This correction was provided provided by Nikolai Kukushkin, January 2007.

Milgrom and Shannon (1994) ascribe the following statement to Veinott (1989).

Theorem A2. *Let* $\{S_\tau\}$ *be a net of nonempty sets that is* weakly ascending*, that is, such that if* $\tau' \geq \tau$*, and* $x \in S_\tau$*,* $x' \in S_{\tau'}$*, then either* $x \vee x' \in S_{\tau'}$ *or* $x \wedge x' \in S_\tau$*. Then there exists a monotone selection* $\{x(\tau)\}$ *from* $\{S_\tau\}$*.*

Counterexample. Let $T = [0,1]$, $S_\tau = (0, 1-\tau]$ for $0 \leq \tau < 1$, and $S_1 = \{1\}$. Let $\tau' \geq \tau$, $x \in S_\tau$, and $x' \in S_{\tau'}$. If $\tau' < 1$, then $x \wedge x' \in S_{\tau'} \subseteq S_\tau$. If $\tau' = 1$, then $x' = 1$, hence $x \wedge x' = x \in S_\tau$. Therefore, $\{S_\tau\}$ is weakly ascending.

Let $\{x(\tau)\}$ be a selection from $\{S_\tau\}$; we have $0 < x(0) \leq 1$. Let $\tau = 0$, and $1 > \tau' > 1 - x(0) \geq 0$; then $x(\tau') \leq 1 - \tau' < x(0)$, i.e., $\{x(\tau)\}$ cannot be monotone.

If an assumption that every S_τ is a chain-complete subset of the same complete lattice is added in Theorem A2, the counterexample will become irrelevant. And this version of the theorem seems sufficient for the proof of Theorem A3 of Milgrom and Shannon.

References

Milgrom, P., and C. Shannon, 1994. Monotone comparative statics. Econometrica 62, 157–180.

Veinott, A.F., 1989. Lattice Programming. Unpublished notes from lectures delivered at John Hopkins University.