f(N)/g(N) \to 0$. Similarly we use $\Theta(g(N))$ for any $f(N)$ satisfying $c_1g(N) < f(N) < c_2g(N)$ for some $0 < c_1 < c_2$. We say $f(N) = O(g(N))$ if $g(N) = o(f(N))$.

B. Propagation characteristics

Our propagation model is the ray tracing model (or spatial channel model) similar to models defined in 3GPP for LTE [11]. We assume that the absolute values of the attenuation and the instantaneous angles of arrival for each of the rays are known perfectly. For simplicity of analysis, the gains are assumed to be equal and the angles of arrival of transmissions from a particular user are assumed to be uniformly distributed around a central angle specific to the particular user. An additional assumption in our analysis is that the receiver antenna array is in the far field and that all the elements of the antenna array are collocated. Note, in particular, that this assumption precludes modeling the nonstationarity across large antenna arrays [12]. While operating at low carrier frequencies may necessitate explicit modeling of this nonstationarity, the use of high carrier frequencies and the resulting small form factors of large antenna arrays makes the colocation and the stationarity assumption closer to reality. Moreover, for simplicity of presentation, in this work, we consider only the azimuth direction, assuming that all transmitters and receivers are at the same elevation. Extensions of this theory to situations with both elevation and azimuth dimensions follow very similar lines and is deferred to future work. The received signal in the uplink is given by

$$y[t] = \sum_{b \in \mathcal{B}} \sum_{p \in \mathcal{L}_b} G_{b,p} e^{j\phi_{b,p}[t]} a(\theta_{b,p}) s_b[t - \tau_{b,p}] + \nu[t],$$

where $\phi_{b,p}$ (assumed to be unknown at the receiver) is uniformly distributed in $[0, 2\pi]$ and $\theta_{b,p}$ are uniformly distributed in $[\mu_i - c, \mu_i + c]$ for some constant $c$ and for $\mu_i$ uniformly distributed in $[0, 2\pi]$. This constant $c$ is a function of the propagation environment and the beamwidth. $a(\cdot)$ is a one dimensional channel response vector satisfying the following: $a(\theta)_m = e^{j2\pi m \frac{\theta - \mu_m}{\Delta \theta}}$, where the subscript $m$ refers to the phase shift in the received path at the $m$-th receiver antenna and the inter-element spacing is assumed to be 1 length unit. The uplink propagation model is depicted pictorially in Fig. 1.

We also consider downlink transmission in our analysis (Fig. 2). Since there is an excess of antennas at the downlink, the beamwidths can be much narrower. However, there does exist multiuser interference from the sidelobes. The system model is thus:

$$y_b[t] = \sum_{b \in \mathcal{B}} G_{b,p} e^{j\phi_b[t]} a_2(\theta_{b,p}) s_b[t] + \nu[t]$$

$$\triangleq \sum_{b \in \mathcal{B}} h_{d,b}[t] s_b[t] + \nu[t],$$

where $a_2(\cdot)$ is the beamforming gain of beam $b$ at direction $\cdot$ and is given by a factor $f(\cdot)$ depending on the distance in the beamspace. This factor is described in more detail in Appendix A. Note also that for each beam $b$, the transmitter can choose any path $p \in \mathcal{L}_b$, denoted by $p(b)$ to send the information symbols along.
Two multipath components very close to each other interfere destructively.

The sidelobes of multipath components with unknown phases far away in the beamspace introduce inter-path interference.

Additive white Gaussian noise at the receiver.

### TABLE I: Reasons behind performance degradation for a finite $N$

<table>
<thead>
<tr>
<th>Outage:</th>
<th>Two multipath components very close to each other interfere destructively.</th>
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<tbody>
<tr>
<td>Inter-path interference (IPI):</td>
<td>The sidelobes of multipath components with unknown phases far away in the beamspace introduce inter-path interference.</td>
</tr>
<tr>
<td>Additive noise:</td>
<td>Additive white Gaussian noise at the receiver.</td>
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C. Transmission and detection schemes

We describe these schemes separately for the uplink and the downlink.

- **Uplink**: The users use equiprobable on-off keying with an average power $2P$. At the receiver, the following statistic is computed over a grid of $\psi$ values in $[0, 2\pi]$:

$$ y_{\text{beamspace}, \psi} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} y_k e^{-i k \psi}. $$

The beamspace decoder either declares an outage or performs single user detection (by using an energy-threshold detector) by looking at all the multipath components $y_{\text{beamspace}, \psi}$ corresponding to a single user’s transmissions (arriving along different angles $\theta$). We do not analyze the performance with joint decoding of multiple users in this work.

- **Downlink**: We assume on-off keying in the beamspace, i.e., the transmitter chooses any path corresponding to the user and uses equiprobable on-off keying along that path with an average power $2P$. Thus the total power used by the transmitter is the same as that used by a user in the uplink. The receiver performs energy level detection.

### III. Metrics and summary of our results

We analyze uncoded communication and detection by considering one of the paths corresponding to a particular beam at an angle $\psi$ in the beamspace, under the assumption that the phases $\phi$ are not known at the detector. In this setup, an error event occurs due to the conditions summarized in Table I.

We observe that in the limit of an infinite $N$, all the multipath components are resolvable, but for any finite $N$ there is always some limit to the resolvability of different multipath components. For each of the conditions identified in Table I, we identify the error event and the corresponding metrics used to quantify it.

- **Outage**: For a given $N$, we fix a distance $d$ (which decreases with increasing $N$) and define an outage event as the event when the distance between any two paths in the beam space is less than $d$. Computing the probability of the outage event is shown in the extended version of the manuscript [13] to be related to the birthday problem [14], where we also state the exact dependence of the outage probability on $N$ and the total number of paths $|\mathcal{P}|$. We have the following lemma.

**Lemma 1.** For a separation $d = o(1)$, the probability of outage vanishes like $\Theta(|\mathcal{P}|^2d)$ if $|\mathcal{P}| = o\left(\sqrt{\frac{d}{N}}\right)$, and approaches a positive constant otherwise.

- **IPI**: If there is no outage, the random phases from $k$ multipath components add up constructively or destructively. In the limit of a large $k$, it produces interference distributed as a Gaussian random variable. The variance of the Gaussian random variable depends on the choice of $d$. From the results of Lemma 1 and considering that the beamwidth of the lobes is at least $\frac{1}{N}$ (Appendix A), one can choose $d = O\left(\frac{|\mathcal{P}|^2}{N}\right)$ for vanishing outage probability.

- **Additive noise**: Since the DFT is a unitary transform, the statistics of the additive noise at each point in the beamspace remains the same. This is the noise floor. The combined effect of the additive noise and the additive interference when the system is not in outage is captured by the following theorem, a formal proof of which is presented in the extended version of the manuscript [13].

**Theorem 1.** The detection error probability in decoding the information in path 1 of beam/user 1 (i.e., $(1, 1)$), for any $|\mathcal{B}|$, satisfies, for a large $N$,

$$ P_b \leq E[Q(\sqrt{NPG(1,1)} - \sum_{(b,p) \neq (1,1)} \sqrt{P}\cos(\bar{\phi}_{b,p})G_{b,p}f(2\pi/\lambda|\sin(\theta_{1,1}) - \sin(\theta_{b,p})|))], $$

where $P$ is the power in each beam, the expectation is with respect to the randomness in the signaling (equiprobable on-off keying) and the phase $\phi_{b,p}$ is uniformly distributed in $[0, 2\pi]$.

The multiplexing gain is defined as the number of beams (independent streams of symbols) $|\mathcal{B}|$ that are discernible at the receiver with a vanishing probability of error. If each beam corresponds to $L$ multipath components and there are $|\mathcal{P}|$ multipath components in total, then the multiplexing gain $|\mathcal{B}|$ satisfies $|\mathcal{B}| = \frac{|\mathcal{P}|}{L}$. The diversity gain is defined as a first order exponent in the error probability of non-outage transmissions in $N$, i.e., diversity gain is defined as a $g(N)$ such that

$$ \lim_{N \to \infty} -\log(\text{Probability of error}) = g(N) = 1. $$

In the next section, we present results about the number of users that can be supported in this system under a vanishing outage and error probability.

### IV. Diversity multiplexing tradeoffs

In this section we describe how many users many be supported in the system. We assume throughout that $G_{b,p} = \frac{1}{N}$. Due to space constraints, we defer the proofs of these results to the extended version of this manuscript [13]. We first state a theorem about the maximum number of users or beams that can be supported with a vanishing outage probability as $N \to \infty$. 

A non-outage event for $k = 3$. Each of the red segments are twice the length of $d$. 
**Theorem 2.** The number of users \( k = |\mathcal{B}| \) that can be supported with a vanishing outage probability satisfies
\[
k = o(\sqrt{N}).
\]

The proof of this theorem follows from the birthday problem and the fact that the separation between paths cannot be less than \( \frac{1}{N} \) for paths to be resolvable. We note that as the number of users \( k \) increases, the \( d \) in Lemma 1 used to declare outage decreases. We now state diversity multiplexing tradeoffs for non outage transmissions if the number of users/beam is less than the above.

**A. Finite number of users**

We first present the results for the uplink. The “noise” in this case is dominated by the fact that the transmissions corresponding to different beams would add up constructively. We thus get the following tradeoff.

**Theorem 3.** The dominating exponent in the error probability of non-outage transmissions satisfies
\[
g(N) = \left( \sqrt{NP/L} - (kL - 1)\sqrt{P/L} \right)^2.
\]

For the downlink, we have similar results except that the transmit power is changed and the interference is due to the \((k-1)\) sidelobes.

**Theorem 4.** The diversity gain for non-outage transmissions satisfies
\[
g(N) = \left( \sqrt{NP/(kL)} - (k-1)\sqrt{P/(kL)} \right)^2.
\]

**B. Number of users \( k \) increasing with \( N \)**

In the uplink, the simultaneous transmissions from all users produce a Gaussian distributed interference. It turns out that for vanishing outage probability, \( d \) cannot be small and \( k \) needs to be \( o(\sqrt{N}) \). In particular, for \( k = O(1) \), we have the following corollary of Lemma 1.

**Corollary 1.** The minimum separation \( d \) used to declare outage satisfies \( d = o(1/k^2) \).

Note that \( d \) cannot be smaller than the width of the main lobe \((\Theta(1/N))\). We then have the following results for the uplink and the downlink.

For the uplink we have:

**Theorem 5.** The diversity gain for non-outage transmissions satisfies
\[
g(N) = \left( \frac{\sqrt{NP/L} - f(1/k^2)\sqrt{P/L}}{\sigma^2 + 1} \right)^2,
\]

where \( \sigma^2 \) is a constant depending on the distribution of the interference induced by randomness in the angles of arrival, phase and signaling distributions.

From Appendix A, for \( \epsilon = 1/k^2 \), we have \( f(\epsilon) = \Theta(k^2/\sqrt{N}) = o(\sqrt{N}) \). In the downlink we have:

**Theorem 6.** The diversity gain for non-outage transmissions satisfies
\[
g(N) = \left( \frac{\sqrt{NP/(kL)} - f(1/k^2)\sqrt{P/(kL)}}{\sigma^2 + 1} \right)^2.
\]

**V. Numerical results**

For this section, unless mentioned otherwise, we consider \(|\mathcal{L}_b| = 4\), i.e., each beam has 4 multipath components. The wavelength \( \lambda = 0.05 \) units (note that the inter-element spacing at the base station antennas is chosen to be 1 unit). The phases \( \phi_{b,p} \) are chosen uniformly at every symbol time, and the gains are fixed. \( c \), the angle spread parameter in the generation of the angles of arrival, is fixed to be \( \pi/3 \).

Since we do not use joint decoding in the uplink or in the downlink, we are not able to cancel the effects of IPI (inter-path interference) along the direction of the beam. We first show the probability of error curves. In the simulations we do not distinguish between outage or non-outage events (for the values shown, outage events were seen to be very rare for a small constant \( d \)). Figure 3a shows the behavior of BER with respect to the number of antennas \( N \) for a single beam in the uplink with a path at 0 radians in the azimuthal plane. Note that the only source of performance degradation is additive noise and the interference from other multipath components from the same beam.

Next we compare how many beams we can support in the uplink for a certain number of antennas subject to a particular error probability bound. We compute detection error performance at position \( \theta_{1,1} = 0 \). For each number of base station antennas \( N \), we plot the number of users \( \mathcal{B} \) such that the average probability of error is less than \( 10^{-2} \). To boost the non-outage detection performance in non-asymptotic regimes, we combine (noncoherently) signals along each of the \( L \) paths associated with the same beam (as shown in the extended version [13], this boosts the diversity gain by a factor of \( L \)).

The resulting estimates are plotted in Figure 3b. The plots support the fact that the number of supported users grows as \( \sqrt{N} \).

**VI. Conclusions**

In this work we consider a ray tracing propagation model with a finite number of multipath components and an uncoded noncoherent communication scheme which does not require knowledge of the instantaneous phase of the channel gains. We assume perfect knowledge of the amplitude of the gains and the angles of arrival for each of the multipath components. Under this assumption we find that, in the limit of a large number of antennas \( N \), we can support up to \( o(\sqrt{N}) \) users both in the uplink and the downlink with a vanishing outage.
and error probability. This is in sharp contrast to noncoherent communications in a rich scattering environment where multiplexing gains are nonexistent. We define the diversity gain as being equal to the dominating exponent in the error probability associated with an equiprobable on-off transmission scheme. Under this definition we derive the diversity-multiplexing tradeoff associated with non-outage communications.

Our results indicate that the spatial resolution inherent in large antenna arrays can not only help us resolve each of the multipath components in beamspace, it can also let us use noncoherent methods to design and construct simple energy-detection based transmission and detection schemes. Considering that only spatial parameters (angle of arrival and the gains) need to be tracked, the potential overhead of parameter extraction for the channel is also much smaller than if the instantaneous phase needs to be tracked. The gains are not limited to noncoherent systems only, keeping track of the instantaneous phase along each multipath component is also made easier by the improved resolution — this has been exploited in coherent architectures in [15], [16].

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APPENDIX A

In this appendix, we show some properties of \( f(x) = \frac{1}{\sqrt{N}} \left| \frac{\sin(N x/2)}{\sin(x/2)} \right| \).

\[ f(x) = 0 \text{ iff } x = 2n\pi \text{ for } n \text{ belonging to the set of non zero integers } \mathbb{Z} \setminus \{0\}. \]

This suggests that the “width” of the mainlobe is given by \( \frac{\pi}{N} \).

\[ \text{For } x \in \left[ \frac{1}{N}, o(1) \right], \left| f(x) \right| \approx \frac{1}{(\sqrt{N} \sin(x/2))} \approx \frac{2}{\sqrt{N} x^2}. \]

REFERENCES


