APPENDIX A

SYMBOLS AND NOTATIONS

In this Appendix we summarize the conventions adopted throughout the book for symbols and notations. Secs. A.1 and A.2 deal with equivalence relations and orders of growth. Sec. A.3 presents notations used in combinatorics and probability. Table A.4 gives the main mathematical notations, and A.5 information theory notations. Table A.6 summarizes the notations used for factor graphs and graph ensembles. Table A.7 focuses on the notations used in message-passing, belief and survey propagation, and the cavity method.

A.1 Equivalence relations

As usual, the symbol $=$ denotes equality. We also use $\equiv$ for definitions and $\approx$ for 'numerically close to'. For instance we may say that the Euler-Mascheroni constant is given by

$$\gamma_{\rm E} \equiv \lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{1}{k} - \log n \right) \approx 0.5772156649\,. \tag{A.1}$$

When dealing with two random variables X and Y, we write $X \stackrel{\rm d}{=} Y$ if X and Y have the same distribution. For instance, given $n+1$ i.i.d. gaussian variables $X_0, \dots, X_n$, with zero mean and unitary variance, then

$$X_0 \stackrel{\rm d}{=} \frac{1}{\sqrt{n}}\,(X_1 + \cdots + X_n)\,. \tag{A.2}$$

We adopted several equivalence symbols to denote the asymptotic behavior of functions as their argument tends to some limit. For sake of simplicity we assume here the argument to be an integer $n \to \infty$. The limit to be considered in each particular case should be clear from the context. We write $f(n) \doteq g(n)$ if f and g are equal 'to the leading exponential order' as $n \to \infty$, i.e. if

$$\lim_{n\to\infty} \frac{1}{n} \log \frac{f(n)}{g(n)} = 0\,. \tag{A.3}$$

For instance we may write

$$\binom{n}{\lfloor n/2 \rfloor} \doteq 2^n\,. \tag{A.4}$$

We write instead $f(n) \sim g(n)$ if f and g are asymptotically equal 'up to a constant', i.e. if

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = C\,, \tag{A.5}$$

for some constant $C \neq 0$. For instance we have

$$\frac{1}{2^n}\binom{n}{\lfloor n/2 \rfloor} \sim n^{-1/2}\,. \tag{A.6}$$

Finally, the symbol $\simeq$ is reserved for asymptoric equality, i.e. if

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 1\,. \tag{A.7}$$

For instance we have

$$\frac{1}{2^n}\binom{n}{\lfloor n/2 \rfloor} \simeq \sqrt{\frac{2}{\pi n}}\,. \tag{A.8}$$

The symbol $\cong$ denotes equality up to a constant. If $p(\,\cdot\,)$ and $q(\,\cdot\,)$ are two measures on the same finite space $\mathcal{X}$ (not necessarily normalized), we write $p(x) \cong q(x)$ if there exists $C > 0$ such that

$$p(x) = C\,q(x)\,, \tag{A.9}$$

for any $x \in \mathcal{X}$. The definition generalizes straightforwardly to infinite sets $\mathcal{X}$: the Radon-Nikodyn derivative between p and q is a positive constant.

A.2 Orders of growth

We used a couple of symbols to denote the order of growth of functions when their arguments tend to some definite limit. For sake of definiteness we refer here to functions of an integer $n \to \infty$. As above, the adaptation to any particular context should be straightforward.

We write $f(n) = \Theta(g(n))$, and say that $f(n)$ is of order $g(n)$, if there exists two positive constants C_1 and C_2 such that

$$C_1\,g(n) \leq |f(n)| \leq C_2 g(n)\,, \tag{A.10}$$

for any n large enough. For instance we have

$$\sum_{k=1}^{n} k = \Theta(n^2)\,. \tag{A.11}$$

We write instead $f(n) = o(g(n))$ if

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0\,, \tag{A.12}$$

For instance

$$\sum_{k=1}^{n} k - \frac{1}{2}n^2 = o(n^2)\,. \tag{A.13}$$

Finally $f(n) = O(g(n))$ if there exist a constant C such that

$$|f(n)| \leq C\, g(n) \tag{A.14}$$

for any n large enough. For instance

$$n^3 \sin(n/10) = O(n^3)\,. \tag{A.15}$$

Notice that both $f(n) = \Theta(g(n))$ and $f(n) = o(g(n))$ imply $f(n) = O(g(n))$. As the last example shows, the converse is not necessarily true.

A.3 Combinatorics and probability

The standard notation is used for multinomial coefficients. For any $n \geq 0$, $l \geq 2$ and $n_1, \dots, n_l \geq 0$ such that $n_1 + \cdots + n_l = n$, we have:

$$\binom{n}{n_1, n_2, \dots, n_l} \equiv \frac{n!}{n_1! n_2! \dots n_l!}\,. \tag{A.16}$$

For binomial coefficients (i.e. for $l = 2$) the usual shorhand is

$$\binom{n}{k} \equiv \binom{n}{k, l-k} = \frac{n!}{k!(n-k)!}\,. \tag{A.17}$$

In combinatorics, certain quantities are most easily described in terms of their generating functions. Given a formal power series $f(x)$, $\mathsf{coeff}\{f(x), x^n\}$ denotes the coefficient of the monomial x^n in the series. More formally

$$f(x) = \sum_n f_n x^n \quad \Rightarrow \quad f_n = \mathsf{coeff}\{f(x), x^n\}\,. \tag{A.18}$$

For instance

$$\mathsf{coeff}\{(1+x)^m, x^n\} = \binom{m}{n}\,. \tag{A.19}$$

Some standard random variables:

- A Bernoulli p variable is a random variable X taking values in $\{0, 1\}$ such that $\mathbb{P}(X = 1) = p$.
- $B(n, p)$ denotes a binomial random variable of parameters n and p. This is defined as a random variable taking values in $\{0, \dots, n\}$, and having probability distribution

$$\mathbb{P}\{B(n,p) = k\} = \binom{n}{k} p^k (1-p)^{n-k}\,. \tag{A.20}$$

- A Poisson random variable X of parameter λ takes integer values and has probability distribution:

$$\mathbb{P}\{X = k\} = \frac{\lambda^k}{k!} e^{-\lambda} \, . \tag{A.21}$$

The parameter λ is the mean of X.

Finally, we used the symbol δ_a for Dirac 'delta function'. This is in fact a measure, that attributes unit mass to the point a. In formulae, for any set A:

$$\delta_a(A) = \mathbb{I}(a \in A) \, . \tag{A.22}$$

A.4 Summary of mathematical notations

$=$	Equal.
$\equiv$	Defined as.
$\approx$	Numerically close to.
$\overset{\mathrm{d}}{=}$	Equal in distribution.
$\doteq$	Equal to the leading exponential order.
$\sim$	Asymptotically equal up to a constant.
$\cong$	Equal up to a normalization constant (for probabilities: see Eq.(14.3)).
$\Theta(f)$	Of the same order as f (see Sec. A.2).
$o(f)$	Grows more slowly than f (see Sec. A.2).
$\text{argmax} f(x)$	Set of values of x where the real valued function f reaches its maximum.
$\lfloor \cdot \rfloor$	Integer part. $\lfloor x \rfloor$ is the largest integer n such that $n \leq x$.
$\lceil \cdot \rceil$	$\lceil x \rceil$ is the smallest integer n such that $n \geq x$.
$\mathbb{N}$	The set of integer numbers.
$\mathbb{R}$	The set of real numbers.
$\beta \downarrow \beta_{\rm c}$	β goes to $\beta_{\rm c}$ through values $> \beta_{\rm c}$.
$\beta \uparrow \beta_{\rm c}$	β goes to $\beta_{\rm c}$ through values $< \beta_{\rm c}$.
$]a, b[$	Open interval of real numbers x such that $a < x < b$.
$]a, b]$	Interval of real numbers x such that $a < x \leq b$.
$\mathbb{Z}_2$	The field of integers modulo 2.
$a \oplus b$	Sum modulo 2 of the two integers a and b.
$\mathbb{I}(\cdot)$	Indicator function: $\mathbb{I}(A) = 1$ if the logical statement A is true, $\mathbb{I}(A) = 0$ if the statement A is false .
$A \succeq 0$	The matrix A is positive semidefinite.

A.5 Information theory

H_X	Entropy of the random variable X (See Eq.(1.7)).
I_{XY}	Mutual information of the random variables X and Y (See Eq.(1.25)).
$\mathcal{H}(p)$	Entropy of a Bernoulli variable with parameter p.
$\mathfrak{M}(\mathcal{X})$	Space of probability distributions over a finite set $\mathcal{X}$.
$\mathfrak{C}$	Codebook.
$\preceq$	BMS(1) $\preceq$ BMS(2): Channel BMS(2) is physically degraded with respect to BMS(1).
$\mathfrak{B}$	Bhattacharya parameter of a channel.

A.6 Factor graphs

$\mathbb{G}_N(k, M)$	Random k-factor graph with M function nodes and N variables nodes.
$\mathbb{G}_N(k, \alpha)$	Random k-factor graph with N variables nodes. Each function node is present independently with probability $N\alpha/\binom{N}{k}$.
$\mathbb{D}_N(\Lambda, P)$	Degree constrained random factor graph ensemble.
$\mathbb{T}_r(\Lambda, P)$	Degree constrained random tree factor graph ensemble.
$\mathbb{T}_r(k, \alpha)$	Shorthand for the random tree factor graph $\mathbb{T}_r(\Lambda(x) = e^{k\alpha(x-1)}, P(x) = x^k)$.
$\Lambda(x)$	Degree profile of variable nodes.
$P(x)$	Degree profile of function nodes.
$\lambda(x)$	Edge perspective degree profile of variable nodes.
$\rho(x)$	Edge perspective degree profile of function nodes.
$\mathsf{B}_{i,r}(F)$	Neighborhood of radius r of variable node i.
$\mathsf{B}_{i\to a,t}(F)$	Directed neigborhood of an edge.

A.7 Cavity and Message passing

$\nu_{i\to a}(x_i)$	BP messages (variable to function node).
$\widehat{\nu}_{a\to i}(x_i)$	BP messages (function to variable node).
Φ	Free-entropy.
$\mathbb{F}(\underline{\nu})$	Bethe free-entropy (as a function of messages).
$\mathbb{F}^{\rm e}(\underline{\nu})$	Bethe energy (as a function of min-sum messages).
$\mathrm{f}^{\rm RS}$	Bethe (RS) free-entropy density.
$Q_{i\to a}(\nu)$	1RSB cavity message/SP message (variable to function node).
$\widehat{Q}_{a\to i}(\widehat{\nu})$	1RSB cavity message/SP message (function to variable node).
$\mathtt{x}$	Parisi 1RSB parameter.
$\mathfrak{F}(\mathtt{x})$	free-entropy density of the auxiliary model counting BP fixed points.
$\Sigma(\phi)$	Complexity.
$\mathbb{F}^{\rm RSB}(Q)$	1RSB cavity free-entropy (Bethe free-entropy of the auxiliary model, function of the messages).
$\mathrm{f}^{\rm RSB}$	1RSB cavity free-entropy density.
$\mathtt{y}$	Zero-temperature Parisi 1RSB parameter ($\mathtt{y} = \lim_{\beta\to\infty}\beta\mathtt{x}$).
$\mathfrak{F}^{\rm e}(\mathtt{y})$	Free-entropy density of the auxiliary model counting min-sum fixed points.
$\Sigma^{\rm e}(e)$	Energetic complexity.
$\mathbb{F}^{\rm RSB,e}(Q)$	Energetic 1RSB cavity free-entropy (Bethe free-entropy of the auxiliary model, function of the messages).
$\mathrm{f}^{\rm RSB,e}$	Energetic 1RSB cavity free-entropy density.

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