

Interpreting the coefficients of loglinear models.

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1) Starting point: Simple things one can say about the coefficients of loglinear models that derive directly from the functional form of the models.

Let's say we have a simple model,

$$1a) \text{Log}(U) = \text{Const} + B_1 X_1 + B_2 X_2 + \dots$$

Where the B's are model coefficients, and the X's are the variables (usually dummy variables) and the U are predicted counts.

When $X_1=0$, we have:

$$1b) \text{Log}(U) = \text{Const} + 0 + B_2 X_2 + \dots$$

and when $X_1=1$

We have

$$1c) \text{Log}(U) = \text{Const} + B_1 + B_2 X_2 + \dots$$

So we can always say, as a simple function, that the coefficient B_1 represents an increase in the log of predicted counts. If $B_1=2$, for instance, we could say that 'this model shows that factor X_1 increases the predicted log count by 2 (all other factors held constant)' because equation 1b- equation 1a = B_1 . This is true but not the most helpful thing to say.

Remembering that $e^0=1$, we can also say, When $X_1=0$, we can exponentiate equation 1b to get

$$1d) U = e^{\text{Const}} e^{B_2 X_2}$$

and when $X_1=1$, we can exponentiate equation 1c to get

$$1e) U = e^{\text{Const}} e^{B_1} e^{B_2 X_2}$$

If we take the ratio of 1e/1d, we get e^{B_1} . If we give B_1 the arbitrary value of 2, $e^2=7.4$, we could say that ' B_1 increases the predicted counts by a factor of 7.4, that when X_1 is true, predicted counts increase by 640% (all other factors being held constant). Alternatively if $B_1=-0.2$, $e^{-0.2}=.82$, we could say that when X_1 is true the predicted counts are reduced by 18%, (all other factors being held constant).

2. Why the interaction terms are really log odds ratios

I have also claimed that interaction coefficients in the loglinear models correspond to log odds ratios. We have demonstrated this in the first homework, and it can be easily demonstrated algebraically.

Let's start with a saturated model for the 2x2 table:

$$\text{Log}(U) = \text{Const} + B_1R + B_2C + B_3RC$$

Where RC is the interaction of the row and column parameters. We can show that B_3 represents the log odds ratio of the interaction between the Row and Column variables.

If we take e^{B_3} , then we have the odds ratio of the Row variable interacted with the Column variable. Take, for example, homework 1, dataset A, the race by occupation table from the 2000 Current Population Survey.

		Race	
		White	Non White
occupation	White Collar	17,216	2,361
	Other	42,012	7,146

We can calculate the Odds Ratio by hand, it is simply the cross product of the 4 cells, $AD/BC=1.24$, and the log odds ratio is $\log(1.24)=0.215$.

We can also calculate the asymptotic standard error of this log odds ratio by hand, and it is square root of $(1/A + 1/B + 1/C + 1/D)=0.025$

The interpretation of the odds ratio is as follows. The odds of being in a white collar job for subjects who are White are $17216/42012=0.41$. The odds of being in a white collar job for non-White subjects is somewhat lower, $2361/7146=0.33$. The odds ratio is simply the ratio of the odds, $0.41/0.33=1.24$. One may say that the 'odds of being in the white collar sector are 24% higher for Whites than for non-Whites', or, equivalently, 'the odds of being White are 24% higher for persons in the white collar sector'. We can also invert the odds ratio. The odds ratio of non-White representation in the white collar sector is $0.33/0.41=0.80$. One might say 'the odds of being in the white collar sector are lower for non-whites by a factor of 0.8', or one might say 'the odds of being in the white collar sector are 20% lower for non-Whites than for Whites.'

It is easy to keep in mind the symmetry of the situation when using the log odds ratio, since the log odds ratio for White representation in the white collar sector is 0.215, and the log odds ratio for non-White representation in the White collar sector is -0.215.

Here is the loglinear model output from STATA for the coefficients of the saturated model for this 2x2 dataset. The race by occupation interaction coefficient is 0.215, and its standard error is 0.025, which is exactly what we calculated by hand for the log odds ratio.

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. desmat: poisson count race*occ
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nr	Effect	Coeff	s.e.
	count		
	race		
1	w	1.771**	0.013
	occ		
2	WC	-1.107**	0.024
	race.occ		
3	w.WC	0.215**	0.025
4	_cons	8.874**	0.012

Why does the interaction coefficient equal the log odds ratio? Here's why.

Let's start with our standard 2x2 table,

		Var 2	
		0	1
Var 1	0	A	B
	1	C	D

If we take the first category as the excluded category (this is an arbitrary decision which has no substantive effect), then the row effect will be value 1 compared to zero, and the column effect will be value 1 compared to zero, and the interaction term will zero everywhere except for the cell where Var 1= Var 2=1. Again, any other reasonable construction of the contrasts will yield the same result.

If we run the saturated model, which fits the data exactly and which is the only model that includes our interaction term, we get the following:

log (A)= const
 log (B)= const +Col effect
 log (C)= const+ Row effect
 log (D)= const+ Col effect + Row effect + Row and Col interaction.

log(A)+log (D)-log(B)-log(C)=Row and Col interaction.

But log(A)+log (D)-log(B)-log(C)=log(AD/BC), which is our log odds ratio

so

Row and Col interaction= $\log(AD/BC)$

That's why the interaction coefficient in our loglinear model is really a log odds ratio.

3) What to say about combinations of coefficients

Now let's say we have many variables in our dataset including: race, occupation, and year. The years in this hypothetical dataset will be 2000, 2001, and 2002. The log odds ratio in 2000 for Whites interacted with white collar jobs is the one piece of true data here, and something we already know, 0.215.

<u>Coefficients:</u>		<u>S.E.</u>	<u>Odds Ratio</u>
race*occ interaction	0.215	0.025	1.24
Year*race*occ			
2000 (comparison category)			
2001	0.1	0.03	1.11
2002	0.15	0.034	1.16
Combining them by addition:			
Year*race*occ			
2000	0.215	a	1.24
2001	0.315	a	1.37
2002	0.365	a	1.44

Note a: The standard errors of the combined coefficients can be obtained by hand if you ask STATA or whatever software you are using to give the variance-covariance matrix of the estimates. $\text{Var}(A+B) = \text{Var}(A) + \text{Var}(B) + 2\text{Cov}(A,B)$. In stata you can use the **lincom** command to give you the value and standard error of any linear combination of coefficients from your most recently estimated model.

So here are a few things you could say about this hypothetical data.

- 1) The odds ratio of overrepresentation of Whites in the white collar sector increased by 11% from 2000 to 2001, and by 16% in 2000-2002.
- 2) In log odds ratio terms, the interaction between race and occupation is 0.215 in 2000, and 0.315 in 2001, and 0.365 in 2002, an increase of 47% from 2000-2001, and an increase of 70% from 2000 to 2002. These increases are a lot larger than one would expect from real data.
- 3) The increases in the overrepresentation of Whites in white collar jobs over time appear to be significant- $0.1/0.03=3.33$ corresponds to a P value of less than 0.05.