AUCTIONS, BIDDING, AND CONTRACTING:
Uses and Theory

Edited by
Richard Engelbrecht-Wiggans
Martin Shubik
Robert M. Stark

The medallion on the cover of this series was designed by the
French contemporary artist Georges Mathieu as one of
a set of medals struck in Paris by the Musée de la Monnaie in 1971. Eighteen
medals were created by Mathieu to "commemorate 18 stages in the
development of western consciousness." The Edict of Milan in 313
was the first, Game Theory, 1944, was number seventeen.

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PRIVATE INFORMATION IN AN
AUCTIONLIKE SECURITIES MARKET[*]

Paul R. Milgrom

Students of auction theory usually restrict the
scope of these analyses to include only those settings
where a single seller offers items for sale or a single
buyer solicits delivery price offers. A large volume
of trading takes place in settings of this kind, but
an even larger volume is conducted in settings where
there are both many buyers and many sellers making bids
and offers. The major securities market and commodities
and options exchanges are mostly of this second kind.

There is a reasonably acceptable economic theory
of securities markets for the hypothetical case where
all investors and traders have identical information.
This theory is an extension of the Arrow-Debreu compet-
itive equilibrium theory, and as such it leads to the
usual kinds of conclusions about the existence and ef-
ficiency of equilibrium. However, like the competitive
equilibrium theory, the securities markets theory has
little to say about how prices are formed. This omis-
sion is particularly disturbing in extensions of the
theory in which some traders have private information.
In those extensions, the equilibrium prices typically
reflect more information than was available (when the
markets opened) to any individual trader. This is a
puzzling prediction of the theory, since <u>someone</u> must
set the prices! Far from explaining this puzzle,
traditional analyses have generated many doubtful
propositions, including the following ones: Traders
with inside information may be unable to profit from
concealing and using that information.[1] Traders'
beliefs about security prices—beliefs that are based
partly on past and present prices—can be extremely
sensitive to small variations in the prevailing prices,
and the prices themselves are so sensitive to small
shifts in demand that a single small trader can some-

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[1]Grossman (1976).

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times completely control price levels.[2] Moreover, equilibrium prices must sometimes reflect information that is not available to any active trader![3]

In contrast to the theory of securities markets and to competitive equilibrium theory, auction theory deals simply and directly with the issues of how prices are determined, how bidders use and benefit from their private information, and how private information affects prices. The insights summarized below, which are gleaned from the study of auctions, suggests some new ideas and approaches for security market theory.

Consider a sealed-bid auction in which a single object is offered for sale to n bidders. In general, the amount a bidder is willing to pay for the object depends on his personal preferences and the characteristics of the object. In a charity auction where a Carribean vacation for two is being sold, each bidder can determine an appropriate value for himself by introspection, and he can then make an intelligent bid based on that value and on his beliefs about how others will bid. In contrast to this case is the case where oil rights on an offshore tract of land are being offered: An appropriate value cannot then be determined by introspection because some major

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[2] Milgrom (1981).

[3] Tirole (1980).

value determining characteristics--the amount of re-
coverable oil, its depth, pressure, etc.--are unknown.
Experienced bidders base their bids on their _estimate_
of the value, but they also bid cautiously to counter-
act a phenomenon known as the _winner's curse_. The
winner's curse is the tendency of a winning bidder
to find that he has overestimated the value of the
prize he wins at auction. The curse arises because
a bidder is more likely to win an auction when he has
overestimated the value of the prize than when he has
underestimated it. (Economists may recognize the win-
ner's curse as a variant of the phenomenon known as
adverse selection.)

Experienced bidders escape the ill effects of
the winner's curse in auctions for oil, gas, and min-
eral rights by reducing their bids to allow a margin
for estimation error and by carefully gathering and
analyzing geological data to reduce that error. In
those settings, it is clear that private information
can be useful to a bidder in choosing his bid.

In addition to its value in decision making,
private information can have a kind of strategic
value in auctions. Bidders facing a well-informed
competitor are particularly susceptible to the win-
ner's curse. Consequently, the mere fact that a bid-
der has good information may lead the other bidders

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to bid more cautiously, a response that benefits the
well-informed bidder.[4]

The question of how much of the bidders' private
information gets reflected in the selling price is a
subtle one. One might think that the price resulting
from a sealed-bid auction cannot reflect more infor-
mation than was available to the winning bidder when
he made his bid, since that bid determines the price.
This reasoning, however, is incorrect. When the bid
made by the winning bidder is known, it can be inferred
that the other n-1 bidders made lower bids. The
information which this inference conveys about the
probable value of the mineral rights was unavailable
to the winning bidder when he made his bid. Thus,
the price resulting from a sealed-bid auction may re-
flect more information than was originally available
to any individual bidder, a prediction which is in-
triguingly close to the predictions of the securities
markets theories.

The essential feature that distinguishes bidding
models from existing securities market models is not
that there is competition on only one side of the
market or that the objects being sold are indivisible
nor even that the number of bidders is generally small:
the essential feature is that the _process_ of price

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[3] Milgrom and Weber (1982a).

determination in bidding models is explicit. By ex-
amining this process, one is able to see how infor-
mation can be used, why it is valuable, and how it
affects prices.

In the remainder of this paper, a simple model
of a competitive securities market is sketched in
which the bid and ask prices are set by a group of
market makers. When all traders have identical in-
formation, the equilibrium bid and ask prices are
constant over time and equal to each other and to
the price predicted by competitive equilibrium theory.
However, when some traders have private information,
the equilibrium ask price always exceeds the bid
price. If the informed traders are speculators with
no transactions motive for trading, then the market
makers lose money to the informed traders but profit
from the uninformed. The spread between the ask and
bid prices generally results in a reduced volume of
trade and an inefficient allocation of securities.
Information has positive value to individual traders
and prices come to reflect some or all of the infor-
mation used by the informed traders.

Let there be a single marketed security offering
a random gross return R . Let each of the n traders
(indexed by i) be endowed with one share of the
security and one unit of a nonstorable commodity money

and let each be risk neutral. Trader i observes
two random variables X_i and Y_i : the variable
X_i represents the utility available to i from im-
mediate consumption and Y_i represents information
about R . The X_i's are assumed to be independent
and identically and non-atomically distributed, and,
initially, they are also assumed to be independent
of $(R, Y_1, \ldots, Y_n)$. To capture the anonymity of
the market, it is further assumed that the joint dis-
tribution of $(R, Y_1, \ldots, Y_n)$ is invariant under
permutations of the indices. The numbers 1 through
n designate the order in which traders arrive at
the market.

The k market makers are assumed to be risk
neutral, and to have sufficient supplies of money
and securities that any $k-1$ of them could accommo-
date the traders' net demands along any price path
in the range of the equilibrium price function com-
puted below. The market makers have no private in-
formation; even the ticker tape (the history of trad-
ing H_{i-1} when trader i arrives) is assumed to be
known by trader i . The market makers alternative
investments yield $X_0 = 1$, and 1 is also in the
support of the distribution of the X_i's .

The market operates as follows. When a customer
arrives, each of the k market makers simultaneously

announces a bid price and an ask price. The trader,
given his private information, his utility of current
consumption and these prices, chooses among three
options: to buy at the lowest ask price, to sell at
the highest bid price, or to stand pat and consume
his endowment. He makes the choice that maximizes
his expected payoff. Borrowing and short-sales are
prohibited.

The competition among the market makers resembles
the well known Bertrand competition in oligopoly theory,
and the Bertrand arguments show that, at equilibrium,
the highest bid price b_1^* is equal to the highest
price at which a market maker can at least break even
in expectation. Similarly, the lowest ask price a_i^*
is the lowest break-even selling price.

If trader i faced bid and ask prices b and
a with $a \geq b$, he would sell at b if
$b \underset{\sim}{X}_i > E[\underset{\sim}{R}|\underset{\sim}{H}_{i-1}, \underset{\sim}{X}_i]$ and he would buy at a if
$a \underset{\sim}{X}_i < E[\underset{\sim}{R}|\underset{\sim}{H}_{i-1}, \underset{\sim}{X}_i]$. Calling the first of these
random events " i sells at b " and the second " i
buys at a ," the equilibrium prices can be written
as follows:

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$$b_i^* = \sup b\{E[R|H_{i-1}, i \text{ sells at } b] > b\}$$

$$= E[R|H_{i-1}, i \text{ sells at } b_i^*] , \quad \text{and}$$

$$a_i^* = \inf a\{E[R|H_{i-1}, i \text{ buys at } a] \leq a\}$$

$$= E[R|H_{i-1}, i \text{ buys at } a_i^*] .$$

Notice that there is a tendency for traders to buy when they believe R is high and to sell when they believe R is low. Thus, the market makers are faced with a variation of the winner's curse. To avoid losing money, the market makers must build a margin of safety into their price quotations. It can be shown that if the density f_i of $\ln X_i$ satisfies $d^2 \ln f_i/dx^2 \leq 0$ and if $(R, X_1, \ldots, X_n)$ are affiliated (see Milgrom and Weber for the definition and properties of "affiliation"), then the equilibrium prices specified above satisfy this requirement; that is, $b_i^* \leq E[R|H_{i-1}] \leq a_i^*$. For example, if the X_i's have lognormal distributions and if the X_i's are independent estimates of R with the monotone likelihood ratio property, then these requirements are satisfied.

There are three cases which serve to illustrate the qualitative predictions of this model. In the first case, the X_i's are degenerate, and convey

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[5] Milgrom and Weber (1982b).

no information about R . It is then plain from the
formulas given above that $a_i^* = b_i^* = E[R|H_{i-1}] = E[R]$:
all trading takes place at the competitive equilibrium
price. In the second case, the X_i's are degenerate
and all equal to 1, but the X_i's are non-degenerate.
Comparing the price formula in this case with the
trader's trading rule, it is clear that no trade can
take place. The equilibrium bid price is set so low
and the ask price so high that all potential traders
are discouraged. The third case arises when we assume
that trader i gets information about R only if he
has no transactions motive for trading with the mar-
ket makers, i.e., only if $X_i = 1$. In this case,
it can be shown that if any trading involving the
informed traders takes place, then these traders im-
pose losses in expectation on the market makers.
Since the market makers break even in expectation
overall, they must profit from trading with the unin-
formed.

The foregoing theory has some testable implica-
tions concerning security prices and trading volumes.
It predicts, for example, that closely held firms
(where there are relatively many traders with private
information) should exhibit larger bid-ask spreads
and less active trading among outsiders than compar-
able firms where insider holdings are small.

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The model described in this paper is very spe-
cialized, but it seems likely that the qualitative
conclusions can be extended to much more general
settings.   Once the process of price determination
is made explicit, many of the contradictions and
apparent paradoxes that plague the traditional
theories of securities markets disappear, and a co-
herent alternative theory begins to emerge.

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