The Economics of Modern Manufacturing: Technology, Strategy, and Organization

By PAUL MILGROM and JOHN ROBERTS*

Manufacturing is undergoing a revolution. The mass production model is being replaced by a vision of a flexible multiproduct firm that emphasizes quality and speedy response to market conditions while utilizing technologically advanced equipment and new forms of organization. Our optimizing model of the firm generates many of the observed patterns that mark modern manufacturing. Central to our results is a method of handling optimization and comparative statics problems that requires neither differentiability nor convexity. (JEL 022)

In the early twentieth century, Henry Ford revolutionized manufacturing with the introduction of his “transfer line” technology for mass production, in which basic inputs are processed in a fixed sequence of steps using equipment specifically designed to produce a single standardized product in extremely large quantities for extended periods of time. Although the specialization of Ford’s factories was extreme—the plants had to be shut down and redesigned when production of the Model T was ended—the transfer line approach influenced generations of industrialists and changed the face of manufacturing (see David A. Hounshell, 1984).

In the late twentieth century, the face of manufacturing is changing again. First, the specialized, single-purpose equipment for mass production which had characterized Ford’s factories is being replaced by flexible machine tools and programmable, multitask production equipment. Because these new machines can be quickly and cheaply switched from one task to another, their use permits the firm to produce a variety of outputs efficiently in very small batches, especially in comparison to the usual image of mass production (Nicholas Valery, 1987). Kenneth Wright and David Bourne (1988) report that in a recent survey of aerospace and other high precision industries 8.2 percent of all batches were of size one and 38 percent were sixteen or less. An Allen-Bradley Company plant making electric controls is reported to be able to switch production among its 725 products and variations with an average changeover time for resetting equipment of six seconds, enabling it to schedule batches of size one with relative efficiency (Tracy O’Rourke, 1988). Even in the automobile industry, flexible equipment has become much more common. Recently,  

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1 Probably no single firm is involved in all the changes we will describe. Nevertheless, there is a definite, discernable pattern of change in technology, manufacturing, marketing, and organizational strategy that characterizes successful “modern manufacturing.” For a description of the technologies involved, see U.S. Congress Office of Technology Assessment, 1984.

2 Optimal batch size can be determined via a standard Economic Order Quantity model, in which the setup costs of switching from one product to another are traded off against the costs of holding the larger average inventories of finished goods that go with longer runs and less frequent changeovers. Optimal batch size is a decreasing function of setup costs and so batch sizes optimally decrease as more flexible machines are introduced.
General Motors' engineers were able, for the first time in company history, to use a regular, producing facility to make pilots of the next year's model cars. The engineers set the equipment to make 1989 models after workers left the factory on Friday afternoon, ran the equipment to manufacture the new models over the weekend, and then reset the equipment to produce 1988 model cars so that regular production could be resumed on Monday morning (Thomas Moore, 1988). In contrast, with the older, less flexible technologies that have been the norm in the industry, changing over to produce the new year's models typically involved shutting down production for weeks.

Flexible equipment and small batch sizes have been accompanied by other changes. Smaller batch sizes are directly associated with a shortening of production cycles and with reductions in work-in-process and finished goods inventories. Shorter product cycles in turn support speedier responses to demand fluctuations and lead to lower back orders. Indeed, a general strategic emphasis on speeding up all aspects of the firm's operations is becoming common (Brian Dumasine, 1989). This is manifested in shorter product-development times, quicker order-processing, and speedier delivery, as well as in producing products faster. Examples abound. General Electric has reduced the design and production time it takes to fill an order for a circuit-breaker box from three weeks to three days, in the process reducing back orders from sixty days to two (Dumasine). The Allen-Bradley plant mentioned above fills orders the day after they are received, then ships them that same day by air express (O'Rourke). Building on early development by Toyota, many manufacturers now plan production jointly with their suppliers and maintain constant communication with them. This allows the downstream firms to replace inventories of components and supplies with "just-in-time" deliveries of needed inputs (James Abegglen and George Stalk, Jr., 1985; Nicholas Valery, 1987). Combining flexible production and low finished goods inventories with reliance on electronic data communications, Benetton maintains inventories of undyed clothing (shirts, scarves, pullovers) and uses nightly sales data gathered at its automated distribution center from terminals in individual stores to determine the colors it should make and where to ship its output (Tom Peters, 1987). Strategies like this one require not only flexible equipment to produce the right products, but short production cycles so that products are available at the right time. The extreme of this line of development is production of previously mass-produced items on a make-to-order basis: Moore reports a widespread rumor that the GM Saturn project will involve cars being custom-built within days of receipt of customers' computer-transmitted individual orders.

The manufacturing firms that adopt these new technologies and methods appear to differ from traditional firms in their product strategies as well. Many firms are broadening product lines, and there is a widespread increased emphasis on quality, both through frequent product improvements and new product introductions, and through reductions in defects in manufacturing. Caterpillar Corporation's $1.2 billion "Plant with a Future" modernization program has been accompanied by a doubling of the size of its product line (Ronald Henkoff, 1988). Rubbermaid insists that 30 percent of its sales should come from the products introduced in the preceding five years, and 3M Company has a similar 25 percent rule for each of its 42 divisions, with 32 percent of its $10.6 billion in 1988 sales actually coming from products less than five years old (Russell Mitchell, 1989). Meanwhile, reports of order-of-magnitude reductions in percentage defects are becoming commonplace.

New organizational strategies and workforce management policies are also part of this complex of changes. Ford has adopted a parallel, team (rather than sequential) approach to design and manufacturing engineering that, in conjunction with CAD/CAM (Computer Aided Design/Computer Aided Manufacturing) techniques, has cut development time on new models by one third (Alex Taylor, III, 1988). AT&T successfully used a similar, multidepartmental team approach in developing its new 4200 cordless phone (Dumasine), as did NCR with
its recently introduced 2760 electronic cash register (Otis Port, 1989b). Lockheed Corp.’s Aeronautical Systems Group has managed to reduce the time for designing and manufacturing sheet-metal parts by 96 percent, from 52 days to 2; the project manager credits organizational changes (including the arrangement of workstations, redefinition of worker responsibilities, and adoption of team approaches) with 80 percent of the productivity gain (Port, 1989a, p. 143; Warren Hausman, 1988). Motorola’s adoption of a pay scheme based on the skills employees acquire (rather than on their job assignments), its elimination of segmented pay categories among production workers, and its giving workers multiple responsibilities (including having production workers do quality inspections) is credited with major improvements in quality (Norm Alster, 1989).

A further auto industry example comes from GM’s massive investments in new technology, which have gone hand-in-hand with new supplier relations and more flexible work arrangements, as well as a broadened product line (General Motors Corporation, 1988).

More generally, Michael J. Piore (1986) provides survey evidence from firms around Route 128 in Boston of wider product lines, shorter product life cycles, greater emphasis on product quality, increased reliance on independent suppliers and subcontractors, and a more flexible organization of work that is supported by new compensation policies. Banri Asanuma, (1988a), has found similar trends among Japanese firms, Valery provides more anecdotal evidence drawn from a wide variety of industries internationally, and earlier Piore and Charles F. Sabel (1984) described related developments among small businesses in Italy and Austria.

A striking feature of the discussions of flexible manufacturing found in the business press is the frequency with which it is asserted that successful moves toward “the factory of the future” are not a matter of small adjustments made independently at each of several margins, but rather have involved substantial and closely coordinated changes in a whole range of the firm’s activities. Even though these changes are implemented over time, perhaps beginning with “islands of automation,” the full benefits are achieved only by an ultimately radical restructuring. Henkoff (p. 74) noted that one of the lessons of Caterpillar’s program was: “Don’t just change selected parts of your factory, as many manufacturers have done. To truly boost efficiency… it’s necessary to change the layout of the entire plant.”

The first lesson that Dumaine drew from studying successful adoption of speed-based strategies was to “start from scratch.” In discussing the adoption of “computer integrated manufacturing” (CIM), Valery (p. 15) stated that “nothing short of a total overhaul of the company’s strategy has first to be undertaken.” And in a parallel fashion, Walter Kiechel III (1988, p. 42) noted: “To get these benefits (of more timely operations), you probably have to totally redesign the way you do business, changing everything from procurement to quality control.”

This paper seeks to provide a coherent framework within which to understand the changes that are occurring in modern manufacturing. We ask, Why are these changes taking place? Is it mere coincidence that these various changes appear to be grouped together, or is there instead some necessary interconnection between them and common driving force behind them? What are the implications of the changes in manufacturing technology for inventory policy, product market strategy, and supplier and customer relations? What are the implications for the “make or buy” and vertical integration decisions and for the structure of business organization more generally?

Our approach to these questions is a price-theoretic, supply-side one involving three elements: exogenous input price changes, complementarities among the elements of the firm’s strategy, and non-convexities. The first element is the effect of technological change in reducing a set of costs. The particular ones on which we focus include: the costs of collecting, organizing, and communicating data, which have been reduced over time by the development of computer networks and electronic data transmission systems; the cost of product design and development, which have fallen with the emergence of computer-aided design; and the
costs of flexible manufacturing, which have declined with the introduction of robots and other programmable production equipment. We take these relative price reductions, whose existence is well documented, to be exogenous.

The direct effect of any of these price changes individually would be to increase use of the corresponding factor: for example, the emergence (i.e., falling cost) of CAD/CAM encourages its adoption, and the reduced cost of designing and beginning production of new products directly increases the attractiveness of expanded product lines and frequent product improvements. However, with several relative prices falling, there are multiple interactions, both among the corresponding technological factors and between them and marketing and organizational variables. These interactions give rise to indirect effects that might in principle be as large as the direct effects, and opposite in sign. Here, the second element of our analysis appears: These indirect effects tend, in the main, to reinforce the direct effects because the corresponding relationships are ones of “complementarity.” Here, we use the term “complements” not only in its traditional sense of a relation between pairs of inputs, but also in a broader sense as a relation among groups of activities. The defining characteristic of these groups of complements is that if the levels of any subset of the activities are increased, then the marginal return to increases in any or all of the remaining activities rises. It then follows that if the marginal costs associated with some activities fall, it will be optimal to increase the level of all of the activities in the grouping.

As an illustration, let us trace some of the indirect effects of a fall in the cost of computer-aided design (CAD) equipment and software that leads to the equipment being purchased. Some CAD programs prepare actual coded instructions that can be used by programmable manufacturing equipment, so one effect of the adoption of CAD may be to reduce the cost of adopting and using programmable manufacturing equipment. Since the prices of that equipment are also falling, the effects of the two price changes on the adoption of that equipment are mutually reinforcing. Of course, CAD also makes it cheaper for the firm to adopt a broader product line and to update its products more frequently. If the firm does so, than an indirect effect is to make it more profitable to switch to more flexible manufacturing equipment that is cheaper to change over. So, this indirect effect reinforces the direct effects of the changing input prices. With short production runs, the firm can economize on inventory costs (such as interest, storage, and obsolescence) by scheduling production in a way that is quickly responsive to customer demand. Such a scheduling strategy increases the profitability of technologies that enable quicker and more accurate order processing, such as modern data communications technologies. So, another indirect effect of falling CAD prices coincides with the falling price of data communication equipment. Thus, CAD equipment, flexible manufacturing technologies, shorter production runs, lower inventories, increased data communications, and more frequent product redesigns are complementary. However, the complementarities do not stop at the level of manufacturing, but extend to marketing, engineering, and organization.

The marketing side of the analysis involves two additional elements besides those already mentioned. First, more frequent setups lower the inventory necessary to support a unit of sales and thus also the marginal cost of output. This encourages lower prices. The second element arises because buyers value fast delivery. If most customers have good alternative sources of supply and only a few are “locked in,” then the resulting relationship between delivery time and demand is convex. In that case, reducing or eliminating production delays makes it profitable to reduce other sources of delay as well. Then, computerized order processing and a fast means of delivery are complementary to the quick responsiveness of the modern factory to new orders.

On the engineering side, as product life cycles become much shorter than the life of the production equipment, it becomes increasingly important to account for the characteristics of existing equipment in designing
new products. At the same time, the emergence of computer-aided design has made it less costly to modify initial designs, to estimate the cost of producing various designs with existing equipment, and to evaluate a broader range of potential designs. These changes have contributed to the growing popularity among U.S. firms of "design for manufacturability," in which products are developed by teams composed of designers, process engineers, and manufacturing managers (Robert Hayes, Steven Wheelwright, and Kim Clark, 1988), and the corresponding practice among Japanese automakers of providing preliminary specifications to suppliers who comment on the proposed design and supply drawings of parts (Asanuma, 1988b)—innovations in engineering organization that contribute to a more efficient use of existing production equipment and manufacturing know-how. Moreover, taking account of the limits and capabilities of production equipment in the design phase makes it easier to ensure that quality standards can be met, and so is complementary to a marketing strategy based on high quality.

The firm's problem in deciding whether to adopt any or all of these changes is marked by important non-convexities. These are first of the familiar sorts associated with indivisibilities: product line sizes are naturally integer-valued. A form of increasing returns also figures in the model, because the marginal impact of increasing the speed with which customers are served increases the service speed. Beyond these, however, the complementarities noted above can be a further source of non-convexities that are associated with the need to coordinate choices among several decision variables. For example, purchase of CAD/CAM technology makes it less costly for a firm to increase its frequency of product improvements, and more frequent product introductions raise the return to investments in CAD/CAM technology. Thus, it may be unprofitable for a firm to purchase a flexible CAD/CAM system without changing its marketing strategy, or to alter its marketing approach without adopting a flexible manufacturing system, and yet it may be highly profitable to do both together. (In contrast, if the value of a smooth concave function at some point in the interior of its domain cannot be increased by a small change in the value of any single variable, then the function achieves a global maximum at that point.) These non-convexities then explain why the successful adoption of modern manufacturing methods may not be a marginal decision.

It is natural to expect the characteristics of the modern manufacturing firm to be reflected in the way the firm is managed and the way it structures its relations with customers, employees, and suppliers. Exploiting such an extensive system of complementarities requires coordinated action between the traditionally separate functions of design, engineering, manufacturing, and marketing. Also, according to transaction costs theories, the increasing use of flexible, general purpose equipment in place of specialized, single purpose equipment ought to improve the investment incentives of independent suppliers (Oliver Williamson, 1986; Benjamin Klein, Robert Crawford, and Armen Alchian, 1978; Jean Tirole, 1986) and to reduce cost of the negotiating short-term contracts (Milgrom and Roberts, 1987) and so to favor short-term contracting with independent suppliers over alternatives like vertical integration or long-term contracting. The supplier relations that mark modern manufacturing firms—involving close coordination between the firm and its independently owned contractors and suppliers—appear to be consistent with these theories, and inconsistent with theories in which joint planning can only take place in integrated firms.

In this essay, we develop a theoretical model of the firm that allows us to explore many of the complementarities in modern manufacturing firms. The non-convexities inherent in our problem makes it inappropriate to use differential techniques to study the effects of changing parameters. Instead, we utilize purely algebraic (lattice-theoretic) methods first introduced by Donald M. Topkis (1978), which provide an exact formalization of the idea of groups of complementary activities. In problems with complementarities among the choice variables this approach easily handles both indivisibilities and non-concave maximands while allowing
sharp comparative statics results. In particular, we give conditions under which the set of maximizers moves monotonically with changes in a (possibly multidimensional) parameter. Because these methods are quite straightforward and would seem to be of broad applicability in economics, but are not well known among economists, we describe them in some detail in Section I.

Our model and its basic analysis are provided in Section II. The firm in our model chooses its price; the length of the product life cycle or frequency of product improvements (a surrogate for quality); its order-receipt, processing, and delivery technologies; various characteristics of its manufacturing and design technologies as reflected in its marginal cost of production and its costs of setups and new product development; its manufacturing plan, including the length of the production cycle (and, implicitly, its inventory and back-order levels); and aspects of its quality control policy, all with the aim of maximizing its expected profits. Using reasonable assumptions about the nature and equipment costs, we find that the complementarities in the system are pervasive. We use the firm’s optimizing response to assumed trends in input prices (the falling costs of communication, computer-aided design, and flexible manufacturing) in the presence of these complementarities to explain both the clustering of characteristics and the trends in manufacturing.

In Section III, we turn our attention to the organizational problems associated with the new technologies. We summarize and review the predictions of the model in the concluding Section IV.

I. The Mathematics of Complementarities

Here we review some basic definitions and results in the mathematics of complementarities. The results permit us to make definite statements about the nature of the optimal solution to the firm’s problem and how it depends on various parameters, even though the domain of the objective function may be non-convex (for example, some variables may be integer-valued) and the objective function itself may be non-concave, non-differentiable, and even discontinuous at some points.

For additional developments and missing proofs, see Topkis.

We first introduce our notation. Let $x, x' \in \mathbb{R}^n$. We say that $x \succeq x'$ if $x_i \geq x'_i$ for all $i$. Define $\max(x, x')$ to be the point in $\mathbb{R}^n$ whose $i$th component is $\max(x_i, x'_i)$, and $\min(x, x')$ to be the point whose $i$th component is $\min(x_i, x'_i)$. This notation is used below to define the two key notions of the theory. The first notion is that of a supermodular function, which is a function that exhibits complementarities among its arguments. The second is that of a sublattice of $\mathbb{R}^n$, a subset of $\mathbb{R}^n$ that is closed under the $\max$ and $\min$ operations and whose structure lets us characterize the set of optima of a supermodular function.

**Definition 1:** A function $f: \mathbb{R}^n \to \mathbb{R}$ is supermodular if for all $x, x' \in \mathbb{R}^n$,

\[
(1) \quad f(x) + f(x') \leq f(\min(x, x')) + f(\max(x, x')).
\]

The function $f$ is submodular if $-f$ is supermodular. Inequality (1) is clearly equivalent to

\[
[f(x) - f(\min(x, x'))] + [f(x') - f(\min(x, x'))] \leq f(\max(x, x')) - f(\min(x, x'));
\]
the sum of the changes in the function when several arguments are increased separately is less than the change resulting from increasing all the arguments together. The inequality is also equivalent to

\[
f(\max(x, x')) - f(x') \geq f(x) - f(\min(x, x'));
\]

increasing one or more variables raises the return to increasing other variables. These reformulations of the defining inequality make clear the sense in which the supermodularity of a function corresponds to complementarity among its arguments.

Note that any function of a single variable is trivially supermodular. This observation serves to resolve various questions about possible relationships between supermodularity and other concepts. However, even in
a multidimensional context, supermodularity is distinct from, but related to, a number of more familiar notions. First, supermodularity has no necessary relation to the concavity or convexity of the function: consider \( f(x_1, x_2) = x_1^a + x_2^b \), which is supermodular for all values of \( a \) and \( b \) but may be either concave or convex (or both or neither). Nor, in the context of production functions, does supermodularity carry implications for returns to scale. For example, the Cobb-Douglas functions \( f(x_1, x_2) = x_1^a x_2^b \) may show increasing or decreasing returns to scale but are supermodular for all positive values of \( a \) and \( b \). This is most easily checked using Theorem 2, below, which states that a smooth function \( f \) is supermodular if and only if \( \partial^2 f / \partial x_i \partial x_j \geq 0 \) for \( i \neq j \). Thus, if \( f \) is supermodular and smooth, then the smooth supermodular function \(-f\) shows weak cost complementarities as defined by William Baumol, John Panzar, and Robert Willig (1982, pp. 74–75). Even without smoothness, it is easily shown that a submodular function that is zero at the origin shows economies of scope as defined by Baumol et al. More generally, submodularity is related to, but distinct from, the notion of subadditivity that figures centrally in the study of cost functions.\(^3\) For example, any function of a single variable is submodular, but obviously not all such functions are subadditive. Meanwhile, the functions on \([0, 1] \times [0, 1]\) given by \( f(x_1, x_2) = 1 + x_1 + x_2 + \varepsilon x_1 x_2 \) are submodular for \( \varepsilon > 0 \), supermodular for \( \varepsilon \geq 0 \), and subadditive for all \( \varepsilon \) sufficiently close to zero in absolute value.

Six theorems about supermodular functions are provided here. The first four together provide a relatively easy way to check whether a given function is supermodular. Theorem 5 indicates how, in a parameterized maximization problem, the maximizer changes with changing parameters, while Theorem 6 characterizes the set of maximizers of a supermodular function. It is Theorem 5 that makes our comparative statics exercises possible.

Let \( x \setminus i \) denote the vector \( x \) with the \( i \)th component removed and let \( x \setminus i, j \) denote \( x \) with the \( i \)th and \( j \)th components removed. Let subscripts on \( f \) denote partial derivatives, for example, \( f_i = \partial f / \partial x_i \), \( f_{ij} = \partial^2 f / \partial x_i \partial x_j \).

**THEOREM 1:** Suppose \( f : \mathbb{R}^n \rightarrow \mathbb{R} \). If for all \( i, j \), and \( x \setminus i, j \), \( f(x_i, x_j, x \setminus i, j) \) is supermodular when regarded as a function of the arguments \( (x_i, x_j) \) only, then \( f \) is supermodular.

**THEOREM 2:** Let \( I = [a_1, b_1] \times \cdots \times [a_n, b_n] \) be an interval in \( \mathbb{R}^n \) with nonempty interior and suppose that \( f : I \rightarrow \mathbb{R} \) is continuous and twice continuously differentiable on the interior of \( I \). Then \( f \) is supermodular on \( I \) if and only if for all \( i \neq j \), \( f_{ij} \geq 0 \).

Theorem 2 is stated above in the form given in Topkis. For our application, we will need a slightly stronger theorem in which the condition that \( f \) is twice continuously differentiable is weakened to the condition that it can be written as an indefinite double integral with a nonnegative integrand. The precise extension is stated and proved in the Appendix.

**THEOREM 3:** Suppose that \( f, g : \mathbb{R}^n \rightarrow \mathbb{R} \) are supermodular functions. Then \( f + g \) is supermodular. If, in addition, \( f \) and \( g \) are nonnegative and nondecreasing, then \( fg \) is supermodular.

**THEOREM 4:** Suppose that \( f : \mathbb{R}^{n+1} \rightarrow \mathbb{R} \) is supermodular and continuous in its first argument. Then for all \( a, b \in \mathbb{R} \), the function \( g : \mathbb{R}^n \rightarrow \mathbb{R} \) defined by \( g(x) = \max_{y \in [a, b]} f(y, x) \) is supermodular.

**PROOF:**

Since \( f \) is continuous in its first argument, the function \( g \) is well defined. For all \( x \) and \( x' \), there exist \( y \) and \( y' \) with \( g(x) = f(y, x) \) and \( g(x') = f(y', x') \). Then,

\[
g(x) + g(x') = f(y, x) + f(y', x')
\]

\[
\leq f(\max(y, y'), \max(x, x')) + f(\min(y, y'), \min(x, x'))
\]

\[
\leq g(\max(x, x')) + g(\min(x, x')).
\]

\(\square\)
In what follows we will be particularly concerned with constrained optimization of supermodular functions, and our results will depend on the constraint set having the right structure or shape, namely, that of a sublattice of \( \mathbb{R}^n \).

**Definition 2:** A set \( T \) is a sublattice of \( \mathbb{R}^n \) if for all \( x, x' \in T \), \( \min(x, x') \in T \) and \( \max(x, x') \in T \).

In our application, the definition of a sublattice represents the idea that if it is possible to engage in high (respectively, low) levels of each of several activities separately, then it is possible to engage in equally high (resp., low) levels of all of the activities simultaneously. Thus, for example, if \( S_1, \ldots, S_n \) are arbitrary subsets of \( \mathbb{R} \), then \( S_1 \times \cdots \times S_n \) is a sublattice of \( \mathbb{R}^n \). However, the product sets are not the only sublattices. The sublattice structure also permits the possibility that some activities can be engaged in at a high level only if the others are also carried out at a high level. For example, if \( x \geq x' \), then \( \{ x, x' \} \) is a sublattice.

**Definition 3:** Given two sets \( S, S' \subset \mathbb{R}^n \), we say that \( S \) is **higher than** \( S' \) and write \( S \succeq S' \) if for all \( x \in S \) and \( x' \in S' \), \( \max(x, x') \in S \) and \( \min(x, x') \in S' \).

**Theorem 5:** Suppose \( f: \mathbb{R}^{n+k} \to \mathbb{R} \) is supermodular and suppose \( T(y) \) and \( T(y') \) are sublattices of \( \mathbb{R}^n \). Let \( S(y) = \text{argmax}(f(z, y) : z \in T(y)) \), and define \( S(y') \) analogously. Then \( y \succeq y' \) and \( T(y) \succeq T(y') \) imply that \( S(y) \succeq S(y') \).

**Proof:**

Let \( x \in S(y) \) and \( x' \in S(y') \) and \( y \succeq y' \) so that \( y = \max(y, y') \) and \( y' = \min(y, y') \). Since \( T(y) \succeq T(y') \), \( \max(x, x') \in T(y) \) and \( \min(x, x') \in T(y') \). From the definitions, \( f(x, y') \geq f(\max(x, x'), y) \) and \( f(x', y') \geq f(\min(x, x'), y') \), but since \( f \) is supermodular, \( f(x, y') + f(x', y') \leq f(\max(x, x'), y') + f(\min(x, x'), y') \) from which the conclusion is immediate.

**Theorem 6:** Suppose \( f: \mathbb{R}^n \to \mathbb{R} \) is supermodular and suppose \( T \) is a sublattice of \( \mathbb{R}^n \). Then the set of maximizers of \( f \) over \( T \) is also a sublattice.

**Proof:**

Apply Theorem 5 with \( y = y' \).

Theorem 5 is particularly important for our application. When its conclusion holds, we shall say that the set of optimizers “rises” as the parameter values increases. What justifies this language? The theorem implies, for example, that if \( x^*(y) \) and \( x^*(y') \) are the unique maximizers given their respective parameter vectors \( y \) and \( y' \) and if \( y \succeq y' \), then \( x^*(y) \succeq x^*(y') \). (For uniqueness implies that \( x^*(y) = \max(x^*(y), x^*(y')) \), from which \( x^*(y) \succeq x^*(y') \) follows.) Alternatively, suppose we assume that \( f \) is a continuous supermodular function that \( T \) is compact sublattice, so that the set of maximizers corresponding to any parameter vector \( y \) is compact. Then, by Theorem 5, there are greatest and least elements \( \bar{x}(y) \) and \( \underline{x}(y) \) in the set of maximizers \( S \). One can show that both \( \bar{x}(y) \) and \( \underline{x}(y) \) are nondecreasing functions of \( y \). (Using Theorem 6 and the definitions, \( \bar{x}(y') \leq \max(\bar{x}(y), \bar{x}(y')) \leq \bar{x}(y) \) and similarly \( \underline{x}(y) \leq \min(\underline{x}(y), \underline{x}(y')) \leq \underline{x}(y') \).)

**II. Complementarities in Production**

We study a model of a multiproduct firm facing a downward sloping demand curve. The firm may be a monopoly or monopolistic competitor. Alternatively, our model may be viewed as a building block for a model of oligopolistic markets.

In the formal model, the firm chooses the levels of the following decision variables:

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<tr>
<th>Variable</th>
<th>Interpretation</th>
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<tbody>
<tr>
<td>p</td>
<td>Price of each product</td>
</tr>
<tr>
<td>q</td>
<td>(Expected) number of improvements per product per period</td>
</tr>
<tr>
<td>a</td>
<td>Order receipt and processing time</td>
</tr>
<tr>
<td>b</td>
<td>Delivery time</td>
</tr>
<tr>
<td>c</td>
<td>Direct marginal costs of production</td>
</tr>
<tr>
<td>d</td>
<td>Design cost per product improvement</td>
</tr>
<tr>
<td>e</td>
<td>Extra set-up costs on newly changed products</td>
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m Number of setups per period
r Probability of a defective batch
s Direct cost of a setup
w Wastage costs per setup

In addition, we denote the number of products by \( n \).

The functional relationships and parameters that complete the model include the demand specification, the specification of the capital costs of different levels of the technological variables, the functional relation linking the average delay between receipt of an order and its being filled to the number of products and of setups, the marginal cost of production, the marginal cost of reworking defectives, the cost of holding inventories, and a time parameter that will proxy for the state of technology and demand. More specifically, we have

<table>
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<th>Parameter</th>
<th>Interpretation</th>
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<tr>
<td>( \rho )</td>
<td>Marginal cost of reworking a defective unit</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Calendar time</td>
</tr>
<tr>
<td>( \iota )</td>
<td>Cost of holding inventory per unit</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Capital costs ( (\kappa = \kappa(a, b, c, d, e, r, s, w, \tau)) )</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Base demand per product ( (\mu = \mu(p, q, n, \tau)) )</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Demand shrinkage with delay time ( (\delta = \delta(t, \tau)) )</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Expected wait for a processed order to be filled ( (\omega = \omega(m, r, n)) )</td>
</tr>
</tbody>
</table>

The total expected wait for an order to be received, processed, filled, and shipped, which determines the value of the shrinkage factor \( (\delta) \) on demand, is \( t = a + \omega + b \), and realized demand is then \( \mu(p, q, n, \tau) \delta(a + \omega + b, \tau) \). Thus, the firm’s payoff function \( \Pi \) is

\[
\Pi(p, q, m, a, b, c, d, e, r, s, w, \tau) = (p - c - rp - \iota/m)n\mu(p, q, n, \tau) \\
\times \delta(a + \omega(m, r, n) + b, \tau) \\
- m(s + w) - nq(d + e) \\
- \kappa(a, b, c, d, e, r, s, w, \tau).
\]

The total profit \( \Pi \) is the operating profit minus the fixed costs associated with machine setups, product redesign, and the purchase of capital equipment.

The first term \( (p - c - rp - \iota/m)n\mu \delta \) is the operating profit. For each unit sold, the firm receives the price \( p \) and pays direct production costs, expected rework costs, and inventory holding and handling costs. In line with the Economic Order Quantity models commonly used for inventory analysis, we treat the average levels of work-in-process and finished goods inventories as being directly proportional to demand and inversely proportional to the number of setups. Similarly, back orders are directly proportional to demand and decreasing in the number of setups. We have used the function \( \delta \), which we take to be uniform across products, to model the cost of back orders; this takes the form of lost demand when delivery is delayed. We have also modeled the firm as setting a uniform price \( (p) \) across the product line, which is reasonable given the symmetry of the products in the model.

The second term \( m(s + w) \) is the cost of the setups, which, as suggested above, consists of the number of setups times the sum of the direct costs plus wastage per setup. The term \( nq(d + e) \) is the cost of redesign over the period, including the extra setup costs on newly altered products. In this term, \( nq \) is the total number of redesigns or improvements.

The last term is \( \kappa \), which is the capital cost of selecting the various technological variables, \( a, b, c, d, e, r, s, \) and \( w \), at any date \( \tau \). Among these technology variables, the order receipt and processing time \( (a) \) is determined by the technology used for communicating orders (mail, express courier, FAX, electronic data communications networks, etc.) and by the means used to handle orders once received (manual entry, computerized order entry systems). Whichever choices are made, there are capital costs involved in setting up the corresponding systems. Similarly, different options exist that determine the speed of delivery \( (b) \) from inventory, and these too have differing capital costs.

Our model allows us to represent many aspects, and tradeoffs in the firm’s choice of manufacturing strategy. For example, the
flexibility of design technology is modeled by the variable \( d \). The introduction of computer aided design (CAD) lowers these marginal costs of redesigning and improving products, but it also involves significant capital expenditures on training, hardware, and software. Both of these effects are captured in our profit function.

Flexibility of manufacturing equipment has a number of aspects, several of which are represented in our model. First, flexibility is often associated with low costs of routinely changing over from producing one good to another. Here, this effect is represented first through the variable \( s \): more flexible equipment means lower setup costs in terms of the downtime and direct labor costs involved in resetting the machines, switching dies, etc. Also, more flexible equipment might involve less wastage (lower \( w \)) per setup. This wastage might be in the form of extra inspection, scrap, rework, and repair costs that are necessary when a changeover is made. The precision of computer aided equipment, “design for manufacturability” (facilitated by CAD-CAM), and similar investments lower these costs. Finally, flexibility might involve costs of changing machinery over to produce new or redesigned products (low values of \( e \)).

The technological quality variable \( r \) captures a somewhat different feature of modern manufacturing methods. Improving quality on this dimension may involve investing in more precisely controlled machinery which may even constantly monitor and adjust itself. It may also improve more prosaic but possibly more significant efforts aimed at changing attitudes toward quality, such as giving workers the ability to stop the production line when a problem arises.

Although we do not explicitly model labor force decisions here, an element of the flexibility of modern manufacturing is associated with broadly trained workers and with work rules that facilitate frequent changes in activities. In this context we may interpret investments in flexibility in terms of worker education and industrial relations efforts, as well as the purchase of physical capital. Certainly, flexibility in the labor force and in the capital equipment are mutually complementary.

Finally, the choice of \( c \), the marginal costs of production, has capital cost implications, if only through investing in learning how to control costs.

Even before we make any assumptions about the form of the unspecified functions \( \kappa, \mu, \delta, \) and \( \omega \), certain complementarities are evident in the model. For example, with more frequent changeovers (higher \( q \)), the returns to more efficient technologies for redesigning products and changing over equipment (higher values of \(-d\) and \(-e\)) will naturally rise and, conversely, more efficient changeover and redesign technologies raise the marginal returns to increasing \( q \). Similarly, an increase in the number of setups per period \( (m) \) and the concomitant reduction in inventories and back orders is complementary with a reduction in the components of set-up costs (increases in \(-s\) and \(-w\)). Technologically, one expects that reduced set-up and changeover costs are bundled together in the new equipment. However, conclusions such as that one require that we make an assumption about the properties of the unspecified function \( \kappa \). To make further statements, we need to make assumptions about properties of all the unspecified functions.

Our assumption about the form of \( \delta \) is the following:

**ASSUMPTION A1:** \( \delta \) is twice continuously differentiable, nonnegative, decreasing and convex in \( t \), nondecreasing in \( \tau \) and submodular in \((t, \tau)\).

The assumption that \( \delta \) is decreasing in \( t \) simply means that increased delay reduces sales, while convexity says that the larger is the delay, the smaller is the marginal impact of additional delay. Inclusion of \( \tau \) allows for a time trend in demand through \( \delta \). This trend must be nonnegative for our results, but it could be trivial. The submodularity assumption means that, as time passes, \( \delta \) becomes weakly more negative or, equivalently, the returns to reducing waiting time, \(-t\), increase weakly. This might come about because the adoption of more modern manufacturing methods by the firm’s customers raises the importance of speedy service to them.
An immediate implication of the convexity assumption in Assumption A1 is that activities that reduce the several components of delay time \((a, b, \text{and } \omega)\) are mutually complementary, as can be easily verified by checking that the corresponding mixed partial derivatives of the profit function are positive. These complementarities may seem surprising, since the three components of waiting time are perfect substitutes for one another in determining the total delay. However, as our analysis shows, the possibility of substituting these elements to achieve a fixed time delay is irrelevant to their assessment as potential complements within the corporate strategy.

To complete our evaluation of the complementarities associated with speed, we must take an assumption about the \(\omega\) function. Generally, we would expect \(\omega\) to be increasing in the probability of a batch requiring reworking \((r)\) and decreasing in the number of setups \((m)\). The one nonobvious element in Assumption A2 below is that \(r\) and \(m\) are complements in determining \(\omega\): an increase in the number of setups (or decrease in batch size) is assumed to raise the impact on delay time of an increase in the probability of a batch being defective. This complementarity may be caused by the more frequent changeovers in the rework facility being required by more frequent changeovers in the main facility. Otherwise, it seems natural to expect the effect to be zero, which is also consistent with our assumption.

**ASSUMPTION A2:** \(\omega = \omega(m, r, n)\) is twice continuously differentiable, decreasing in \(m\), increasing in \(r\) and supermodular in \(m\) and \(r\), given \(n\). That is, \(\omega_m \leq 0\), \(\omega_r \geq 0\), and \(\omega_{mr} \geq 0\).

As a consequence of (A1) and (A2), \(-a, -b, m, \text{and } -r\) are mutually complementary in increasing demand.

To complete our analysis of the marketing aspects of strategy, the form of the demand function \(\mu\) must be restricted. We make two assumptions. The first is a standard one:

**ASSUMPTION A3:** \(\mu\) is twice continuously differentiable, increasing in \(q\), and decreasing in \(p\), while operating profits, defined by \((p - c - rp - \tau)\delta(a + \omega + b, \tau),\) are a strictly quasi-concave function of \(p\).

The first part asserts innocuously that consumers prefer lower prices and higher quality. Since we will hold \(n\) fixed, we need make no assumptions on its effect, although it would be natural to assume that \(n\mu\) is increasing in \(n\). The assumption that demand is quasi-concave in prices is standard. The nonstandard part of our assumption about demand is contained in Assumption A4:

**ASSUMPTION A4:** \(\mu(p, q, n, \tau)\) is non-decreasing in \(\tau\) and supermodular when regarded as a function of \(-p, q\) and \(\tau\), for given \(n\).

A4 is a complicated assumption. It would be satisfied, for example, by a multiplicatively separable specification of demand, \(\mu = A(p)B(q)C(\tau)\), as well as by additively separable demand \(A(p) + B(q) + C(\tau)\), and \(\lambda < 0, \lambda' > 0, \text{and } C' \geq 0\) in both cases. It asserts that the quantity demanded becomes (weakly) more sensitive to price and quality with passing time, and that at higher quality levels the quantity demanded is more sensitive to price changes. Again, we emphasize that we allow demand to be independent of \(\tau\), but if a dependence exists, it should not be such as to offset the supply-side effects of technological progress embodied in the effect of \(\tau\) in the \(\kappa\) function.

Our final assumption is the following one, on the \(\kappa\) function. It is key because it embodies the presumed technological changes in the capital goods industries supplying the firm that are the basis for our arguments:

**ASSUMPTION A5:** \(\kappa(-a, -b, -c, -d, -e, -r, -s, -w, \tau)\) is submodular.

This assumption is stated in terms of the negatives of the natural decision variables because \(a\) and \(b\) decrease with improved communication systems, better data transmission, entry, storage, manipulation, and retrieval systems, and speedier delivery methods, and the other choice variables decrease with improved design, manufacturing, quality, and cost-control technologies, that is, with increases in \(\tau\).
Conceptually, A5 has two parts. The first is our assumption about the time path of exogenous technological change: the incremental capital costs of modern technologies for communication, delivery, design, and flexible production are falling over time. Assuming differentiability of $\kappa$, so that Theorem 2 applies, these trends are captured in the inequalities $\frac{\partial^2 \kappa}{\partial x \partial \tau} \leq 0$, $x = -a, -b, -c, -d, -e, -r, -s, -w$: technological change among capital equipment suppliers lowers the costs over time of the firm’s increasing delivery speeds, using more flexible manufacturing methods, reducing the probability of defects, reducing costs of redesign and controlling production costs. Notice that we require no assumptions about the relative rates at which these prices are falling, because all these price changes will turn out to have mutually reinforcing effects.

The second part of A5 concerns the interrelationships among investments in the new technologies. For example, assuming that the mixed partial derivative of $\kappa$ with respect to $-d$ and $-e$ is positive means that the level of investment in flexible equipment necessary to reduce extra set-up costs by a given amount is reduced by investments in flexible design equipment. Of course, if the technologies were completely separable, the condition would be met but, as we argued in the introduction, separability is not a realistic assumption. The cost of instituting both computer-aided design and flexible machining systems (FMS) to achieve given levels of design and set-up costs is generally less than the sum of the costs of instituting the two separately because the CAD equipment may provide set-up instructions readable by the FMS machinery, eliminating the costly step of encoding (possibly with error) the design instructions into a form readable by the flexible machine. Other complementarities in the physical equipment are similarly represented in $\kappa$. Thus, CAD makes it less costly to reduce defects by making it much cheaper to design products that are easily manufactured, while a computerized order-entry system can eliminate the need to transcribe order information into a form readable by the manufacturing computers, saving costs and reducing errors.

Other interactions, based on substitute uses of resources, could work against our complementary assumptions. For example, if the firm faces a fixed capital budget or a rising cost of capital with increased levels of investment, or if there are constraints on space or personnel and computer based systems for communication and design compete for these resources, then an investment in lowering a would raise the cost of investments to lower $d$. The second part of A5 is the hypothesis that the technological complementarities we have identified are larger than the effects of any constraints on the resources that the systems must share.

Note that throughout the analysis, we are holding the rework cost parameter $\rho$ and the inventory holding cost parameter $\iota$ fixed.

The problem of maximizing $\Pi$ is not amenable to standard, calculus-based techniques. First, although demand is assumed to be a quasi-concave in price, we have made no other concavity assumptions. Indeed, the assumed convexity of $\delta$ means that profit, exclusive of capital costs, is actually convex in the total delay between placing an order and receiving shipment, and since A5 places no restrictions on the concavity or convexity of $\kappa$, $\Pi$ may well be convex in $-a, -b, m$ and $-r$ over some ranges. In this case, satisfaction of a first-order condition identifies a (local) minimum with respect to the variable in question. Moreover, it is natural to take $m$ to be integer-valued of the form, $nk$, where $k$ is the number of production cycles per period and $n$ is the number of products. However, the methods developed in the previous section are applicable here, once we have the necessary complementarities. In this, A5 plays a major role.

We are now ready to state and prove our main results. The idea is to use Theorem 2 to show that the firm’s objective function is supermodular in the firm’s (sign-adjusted) decision variables and that consequently, by Theorems 5 and 6, the set of optimizers forms a sublattice that moves up over time. However, it is not true that $\Pi$ is a supermodular function of all its arguments, because the mixed partial derivatives of that function in price and the determinants of waiting time have the wrong sign when the
price is set too low. To get around that difficulty, we consider the optimized value of profit with respect to price where the price is restricted by a lower bound $P$. (See Figure 1.) Letting this bound replace the price as the choice variable in our problem, it is apparent that this change of variables leaves the optimal values of the non-price variables unchanged. By A3, for any fixed values of the other decision variables and parameters, the corresponding optimal value of $p$ is unique, and the highest optimal value of $P$ equals the optimal price $p$. Moreover, as we now show, the new function is super-modular in the sign-adjusted decision variables.

**THEOREM 7:** Assume A1 through A5. Then the function

$$\pi(-P, q, m, -a, -b, -c, -d, -e, -r, -s, -w, \tau)$$

$$\equiv \max_{p \geq P} \Pi(p, q, m, a, b, c, d, e, r, s, w, \tau)$$

is supermodular on the sublattice of $R^n$ defined by the restrictions that all the decision variables be nonnegative.

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**PROOF:**

By Theorem 3 and A5, it is enough that the function $\pi + \kappa$ be supermodular. If $\pi + \kappa$ were twice continuously differentiable everywhere, then by Theorem 2 it would be sufficient to check that all the cross-partial derivatives of $\pi + \kappa$ are nonnegative. The verification is routine, except that $\pi + \kappa$ may have no second derivatives on the set of points in the domain where $\partial^2(\Pi + \kappa)/\partial p^2 = 0$. However, it is straightforward to verify that the slightly weaker assumptions of Theorem 2* in the Appendix are satisfied, so $\pi + \kappa$ is supermodular, as we had required. □

**THEOREM 8:** Assume A1 through A5 and that $\kappa$ is continuous. Let the individual decision variables each be constrained to lie in a compact set consistent with the nonnegativity requirement, so that together they lie in a compact set that is a sublattice. Then the set of maximizers of $\pi$ is a compact sublattice which rises with $\tau$.

**PROOF:**

Apply Theorems 6 and 7. Note that this result allows us to restrict the decision variables to be integer-valued and to limit the number of available technologies to some finite set. The supermodularity of the functional form $\pi + \kappa$ is verified by investigating its derivatives on continuous intervals, and the restriction to a compact sublattice is imposed later in a way that permits a restriction to discrete choices.

The key conclusion is that the sign-adjusted decision variables all rise over time. Thus, as time passes, one expects to see a pattern of the following sort linking changes in a wide range of variables:

- Lower Prices,
- Lower Marginal Costs,
- More Frequent Product Redesigns and Improvements,
- Higher Quality in Production, Marked by Fewer Defects,
- Speedier Communication with Customers and Processing of Orders,
- More Frequent Setups and Smaller Batch Sizes, with Correspondingly Lower Levels of Finished-Goods and Work-In-Process
Inventories and of Back Orders per Unit Demand,
• Speedier Delivery from Inventory,
• Lower Setup, Wastage, and Changeover Costs,
• Lower Marginal Costs of Product Redesign.

The conclusion in Theorem 6 that the set of optimizers forms a sublattice implies that if at any time there are multiple solutions to the optimization problem, then there is a highest and a lowest optimal solution in the vector inequality sense. Further, comparing any two firm’s choices, if these differ, then the choice of selecting the higher, “more modern” level for each decision variable is also optimal, as is the vector made up of the term-by-term minimal values of the two firm’s choices. More typically, however, we might expect a unique solution.

In any case, the chosen levels move up together over time in response to the falling costs of faster communications, more flexible production, and more frequent redesign.

The model we have presented is a static one, but it is nevertheless suggestive about the nature of the path to the modern manufacturing strategy. Specifically, it suggests that even if the changes that take place in the environment—especially the falling cost of the equipment used under the modern manufacturing strategy—happen gradually, the adoption process may be much more erratic, for two reasons. First, there are non-convexities, which mean that the optimum may shift discontinuously, with the profit-maximizing levels of the whole complex of variables moving sharply upward. This makes it relatively unprofitable to be stuck with a mixture of highly flexible and highly specialized production equipment. One does not necessarily expect to find that the adoption of the new equipment is sudden; it may still be desirable to iron out the wrinkles in the new technology with an initial small scale adoption. What the theory suggests we should not see is an extended period of time during which there are substantial volumes of both highly flexible and highly specialized equipment being used side-by-side. Then, once the adoption is well underway, it should proceed rapidly, with increasing momentum.

Second, there are the complementarities, which make it relatively unprofitable to adopt only one part of the modern manufacturing strategy. The theory suggests that we should not see an extended period of time during which one component of the strategy is in place and the other components have barely begun to be put into place. For example, we should not see flexible equipment used for a long period with unchanging product lines.

The conclusion of Theorem 8 that firms will increase quality in the sense of reducing the probability $r$ of a defective batch is worth further comment. Many observers have noted a focus on increased quality of output among modern manufacturing firms. One would expect that design for manufacturability would result directly in lower defect rates. However, the complementarities displayed in the model provide a second, less obvious incentive for increased quality. Decreases in the probability of defects are strictly complementary with increases in $m$ through the effect on operating profit: demand grows with increases in $m$, and this increases the return to lowering costs by reducing the probability of reworking.5

Recall that we have held $n$, the number of products, fixed throughout this analysis. Inspection of the profit function in light of the arguments in Theorem 7 should make the necessity of doing this clear: neither $n$ nor its negative are naturally complementary with the other decision variables. This shows up most clearly in the cost of redesign term, $-nq(d + e)$, where increases in $n$ make decreases in $d$ and $e$ more attractive but increases in $q$ less attractive. There are further potential complications through the demand term, and so without very special assumptions we cannot include $n$ in the cluster of complements.

That we cannot include the number of products is somewhat surprising: surely broader product lines would seem to be

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5Note too that decreases in $r$ are also strictly complementary with increases in the other quality variable, $q$, as well as with decreases in the delay in communicating with customers and processing their orders (a) and in the time to deliver the inventory (b).
complementary with reduced set-up costs, and this intuition has in fact been verified in simpler models (see Xavier de Groote, 1988). However, the ambiguity surrounding \( n \) in richer models appear to reflect something real. On the one side, there are numerous examples of firms massively broadening their product lines with the adoption of modern manufacturing methods, and some of these were cited above. On the other, anecdotal evidence (for example, James B. Treece, 1989) as well as both the discussions of the “focused factory” found in the literature on manufacturing strategy (for example, David A. Garvin, 1988, especially Ch. 8) and some formal statistical analysis (Mihkel Tombak and Arnoud De Meyer, 1988) point to firms having reason to narrow their product lines when shifting to more modern manufacturing patterns and of their acting to do so.

III. Manufacturing and Organization

How is a manufacturing firm most efficiently organized and managed? Several of the trends analyzed in Section II have a direct bearing on this question. First, consider the complementarities that exist between the various functions in the firm: marketing, order-processing, shipping, engineering, and manufacturing. If the firm’s problem were smooth and concave (despite the complementarities) and its environment were stationary and if the optimum is not on the boundary of the feasible set, the complementarities would not pose a serious organizational problem: if none of the managers controlling the individual functions can find a small change that raises the firm’s expected profits, then there is no coordinated change — large or small — that can raise profits. However, in our non-concave problem, it is possible that only coordinated changes among all the variables will allow the firm to achieve its optimum. Non-convexities and significant complementarities provide a reason for explicit coordination between functions such as marketing and production.\(^6\)

\(^6\)A similar point is made by de Groote, who investigates a different model of complementarities between marketing and manufacturing.

(Extension of the methods in this paper to a game-theoretic context can be used to model this coordination problem and the role of the central coordinator: see Milgrom and Roberts, 1989.)

Even without non-convexities, significant complementarities in a rapidly changing environment provide another reason for close coordination between functions. Think of the managerial planning process as an algorithm to seek the maximum of the profit function. Successful performance in the face of rapid environmental change requires the use of fast algorithms (for example, Newton’s method), and these require a coordinated choice of the decision variables that recognizes the interactions among these variables in the profit function.

Second, suppose that the organization being modeled is one where sales are made through several different stores. If the optimal speed of order-processing (a) jumps down, it may be desirable that all the stores install computerized systems linked to the manufacturing facility to track orders and sales. If there are fixed costs or other economies of scale in the computer system, then it is important that all, or nearly all, of the stores participate. However, unless all the costs and benefits of the change accrue to one agent, there arises a standard public goods, free-rider problem. Eliciting efficient cooperation from the store owners could be expensive and may provide a reason for vertical ownership of the distribution channel.

Third, Oliver Williamson and Klein, Crawford, and Alchian (1978) have argued that the advantages of increased vertical governance grow as assets become increasingly specialized. This occurs, it is argued, because the returns from specialized investments are vulnerable to appropriation. Then, as Williamson and Jean Tirole (1986) have argued, fear of appropriation causes insufficient investment to be made or, as we have argued (Milgrom and Roberts, 1987), it encourages the parties to waste resources by investing in bargaining position. Following this line of argument, let us equate “specialization” of assets with inflexibility of retooling to produce different products, so that it may be measured by \( e \). The net costs of
governance, bargaining, and deterred or distorted incentives are $\gamma(-v,-e)$, where $v$ is a vector measure of the extent or complexity of vertical governance. We formalize a version of the hypothesis that increased flexibility of assets reduces the marginal value of governance activities with:

**ASSUMPTION A6:** The function $\gamma(-v,-e)$ is sub-modular.

**THEOREM 9:** Assume that A1–A6 hold and consider the profit function:

$$\pi(-p,m,q,-a,-b,-c,-d,-e,-r,-s,-w,\tau) - \gamma(-v,-e).$$

Let each decision variable be constrained as in Theorem 8. Then the set of optimizers of $\pi - \gamma$ is a sublattice and rises with $\tau$.

**PROOF:**
A direct consequence of Theorems 3, 5, 6, and 7, and A6.

Thus, given Assumptions A1–A6 another predicted attribute in the characteristic cluster for flexible manufacturing companies is low vertical governance, for example, the extensive use of independently owned suppliers and subcontractors. This characteristic is an especially interesting one, given the usual conception of the difference between internal and market organization. Although uncertainty is not formally part of our model, running this sort of "tight," low inventory operation with frequent redesigning of products in a world of uncertainties would surely require close coordination and communications with suppliers. Yet according to our theory, the modern firm—despite its close relationships with suppliers and customers—will have little formal vertical governance.

Economists sometimes emphasize the need for close communication in the presence of supply or demand uncertainty as a reason for vertical integration (for example, Kenneth Arrow, 1975). If we were to formulate this alternative hypothesis using a submodular governance cost function $\lambda(m,v)$, we would arrive at the conclusion that $v$ increases over time and that more extensive vertical governance is part of the cluster of characteristics of a modern manufacturing firm. The anecdotal evidence contained in press reports suggests to us that this conclusion is wrong, and that the former hypothesis A6 is the better one.

### IV. Conclusion

The cluster of characteristics that are often found in manufacturing firms that are technologically advanced encompasses marketing, production, engineering, and organization variables. On the marketing side, these firms hold down prices while emphasizing high quality supported by frequent product improvements. Customers orders are filled increasingly quickly, with back-order levels being systematically reduced. In terms of technology, modern manufacturing firms exploit rapid mass data communications, production equipment with low setup, wastage, and retooling costs, flexible design technologies, product designs that use common inputs, very low levels of inventories (of both work in process and finished goods), and short production cycle times. They also seem to push differentially to increase manufacturing quality and, simultaneously, to control variable production costs. At the engineering and organizational levels, there is an integration of the product and process engineering functions and an extensive use of independently owned suppliers linked with the buying firm by close communications and joint planning.

We have argued in this paper that this clustering is no accident. Rather, it is a result of the adoption by profit-maximizing firms of a coherent business strategy that exploits complementarities, and the trend to adopt
this strategy is the result of identifiable changes in technology and demand. Our formal model includes eleven decision variables from the claimed cluster of complements plus a parameter to account for the passage of time. There are thus 66 potential cross effects among the twelve variables, and all of these are nonnegative: there are extensive complementarities in marketing, manufacturing, engineering, design, and organization that make it profitable for a firm that adopts some of these characteristics to adopt more. We have also argued that the non-convexities in the problem mitigate against any smooth distribution of these characteristics among firms. For this reason, we are hopeful that empirical work will provide evidence of distinctly separated clusters of firm characteristics as support for our theory. Given our assumptions about time trends in prices, we also expect to find an increasing proportion of manufacturing firms adopting the modern manufacturing strategic cluster that we have described.

APPENDIX

THEOREM 2*: Let $I = [a_1, b_1] \times \cdots \times [a_n, b_n]$ be an interval in $\mathbb{R}^n$ with nonempty interior and let $f: I \to \mathbb{R}$. Suppose that for every pair of arguments $ij$, there exists a function $f_{ij}: I \to \mathbb{R}$ such that $f$ is the indefinite integral of $f_{ij}$. That is, for fixed $x_{-ij}$ and for $x'_{ij} > x_{ij}$,

$$f(x', x_{-ij}) = f(x_{ij}, x_{-ij}) + \int_{x_{ij}}^{x'_{ij}} f_{ij}(x_{ij}, x_{-ij}) \, dx_{ij}$$

If each $f_{ij}$ is nonnegative, then $f$ is supermodular on $I$.

Remark 1: In our application, $f$ is continuous on $I$ and twice continuously differentiable on a set $S$ with $\nabla^2 f / \nabla x_i \nabla x_j \geq 0$ on $S$. Moreover, for all $x_{-ij}$, the set $(I - S) \cap \{x \mid x_{ij} = x_{-ij}\}$ is a curve. So, taking $f_{ij} = \nabla^2 f / \nabla x_i \nabla x_j$ where defined and $f_{ij} = 0$ elsewhere, Theorem 2* implies that $f$ is supermodular.

PROOF:

In view of Theorem 1, it suffices to establish the conclusion for the case $n = 2$. Given any two unordered points $x$ and $x'$ with, say, $x_1 > x'_1$ and $x_2 < x'_2$,

$$f(\max(x, x')) + f(\min(x, x')) - f(x) - f(x') = \int_{x'_1}^{x_1} \int_{x'_2}^{x_2} f_{12}(s, t) \, ds \, dt \geq 0,$$

from which it follows that $f(x) + f(x') \leq f(\max(x, x')) + f(\min(x, x'))$.

\[ \square \]

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