Prob 1 sketch

We begin with the assumption that I do not bet money after I reach target.

(a) Let $X_n$ be the bold MG $\Rightarrow X_0$ exists

$$m = \sum_{k=0}^{m+1} P(X_0 = k) \Rightarrow (m+1)P(X_0 = m+1) \leq \frac{m}{m+1}$$

This is achieved by betting 1 at each round. (Gambler's ruin)

(b) Same, success prob $= \frac{m}{k^1}$

(c) No. Consider the above strategy vs the strategy that plays $\$1$ every $L$ steps for $L$ large.
Prob 3 sketch

(a)  

\begin{align*}
\text{series} & \quad b = e' \\
\frac{1}{C_1} & \quad \frac{1}{C_2} \\
\text{match currents} & \quad i^{(1)} \text{ on (1), } \frac{i^{(2)}}{C_1} \text{ on (2)} \\
\text{so that there is a unit current} & \text{ and eqs are satisfied at } b = e' \\
\text{voltage drop} & \quad V(e) - V(b) = \frac{1}{C_1} + \frac{1}{C_2} \\
\text{because we scaled current} & \quad = V(e) - V(b) + V(e') - V(b) \\
\text{=> total conduct} & \quad \frac{i}{V(e) - V(b)} = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)
\end{align*}
(b) Construct a unit current

\[ \frac{1}{d(d-1)} \]

At level \( \ell \) \( n \) current on each edge is \( \frac{1}{d(d-1)^{n-1}} \).

\[ V(0) - V(n) = \sum_{k=1}^{n} \frac{1}{d(d-1)^{k-1}} \]

\[ = \frac{1 - (d-1)^{-n}}{d(1 - (d-1)^{-1})} \]

\[ R_{e,B_n} = \frac{V(0) - V(n)}{\text{hit.current}} = \frac{1 - (d-1)^{-n}}{d(1 - (d-1)^{-1})} \]

\[ R_{e \leftrightarrow \infty} = \frac{d-1}{d(d-2)} \quad \& < \infty \]

(c) We showed in class that SRW transient if \( R_{v \leftrightarrow \infty} < \infty \).
(d) By monotonicity (also proved in class) \( R \) effective resistance decreases by adding edges (in alternative: repeat computation at point (b)).

(e) Repeat computation at (b).

(f) take \( \phi_n = 2 \) \( \forall \alpha \) for \( n \neq 2^k \ k \in \mathbb{N} \) and \( d_n = 3 \) for \( n = 2^k \ k \in \mathbb{N} \).