Constellation Design in Noncoherent Massive SIMO Systems

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Massive SIMO systems

Benefits of many receive antennas

With lots of antennas we have
- large beamforming gains
- interference reduction
- reduced path loss
Massive SIMO systems

Benefits require precise CSI

This is compromised by
- noisy channel estimates
- inaccurate channel reciprocity
- pilot contamination
To avoid these issues, we analyze Massive SIMO without *instantaneous* CSI?
The setup

System model

- One shot transmission
- i.i.d. unit energy channel coefficients
- Unit energy noise
- Instantaneous CSIR and CSIT unknown
- **Energy detectors** at the receiver
The setup

Available information for fading and noise

We consider three cases:

- Exact distribution
- The first few moments
- The moments upto bounded uncertainty
The setup

\[ x \in \mathbb{C} \rightarrow \tilde{y} \in \mathbb{C}^n \rightarrow y = \frac{\|\tilde{y}\|^2}{n} \]

Our goal

To minimize BER or the error exponent

\[
\text{Error exponent} \triangleq \lim_{n \to \infty} -\frac{\log (\text{BER})}{n}
\]
Transmission and decoding: scheme 1

Energy decoders, so phase information lost

Transmit constellation $C$
Amplitude modulation (nodes are Tx energies)

Decoding regions for $y$
Transmission and decoding: scheme 1

Energy decoders, so phase information lost

Rate function $I(\cdot)$

$y$ concentrates around $p_i + \sigma^2$. $I(\cdot)$ captures “tail”

$$I_{i,R}(d) \triangleq \lim_{n \to \infty} \frac{-\log \left( \frac{\text{Prob}(y - p_i - \sigma^2 > d)}{n} \right)}{n}$$

We see

Larger energies more likely to be in error
Transmission and decoding: scheme 1

Energy decoders, so phase information lost

Can we make the constellation better?
Yes, by equalizing error exponents
Towards equalizing error exponents

\[ I(d) \approx w_i d^2 \text{ for small } d \]

\( w_i \) depends only on first four moments
Design for equal error exponents: scheme 2

- Fading, noise distribution not needed
- First four moments needed
- Efficient algorithms to optimize constellation

Optimized transmit constellation $C_2$

Histogram of received $y$

Larger energies further apart than in scheme 1
Design for equal error exponents: scheme 2

- Fading, noise distribution not needed
- First four moments needed
- Efficient algorithms to optimize constellation

Need precise knowledge of statistics
What if that information is not available?
Robust design: scheme 3

Moments not known, bounded uncertainty ($a$ dB)

- For small uncertainty, error exponent worse
- For large uncertainty, eventually hit error floor

Transmit constellation $C_3^{(a)}$

Histogram of received $y$ for small uncertainty

Lower energies are further apart than in scheme 2
Robust design: scheme 3

Moments not known, bounded uncertainty ($a$ dB)

- For small uncertainty, error exponent worse
- For large uncertainty, eventually hit error floor

Histogram of received $y$ for large uncertainty

We hit an error floor
Numerical plots

- SIMO systems today have 100 antennas
- dB units used for $K$ and SNR

Channel, noise parameters

- Channel coefficients are Rician with factor $K$
- Channel power gain is unity
- Noise energy is $\sigma^2$ ($\text{SNR} = \frac{1}{\sigma^2}$)
Optimized design

Number of antennas required for SER = 10^{-4}

![Graph showing the number of receiver antennas vs. constellation size for different channel conditions.]

- $C_2, (10, 20)$ channel
- $C_2, (10, 10)$ channel
- $C, (10, 10)$ channel
Optimized design

Number of antennas required for $\text{SER} = 10^{-4}$

Great performance with small no. of antennas
Robust design

Effects of channel mismatch

Severe performance loss without robust design
Robust design

Robust versus non robust with no uncertainty

Performance penalty of robust design is small
Noncoherent energy based communication systems have excellent performance and are robust!