Real time control of non-convex and stochastic energy resources

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1 Introduction

We consider the problem of controlling an ensemble of heterogeneous energy resources connected to an electrical power distribution network at the same point of common coupling (PCC). A PCC could be the connection between a house containing various energy resources like appliances, air conditioning, battery storage, and rooftop solar photo-voltaic (PV) with the grid, or it could represent the aggregation of geographically dispersed resources on a distribution system as a Virtual Power Plant (VPP). We are interested in exploring solutions for *real-time* control of realistic energy resources, as opposed to solving the offline optimal scheduling problem (e.g. [MKF+10]). In addition, we are interested in including resources that have high uncertainty (such as solar PV systems and wind turbines) and those that have non-convex feasible sets (such as thermostatically controlled loads like air conditioning). Finally, we will explore an extension of [BBLB17] to include both real and reactive power management.

2 Methodology

2.1 Problem Statement

Inspired by the work from [BBLB17], we formulate the non-convex aggregate set point tracking problem as

$$\begin{array}{ll} \underset{\mathbf{p}^k}{\text{minimize}} & \sum_{i=1}^{N} C_i^{(k)}(P_i^{(k)}) \\ \text{subject to} & P_i^{(k)} \in \mathcal{Y}_i^k \qquad i = 1, \ldots, N \\ & \sum_{i=1}^{N} P_i^{(k)} = P_{\text{pcc}}^{(k)} \end{array}$$

where the function $C_i^{(k)} : \mathbf{R^2} \to \mathbf{R}$ characterizes the performance cost of resource i at time step k and is assumed to be convex. The vector $\mathbf{p}^k = \left\{P_i^{(k)}\right\}_{i=1}^N$ consists of the real and reactive power set points of each resource i at time k. Given N resources, each resource's power set point $P_i^{(k)}$ belongs to its feasible set $\mathcal{Y}_i^{(k)}$, which is compact but may not be convex. $P_{\text{pcc}}^{(k)}$ is the target aggregate set-point. This formulation of the problem has two challenges. First, the feasible sets of energy resources may be non-convex or discrete. Second, due to communication delays between the resources and the controller, it is not reasonable to assume that the aggregator knows the current feasible set of each resource. *Our approach* includes: (a) the convex relaxation and error diffusion method presented in [BBLB17]; (b) the extension to active and reactive power management (multi-variate error diffusion and optimization); (c) the modeling of additional energy resources not considered in [BBLB17], including generator/fuel-cell backup power, generic discrete load, and realistic solar resource data. The main challenges are: (a) Developing new resource models that realistically describe the feasible sets extended to $\mathbf{R}^2$ and cost functions for the devices listed above. (b) Extending the error diffusion algorithm to the multi-variate case. (c) Developing numerical case studies for realistic and robust evaluation of the algorithm's potential.

2.2 Convex Relaxation and Error Diffusion

We formulate the relaxed convex optimization problem for each time step as follows:

$$
\begin{aligned}
\underset{\mathbf{p}^k,\epsilon}{\text{minimize}} \; & \sum_{i=1}^{N} C_i^{(k)}(P_i^{(k)}) + \mu\epsilon \\
\text{subject to } \; & P_i^{(k)} \in \mathbf{conv}\, \mathcal{Y}_i^{(k-1)}, \quad i = 1, ..., N \\
& P_{\text{pcc}}^{(k)} - \epsilon \leq \sum_{i=1}^{N} P_i^{(k)} \leq P_{\text{pcc}}^{(k)} + \epsilon \\
& \epsilon \geq 0
\end{aligned}
$$

where μ is a hyperparameter that penalizes the aggregated tracking error ϵ at time step k. The term $\mathbf{conv}\, \mathcal{Y}_i^{(k-1)}$ is the one-step delayed convex hull of the current feasible set $\mathcal{Y}_i^k$. Once we obtain the solution $\mathbf{p}^k = [P_1^{(k)}, P_2^{(k)}, \ldots, P_N^{(k)}]$, we project the solutions onto the actual feasible sets for each energy resource.

To close the loop between the relaxed control policy and the actual behavior of the energy resources, we use the method of *error-diffusion* to dynamically adjust the control decisions. This approach is motivated by the requirement that power balance within the system should be maintained on average over iterations of the control problem. Characterizing the average power setpoint over a given period is useful since it captures how much energy is consumed or generated during that period. The general procedure of error diffusion involves two phases: for each resource, 1) calculate the accumulated error, and 2) adjust the requested power by the accumulated error before projecting onto the current feasible set. The specific steps are illustrated as follows. Let the error at the time step k be $e_i^{(k)}$ for resource i, where $e_i^{(k)} \in \mathbf{R}^d$ and d represents the dimensions of the corresponding resource setpoint. The first step error is $e_i^0 = \mathbf{0}$, then for $k = 1, 2, \ldots,$ we have

$$
e_i^{(k)} = \sum_{\tau=1}^{k} (\tilde{P}_i^\tau - P_i^\tau) = e_i^{(k-1)} + \tilde{P}_i^{(k)} - P_i^{(k)},
$$

where $P_i^{(k)} \in \mathbf{conv}\, \mathcal{Y}_i^{(k)}$, and $\tilde{P}_i^{(k)} \in \mathcal{Y}_i^{(k)}$. We want to apply the projection operation to get $\tilde{P}_i^{(k)}$ in the non-convex feasible set from $P_i^{(k)}$. This is obtained by setting

$$
\tilde{P}_i^k = \mathbf{proj}_{\mathcal{Y}_i^{(k)}}(P_i^k - e_i^{k-1}).
$$

When $d = 1$, $\mathbf{proj}_{\mathcal{Y}}(x) = \arg\min_{y\in\mathcal{Y}} |x - y|$ given x and y are scalars. It has been proven that (Theorem 1 in [BBLB17]), if $\|e_i^k\|$ is bounded, then $\|\frac{1}{k}\sum_{\tau=1}^{k} \tilde{P}_i^{\tau} - \frac{1}{k}\sum_{\tau=1}^{k} P_i^{\tau}\| \to 0$ as k goes to ∞. In the case when d is high dimensional, the theoretical convergence guarantee is proved in [BBB16]. The specific projection method used depends on the model of the energy resource. For convex but rapidly changing feasible sets (like PV systems), this is the Euclidean projection in $\mathbf{R}^2$. For a collection of discrete loads, this is a nearest neighbors search over the set of feasible points. It has been discovered in both our experiments and in simulations from [BBLB17] that finding the closest point of the relaxed solution in the feasible set without error diffusion, i.e. $\tilde{P}_i^k = \mathbf{proj}_{\mathcal{Y}_i^k}(P_i^k)$, can lead to the aggregator drifting away from the setpoint on average over time. Closed-loop control with error diffusion is analogous to the proportional-integral (PI) control, where the "P" part is handled by the optimization and projection, and the "I" part is captured by accumulating the error.

3 Implementation

We have developed a modular and scalable API that allows a user to easily add new resource models to run numerical studies on various collections of resources. To accomplish this, we split the algorithm operation into two main components: a master aggregate optimization and general resource objects. This provides separation of resource specific constraints and general problem constraints. The communication between the two components and flow of the algorithm is illustrated in **Figure 1**.

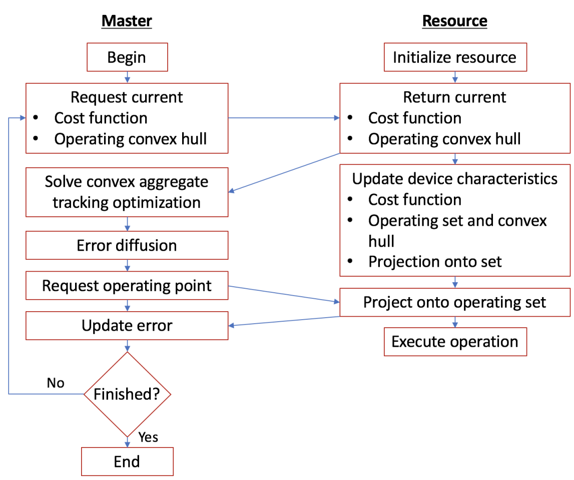
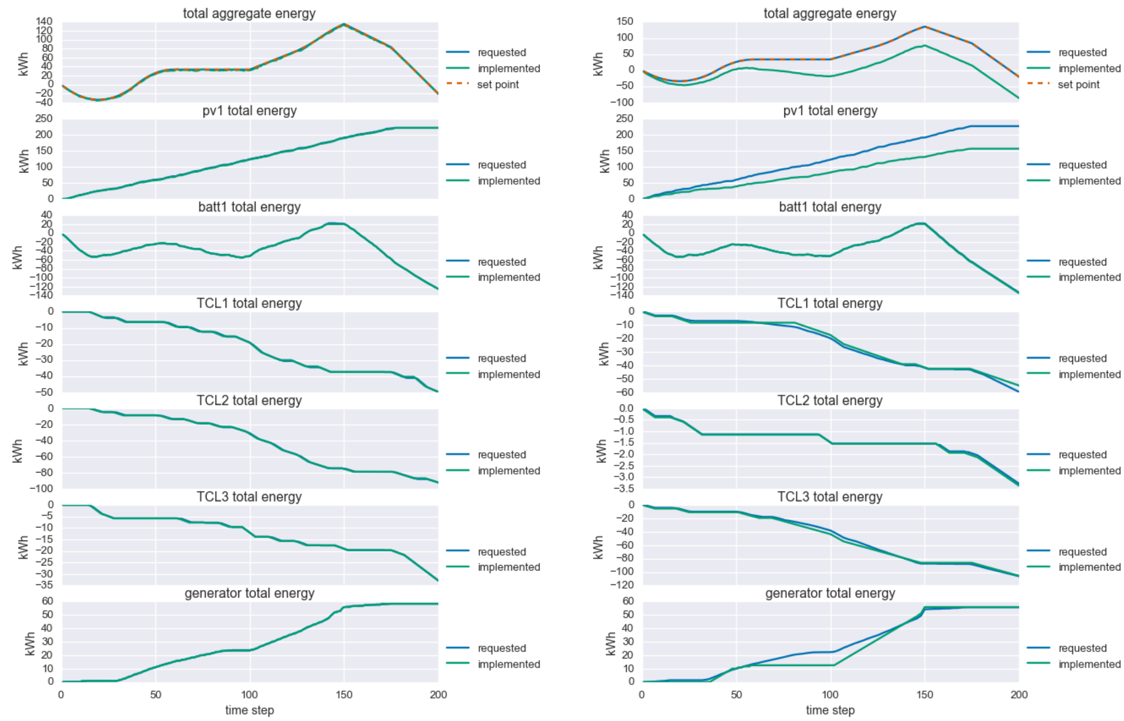


Figure 1: Flow of Algorithm Between Master and Individual Resources

The master class includes the aggregate tracking convex optimization and the error diffusion algorithm. The resource class includes the cost function, operating set, convex hull of the operating set, and projection of the convex optimization output onto the true operating set of each resource. Notice how the algorithm captures the realistic aspect that observations are delayed by one step from the implementation. As seen, the resource updates its characteristics after sending them to the master controller. It then projects the operating point



(a) **(b)**

Figure 2: Panel (a) shows the energy of the collection of resources with error diffusion; Panel (b) shows the the same simulation without error diffusion.

from the master onto the updated operating set, modeling the delays due to communication and computation.

Along with the master controller, we have implemented solar photovoltaic (PV), a generic discrete load or thermostatically controlled load (TCL), a basic battery model, and a basic real power generator to be used in our numerical studies. All code is publicly hosted here: `https://github.com/tnavidi1/EE364b_project`. The data are under NDA (PG&E and SunPower) and cannot be hosted publicly.

4 Case Studies

4.1 Microgrid

Our first case study represents a microgrid, which consists of an isolated collection of generators and loads. In this study, we include a PV system, battery, diesel generator, and three discrete loads (e.g. variable frequency fans, refrigerator, water heater, etc), which must track a power setpoint representing uncontrollable resources in the microgrid. We we set $\mu = 10^3$, so the resulting error term ϵ is small in the convex optimization problem. **Figure 2a** displays the energy of the collection of resources with error diffusion, while the **Figure 2b** depicts the same simulation, but without error diffusion. Our results show the aggregate

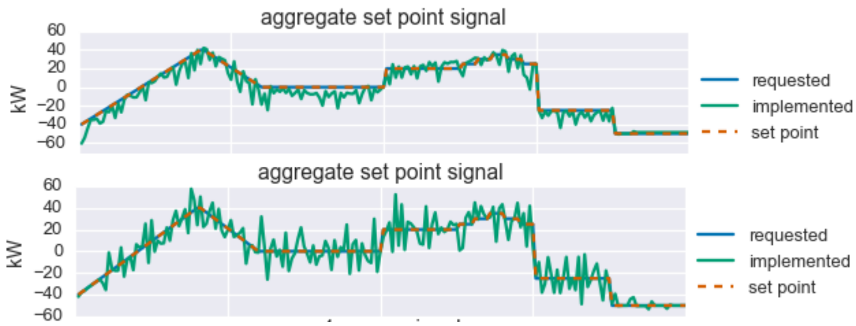
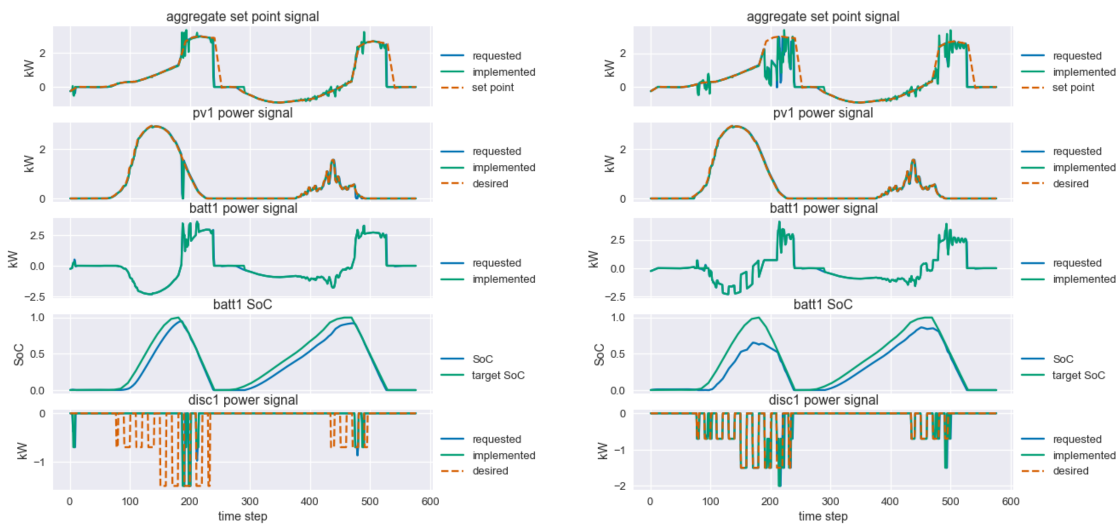


Figure 3: Aggregate Signal Tracking Power. The top panel shows the simulation without error diffusion. The bottom panel shows the same simulation yet with error diffusion

signal drifts from the set point on average over time without error diffusion as seen in the total aggregate energy plot. Specifically, resources with high variability, like the PV, or with non-convex operating sets, like the generator, cause a significant deficit of energy when controlling without error diffusion. However, when operating with error diffusion, they are able to track their respective set points on average nearly perfectly. From the perspective of the aggregate power setpoint, **Figure 3** shows the difference between the simulation with and without error diffusion. The implemented power is often below the requested one in the scenario without error diffusion. The error diffusion removes the bias between the total requested and implemented power at the PCC. The increase in oscillations due to the error compensation and the impact on microgrid stability is something we would like to address in future work.

4.2 Virtual Power Plant

Our second case study involves the simulation of our controller acting as the bottom layer of a virtual power plant control system. A virtual power plant (VPP) is a large collection of energy resources that, when aggregated, can participate in energy markets to buy and sell electricity on the grid. The bottom layer controller is responsible for disaggregating the control signals determined by the higher level planning controllers in real time. To generate the higher level control signals, we simulated the operation of a VPP that represents the aggregation of the IEEE 123 bus test case. We used real PV data and residential load data aggregated to match the demand of the IEEE test case. We assumed a solar and storage penetration equal to 30% of the total energy demanded. The price of electricity we used is the standard time of use rate that is popular in California. The higher level controller solves an optimization problem where the objective is to minimize the cost of electricity while maintaining voltages and keeping the aggregation of controllable resources in their predetermined operating regime. This optimization is performed at an hourly time resolution over a 2 day horizon with forecasts to predict the future value of PV and demand. The output of the optimization is the aggregate set point signal used for our simulations. The higher level controller also determines the target state of charge for the battery. Our controller then uses linear interpolation to up-sample the signal to a 15 minute resolution, and disaggregates this



(a) **(b)**

Figure 4: Panel (a) shows power tracking for a VPP focused on aggregate performance; Panel (b) shows the the same simulation, but the objective of the VPP is to focus on individual resource performance.

signal across a PV, battery, and discrete controllable load. **Figure 4a** represents the case where the VPP would like to match the aggregate signal at the expense of individual device operation, while **Figure 4b** shows the same simulation, but the VPP prefers satisfying the individual resources over the aggregate. This demonstrates our algorithms ability to easily consider priority levels among resources and the aggregate. In the interest of space, we have omitted the reactive power curves, but the performance is similar to the real power curves. The computation time on a personal computer was less than .03 seconds per time step, which demonstrates the capability for real time control. Future work in this area will consider controlling for voltage and frequency of the system in real time in addition to the control present in the higher level planning algorithm.

$$
\begin{aligned}
\underset{c,d}{\text{minimize}} \quad & \sum_{t=1}^{T} \lambda_e(t)(P_t + c_t - d_t) + \lambda_d \max(P + c - d, P_0^{max}) \\
& + \lambda_b \sum_{t=1}^{T} (c_t + d_t) \\
\text{subject to} \quad & 0 \leq c \leq c_{max} \\
& 0 \leq d \leq d_{max} \\
& Q_t = \gamma_l Q_{t-1} + \gamma_c c_t - \gamma_d d_t \\
& Q_{min} \leq Q \leq Q_{max}
\end{aligned}
$$

References

[BBB16] Andrey Bernstein, Niek J Bouman, and Jean-Yves Le Boudec. Real-time minimization of average error in the presence of uncertainty and convexification of feasible sets. *arXiv preprint arXiv:1612.07287*, 2016.

[BBLB17] Andrey Bernstein, Niek Bouman, and Jean-Yves Le Boudec. Real-time control of an ensemble of heterogeneous resources. In *Proceedings of the 56th IEEE Conference on Decision and Control*, number EPFL-CONF-230209, 2017.

[MKF+10] Hugo Morais, Péter Kádár, Pedro Faria, Zita A. Vale, and H. M. Khodr. Optimal scheduling of a renewable micro-grid in an isolated load area using mixed-integer linear programming. *Renewable Energy*, 35(1):151–156, 2010.