Income and Wealth Distributions Along the Business Cycle: Implications from the Neoclassical Growth Model

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Abstract

This paper studies the business cycle dynamics of income and wealth distributions in the context of the neoclassical growth model where agents are heterogeneous in initial wealth and non-acquired skills. Our economy admits a representative consumer which enables us to characterize distributive dynamics by the evolution of aggregate quantities. We show that inequality in both wealth and income follow a countercyclical pattern: the former is countercyclical because labor income is more sensitive to the business cycle than capital income, while the latter is countercyclical due to the wealth-distribution effect. We find that the predictions of the model about the income distribution dynamics accord well with the U.S. data.

KEYWORDS: Neoclassical growth model, heterogeneous agents, aggregation, business cycle, income and wealth distributions, inequality

1 Introduction

This paper examines the business cycle behavior of income and wealth distributions in the context of the standard neoclassical stochastic growth model. We assume that the economy is populated by heterogeneous agents who differ in initial wealth and non-acquired skills. Under the assumptions of complete markets and identical homothetic individual preferences, our economy admits a representative consumer in the sense of Gorman (1953), so that income and wealth distributions are irrelevant for macroeconomic performance. In contrast, aggregate fluctuations fully determine the evolution of income and wealth distributions over the business cycle. We therefore focus on the role of aggregate fluctuations in distributive dynamics.[1]

We show that, at any point in time, the income and wealth distributions in our economy can be represented as a linear combination of the skill distribution and the initial wealth distribution. During expansions, the weights of the skill distribution in the income and wealth distributions increase over those of the initial wealth distribution. During recessions, the reverse holds. The empirical evidence indicates that the skill differentials across agents are, on average, lower than the wealth differentials, which leads us to conclude that inequality in income and wealth is countercyclical. That is, expansions are equalizing, and recessions are disequalizing.

In our economy, countercyclical behavior of wealth inequality can be understood by looking at how the wealth of different individuals is affected by business cycle fluctuations. A positive technology shock raises both capital income and labor income. However, in percentage terms, the former increases less than the latter. For "rich, low-productive" agents (i.e., those whose wealth share is high in relation to their skill share), labor income represents a small fraction of wealth, so that an increase in it has little impact on wealth. In contrast, for "poor, high-productive" agents, labor income constitutes a large fraction of wealth, so that its increase raises wealth considerably. As a result, "rich, low-productive" agents decrease their wealth shares, and "poor, high-productive" agents increase their wealth shares, and

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[1]Chatterjee (1994) and Caselli and Ventura (2000) also study the implications of a neoclassical growth model for income and wealth distributions employing the aggregation theory. These papers, however, concentrate on deterministic distributive dynamics over the process of economic development and neglect business cycle fluctuations. Maliar and Maliar (2001, 2003a) consider stochastic versions of the neoclassical growth model, but these papers focus on the role of distributions for aggregate dynamics.

wealth inequality therefore decreases. In turn, countercyclical behavior of income inequality is due to the wealth-distribution effect: a more equal distribution of wealth across agents leads to a more equal distribution of capital gains, which reduces income inequality.

We test the model's predictions on the business cycle dynamics of income distribution with U.S. data. First, we study the time-series properties of the distances between income and skill distributions and between income and wealth distributions. We obtain that the former distance is countercyclical while the latter is procyclical. We interpret these findings as evidence that supports the model's prediction that during expansions income distribution moves towards skill distribution, and during recessions it moves towards wealth distribution. Secondly, we calibrate and simulate the model, as is typically done in real business cycle literature. We find that the model can account for the degrees of income inequality and for the comovements of income inequality indices (quintiles and the Gini coefficient) with output, however, it underpredicts the volatility of income inequality indices relative to that shown by U.S. data.

To our minds, the main value of our exercise consists of benchmarking: we consider the simplest possible complete-market neoclassical economy where the property of aggregation allows us to analytically characterize the business cycle behavior of distributions. The benchmark can be compared with more sophisticated, realistic models which do not admit aggregation results and analytical characterization. This can provide an answer to the question: "How much is gained in terms of understanding the business cycle behavior of distributions by introducing new features in the model?" For example, Castañeda, Díaz-Giménez and Ríos-Rull (1998) study the business cycle dynamics of income distribution in a neoclassical framework with idiosyncratic shocks, incomplete markets and restrictions on borrowing. Unlike our model, theirs can account for the size of fluctuations in the income inequality indices in the U.S. data. Given the results of Castañeda et al. (1998), we conjecture that the above features of their model are essential for generating the appropriate volatility of income inequality.

The rest of the paper is laid out as follows: Section 2 formulates the model and summarizes the aggregation results. Section 3 studies the model's distributional implications. Section 4 discusses the empirical results, and Section 5 concludes.

2   The model

We consider a heterogeneous agents variant of the standard neoclassical stochastic growth model. Time is discrete and the horizon is infinite, $t \in T$, where $T = \{0, 1, ...\}$. The economy consists of a representative production firm and a set of infinitely-lived agents S. The measure of agent s in the set S is denoted by ds. The total measure of agents is one, $\int_S ds = 1$. There is a complete set of markets, i.e., agents are permitted to trade state-contingent claims to next-period output.

The representative firm owns the production technology, which is given by a constant return-to-scale Cobb-Douglas function, $y_t = \theta_t k_t^{\alpha} h_t^{1-\alpha}$, where y_t is output; k_t and h_t are the aggregate inputs of capital and labor, respectively; $\alpha \in (0, 1)$; and θ_t is an exogenous technology shock. The shock follows a first-order Markov process with a transitional probability given by $\Pr \{\theta_{t+1} = \theta' \mid \theta_t = \theta\}_{\theta', \theta \in \Theta}$, where Θ denotes the set of all the possible realizations of technology shocks. The firm maximizes period-by-period profits by choosing demands for capital and labor. The profit-maximizing conditions of the firm imply that the real return on capital, r_t, and the real wage, w_t, are equal to the marginal products of capital and labor inputs, respectively, i.e., $r_t = \alpha \theta_t k_t^{\alpha-1} h_t^{1-\alpha}$ and $w_t = (1 - \alpha) \theta_t k_t^{\alpha} h_t^{-\alpha}$.

We assume that the agents are endowed with one unit of time and that they do not value leisure.[2] Hence, the agents supply their time endowment inelastically to the market. Further, the agents are heterogeneous in initial wealth and non-acquired skills. The skills of agent s reflect the number of efficiency hours e^s that correspond to one physical hour worked by that agent. Note that individual skills are assumed to be constant over time and across states of nature. For the sake of convenience, we normalize the average level of skills to one, $\int_S e^s ds = 1$, so that the aggregate labor input is also equal to one, $h_t = 1$ for all t.

The period utility function of agent s is of the Constant Relative Risk Aversion (CRRA) type. The agent solves the following intertemporal utility-maximization problem:

$$\max_{\{c_t^s, k_{t+1}^s, m_{t+1}^s(\theta)\}_{\theta \in \Theta, t \in T}} E_0 \left[\sum_{t=0}^{\infty} \delta^t \frac{(c_t^s)^{1-\eta} - 1}{1 - \eta} \right] \tag{1}$$

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[2] The assumption that agents do not value leisure simplifies the analysis considerably. However, all the implications of our subsequent analysis carry over to the case of valued leisure, see Maliar, Maliar and Mora (2003).

subject to

$$c_t^s + k_{t+1}^s + \int_\Theta p_t(\theta) m_{t+1}^s(\theta) \, d\theta = (1 - d + r_t) k_t^s + m_t^s(\theta_t) + w_t e^s, \quad (2)$$

where the initial endowment $[(1 - d + r_0) k_0^s + m_0^s(\theta_0)] > 0$ is given. Here, E_t denotes the conditional expectation; c_t^s is consumption; k_{t+1}^s is the capital stock; $\{m_{t+1}^s(\theta)\}_{\theta \in \Theta}$ is the portfolio of state-contingent claims; $p_t(\theta)$ is the price of a claim that entitles the agent to the payment of one unit of consumption goods in period $t + 1$ if state θ occurs; $d \in (0, 1]$ is the depreciation rate of capital; $\delta \in (0, 1)$ is the discount factor; and finally, $\eta > 0$ is the coefficient of risk aversion.

We define an agent's wealth, Z_t^s, as the value of her end-of-period asset portfolio, expressed in terms of current consumption good,

$$Z_t^s \equiv k_{t+1}^s + \int_\Theta p_t(\theta) m_{t+1}^s(\theta) \, d\theta. \quad (3)$$

A competitive equilibrium is defined as a sequence of contingency plans for the consumers' allocation $\{c_t^s, Z_t^s\}_{t \in T}^{s \in S}$, for the firm's allocation $\{k_t\}_{t \in T}$ and for the prices $\{r_t, w_t, p_t(\theta)\}_{\theta \in \Theta, t \in T}$ such that, given the prices, the sequence of plans for the consumers' allocation solves each agent's utility maximization problem (1), (2); the sequence of plans for the firm's allocation makes the rental price of each input equal to its marginal product; all markets clear:

$$k_t = \int_S k_t^s ds, \qquad \int_S m_{t+1}^s(\theta) \, ds = 0; \quad (4)$$

and the economy's resource constraint is satisfied:

$$c_t + k_{t+1} = (1 - d) k_t + \theta_t k_t^\alpha, \quad (5)$$

where $c_t \equiv \int_S c_t^s ds$ is the aggregate consumption. Moreover, the equilibrium plans must be such that $c_t^s \geq 0$ for all s, θ, t, and w_t, r_t, $k_t \geq 0$ for all θ, t. We assume that equilibrium exists and that it is interior and unique.

Under our assumptions, there exists a representative consumer in the sense of Gorman (1953). This fact allows us to characterize the equilibrium in the heterogeneous agents economy in a simple way.

Proposition 1 *For the economy $(1) - (5)$, we have:*
i). The aggregate dynamics $\{c_t, k_{t+1}\}_{t \in T}$ are described by the representative

consumer model,

$$\max_{\{c_t, h_t, k_{t+1}\}_{t \in T}} E_0 \left[\sum_{t=0}^{\infty} \delta^t \frac{c_t^{1-\eta} - 1}{1-\eta} \right] \quad subject\ to \quad (5)\,; \qquad (6)$$

ii). Consumption of agents, $\{c_t^s\}_{t \in T}^{s \in S}$, satisfies

$$c_t^s = c_t f^s, \qquad (7)$$

where $\{f^s\}^{s \in S}$ is a set of positive numbers with $\int_S f^s ds = 1$; [3]
iii). Wealth of agents, $\{Z_t^s\}_{t \in T}^{s \in S}$, satisfies the lifetime budget constraints,

$$Z_t^s = E_t \left[\sum_{\tau=t+1}^{\infty} \delta^{\tau-t} \frac{c_\tau^{-\eta}}{c_t^{-\eta}} \left(c_\tau^s - w_\tau e^s \right) \right]. \qquad (8)$$

Proof. See Maliar and Maliar (2001), Appendices A and B. ■

Note that the variable "wealth" defined in (3) and given by (8) is financial wealth. Chatterjee (1994) uses a different concept of wealth, namely, lifetime wealth, which is given by the sum of financial wealth and the discounted lifetime labor income. He studies the evolution of lifetime wealth in the context of a deterministic neoclassical growth model where initial endowment is the only source of heterogeneity. The results of Chatterjee (1994) imply that if preferences are homothetic, then the distribution of lifetime wealth is constant over time and independent of aggregate state variables. Proposition 1 shows that this result is also true for our stochastic model. Indeed, formula (7) implies that the distribution of consumption in our model is constant over time and independent of the aggregate state variables, and formula (8) shows that the same applies to the distribution of lifetime wealth.

With the result of Proposition 1, we can find equilibrium in the heterogeneous agents economy $(1) - (5)$ in two steps: first, solve for the aggregate quantities from the representative consumer model (6) and secondly, restore the individual quantities from (7), (8). In the next section, we employ the representation $(6) - (8)$ to derive some useful analytical results regarding the evolution of the income and wealth distributions in the model.

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[3]The function $\{f^s\}^{s \in S}$ is related to the welfare weights $\{\lambda^s\}^{s \in S}$ in the associated planner's problem by $f^s = \frac{(\lambda^s)^{1/\eta}}{\int_{s \in S} (\lambda^s)^{1/\eta} ds}$.

3   Distributional implications of the model

Before analyzing the model's distributional implications, we should highlight certain aspects of the data. We focus on the empirical facts of income distribution dynamics because empirical evidence about evolution of wealth distribution is rather scarce. We restrict our attention to distributional regularities observed in the U.S. economy because our subsequent empirical study is carried out using U.S. data.

Income inequality in the U.S. economy displays both low-frequency movements and short-run (business cycle) fluctuations. As regards the long-run trend, income inequality falls during the first half of the 20th century but increases in the 1970's and 1980's (see, e.g., Caselli and Ventura, 2000, Piketty and Saez, 2003). The business cycle behavior of the U.S. income inequality is documented by, e.g., Castañeda, Díaz-Giménez and Ríos-Rull (1998). Using the Current Population Survey data, they calculate the correlations between output and the income shares of different income groups. For the lowest three quintiles, the income shares are procyclical, and the correlation of the income share with output monotonically decreases from the first to the third quintile. For the fourth quintile and for the next 15% of the population, the income shares are countercyclical. Finally, for the top 5% income earners, the income share is acyclical. Similar regularities in the business cycle dynamics of U.S. income quintiles are observed by Dimelis and Livada (1999) from U.S. Current Population Report data. The latter paper reports that the aggregate inequality measures of U.S. income distribution such as the Gini and Theil coefficients are weakly countercyclical.[4]

Our model cannot explain long-run inequality trends observed in the data (it produces no such trends by construction). However, the model is capable of generating non-trivial dynamics of income and wealth distributions over the business cycle. We therefore focus on the business cycle movements of these distributions.

We start by analyzing the model's implications for wealth distribution

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[4]The first empirical studies on the determinants of business cycle dynamics of income inequality date back to Mendershausen (1946) and Kuznets (1953) who found that U.S. income inequality followed a countercyclical pattern in the interwar period. Further evidence about cyclical properties of income inequality was provided by, e.g., Minarik (1979), Blank (1989). See Parker (1999) for a survey of this stream of literature.

dynamics. Consider the share of the total wealth held by agent s,

$$z_t^s \equiv \frac{Z_t^s}{\int_S Z_t^s ds} = \frac{k_{t+1}^s + \int_\Theta p_t(\theta) m_{t+1}^s(\theta) d\theta}{k_{t+1}}. \tag{9}$$

The fact that $\int_S Z_t^s ds = k_{t+1}$ follows from the market clearing condition for claims in (4). It turns out that there is a simple formula that characterizes the evolution of the wealth distribution in our economy. Specifically, we have the following proposition:

Proposition 2 *For all t, $v \geq 0$, we have*

$$z_t^s = \xi_{t,v} z_v^s + \left(1 - \xi_{t,v}\right) e^s, \tag{10}$$

where $\xi_{t,v}$ is defined by

$$\xi_{t,v} \equiv \frac{k_{v+1} E_t \left[\sum_{\tau=t+1}^\infty \delta^{\tau-t} \frac{c_\tau^{-\eta}}{c_t^{-\eta}} c_\tau \right]}{k_{t+1} E_v \left[\sum_{\tau=v+1}^\infty \delta^{\tau-v} \frac{c_\tau^{-\eta}}{c_t^{-\eta}} c_\tau \right]}. \tag{11}$$

Proof. See Appendix. ∎

According to (10), the wealth distribution in a period t can be represented as a linear combination of the wealth distribution in any other period v and the skill distribution. The movements of the variable $\xi_{t,v}$ capture the entire effect of the aggregate dynamics on the wealth distribution.

One straightforward implication of our analysis is that any wealth distribution can be supported in the steady state. Indeed, if the representative consumer economy (6) starts in the steady state, then we have that $\xi_{t,v} \equiv 1$ for all t, v and, therefore, the initial wealth distribution will be perpetuated, i.e., $z_t^s = z_0^s$ for all t and s. Another case in which the model has trivial implications with regard to the evolution of wealth distribution is when the initial wealth distribution coincides with the skill distribution. In such a case, the wealth distribution will always be the same, independently of the movements of the variable $\xi_{t,v}$.

Consider $v = 0$ and define $\xi_t \equiv \xi_{t,0}$. In Section 4.2, we show by simulation that the variable ξ_t moves countercyclically in the model. The intuition

behind this result can be seen in the example of the logarithmic utility function, $\lim_{\eta \to 1} \frac{c_t^{1-\eta}-1}{1-\eta} = \log(c_t)$. Under this assumption, expression (11) takes a simple form:

$$\xi_t = \frac{c_t/c_0}{k_{t+1}/k_1}. \tag{12}$$

Suppose that the economy experiences a positive technology shock in period t. On impact, both consumption, c_t, and capital, k_{t+1}, of the representative agent increase. However, given that the agent is risk-averse, consumption increases less than capital, so ξ_t goes down.

The fact that the variable ξ_t moves countercyclically implies that in our model, the agent's s wealth share, z_t^s, increases (decreases) during expansions, if her initial wealth endowment is lower than her skills, $z_0^s < e^s$ (higher than her skills, $z_0^s > e^s$). Unfortunately, we cannot test this prediction of the model because, as we have said, there is no reliable empirical evidence on the dynamics of the wealth distribution over the business cycle.

We now focus on the dynamics of income distribution. We define the individual's income, Y_t^s, as the sum of the returns on her asset portfolio and her labor earnings expressed in terms of current consumption good,

$$Y_t^s \equiv r_t k_t^s + m_t^s(\theta_t) + e^s w_t. \tag{13}$$

It follows from definition (13) that individual income depends on the composition of the agent's asset portfolio, i.e., on how much capital and how many units of claims of each type $\theta \in \Theta$ were purchased by the agent in the previous period. Note that in our economy the equilibrium composition of the individual asset portfolio is not uniquely determined.[5] As a result, there is indeterminacy in individual income.

This indeterminacy is due to the assumption of complete markets. In our economy, the agents are not concerned about how much income they receive in each period, but rather about how much income they receive over their lifetime. Consequently, the agents are indifferent between sequences of asset portfolios as long as they lead to the same expected lifetime payoff. To overcome the problem of indeterminacy, we need to impose some additional restrictions on the composition of the agents' portfolios. The identifying

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[5]The reason for the indeterminacy is that there are more assets traded in equilibrium (Θ types of claims and the capital stock) than there are states in the economy (Θ). As a consequence, one of the assets will be always a linear combination of the others.

restriction we use is that state-contingent claims are not traded, so that only capital stock is in operation. For each given agent, such a restriction is consistent with the utility maximization. Indeed, following the steps in Appendix A of Maliar and Maliar (2001), it can be shown that an agent who holds only capital stock faces the same lifetime budget constraint as one who holds both capital stock and state-contingent claims.

Let y_t^s be the share of the total income held by agent s,

$$y_t^s \equiv \frac{Y_t^s}{\int_S Y_t^s ds}. \tag{14}$$

We obtain the following result for income distribution.

Proposition 3 *For all $t \geq 1$ and $v \geq 0$, we have*

$$y_t^s = \vartheta_{t,v} z_v^s + (1 - \vartheta_{t,v}) e^s, \tag{15}$$

where $\vartheta_{t,v}$ is defined by

$$\vartheta_{t,v} \equiv \alpha \xi_{t-1,v}. \tag{16}$$

Proof. See Appendix. ∎

Hence, as occurs for wealth distribution, the income distribution in our economy is given by a linear combination of the wealth distribution in a certain period v and the skill distribution. Here, $\vartheta_{t,v}$ is the only aggregate variable which is needed to fully characterize the evolution of the income distribution. In fact, the steady state value of $\vartheta_{t,v}$ is around $\alpha \in (0, 1)$, which implies that near the steady state $\vartheta_{t,v}$ lies in the unit interval. We can therefore state that during expansions income distribution moves towards skill distribution, $\{e^s\}^{s \in S}$, while during recessions it moves toward wealth distribution, $\{z_v^s\}^{s \in S}$.[6]

Let us set $v = 0$ and denote $\vartheta_t \equiv \vartheta_{t,v}$. Since $\vartheta_t = \alpha \xi_{t-1}$, today's shock does not affect income inequality today but only tomorrow. As output fluctuations are persistent and the variable ξ_t is countercyclical, we conjecture that the correlation coefficient $corr(\vartheta_t, y_t) = corr(\xi_{t-1}, y_t)$ is negative, i.e.,

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[6]The same does not apply, however, to wealth distribution, because the variable $\xi_{t,v}$ does not, in general, belong to the unit interval: it is equal to 1 in period $t = v$, and it is typically smaller (larger) than 1 when the economy expands (contracts).

that the variable ϑ_t is also countercyclical. In Section 4.2, we also show this fact by simulation.

Countercyclical behavior of ξ_t and ϑ_t implies that wealth and income inequality in our economy is countercyclical. Indeed, the weights of the initial wealth distribution, given by ξ_t and ϑ_t in (10) and (15), respectively, decrease during expansions and increase during recessions. The opposite is true for the weights of the skill distribution, $1 - \xi_t$ and $1 - \vartheta_t$. The (initial) wealth distribution in the data, however, is more unequal than the skill distribution. For example, Quadrini and Ríos-Rull (1997) report that in the 1992 Survey of Consumption Financing data set, the wealth shares of the bottom 40% and the top 1% of the population are 2.2% and 28.2%, respectively, while the corresponding earnings (skill) shares are 10.3% and 14.1%, respectively. With the above empirical regularity, expansions (recessions) have an equalizing (disequalizing) effect on the income and wealth distributions.

Our results about the business cycle dynamics of wealth and income distributions are closely related to those of Caselli and Ventura (2000) who study the transitional dynamics of wealth and income distributions in the context of a continuous time growth model where agents are heterogeneous in initial endowments and skills. Caselli and Ventura (2000) also derive formulas that characterize the individual shares of wealth and income in terms of aggregate variables and the distribution of individual characteristics. For the Ramsey model with logarithmic utility function and the Cobb-Douglas production function, they demonstrate that during the transition from below toward the steady state, agents with a low capital-skill ratio tend to improve their relative position in the cross-section of wealth and income, i.e., there is convergence in wealth and income between agents. Their result that wealth and income inequality decrease over the process of economic development is parallel to our result that wealth and income inequality decrease in response to a positive technology shock.

We shall now analyze the mechanism behind the countercyclical behavior of inequality in the model. Formulas (10) and (15) are not suitable for this purpose because they characterize the evolution of the income and wealth distributions in terms of aggregate variables, and do not reveal what happens at the individual level. We therefore focus on the decisions of heterogeneous agents. By using budget constraint (2), we can re-write the wealth share of

agent s as follows:

$$z_t^s = \frac{k_{t+1}^s}{k_{t+1}} = \frac{(1-d+r_t)\,k_t z_{t-1}^s + w_t e^s - c_t^s}{(1-d+r_t)\,k_t + w_t - c_t}. \tag{17}$$

Suppose that the economy experiences a positive technology shock in period t. As seen from (17), a positive technology shock has three effects, namely, it increases the interest rate, wage and consumption. The former two effects increase wealth whereas the last effect reduces it, so the total effect depends on their relative sizes. Using the fact that $r_t = \alpha\theta_t k_t^{\alpha-1}$ and $w_t = (1-\alpha)\,\theta_t k_t^\alpha$, we can compute the implied percentage changes in capital income and labor income, $\frac{\partial \log\left[(1-d+r_t)k_t z_{t-1}^s\right]}{\partial \log \theta_t} = \frac{r_t}{1-d+r_t} < 1$ and $\frac{\partial \log(w_t e^s)}{\partial \log \theta_t} = 1$, respectively. Furthermore, from (7), we have that a percentage increase in consumption is the same for all agents, $\frac{\partial \log(c_t^s)}{\partial \log(\theta_t)} = \frac{\partial \log(c_t f^s)}{\partial \log(\theta_t)} = \frac{\partial \log(c_t)}{\partial \log(\theta_t)}$. Since labor income increases more in percentage terms than capital income does, "rich, low-productive" agents (i.e., those with $z_{t-1}^s > e^s$) increase their wealth less than "poor, high-productive" agents (i.e., those with $z_{t-1}^s < e^s$). This is precisely the mechanism that accounts for a reduction in wealth inequality in period t.

We now turn to income distribution. Under the assumption of the Cobb-Douglas production function, we have

$$y_t^s = \alpha z_{t-1}^s + (1-\alpha)\,e^s. \tag{18}$$

As formula (18) shows, individual income shares (and hence, income inequality) are not immediately affected by the shock. However, the t-period wealth distribution affects the $t+1$-period income distribution because wealth accumulated in t determines capital income in $t+1$. After a positive shock in period t, the income shares of agents with $z_{t-1}^s > e^s$ $\left(z_{t-1}^s < e^s\right)$ decrease (increase) in period $t+1$, i.e., the income distribution $\left\{y_{t+1}^s\right\}^{s\in S}$ moves in the direction of the skill distribution $\{e^s\}^{s\in S}$. Thus, income inequality falls in period $t+1$, as a consequence of a positive shock in period t. The countercyclical movement of income inequality in our model is therefore due to the wealth-distribution effect.[7]

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[7]In the context of their incomplete-market model, Castañeda et al. (1998) describe a different mechanism allowing the business cycle dynamics of income inequality to be explained, namely, they advocate the importance of unemployment spells and cyclically-moving factor shares.

The model's prediction that income inequality is countercyclical agrees with the previously discussed findings of Dimelis and Livada (1999), that the Gini and Theil coefficients of the U.S. income distribution are weakly countercyclical. The empirical evidence documented by Castañeda et al. (1998) indicates, however, that expansions have an ambiguous effect on income inequality. Specifically, inequality between the bottom and middle deciles goes down, while inequality between the middle and top deciles goes up. If the top-income group is excluded from the sample, the behavior of income inequality is countercyclical, as predicted by our model.[8] The empirical regularities discussed provide indirect evidence in support of relation (15) which, according to our model, describes the evolution of income distribution over the business cycle. In the following section, we test this relation directly by using household data.

4 Empirical analysis

In this section, we test the model's predictions on the business cycle dynamics of income distribution with U.S. data. We first test the model's key implication that income distribution moves towards skill distribution during expansions, and towards the (initial) wealth distribution during recessions. We then study the quantitative predictions of the model by simulation, as is typically done in real business cycle literature.

4.1 The distance between the distributions

To examine whether income distribution moves towards skill distribution during expansions, and towards the initial wealth distribution during recessions, we investigate the business cycle behavior of the distances between income and skill distributions and between income and wealth distributions. As a measure of the distance between two functions $G_1 : U \subseteq \mathbb{R} \to \mathbb{R}$ and $G_2 : U \subseteq \mathbb{R} \to \mathbb{R}$, we use the Kolmogorov distance (see, e.g., Shorack and Wellner, 1986, Section 2.1)

$$D\left(G_1, G_2\right) \equiv \sup_{u \in U} \left|G_1\left(u\right) - G_2\left(u\right)\right|. \tag{19}$$

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[8]The top-income group consists of executives who get high bonuses during expansions. Our model is obviously too simple to account for such evidence.

To characterize the empirical distributions of income, wealth and skills, we use two alternative representations, the Lorenz curve and the cumulative distribution function. Let $\{x^s\}^{s \in S}$ be a discrete and finite set. The Lorenz curve at points j/S, $j = 0, ..., S$, is defined by $L_{\{x^s\}}(0) = 0$ and $L_{\{x^s\}}(j/S) = \sum_{s=1}^{j} x^{(s)} / \sum_{s=1}^{S} x^s$, where $\{x^{(s)}\}^{s \in S}$ is the ascending sequence composed of the elements of $\{x^s\}^{s \in S}$. The Lorenz curve in other points of the interval $[0, 1]$ is obtained by linear interpolation. The cumulative distribution function $F_{\{x^s\}} : \mathbb{R} \to [0, 1]$ is defined by $F_{\{x^s\}}(u) \equiv \frac{1}{S} \sum_{s=1}^{S} I(x^s \leq u)$, where $I(A)$ is the indicator function, which is equal to 1 if event A occurs, and to 0 otherwise.

We interpret an agent in the model as a household in the data. Thus, the agent's income and wealth are those of the household. The agent's skills are proxied by the wage of the household's head.

We use the household data on the U.S. economy from the Panel Study of Income Dynamics (PSID) for 1967-1991. We take the following variables: the yearly household money income, Y_t^s; the average hourly wage of the head, W_t^s; and the household wealth available from the PSID for the years 1984 and 1989, Z_{1984}^s and Z_{1989}^s, respectively. We report only the results for the distances between income distribution and the 1989 wealth distribution, $\{z_{1989}^s\}^{s \in S}$. The results obtained with the 1984 wealth distribution are similar.

The skill distribution is assumed to be time-invariant in the model. However, it changes in the data over time. To compute the distance between the income and skill distributions, we therefore explore two alternatives: one in which the skill distribution changes over time (in each period t, it is proxied by t-period wage distribution), and another in which the skill distribution is the same for all periods (it is represented by the distribution composed of the data on wages over the entire sample period). We find that the distance between income and skill distributions displays similar business cycle properties in both cases. We report only the results obtained with the time-varying skill distribution, $\{e_t^s\}^{s \in S}$.

We also investigate whether the model's distributional implications hold for different income groups. Specifically, in each period t, we split the sample into five equal-sized income groups, i.e., the poorest 20%, the next 20%, etc. We denote these groups by $[0, 0.2]$, $[0.2, 0.4]$, $[0.4, 0.6]$, $[0.6, 0.8]$, $[0.8, 1.0]$ and refer to them as quintiles. Furthermore, to examine the evolution of the income shares of top-income earners, we consider three upper-income

groups separately, i.e., $[0.9, 0.95]$, $[0.95, 0.99]$, $[0.99, 1.0]$. For each income group $[b_1, b_2]$, we construct the Lorenz curves of the income and skill distributions, denoted by $L_{\{y_t^s\}}^{[b_1,b_2]}$ and $L_{\{e_t^s\}}^{[b_1,b_2]}$, respectively, and compute the distance between them, $D\left(L_{\{y_t^s\}}^{[b_1,b_2]},\ L_{\{e_t^s\}}^{[b_1,b_2]}\right)$.[9] The results obtained with the cumulative distribution functions are similar to those obtained with the Lorenz curves and are therefore not reported.

To assess the behavior of the distances over the business cycle, we study their comovement with output, y_t, which we define as real GDP. We take the GDP series from the website of the Federal Reserve Bank of Saint-Louis. We log and detrend the time series for distances and output by using the Hodrick-Prescott filter with a smoothing parameter of 100. This value of the smoothing parameter is chosen following Castañeda et al. (1998).

Figure 1 and Figure 2 plot the computed time series for the distances between the income and wealth distributions and between the income and skill distributions, respectively. In the left- and right-hand columns, we provide the undetrended series and the corresponding cyclical components, respectively. To show the business cycle behavior of the distance series, in the right-hand columns we also provide the detrended output series. Firstly, we note that the distances computed with the Lorenz curves and those computed with the cumulative distribution functions follow similar patterns, both in the long run and over the business cycle.

Regarding long-run behavior, we observe that income distribution moves from a relatively equal skill distribution to a relatively unequal wealth distribution, which implies that income inequality rises over the sample period. As pointed out before, long-run tendencies like this cannot be explained in the context of our stationary model.

As far as business cycle dynamics are concerned, we detect a pronounced countercyclical pattern of the distance between income and skill distributions. In the case of the distance between income and wealth distributions, the evidence is not so strong, although a certain procyclical pattern can be perceived. To make a better judgement, we compute the sample correlations between the detrended distance and output series. The results are reported in Table 1. As can be seen, the correlation of output with the distance be-

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[9] We cannot proceed by computing the time series for distances between the income and wealth distributions of different income groups the same way. Given that our income groups are composed of distinct households in different periods, we can only compute such distances for 1984 and 1989, for which years data on wealth are available.

tween the income and skill distributions is about -0.6, while the correlation of output with the distance between the income and wealth distributions is about 0.2. Taken as a whole, these findings are favorable to our theoretical model.

In Figure 3 and Figure 4, we plot the distance series corresponding to the income quintiles and the three upper-income groups, respectively. In Table 1, we report the correlations of the distances with output. In general, the results for the income groups are similar to those for the entire sample: the distances show an upward long-run trend and move countercyclically. The only exception is the $[0.9, 0.95]$ income group for which the distance between the income and skill distributions is weakly procyclical.[10]

Finally, we verify the robustness of our findings with respect to the concept of distance used. We specifically repeat all the previous computations using an alternative measure of distance, i.e., the Wasserstein distance, defined as $\left[\int_U \left\{ G_1 \left(u \right) - G_2 \left(u \right) \right\}^2 du \right]^{1/2}$ (see, e.g., Shorack and Wellner, 1986, Section 2.6). The results we obtain with the Wasserstein distance prove to be very close to those obtained with the Kolmogorov distance.

4.2   Simulation results

We next assess the distributional implications of the model by simulation. We calibrate the aggregate parameters of the model in the standard way for the real business cycle literature. To be specific, we set the model's period at one quarter, and we set the following values of the parameters: the capital share in production is $\alpha = 0.36$, the discount factor is $\delta = 0.993$, the depreciation rate of capital is $d = 0.0217$, the persistence of technology shock is $\rho = 0.95$, and the standard deviation of shock is $\sigma = 0.00712$. Under this parameter choice, the model is consistent with the key observations on the U.S. economy, see Maliar and Maliar (2003a). We consider three alternative values for the coefficient of risk aversion such as $\eta \in \{0.5, 1, 5\}$. To solve the model, we use den Haan and Marcet's (1990) version of the parameterized expectation algorithm. In order to enforce convergence, we bound the solution along iterations, as described in Maliar and Maliar (2003b).

Once the aggregate solution is computed, we convert the quarterly aggregate series into the corresponding yearly series. We then construct the

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[10]The procyclical behavior of the distance here is presumably related to the previously mentioned fact that executives get high bonuses during expansions.

variable $\vartheta_{t,v}$ and restore the evolution of the income distribution by using (15). To parameterize the skill and wealth distributions in the model, we use the 1989 PSID data. We remove households whose head has no labor income and whose wealth is negative. In Table 2, we illustrate the properties of the skill and wealth distributions by reporting their quintiles.

In Table 3, we provide the first- and the second-moment properties of the income distribution produced by the model. For the sake of comparison, we also report the corresponding statistics on the U.S. income distribution which we compute from the PSID data set over the 1967-1991 period. The model's statistics are sample averages of the corresponding variables computed for each of 500 simulations. Each simulation has a duration of 25 periods, as do the time series for the U.S. economy. Numbers in brackets are the sample standard deviations of the corresponding statistics. The second moments for both the U.S. and the artificial economies are computed after logging and detrending the corresponding variables by using the Hodrick-Prescott filter with a smoothing parameter of 100. It turns out that the model's predictions about income distribution are not particularly affected by the coefficient of risk aversion assumed, so our subsequent discussion applies to all the cases considered.

As the first panel of Table 3 shows, the quintiles and the Gini coefficient of the income distribution in our model are close to those observed in the data. In this respect, our model performs better than that of Castañeda et al. (1998), which considerably understates the degrees of income inequality. Our model is successful in reproducing the first-moment properties of the U.S. income distribution because under the assumption of complete markets, it exactly matches the U.S. wealth distribution; the large differences in wealth (which can be appreciated from Table 2) lead to large differences in income across agents. In contrast, the incomplete-market model of Castañeda et al. (1998) generates wealth distribution endogenously; the predicted differences in wealth across agents are however too small and, as a consequence, the income differences are also understated.[11]

Furthermore, as is seen from the second panel of the table, our model produces too low volatility of the income inequality indices compared to that in the U.S. economy. This is not surprising given that there is just one

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[11]The incomplete-market setup that can generate realistic degrees of wealth inequality is a version of Krusell and Smith's (1998) model where agents are heterogeneous in their patience. It would be interesting to study the business cycle behavior of distributions in such a model.

source of uncertainty in our model, which is aggregate productivity shocks; moreover, in the presence of complete markets, agents can perfectly insure themselves against aggregate uncertainty. The introduction of idiosyncratic shocks together with incomplete markets could help us to improve on the above shortcoming.

Finally, as the third panel of the table indicates, our model can generate correlations of the income inequality indices with output, which are in line with their empirical counterparts. Under all the paramaterizations considered, these correlations follow an identical pattern: their absolute values practically coincide with the value of the correlation coefficient between ϑ_t and output; the signs are positive for the first four income quintiles and are negative for the last income quintile; and finally, the sign is negative for the Gini coefficient. We can understand this pattern with the results of Proposition 3 and formula (15). The evolution of income distribution in our model is driven by just one aggregate variable, ϑ_t, which explains the same size of the correlations. As far as the signs of the correlations are concerned, condition (15) implies that for each agent s, we have

$$corr\left(y_t^s, y_t\right) = sign \mid z_0^s - e^s \mid corr\left(\vartheta_t, y_t\right). \tag{20}$$

where $sign \mid x \mid$ is the sign of a variable x. Given that ϑ_t moves countercyclically, according to formula (20), the sign of $corr\left(y_t^s, y_t\right)$ for each agent s is opposite to $sign \mid z_0^s - e^s \mid$, i.e., y_t^s is procyclical (countercyclical) whenever $z_0^s < e^s$ $(z_0^s > e^s)$. As appears from Table 2, it is, on average, true that $z_0^s < e^s$ for the first four income quintiles, and that $z_0^s > e^s$ for the last income quintile. This explains why the correlations for the first four quintiles are positive whereas the correlation for the last quintile is negative. The fact that the correlation of the Gini coefficient with output is negative is due to countercyclical behavior of income inequality in our model.

5   Conclusion

This paper studies the business cycle behavior of income and wealth distributions in a heterogeneous agents version of the standard neoclassical growth model. Heterogeneity is in two dimensions: initial endowment and non-acquired skills. We show that if markets are complete and agents have identical preferences of the CRRA type, the evolution of the income and wealth distributions in the model is fully characterized by the dynamics of

the associated representative consumer setup. This result implies that the income distribution approaches the skill distribution and the initial wealth distribution during expansions and recessions, respectively, which suggests that income inequality follows a countercyclical pattern. We find that this implication of the model agrees well with U.S. data. Furthermore, we find that a calibrated version of the model can account for the key features of the business cycle behavior of the income inequality indices in the U.S. economy.

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6   Appendix

In this section, we provide the proofs of Propositions 2 and 3 in the main text.

Proof of Proposition 2   By introducing variables z_t^s and k_{t+1} and by substituting (7) in recursive constraint (8), we obtain

$$z_t^s k_{t+1} = E_t \left[\sum_{\tau=t+1}^{\infty} \delta^{\tau-t} \frac{c_\tau^{-\eta}}{c_t^{-\eta}} \left(c_\tau f^s - w_\tau e^s \right) \right]. \tag{21}$$

Formula (21) can be re-written as

$$z_t^s k_{t+1} = E_t \left[\sum_{\tau=t+1}^{\infty} \delta^{\tau-t} \frac{c_\tau^{-\eta}}{c_t^{-\eta}} \left(c_\tau \left(f^s - e^s \right) + \left(c_\tau - w_\tau \right) e^s \right) \right]. \tag{22}$$

Summing (21) over the set of agents yields

$$k_{t+1} = E_t \left[\sum_{\tau=t+1}^{\infty} \delta^{\tau-t} \frac{c_\tau^{-\eta}}{c_t^{-\eta}} \left(c_\tau - w_\tau \right) \right]. \tag{23}$$

After combining (22) and (23), we have

$$z_t^s = e^s + \frac{\left(f^s - e^s \right)}{k_{t+1}} E_t \left[\sum_{\tau=t+1}^{\infty} \delta^{\tau-t} \frac{c_\tau^{-\eta}}{c_t^{-\eta}} c_\tau \right]. \tag{24}$$

As condition (24) is to be satisfied for all t, we can write the same condition for a period $v \neq t$. By combining (24) written for t and v, we can eliminate the term $\left(f^s - e^s \right)$. This gives us equations (10) and (11) in the main text.

Proof of Proposition 3   Under the assumption of the Cobb-Douglas production function, the dynamics of the income share are described by (18). Furthermore, according to (10), the individual wealth share at $t-1$ is given by

$$z_{t-1}^s = \xi_{t-1,v} z_v^s + \left(1 - \xi_{t-1,v} \right) e^s. \tag{25}$$

By substituting (25) into (18), we obtain equations (15) and (16).

Table 1. Selected second moment properties of the distances between the income and skill distributions and between the income and wealth distributions in the U.S. economy.

x_t	σ_x/σ_y	$corr\,(x_t, y_t)$
Total sample		
$D\left(L_{\{y_t^s\}}, L_{\{e_t^s\}}\right)$	2.9714	-0.6387
$D\left(F_{\{y_t^s\}}, F_{\{e_t^s\}}\right)$	1.9159	-0.6121
$D\left(L_{\{y_t^s\}}, L_{\{z_{1989}^s\}}\right)$	0.3657	0.1916
$D\left(F_{\{y_t^s\}}, F_{\{z_{1989}^s\}}\right)$	0.7183	0.1806
Income quintiles		
$D\left(L_{\{y_t^s\}}^{[0,0.2]},\ L_{\{e_t^s\}}^{[0,0.2]}\right)$	2.2757	-0.2900
$D\left(L_{\{y_t^s\}}^{[0.2,0.4]},\ L_{\{e_t^s\}}^{[0.2,0.4]}\right)$	2.5517	-0.6080
$D\left(L_{\{y_t^s\}}^{[0.4,0.6]},\ L_{\{e_t^s\}}^{[0.4,0.6]}\right)$	2.1854	-0.4963
$D\left(L_{\{y_t^s\}}^{[0.6,0.8]},\ L_{\{e_t^s\}}^{[0.6,0.8]}\right)$	1.6651	-0.3951
$D\left(L_{\{y_t^s\}}^{[0.8,1]},\ L_{\{e_t^s\}}^{[0.8,1]}\right)$	2.3840	-0.2888
Top income groups		
$D\left(L_{\{y_t^s\}}^{[0.9,0.95]},\ L_{\{e_t^s\}}^{[0.9,0.95]}\right)$	2.2877	0.1401
$D\left(L_{\{y_t^s\}}^{[0.95,0.99]},\ L_{\{e_t^s\}}^{[0.95,0.99]}\right)$	3.8590	-0.4786
$D\left(L_{\{y_t^s\}}^{[0.99,1]},\ L_{\{e_t^s\}}^{[0.99,1]}\right)$	10.5284	-0.1039

Notes: σ_x is the standard deviation of a variable x_t; $corr\,(x_t, y_t)$ is the correlation coefficient between variables x_t and y_t. All the statistics are computed after logging and detrending the corresponding variables by using the Hodrick-Prescott filter with a smoothing parameter of 100.

Table 2. Shares of income and wealth owned by five quintiles of income holders in 1989.

The income quintile, %	$[0-20]$	$[20-40]$	$[40-60]$	$[60-80]$	$[80-100]$
The skills share, %	8.2630	13.2099	18.0355	23.5046	36.9944
The wealth share, %	2.3860	6.0518	9.1639	14.9740	67.4177

Notes: Statistics in the first row are income quintiles; statistics in the second the third rows are the percentage shares of skills and wealth, respectively.

Table 3. Selected statistics for the U.S. and the artificial economies.

x_t	Model: $\eta = .5$	Model: $\eta = 1$	Model: $\eta = 5$	U.S. economy
First-moment properties: the mean, μ_x				
$[0, 20]$	4.7790 (0.0107)	4.7645 (0.0111)	4.7458 (0.0145)	4.0960
$[20, 40]$	9.0414 (0.0168)	9.0187 (0.0174)	8.9893 (0.0228)	9.7224
$[40, 60]$	13.6356 (0.0221)	13.6057 (0.0229)	13.5672 (0.0299)	16.1210
$[60, 80]$	20.0200 (0.0230)	19.9888 (0.0238)	19.9487 (0.0310)	24.3527
$[80, 100]$	52.5349 (0.0726)	52.6333 (0.0753)	52.7601 (0.0984)	45.6092
$Gini$	0.4733 (0.0008)	0.4743 (0.0008)	0.4757 (0.0011)	0.5855
ξ_t	1.0183 (0.0062)	1.0265 (0.0065)	1.0373 (0.0079)	–
ϑ_t	0.3664 (0.0021)	0.3692 (0.0021)	0.3727 (0.0028)	–
Second-moment properties: the volatility, σ_x/σ_y				
$[0, 20]$	0.1406 (0.0211)	0.1990 (0.0214)	0.1350 (0.0204)	1.1827
$[20, 40]$	0.1166 (0.0176)	0.1651 (0.0179)	0.1120 (0.0170)	0.4904
$[40, 60]$	0.1018 (0.0153)	0.1436 (0.0156)	0.0972 (0.0147)	0.3021
$[60, 80]$	0.0722 (0.0108)	0.1018 (0.0109)	0.0689 (0.0102)	0.3057
$[80, 100]$	0.0867 (0.0131)	0.1219 (0.0133)	0.0823 (0.0122)	0.3688
$Gini$	0.1044 (0.0158)	0.1469 (0.0160)	0.0922 (0.0147)	0.2662
ξ_t	0.3703 (0.0592)	0.5138 (0.0615)	0.3403 (0.0563)	–
ϑ_t	0.3526 (0.0537)	0.4920 (0.0545)	0.3302 (0.0477)	–
Second-moment properties: the correlation coefficient, $corr\,(x_t, y_t)$				
$[0, 20]$	0.2663 (0.0424)	0.2391 (0.0349)	0.1064 (0.0519)	0.4004
$[20, 40]$	0.2668 (0.0422)	0.2393 (0.0347)	0.1068 (0.0514)	0.3602
$[40, 60]$	0.2673 (0.0421)	0.2401 (0.0344)	0.1080 (0.0517)	0.2695
$[60, 80]$	0.2668 (0.0424)	0.2399 (0.0348)	0.1081 (0.0527)	−0.0039
$[80, 100]$	−0.2680 (0.0420)	−0.2409 (0.0343)	−0.1098 (0.0525)	−0.3800
$Gini$	−0.2679 (0.0420)	−0.2408 (0.0343)	−0.1096 (0.0524)	−0.3847
ξ_t	−0.7869 (0.0459)	−0.8740 (0.0388)	−0.7585 (0.0421)	–
ϑ_t	−0.2701 (0.0413)	−0.2432 (0.0335)	−0.1143 (0.0533)	–

Notes: μ_x and σ_x are the mean and the standard deviation of a variable x_t, respectively; $corr\,(x_t, y_t)$ is the correlation coefficient between variables x_t and y_t. The second moments are computed after logging and detrending the corresponding variables by using the Hodrick-Prescott filter with a smoothing parameter of 100.

Figure 1. The distance between the income and skill distributions in the U.S economy.
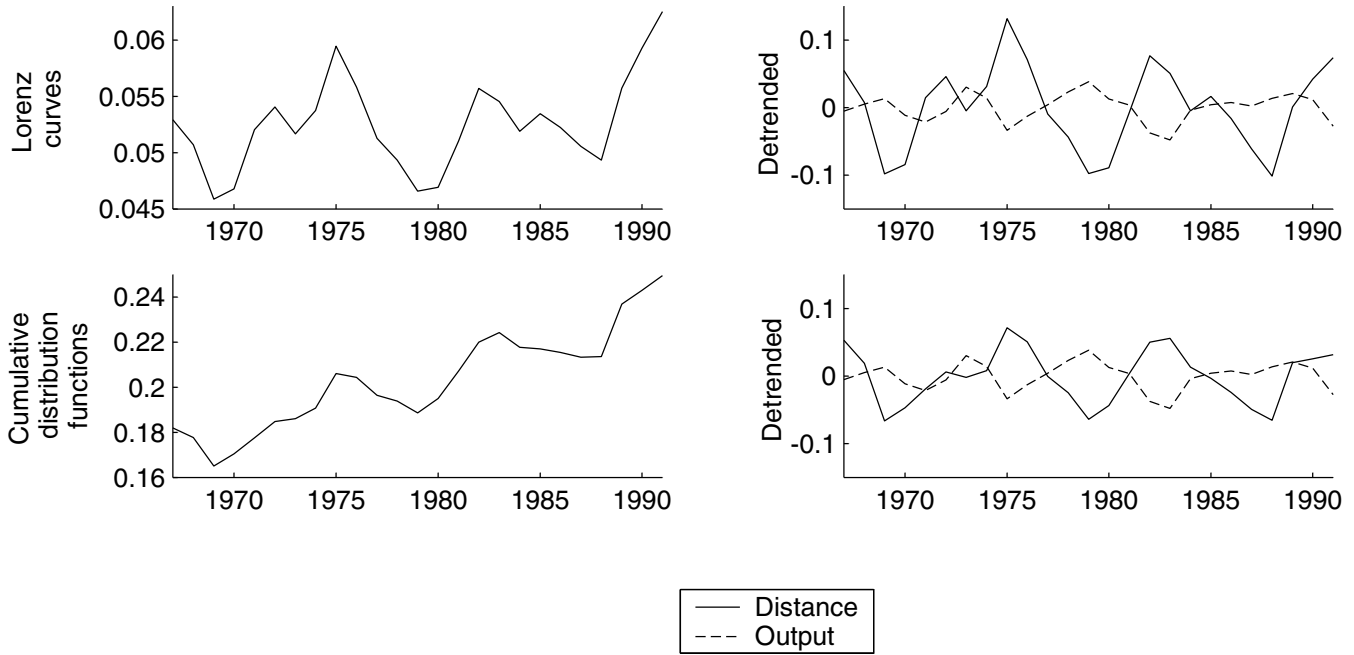
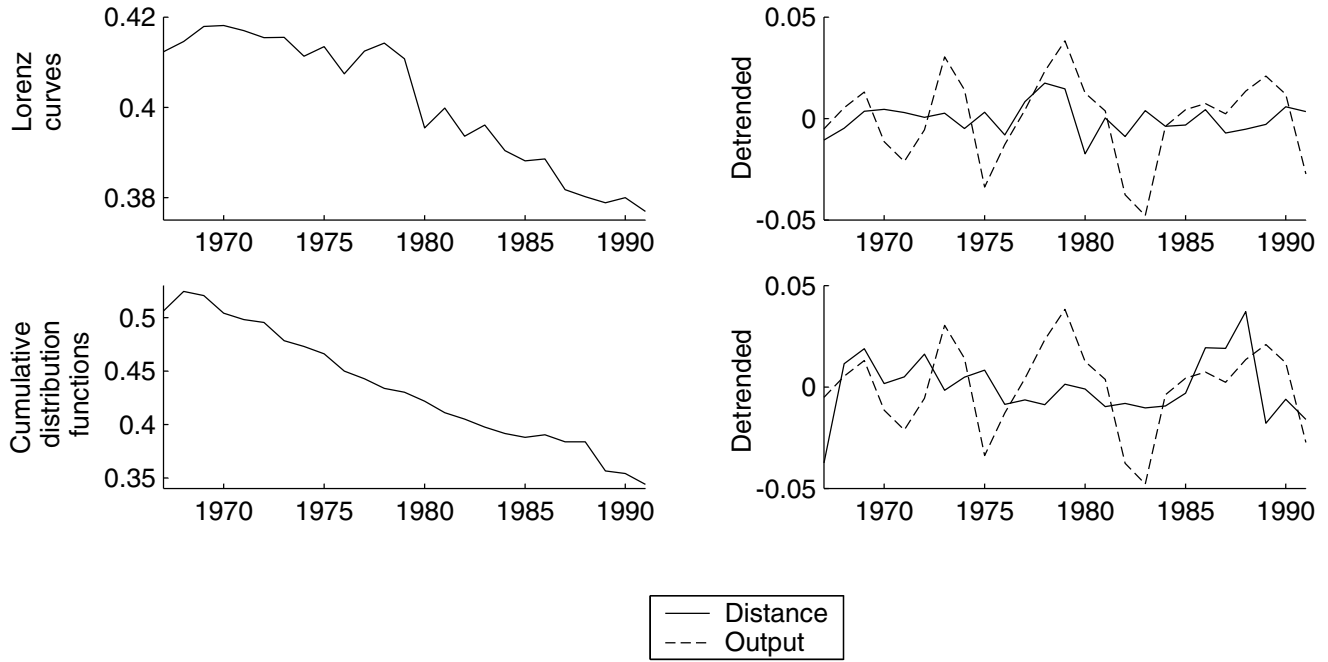Figure 2. The distance between the income and wealth distributions in the U.S economy.

Figure 3. The distance between the Lorenz curves of the income and skill distributions
in the U.S economy: income quitiles.
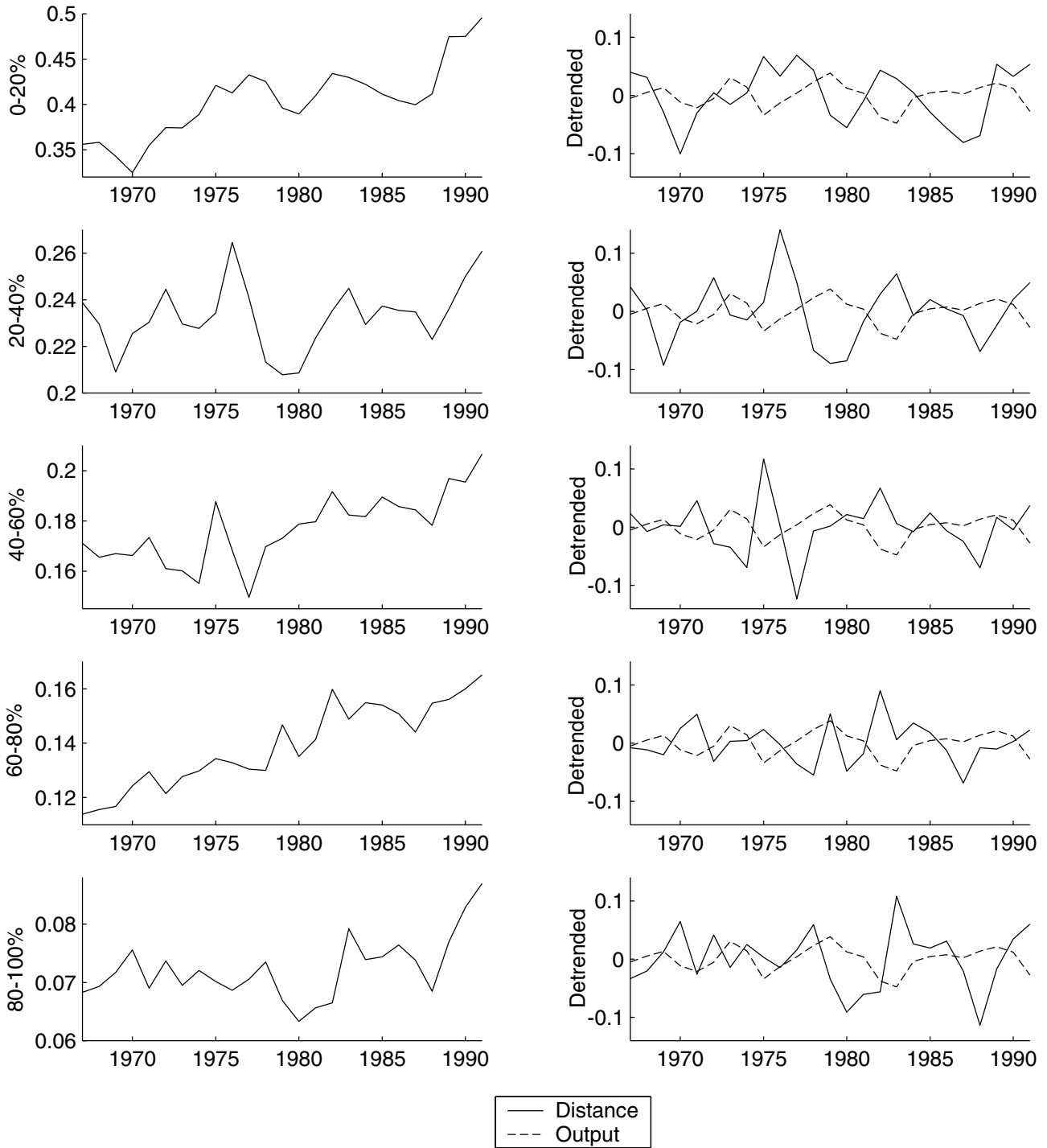

Distance
Output

Figure 4. The distance between the Lorenz curves of the income and skill distributions
in the U.S economy: top income groups.