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*Differential Responses of Labor Supply across Productivity Groups**

There is a substantial amount of microeconomic evidence documenting differential responses of labor supply across productivity groups. In particular, more productive individuals: (*i*) enjoy a higher employment rate, (*ii*) have a lower volatility of employment and (*iii*) spend less time working at home. This paper constructs a real business cycle model with permanent heterogeneity in individual productivity. We calibrate the model with five productivity groups to match key aggregate features of the U.S. economy. We find that the model delivers most of the properties of the data.

1. Introduction

There is a substantial amount of evidence at the micro level documenting differential responses of labor supply across productivity groups. In particular, more productive individuals: (*i*) enjoy a higher employment rate, (*ii*) have a lower volatility of employment and (*iii*) spend less time working at home.

Concerning fact (*i*), in the U.S. the male unemployment rate for professional, technical and managerial workers is 1.8%, while the same rate for laborers is 10%; in the U.K., male unemployment rates for non-manual and for unskilled labor are 2.3% and 18.7% respectively (see Johnson and Layard 1986, Tables 16.5, 16.7). Similar tendencies are observed for other data sets; see, for example, Moscarini (1995).

Many empirical studies provide evidence in support of fact (*ii*). For instance, Ríos-Rull (1993) illustrates this fact by using age as a proxy for individual productivity, Rosen (1968) by looking at a particular industry and Kydland (1984) by studying the educational levels of employees. Using data for different age-sex groups, Hansen (1993) constructs labor-input series

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where individual workers are weighted by relative hourly earnings. He finds that the resulting efficiency units series display smaller fluctuations than the physical hours series. One will expect this result if low skilled workers represent a larger fraction of the labor force during the expansions than during the recessions.

Fact (*iii*) is documented by Ríos-Rull (1993). He partitions the PSID data on U.S. households into five productivity groups and calculates the average hours worked at home by each group. The results indicate that the workers with the highest productivity level work almost three times less at home than workers from the lowest productivity group. He also provides some evidence on the difference in hours of home work by age-groups; given that old workers are less productive, more hours worked at home for this group also supports this fact.

This paper constructs a simple model with heterogeneous agents which explains the above facts. Most features of our setup are standard in the real business cycle (RBC) literature. In particular, we assume rational expectations, endogenous production, a competitive environment, full information, a stochastic technology and complete markets. The economy is populated by agents who have different abilities in producing the output good, and it is assumed that the differences in productivity across agents are permanent. We assume that agents can work in the market only a fixed number of hours but are free to choose the amount of time devoted to working at home. The labor choice is modeled as in Hansen (1985) and Benhabib, Rogerson and Wright (1991).

Besides the standard features, our model has an important new element. We assume that individuals can affect the probability with which they receive a job offer through (costly) variations in search intensity. This modification allows us to overcome the problem of nonuniqueness of equilibria in Hansen's (1985) model when applied to heterogeneous agents settings.[1] We introduce the cost in terms of time; that is, we assume that the probability of being employed is determined by time spent on job search. The more individuals search, the higher is the probability of them finding a job. In terms of Hansen's lotteries, this labor arrangement is equivalent to par-

[1]Nonuniqueness can occur in the heterogeneous agents model for the following reason. Consider Hansen's model with homogeneous agents at the steady state and assume, as an example, that the probability of employment is 1/2. If agents do not discount the future, an agent's two-period expected utility is $2 \cdot [u^e/2 + u^u/2]$, where u^e, u^u are utilities in two states. Observe that there is another allocation that gives the same two-period utility: in the first period a half of the population work and the other half is on vacation, and in the second, the groups interchange, that is, $[1 \cdot u^e + 0 \cdot u^u] + [0 \cdot u^e + 1 \cdot u^u]$. In principle, the two allocations can not be ranked. Hansen (1985) exploits the homogeneity of agents and picks up only the symmetric allocation. A similar argument can not be applied in the heterogeneous agents case.

ticipating in employment lotteries whose probabilities of success depend on the intensity of search.

There is a large body of literature incorporating heterogeneity in productivity in the RBC models. For example, Cho and Rogerson (1988) and Prasad (1996) construct models with two-member families in which the family members differ in skills. Cho (1995) develops a version of neoclassical model with temporary heterogeneities in individual productivity, and Merz (1996) introduces idiosyncratic productivity shocks in a model with matching frictions. These papers demonstrate that incorporating the heterogeneity in productivity helps to improve on aggregate predictions of the existing models. Also, they show that the RBC models can produce fluctuations of physical units of labor which are larger than those of efficiency units and, thus, can account for the empirical findings of Hansen (1993). This literature, however, does not provide a framework for studying differences in labor decisions across productivity groups and, consequently, does not make it possible to explain the stylized facts outlined above.

Relatively few papers consider models which provide a way to work both at the aggregate and at the individual levels. Kydland (1984) introduces two types of agents, skilled and unskilled, in a standard divisible labor setup. He finds that the volatility of working hours is lower for skilled workers than for unskilled. Ríos-Rull (1993) considers the two-period overlapping generations model with perfectly divisible labor and home production. In his setup, ex ante homogeneous agents acquire skills and thus make different labor choices. The model predicts that skilled individuals work more hours in the market and less at home than unskilled ones. However, his model fails to account for a lower volatility of market hours of skilled workers over the business cycle.

In general, solving RBC models with heterogeneous agents is a complicated task. To simplify the solution procedure, we exploit results from aggregation theory. To be precise, starting from the individual maximization problems, we derive relationships which describe the economy's aggregate behavior in terms of aggregate variables and known productivity parameters. The property of aggregation allows us to solve the model with several agents at low computational costs.

We calibrate the model with five heterogeneous consumers to match key aggregate features of the U.S. economy. The results from simulations show that the model is successful at replicating the stylized facts outlined in the beginning. Specifically, it predicts that high productive agents have a higher employment rate, experience lower fluctuations in employment and work less at home. Also, the model does reasonably well at reproducing cyclical behavior of macroeconomic aggregates in the U.S. economy.

The paper is organized as follows. Section 2 describes the economy and defines the equilibrium. Section 3 derives the equilibrium conditions. Section 4 discusses calibration and simulation procedures. Section 5 reports the results from simulations. Section 6 concludes.

2. The Model

The economy consists of S types of infinitely lived heterogeneous consumers, an output producing firm and an insurance company. The share of a type $s \in S$ in the total population is $d\mu_s$, $\int_S d\mu_s = 1$. Within each type there is a continuum of identical consumers with names on the unit interval. Agents are heterogeneous across types with respect to their labor productivity. The distribution of productivity parameters, $\{e_s\}_{s \in S}$, is exogenously given and does not change with time; for convenience, we assume $\int_S e_s d\mu_s = 1$.

The representative firm runs a production technology with two inputs, capital, k_t, and efficiency labor, n_t, both of which it rents from households. The production is subject to a multiplicative technology shock, θ_t. The firm maximizes period-by-period profits

$$\max_{\{k_t, n_t\}} \pi_t^p = \theta_t f(k_t, n_t) - r_t k_t - w_t n_t \ , \tag{1}$$

where r_t and w_t are the interest rate and real wage, respectively. The production function f has constant returns to scale, is concave, continuously differentiable, strictly increasing with respect to both arguments and satisfies the Inada conditions; θ_t follows a first-order Markov process. The initial level of technology θ_0 is given.

A representative consumer of type $s \in S$ (further, consumer, agent, etc.) maximizes expected life-time utility discounted at the rate $\delta \in (0, 1)$ by choosing leisure and consumption of market and home-made goods. The agent owns the capital stock and rents it to the firm. The capital depreciates at the rate $d \in (0, 1]$. In the beginning of each period, the agent is jobless. To find a job, (s)he needs to search. Job opportunities come at random, depending on individual search time and simple luck. "Good" luck means that the agent gets a job and supplies a fixed number of hours, $\bar{n}$, in exchange for the efficiency wage. In the case of "bad" luck, (s)he does not work in the market. Whether the agent works in the market or not, (s)he can work at home.

Markets are complete; that is, the agent can insure himself against unemployment as well as against aggregate uncertainty. In the beginning of each period, (s)he buys unemployment insurance. In the same period, the insurance contract pays out one unit of consumption if the agent is unemployed and zero otherwise. A one-period-ahead contingent claim which allows the agent to insure against the aggregate productivity shock $\theta' \in \Theta$ pays one unit of consumption good in period $t + 1$ if the shock $\theta_{t+1} = \theta'$ and nothing otherwise; here, Θ denotes the set of all possible realizations of the technology shock.

Therefore, the problem solved by the agent is the following:

$$\max_{\{x_{ts}\}} E_0 \sum_{t=0}^{\infty} \delta^t \{\varphi(\pi_{ts})U(c_{ts}^{me}, c_{ts}^{he}, l_{ts}^{e}) + (1 - \varphi(\pi_{ts}))U(c_{ts}^{mu}, c_{ts}^{hu}, l_{ts}^{u})\} , \quad (2)$$

subject to

$$\begin{aligned} c_{ts}^{me} + k_{t+1s}^{e} + p_{ts}y_{ts} + \int_{\Theta} q_t(\theta)m_{t+1s}^{e}(\theta)d\theta = \\ k_{ts}(1 - d + r_t) + \bar{n}e_s w_t + m_{ts}(\theta_t) ; \end{aligned} \quad (3)$$

$$\begin{aligned} c_{ts}^{mu} + k_{t+1s}^{u} + p_{ts}y_{ts} + \int_{\Theta} q_t(\theta)m_{t+1s}^{u}(\theta)d\theta = \\ k_{ts}(1 - d + r_t) + y_{ts} + m_{ts}(\theta_t) ; \end{aligned}$$

$$\begin{aligned} l_{ts}^{e} &= 1 - \bar{n} - h_{ts}^{e} - \pi_{ts} ; \\ l_{ts}^{u} &= 1 - h_{ts}^{u} - \pi_{ts} ; \end{aligned} \quad (4)$$

$$\begin{aligned} c_{ts}^{he} &= g(h_{ts}^{e}) ; \\ c_{ts}^{hu} &= g(h_{ts}^{u}) ; \end{aligned} \quad (5)$$

where $\{x_{ts}\} = \{\pi_{ts}, c_{ts}^{mj}, h_{ts}^{j}, k_{t+1s}^{j}, y_{ts}, \{m_{t+1s}^{j}(\theta)\}_{\theta\in\Theta}\}_{t\in T}^{j\in\{e,u\}}$. Here, the superscript $j \in \{e, u\}$ refers to employed and unemployed states; l_{ts}^{j}, c_{ts}^{mj} and c_{ts}^{hj} denote leisure and consumption of market and home-produced goods chosen by the agent in state j. The variables k_{t+ls}^{j}, y_{ts}, $\{m_{t+1s}^{j}(\theta)\}_{\theta\in\Theta}$ denote individual holdings of capital, unemployment insurance and contingent claims, respectively. The price of one unit of unemployment insurance is p_{ts}, and the price of a contingent claim $\theta \in \Theta$ in period t is given by $q_t(\theta)$. The home technology is represented by the function $g(h_{ts}^{j})$, where h_{ts}^{j} is the time spent by the individual on working at home. Variable π_{ts} denotes the time dedicated to searching for a job in period $t \in T$. This time determines the probability of employed and unemployed states, $\varphi(\pi_{ts})$ and $(1 - \varphi(\pi_{ts}))$. The function φ satisfies $\varphi' > 0$, $\varphi'' < 0$; that is, we assume that higher individual search efforts increase the probability of getting the job but at a diminishing rate. The function U is concave, strictly increasing and twice continuously differentiable in all arguments. The expectations operator, E_0, takes into account that the technology is stochastic. Initial holdings of capital and contingent claims, k_0 and m_0 (θ_0), are given.

The insurance company maximizes period-by-period expected profits with respect to insurance holdings of each type

$$\max_{\{y_{ts}\}s\in S} \pi_t^I = \int_S y_{ts} p_{ts} d\mu_s - \int_S (1 - \varphi(\pi_{ts})) y_{ts} d\mu_s \,. \tag{6}$$

In other words, we assume that agents' searching time can be perfectly and costlessly monitored by the company.[2] This insurance company is an extension of Hansen's (1985) risk-sharing arrangement to the heterogeneous case.

DEFINITION. *A competitive equilibrium is a set of contingency plans for individual allocations,* $\{\pi_{ts}, c_{ts}^{mj}, h_{ts}^j, k_{t+1s}^j, y_{ts}, \{m_{t+1s}^j(\theta)\}_{\theta\in\Theta}\}_{t\in T, s\in S}^{j\in\{e,u\}}$*; the factors of production,* $\{k_t, n_t\}_{t\in T}$*; prices for the factors of production,* $\{r_t, w_t\}_{t\in T}$*; prices for unemployment insurance,* $\{p_{ts}\}_{t\in T, s\in S}$ *and prices for contingent claims,* $\{q_t(\theta)\}_{t\in T,\theta\in\Theta}$ *such that given the prices, all agents maximize their utilities (2) subject to (3)–(5), the firm maximizes its profit (1), the insurance company maximizes its profit (6) and all markets clear. We assume that in equilibrium all model variables are nonnegative and the probabilities satisfy* $0 \leq \varphi(\pi_{ts}) \leq 1$ *for all* $s \in S$ *and* $t \in T$.

3. Analytic Results

It is a well-established fact that in an economy without distortions and with complete markets, a competitive equilibrium allocation belongs to the set of Pareto optimal allocations (First Welfare Theorem) and any Pareto optimal allocation can be supported as a competitive equilibrium with transfers (Second Welfare Theorem). It is also known that each Pareto optimal allocation is a solution to the so-called planner's problem which is the problem of maximizing the weighted sum of individual utilities subject to economy's resource constraint. These results imply that the equilibrium in a heterogeneous model like ours can be computed by using the following iterative procedure: fix weights on individual utilities, solve the planner's problem and use the solution to check whether the assumed weights are consistent with the individual life-time constraints; iterate on the above steps until the fixed point weights are found. More details on this algorithmic procedure can be found, for example, in Garcia-Mila, Marcet and Ventura (1995).

[2]If search efforts were not observable, then moral hazard problem would arise. This issue is difficult to model, however, in a dynamic economy like ours; we leave it for future research.

The above algorithm is costly in terms of computational time because each iteration requires finding a solution to a dynamic stochastic model. Moreover, the cost increases with the number of agents in the economy because having more agents implies more parameters (weights) to iterate on. Consequently, the application of this algorithm is in practice very limited.

The solution procedure simplifies substantially if the aggregate dynamics of a heterogeneous economy do not depend on wealth distribution. This case is known as perfect or Gorman's aggregation. Under perfect aggregation one can, first, solve for aggregate quantities and then recover individual variables from the aggregate solution. This property makes it possible to extend the model to include any number of agents at no additional computational cost compared to the representative agent case. For the reasons discussed above, we restrict our attention only to a version of the model which is compatible with aggregation. Specifically, we assume:

- A1: all agents have identical momentary utilities of the form

$$U(c^m, c^h, l) = \frac{[(c^m + c^h)^\gamma \, l^{1-\gamma}]^{1-\sigma} - 1}{1 - \sigma} ; \tag{7}$$

- A2: the home production function is $g(h) = Ah$;
- A3: the function φ has the form

$$\varphi(\pi) = \beta_1 + \beta_2 \ln(1 + \beta_3 \pi) , \qquad \beta_1, \beta_2, \beta_3 > 0 , \tag{8}$$

where β_1, β_2, β_3 are some parameters.

Let us comment on these assumptions. According to A1, market and home consumption goods are perfect substitutes. This assumption is obviously restrictive, but it is consistent with U.S. data. Eichenbaum and Hansen (1990) find that one cannot reject the hypothesis about perfect substitutability between market goods and services from consumer durables (which can be interpreted as home-made goods). Further, following Ríos-Rull (1993), in A2 we assume that the home technology is linear in labor (that is, it does not require inputs of capital) and that all individuals are equally productive working at home. This captures two important features of home activities that most of home work is labor intensive and that agents with different market productivities have roughly the same skills in preparing meals, cleaning, child care, etc. Finally, according to A3, the search technology is given by the flexible functional form (8), where the parameter β_1 corresponds to the probability of becoming employed if search time is zero and the parameters β_2 and β_3 reflect how the initial probability increases due to search. Under (8), the inverse of the first derivative of the function φ is linear in π; this property helps us to achieve aggregation.

The individual optimality conditions are derived in the appendix. It is also shown that risk averse agents will choose to insure themselves fully against unemployment and that employed and unemployed agents of the same type will always hold the same amount of capital and contingent claims. Here, we summarize the individual optimality conditions which we obtain under A1–A3

$$\pi_{ts} = \beta_2 \bar{n}(A^{-1} e_s w_t - 1) - \beta_3^{-1} ; \tag{9}$$

$$c_{ts}^{mu} + Ah_{ts}^{u} = c_{ts}^{me} + Ah_{ts}^{e} \equiv c_{ts} ; \tag{10}$$

$$1 - h_{ts}^{u} - \pi_{ts} = 1 - \bar{n} - h_{ts}^{e} - \pi_{ts} \equiv l_{ts} ; \tag{11}$$

$$\gamma A \cdot l_{ts} = (1 - \gamma) \cdot c_{ts} ; \tag{12}$$

$$c_{ts}^{-\sigma} = \delta E_t \left[(1 - d + r_{t+1}) c_{t+1s}^{-\sigma} \right] ; \tag{13}$$

$$c_{ts} = \frac{c_t \lambda_s^{1/\sigma}}{\int_S \lambda_s^{1/\sigma} \, d\mu_s} ; \tag{14}$$

$$\begin{aligned} \varphi(\pi_{ts}) c_{ts}^{me} + (1 - \varphi(\pi_{ts})) c_{ts}^{mu} + k_{t+1s} + \int_\Theta q_t(\theta) m_{t+1s}(\theta) d\theta \\ = k_{ts}(1 - d + r_t) + \varphi(\pi_{ts}) \bar{n} e_s w_t + m_{ts}(\theta_t) ; \end{aligned} \tag{15}$$

where $c_t \equiv \int_S c_{ts} d\mu_s$ is total (market plus home) aggregate consumption and λ_s is the weight on utility of individual s in the associated planner's problem.

Let us briefly discuss the individual optimality conditions and analyze some of the model's implications at the individual level.

Equation (9) is informative and helps in understanding several properties of the model. First, it shows how innovations to technology induce fluctuations in the labor market. In particular, a positive shock increases the return to working in market sector compared to that in home sector and, in response, agents choose to search more for a market job. This implies that the level of employment increases. Second, the condition demonstrates that the model can account for fact (*i*) discussed in the introduction. According to (9), workers with high productivity always devote more time to searching and, therefore, always have a higher employment rate than workers whose productivity is low. Finally, the condition indicates that the model's predictions are consistent with the empirical regularity (*ii*). Indeed, using (8) and (9), one can show that $\partial(\partial\varphi_{ts}/\partial w_t)/\partial e_s < 0$. This inequality implies that the level of employment of highly productive agents is less responsive to wage fluctuations than that of the agents whose productivity is low or, in other words, that the volatility of employment in our model decreases with the productivity level.

It is a well-known fact that Hansen's (1985) model has one undesirable property: if leisure is a normal good, the unemployed agent enjoys a higher level of utility than the employed does. It happens because, in equilibrium, employed and unemployed agents have the same consumption level, but unemployed have a higher level of leisure. Our model does not have this implication: according to (10) and (11), total consumption and leisure of employed and unemployed agents of the same type are equal, and, thus, both enjoy the *same* level of utility.

Equations (12) and (13) determine the marginal rate of substitution between current consumption and leisure and between current and expected future consumption respectively. Further, condition (14) states that total consumption of each individual is a constant share of total aggregate consumption. This result is a consequence of complete markets under which the ratio of marginal utilities of any two agents remains constant over time. Finally, due to perfect risk sharing, agents of the same type face the same budget constraint (15) in both employed and unemployed states.

Using individual optimality conditions, we can derive the following set of restrictions on the aggregate variables of the economy

$$c_t^{-\sigma} = \delta E_t[(1 - d + r_{t+1})c_{t+1}^{-\sigma}] , \tag{16}$$

$$\pi_t \equiv \int_S \pi_{ts} d\mu_s = \beta_2 \bar{n}(A^{-1}w_t - 1) - \beta_3^{-1} , \tag{17}$$

$$\varphi_t \equiv \int_S \varphi(\pi_{ts}) d\mu_s = \beta_1 + \beta_2 \int_S \ln\left[\beta_2\beta_3\bar{n}\left(\frac{e_s w_t}{A} - 1\right)\right] d\mu_s , \tag{18}$$

$$n_t \equiv \bar{n} \int_S \varphi(\pi_{ts}) e_s d\mu_s = \bar{n}\beta_1 + \bar{n}\beta_2 \int_S \ln\left[\beta_2\beta_3\bar{n}\left(\frac{e_s w_t}{A} - 1\right)\right] e_s d\mu_s , \tag{19}$$

$$c_t/\gamma - A(1 - \pi_t) + A\bar{n}\varphi_t + k_{t+1} = \theta_t f(k_t, n_t) + k_t(1 - d) , \tag{20}$$

where $k_t \equiv \int_S k_{ts} d\mu_s$. Condition (16) results from (13) and (14). Equations

(17)–(19) follow after substituting (9) into the definitions of the corresponding variables. The aggregate resource constraint (20) is obtained after integrating the individual budget constraint (15), substituting conditions (10)–(12) and using the fact that, in equilibrium, aggregate holdings of contingent claims are equal to zero.

Equations (16)–(20) and the prices, $r_t = \theta_t \partial f/\partial k_t$ and $w_t = \theta_t \partial f/\partial n_t$, determine uniquely the equilibrium aggregate quantities $\{c_t, n_t, k_{t+1}, \pi_t, \varphi_t\}$, provided that the equilibrium exists and is unique. Observe that none of these conditions depends on individual variables. Precisely because of this fact we can solve the model without iterating on the utility weights.

Once the equilibrium aggregate quantities $\{c_t, n_t, k_{t+1}, \pi_t, \varphi_t\}$ are known, individual variables are simple to recover. Individual search time and the probability of being employed can be found using (9) and (8), respectively. To compute the remaining variables, we need to use the individual life-time budget constraint (this condition is derived in the appendix).

$$E_0 \sum_{t=0}^{\infty} \delta^t \frac{U_1(c_{ts}, l_{ts})}{U_1\ (c_{0s}, l_{0s})} [\varphi(\pi_{ts})c_{ts}^{me} + (1 - \varphi(\pi_{ts}))c_{ts}^{mu} - \varphi(\pi_{ts})\bar{n}e_s w_t] = k_0' , \quad (21)$$

where U_1 is marginal utility of consumption and $k_0' = k_0(1 - d + r_0) + m_0(\theta_0)$. This condition restricts the expected discounted value of life-time difference between consumption and labor income to be equal to initial endowment. Substituting (10)–(12), (14) in (21) and rearranging the terms, we obtain

$$\frac{\lambda_s^{1/\sigma}}{\int_S \lambda_s^{1/\sigma}\, d\mu_s} = \frac{k_0' - E_0 \sum_{t=0}^{\infty} \delta^t (c_0/c_t)^{\sigma} \ [\varphi(\pi_{ts})\bar{n}(A - e_s w_t) - A(1 - \pi_{ts})]}{E_0 \sum_{t=0}^{\infty} \delta^t (c_0/c_t)^{\sigma}\ (c_t/\gamma)} . \quad (22)$$

This equation makes it possible to compute the individual utility weights. Given the weights, we can recover individual total consumption from (14) and subsequently, restore market and home consumption using (10)–(12).

4. Calibration and Simulation Procedures

We now move on to calibrate the model to be able to carry out quantitative experiments. The calibration of many parameters is standard (see

TABLE 1. *The Distribution of Productivity and Employment in the U.S. Economy*

	Group				
Variable	1	2	3	4	5
Productivity, e_s	0.415	0.694	0.887	1.144	1.859
Employment, φ_s	0.846	0.905	0.920	0.924	0.925

Source: Castañeda, Díaz-Giménez and Rios-Rull (1995, Table 8); the average productivity is normalized to 1.

e.g. Cooley and Prescott's 1995, account of the calibration procedure). Because of the heterogeneous agents setup, however, we need to calibrate some further parameters on individual characteristics. In particular, we are to choose the number of heterogeneous agents in the model and their productivity levels.

Castañeda, Díaz-Giménez and Ríos-Rull (1995) and Ríos-Rull (1993) divide the Panel Study of Income Dynamics (PSID) sample for 1969–1982 in five equally-sized groups according to the individual wages and computed the groups' averages of several individual variables including the wage, the level of employment, the standard deviation of employment and hours worked at home. We use the results of these studies for calibrating the model and also for testing the validity of the model's predictions. Given that the data are computed for five groups, in a subsequent paper we consider a version of the model with five heterogeneous agents. We will use the wage as a proxy for productivity.

Table 1 reproduces the levels of productivity and employment by groups. The data in Table 1 allow us to compute aggregate employment, $\varphi = \sum_{s=1}^{5} \varphi_s$, and aggregate labor input, $n = \sum_{s=1}^{5} \varphi_s e_s$. We will calibrate the model so that in the steady state it reproduces these two moments.

We assume that market output, y_t^m, is produced according to the Cobb-Douglas production function, $y_t^m = \theta_t k_t^{\alpha} n_t^{1-\alpha}$, and that the technology shock follows the law of motion $\ln \theta_t = \rho \ln \theta_{t-1} + \epsilon_t$, where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$; the autocorrelation coefficient, ρ, and the standard deviation, σ_ϵ, are equal to 0.95 and 0.01 respectively. Aggregate output produced at home is $y_t^h = Ah_t$, where $h_t \equiv \sum_{s=1}^{5} (\varphi(\pi_{ts}) h_{ts}^e + (1 - \varphi(\pi_{ts}))\, h_{ts}^u) d\mu_s$.

To make our results comparable to existing studies, we use standard parameters whenever it is possible. The values (d, δ, α, h, $\bar{n}$) are borrowed from Benhabib, Rogerson and Wright (1991), where h denotes the steady-state level of average home hours. The ratio of net investment to output, i/y^m, is set to 0.25, the value which is used in the RBC models without home

TABLE 2. *The Model's Parameters*

Parameter	d	δ	α	h	$\bar{n}$	n	φ	i/y^m	π
Value	0.025	0.99	0.36	0.28	0.33	0.302	0.904	0.25	0.024

production, see, for example, Cooley and Prescott (1995). This is done because in our case home technology does not require capital. Given that in the steady state investment is used to cover depreciation of capital, $i = dk$, the assumed value i/y^m implies the capital-to-output ratio $k/y^m = 10$. The latter is roughly consistent with the estimate of capital to output ratio in the U.S. economy obtained by Christiano and Eichenbaum (1992) ($k/y^m = 10.62$).

To calibrate average search time π, we use the following considerations. Barron and Gilley (1981) estimate the time spent by the typical unemployed individual on-the-job-search as approximately eight and two-third hours per week. The results of Arellano and Meghir (1992), and Burgess and Low (1992) suggest that about one-third of employed agents participate in on-the-job search. Assuming that both the employed and unemployed have the same intensity of search, these numbers imply that the average agent spends about 2.4% of his discretionary time (total time minus personal care) on job search.

Table 2 summarizes the parameters which are fixed for all simulations.

The remaining parameters to choose are (σ, A,γ, β_1, β_2, β_3). Regarding the coefficient of risk aversion, σ, we consider two different values, namely, 1.0 and 5.0. Further, using the properties of the Cobb-Douglas production function and Equations (10)–(12), one can derive the following relationships:

$$A = \frac{y^h/y^m}{h/n} \cdot \left(\frac{1/\delta - 1 + d}{\alpha} \right)^{\alpha/(\alpha - 1)},$$

$$\gamma = \frac{1 - i/y^m + y^h/y^m}{1 - i/y^m + y^h/y^m \cdot (1 - \pi - \varphi \cdot \bar{n})/h},$$

where y^h/y^m denotes the ratio of home output to market output. Given y^h/y^m, these formulas provide a basis for calibrating A and γ. To calibrate the ratio y^h/y^m, we use the results of existing studies. Eisner (1988) provides a summary of the literature measuring the magnitude of the home production and reports estimates of the ratio y^h/y^m in the interval of (0.2, 0.5). Benhabib, Rogerson and Wright (1991) argue that in a model without government taxation, the relative size of the home production may not be too high. Consequently, they use the ratio 0.26. Presumably, in our case, this ratio might

TABLE 3. *The Model's Parameters*

Case/Parameter	γ	A	β_1	β_2	β_3
$y^h/y^m = 0.15$	0.808	0.599	0.863	0.0300	173.8
$y^h/y^m = 0.20$	0.770	0.804	0.805	0.0387	758.6

be even lower since we assume that the home technology does not require capital. Based on this, we consider two alternative values, 0.15 and 0.20.

We are left to calibrate the parameters of the search technology (8). Evaluating Equations (17), (18), (19) in the steady state and substituting the values (π, φ, n), we obtain the system of three equations with three unknowns, (β_1, β_2, β_3). The solution to this system gives us the values of the search parameters.

Table 3 reports the parameters (A, γ, β_1, β_2, β_3) computed under two values of the home output to market output ratio.

For all numerical experiments, we set the initial aggregate capital, k_0, equal to the steady-state value and assume that the initial technology shock is $\theta_0 = 1$.

We solve for aggregate quantities $\{c_t, n_t, k_{t+1}, \pi_t, \varphi_t\}$, which satisfy equations (16)–(20) by using the parameterized expectations algorithm, see, for example, Den Haan and Marcet (1990). The length of simulations is 10000; the conditional expectation in (16) is parameterized by a second-order exponentiated polynomial. To find utility weights, we approximate the conditional expectations in (22) by the corresponding averages which are computed across 400 simulated data sets of the length 10000. The statistics in Table 4 and Table 5 are the averages of the corresponding variables. The averages are computed across 400 simulated data sets of the length 115. Numbers in parenthesis are standard deviations of the statistics. Before computing the second moments of aggregate variables, we log and detrend the series by using the Hodrick-Prescott filter under the standard penalty for variation in quarterly data, 1600.

5. Simulation Results

This section analyses quantitative implications of the model. First, we focus on aggregate dynamics. After that we turn to the predictions at the individual level.

Aggregate Predictions

In Table 4, we report the first and the second moments of aggregate variables generated by the representative agent (RA) version of the model

TABLE 4. *Selected Statistics for U.S. and Artificial Economies*

Statistic	RA model: $y^h/y^m = 0.20$, $\sigma = 1.0$	Heterogeneous model: $y^h/y^m = 0.15$, $\sigma = 1.0$	Heterogeneous model: $y^h/y^m = 0.20$, $\sigma = 1.0$	Heterogeneous model: $y^h/y^m = 0.20$, $\sigma = 5.0$	RA model (BRW)[a]	U.S. economy[a]
	First moments					
y^h/y^m	0.207	0.152	0.202	0.201	0.260[c]	0.2–0.5[c]
k/y^m	10.279	10.256	10.262	10.358	–	10.00[c]
i/y^m	0.255	0.256	0.256	0.260	–	0.250[c]
φ	0.906	0.904	0.904	0.904	–	0.904[c]
h	0.284	0.283	0.282	0.282	0.280[c]	0.280[c]
π	0.025	0.024	0.024	0.024	–	0.024[c]
	Percentage standard deviations					
σ_k	0.350	0.355	0.362	0.326	0.393	0.661
	(0.074)	(0.076)	(0.075)	(0.068)		
σ_y^m/φ	0.608	1.244	1.213	1.192	0.667	0.905
	(0.080)	(0.164)	(0.150)	(0.150)		
σ_i	4.092	4.155	4.292	3.742	4.668	4.907
	(0.533)	(0.539)	(0.530)	(0.482)		
σ_c^m	0.589	0.424	0.433	0.479	0.872	0.853
	(0.094)	(0.072)	(0.071)	(0.062)		
σ_h	2.445	0.576	0.574	0.644	1.197	
	(0.408)	(0.090)	(0.086)	(0.086)		
σ_n	–	0.057	0.095	0.093	–	–
		(0.008)	(0.012)	(0.013)		

σ_φ	0.813	0.067	0.128	0.124	1.283[b]	1.496[b]
	(0.109)	(0.009)	(0.018)	(0.019)		
σ_π	38.598	2.091	1.994	1.951	–	–
	(22.808)	(0.278)	(0.247)	(0.246)		
σ_y^m	1.420	1.310	1.341	1.315	1.710	1.740
	(0.187)	(0.173)	(0.165)	(0.165)		
			Correlations with output			
$corr(k, y^m)$	0.118	0.098	0.109	0.014	0.090	0.280
	(0.071)	(0.065)	(0.066)	(0.067)		
$corr(y^m/\varphi, y^m)$	1.000	1.000	0.999	1.000	0.750	0.510
	(0.000)	(0.000)	(0.000)	(0.000)		
$corr(i, y^m)$	0.982	0.986	0.984	0.996	0.940	0.960
	(0.005)	(0.004)	(0.004)	(0.001)		
$corr(c^m, y)$	0.907	0.844	0.816	0.981	0.690	0.760
	(0.015)	(0.022)	(0.026)	(0.005)		
$corr(h, y^m)$	−0.991	−0.927	−0.928	−0.992	−0.760	–
	(0.005)	(0.013)	(0.014)	(0.002)		
$corr(n, y^m)$	–	0.999	0.999	0.999	–	–
		(0.000)	(0.000)	(0.002)		
$corr(\varphi, y^m)$	0.999	0.999	0.999	0.997	0.940[b]	0.860[b]
	(0.000)	(0.000)	(0.000)	(0.003)		
$corr(\pi, y^m)$	0.676	0.999	0.999	0.999	–	–
	(0.327)	(0.000)	(0.000)	(0.000)		

NOTES: [a]Source: Benhabib, Rogerson and Wright (1991, table 1).
[b]These statistics are computed using physical hours worked.
[c]Source: see discussion in Section 4.

TABLE 5. *The Distributions of Individual Variables in Artificial and U.S. Economies*

Group	Employment	St. deviation of employment (%)	Time worked at home[a]	Market consumption[a]	Search time
	Model economy: $y^h/y^m = 0.15$				
1	0.866	0.18	0.316	0.817	0.0006
2	0.896	0.14	0.295	0.829	0.0115
3	0.907	0.13	0.284	0.836	0.0191
4	0.917	0.12	0.270	0.844	0.0292
5	0.935	0.11	0.236	0.865	0.0572
	Model economy; $y^h/y^m = 0.20$				
1	0.834	0.63	0.325	0.808	0.0015
2	0.895	0.22	0.295	0.833	0.0120
3	0.911	0.18	0.282	0.843	0.0193
4	0.927	0.16	0.267	0.855	0.0290
5	0.951	0.13	0.232	0.883	0.0559
	U.S. economy[b]				
1	0.846	2.28	0.394	–	–
2	0.905	2.21	0.351	–	–
3	0.920	1.92	0.282	–	–
4	0.924	1.74	0.213	–	–
5	0.925	1.37	0.160	–	–

NOTES: [a]Time worked at home and market consumption are group's averages, the group's average of a variable x_{ts} is defined as $x^e_{ts}\varphi(\pi_{ts}) + x^u_{ts}(I - \varphi(\pi_{ts}))$.

[b]Source (except for time worked at home): Castañeda, Díaz-Giménez and Ríos-Rull (1995, Table 8).

Source for time worked at home: Ríos-Rull (1993, Table 2); the average time is normalized to 0.28.

and by the heterogeneous agent version of the model under three alternative sets of the parameters. For comparison, we also provide the predictions of the standard representative agent model with home production and the corresponding statistics for the U.S. economy.

At the aggregate level, most of the properties of the heterogeneous model are similar to what is found in standard RBC models with homogenous agents. Similar to the model of Benhabib, Rogerson and Wright (1991), our model produces a negative cross-correlation of market output with hours worked at home. Further, the model predicts that efficiency hours are less volatile than employment, implying that the low productive workers represent a larger fraction of labor force during the expansion than during the recession. This implication is in agreement with the empirical finding of Hansen (1993). Comparing the cases $\sigma = 1.0$ and $\sigma = 5.0$ shows that an increase in the coefficient of risk aversion does not affect significantly the aggregate predictions of the model, except for the correlation between capital and output, which becomes too low.

The main shortcoming of the model is the small fluctuations of employment over the business cycle. The results imply that under the benchmark value of $y^h/y^m = 0.20$, the volatility of employment is 0.128, which is about 10 times less than the empirical counterpart. Comparison of the cases $y^h/y^m = 0.15$ and $y^h/y^m = 0.20$ shows that an increase in the ratio y^h/y^m improves on this statistic. In our model, however, this ratio cannot be too high. The reason is that a high ratio y^h/y^m also implies a high return to home hours. This can result in that individuals with low productivity have a return to working at home which exceeds the market wage and, therefore, they will choose to not work in the market at all. This does not seem like an entirely convincing explanation for unemployment.

The implied low volatility of labor market variables is not particularly related to the present model but is instead a more generic property of heterogeneous agents models. For example, in the overlapping generations model with heterogeneous agents considered by Ríos-Rull (1993), the standard deviation of hours is 0.089 (see Ríos-Rull's Table 7), which is even further away from the empirical estimates than our results. In the two-agents version of the standard neoclassical model studied by Garcia-Mila, Marcet and Ventura (1995), this statistic is 0.001 (see their Table 9). In fact, the last paper argues that if the heterogeneous model is calibrated to match cross sectional observations, then such models have more difficulties in accounting for time series stylized facts than a similar representative agent setup.

Our results confirm this conjecture. As we see from the table, the representative agent version of our model ($e_s = 1$) can generate the standard deviation of employment equal to 0.813 which is reasonably close to the corresponding empirical value. The improvement relative to the heterogeneous agents case is due to a single difference in the calibration procedure, which is the choice of the parameter β_2. Specifically, in the representative agent case, $\varphi\bar{n} = n$ and, consequently, two of the restrictions (17)–(19) used for calibrating βs in the heterogeneous model become identical. Therefore, we set β_2 to an arbitrary value, namely, 0.80, and find β_1 and β_3 from the remaining two conditions; this gives us $(\beta_1, \beta_2, \beta_3) = (0.87, 0.80, 2.04)$. It turns out, however, that these values cannot be assumed for calibrating the heterogeneous model because they imply negative search time for low productivity groups. This simple exercise indicates that the set of the parameters which is consistent with cross sections can be very different from the one under which the model has the best chance to account for time series facts.

Another deficiency of the model is the degree to which employment and productivity (y^m/φ) are correlated with output. In the model these correlations are nearly perfect while in the data they are substantially lower. This failure is not surprising given that most of the existing RBC models dramatically exaggerate these statistics (for a discussion see, for example, Christiano and Eichenbaum 1992). Our results indicate that this problem cannot be resolved within our simple framework.

Individual Predictions

In Table 5, we report the levels of employment, the standard deviations of employment, time worked at home, market consumption and search time for the five productivity groups predicted by the model under two alternative values of the home output to market output ratio. For comparison, we also provide the corresponding quantities in the U.S. economy. We report only the case $\sigma = 1.0$ since the case $\sigma = 5.0$ implies practically identical results.

The model can successfully account for a number of the moments of individual variables. It predicts that high productive individuals search more, and, as a result, have a higher employment rate. Furthermore high productive agents experience lower fluctuations in employment, work less at home and consume more. All of these predictions are in line with the empirical evidence. Notice also that the levels of employment are close to those in the data while the standard deviations of employment are somewhat lower than the empirical values. These results are very promising and imply that key features of the individual data can be accounted for by this rather simple heterogeneous agents model.

Regarding the time worked at home, Ríos-Rull's (1993) estimate of average annual home hours is equal to 461, which in terms of normalized to unity discretionary time corresponds to 0.075. Benhabib, Rogerson and Wright (1991) argue that the average share of time worked at home is substantially higher, namely, 0.28. Given that we assume the latter value for calibrating the model, we normalize the data of Ríos-Rull (1993) respectively. As we see, the model can successfully account for the corrected distribution of home hours.

One problem is that the model generates unrealistically little cross-group variability in consumption and, consequently, in welfare levels (up to few percents). This shortcoming is due to the assumption that all individuals have the same initial wealth. Indeed, in our model, market consumption increases with the agent's utility weight, which, in turn, is an increasing function of the initial wealth. As in the micro data, the correlation between the level of productivity and wealth is positive;[3] therefore, the variability of consumption will rise once the heterogeneity in initial wealth is introduced. While we can computationally handle such heterogeneities very easily, we did not include them because of the lack of empirical evidence. Thus, it is somewhat unclear whether the model is consistent or inconsistent with the data along this dimension.

Finally, the model implies that time spent by agents on job search increases across productivity groups from several minutes to more than one hour per day. In fact, the implications of the model with respect to search are difficult to test because most of the existing data sets on the individual behavior do not provide the corresponding data. Several empirical papers construct measures of the intensity of search and use them for analyzing the relation between search, employment and productivity. Barron and Gilley (1981) find that the level of employment is positively related to search. Barron and Mellow (1979) analyze the determinants of search intensity for unemployed workers and find a strong positive effect of education; Arellano and Meghir (1992) report the same tendency for the employed. The relation between search and wages depends on the employment status: for unemployed, past wages have a positive effect on search intensity (Barron and Mellow 1979); for the employed, the effect of wages is negative (Arellano and Meghir 1992).

In short, the model's predictions at the individual level are consistent with all empirical regularities except for a negative effect of wages on the search intensity of the employed. Arellano and Meghir (1992) argue that the last tendency reflects the fact that search time has a higher opportunity cost for workers whose wages are high than for those whose wages are low. Provided that their inference is correct, the failure of the model results from the assumption of the one-period labor contracts. Indeed, in our model the agents may choose not to work in the market instead of searching because they get unemployed at the end of each period; all of them have the same opportunity cost of search, which is home work. Introducing a possibility of long-term labor contracts would presumably help to improve on the model's predictions along this dimension.

6. Concluding Comments

We have analyzed a quantitative general equilibrium model with permanent heterogeneity in productivity with the aim of explaining differential responses of labor supply across productivity groups. The simulation results show that the model is successful in reproducing most of the key features

[3]See, for example, Garcia-Mila, Marcet and Ventura (1995).

of the data. In particular, at the individual level, which is our special matter of interest, it can account for the stylized facts which we outline in the introduction. At the aggregate level, it can generate the cyclical behavior of aggregate quantities, which is reasonably close to that in the U.S. economy. Yet, the heterogeneous version of our model does not produce better aggregate predictions than the associated representative agent setup. This result suggests that introducing heterogeneity is not a necessary condition for a model to be successful in explaining macroeconomic fluctuations in the real world economies. We also stress the computational ease with which our analysis was carried out. This computational aspect of the analysis was obtained due to the use of aggregation theory.

The model failed along some dimensions and this provides valuable insights into the main avenues for future research. Concerning aggregate predictions, it produces too little volatility of employment. As we have already pointed out, this shortcoming is partly due to heterogeneity. Specifically, it is more difficult to match both time series and cross sectional facts than only time series facts as in the representative agent case. To some extent, the lack of volatility of aggregate employment is due to the linear home technology. Presumably, introducing a more general production function for home goods would improve the model's performance. The main shortcoming at the individual level is that the model generates a positive correlation between productivity and search time for all workers while in the data this correlation is positive for the unemployed but negative for the employed. This deficiency is attributed to a simplified structure of the labor markets. A reasonable guess is that the inclusion of a possibility of long-term contracts will result in that highly productive employed workers have higher opportunity cost of search and, therefore, a lower intensity of search than workers with low productivity. Such extensions will be considered in future research.

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Appendix

To derive the individual optimality conditions, we use the value function representation of the agent's problem

$$\begin{aligned}\max_{\{x_{ts}\}} V_s(k_t, k_{ts}, \{m_{ts}(\theta)\}_{\theta\in\Phi}, \theta_t) = \varphi(\pi_{ts}) & [U(c_{ts}^{me} \\ & + Ah_{ts}^{e}, l_{ts}^{e}) \\ & + \delta E_t V_s\ (k_{t+1}, k_{t+1s}^{e}, \{m_{t+1s}^{e}(\theta)\}_{\theta\in\Theta}, \theta_{t+1})] \\ & + (1 - \varphi(\pi_{ts}))\ [U(c_{ts}^{mu} \\ & + Ah_{ts}^{u}, l_{ts}^{u}) \\ & + \delta E_t V_s\ (k_{t+1}, k_{t+1s}, \{m_{t+1s}^{u}(\theta)\}_{\theta\in\Theta}, \theta_{t+1})]\end{aligned} \tag{23}$$

$$\text{s.t. } (3)\ ,\ (4)\ ,$$

where $\{x_{ts}\} = \{\pi_{ts}, c_{ts}^{mj}, h_{ts}^{j}, k_{t+1s}^{j}, y_{ts}, \{m_{t+1s}^{j}(\theta)\}_{\theta\in\Theta}\}_{t\in T}^{j\in\{e,u\}}$ and V_s is the value function of agent $s \in S$.

The first-order conditions for unemployment insurance holdings, capital, hours worked at home, and holdings of contingent claims in employed and unemployed states respectively are

$$\varphi(\pi_{ts})p_{ts}U_1(c_{ts}^{e}, l_{ts}^{e}) = (1 - \varphi(\pi_{ts}))\ (1 - p_{ts})U_1(c_{ts}^{u}, l_{ts}^{u})\ ; \tag{24}$$

$$U_1(c_{ts}^{e}, l_{ts}^{e}) = \delta E_t \frac{\partial V_s(k_{t+1}, k_{t+1s}^{e}, \{m_{t+1s}^{e}(\theta)\}_{\theta\in\Theta}, \theta_{t+1})}{\partial k_{t+1s}^{e}}\ ,$$

$$U_1(c_{ts}^{u}, l_{ts}^{u}) = \delta E_t \frac{\partial V_s(k_{t+1}, k_{t+1s}^{u}, \{m_{t+1s}^{u}(\theta)\}_{\theta\in\Theta}, \theta_{t+1})}{\partial k_{t+1s}^{u}}\ ; \tag{25}$$

$$AU_1(c^e_{ts},\ l^e_{ts}) + U_2(c^e_{ts},\ l^e_{ts}) = 0,$$

$$AU_1(c^u_{ts},\ l^u_{ts}) + U_2(c^u_{ts},\ l^u_{ts}) = 0; \tag{26}$$

$$U_1(c^e_{ts},\ l^e_{ts})q_t(\theta_{t+1}) = \frac{\partial V_s(k_{t+1},\ k^e_{t+1s},\ \{m^e_{t+1s}(\theta)\}_{\theta\in\Theta},\ \theta_{t+1})}{\partial m^e_{t+1s}(\theta_{t+1})}P(\theta_{t+1},\ \theta_t)\ ,$$

$$U_1(c^u_{ts},\ l^u_{ts})q_t(\theta_{t+1}) = \frac{\partial V_s(k_{t+1},\ k^u_{t+1s},\ \{m^u_{t+1s}(\theta)\}_{\theta\in\Theta},\ \theta_{t+1})}{\partial m^u_{t+1s}(\theta_{t+1})}P(\theta_{t+1},\ \theta_t)\ ; \tag{27}$$

where $c^j_{ts} = c^{mj}_{ts} + Ah^j_{ts}$ is the agent's total consumption; U_i refers to the derivative of the utility function with respect to the ith argument. Notice that, given that the shock follows a first-order Markov process, the probability distribution of θ_{t+1}, $P(\theta_{t+1}, \theta_t)$, depends only on the previous period shock, θ_t, and not on the whole history of the economy.

The equilibrium price of insurance is $p_{ts} = (1 - \varphi_{ts}\ (\pi_{ts}))$. This together with (25) gives the risk sharing condition

$$U_1(c^e_{ts},\ l^e_{ts}) = U_1(c^u_{ts},\ l^u_{ts})\ . \tag{28}$$

From the last equality and conditions (26) it follows that

$$U_2(c^e_{ts},\ l^e_{ts}) = U_2(c^u_{ts},\ l^u_{ts})\ . \tag{29}$$

Equations (25), (27) and (28) imply that the holdings of capital and contingent claims in both states are the same, for example, $k^e_{t+1s} = k^u_{t+1s}$ and $m^e_{t+1s}\ (\theta_{t+1}) = m^u_{t+1s}\ (\theta_{t+1})$. Substituting these results into the state contingent constraints (3) gives the equilibrium holdings of unemployment insurance

$$y_{ts} = \bar{n}e_s w_t - c^{me}_{ts} + c^{mu}_{ts}\ . \tag{30}$$

Finding $\partial V_s/\partial k_{ts}$, updating it and combining the resulting condition with (25) and (28), we obtain the standard intertemporal condition

$$U_1(c^e_{ts},\ l^e_{ts}) = \delta E_t\ [(1 - d + r_{t+1})U_1(c^e_{t+1s},\ l^e_{t+1s})]\ . \tag{31}$$

Similarly, finding $\partial V_s/\partial m_{ts}\ (\theta_t)$, updating it and using (27) and (28), we get

$$\delta U_1(c^e_{t+1s},\ l^e_{t+1s})\cdot P(\theta_{t+1},\ \theta_t) = U_1(c^e_{ts},\ l^e_{ts})\cdot q_t(\theta_{t+1})\ . \tag{32}$$

This condition implies that the ratio of marginal utility of any two agents s, $s' \in S$ is constant over time and can be represented as

$$\frac{U_1(c_{ts}^e,\ l_{ts}^e)}{U_1(c_{ts'}^e,\ l_{ts'}^e)} = \frac{\lambda_{s'}}{\lambda_s}, \tag{33}$$

where λ_s is the agent-specific parameter.[4] Using condition (30) and the results that $k_{t+1s}^e = k_{t+1s}^u$ and $m_{t+1s}^e(\theta_{t+1}) = m_{t+1s}^u(\theta_{t+1})$, we can replace the state contingent constraints (3) by a single one

$$\varphi(\pi_{ts})c_{ts}^{me} + (1 - \varphi(\pi_{ts}))c_{ts}^{mu} + k_{t+1s} + \int_\Theta q_t(\theta)m_{t+1s}(\theta)d\theta = k_{ts}(1 - d + r_t) + \varphi(\pi_{ts})\bar{n}e_s w_t + m_{ts}(\theta_t). \tag{34}$$

Therefore, the agent faces the same constraint (34) independently on his employment status. Maximization of (23) subject to (4) and (34) with respect to π_{ts} gives

$$U(c_{ts}^e,\ l_{ts}^e) - U(c_{ts}^u,\ l_{ts}^u) + U_1(c_{ts}^e,\ l_{ts}^e)\ (\bar{n}e_s w_t - c_{ts}^{me} + c_{ts}^{mu}) = \frac{U_2(c_{ts}^e,\ l_{ts}^e)}{\varphi'(\pi_{ts})}. \tag{35}$$

Under the functions U and φ given in (7) and (8), conditions (35), (28), (29), (26), (31), (33) and (34) can be rewritten as (9)–(15) respectively.

Let us derive the individual expected life-time budget constraint. Multiplying (34) by $\delta U_1(c_{ts}^e,\ l_{ts}^e)/U_1(c_{t-1s}^e,\ l_{t-1s}^e)$, substituting (32) and taking conditional expectation, E_{t-1}, from both sides, we get

$$k_{ts} + E_{t-1}\left[\delta\frac{U_1(c_{ts}^e,\ l_{ts}^e)}{U_1(c_{t-1s}^e,\ l_{t-1s}^e)}m_{ts}(\theta_t)\right] =$$

$$E_{t-1}\left[\delta\frac{U_1(c_{ts}^e,\ l_{ts}^e)}{U_1(c_{t-1s}^e,\ l_{t-1s}^e)}\ (c_{ts}^{me}\varphi(\pi_{ts}) + (1 - \varphi(\pi_{ts}))c_{ts}^{mu} - \varphi(\pi_{ts})\bar{n}e_s w_t)\right] +$$

$$E_{t-1}\left\{\delta\frac{U_1(c_{ts}^e,\ l_{ts}^e)}{U_1(c_{t-1s}^e,\ l_{t-1s}^e)}\left[k_{t+1s} + E_t\left(\delta\frac{U_1(c_{t+1s}^e,\ l_{t+1s}^e)}{U_1(c_{ts}^e,\ l_{ts}^e)}m_{t+1s}\right)\right]\right\}.$$

Starting from $t = 0$, we use this condition to substitute recursively for future variables. Applying the law of iterative expectations to the resulting equation, we obtain the expected life-time budget constraint (21).

[4]In fact, λ_s is agent's utility weight in the associated planner's problem.