Some notes on basic mediation tests

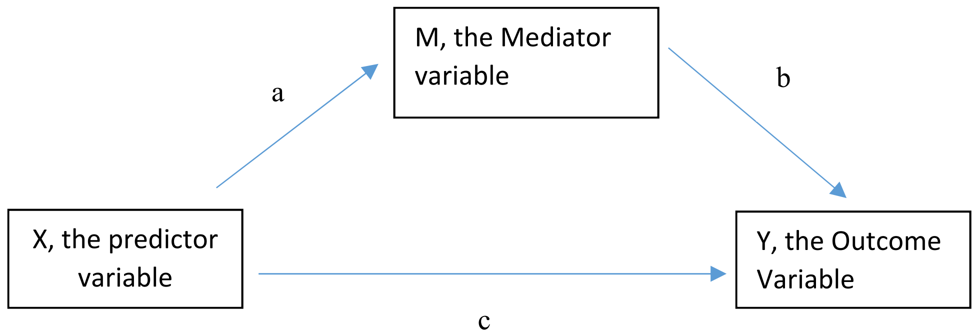


Figure 1: the basic mediation system

* A basic mediation system has two equations, one predicting M, and one predicting Y. In simplest form where both equations are linear probability models (LPM), our friendly OLS regression system, the two equations look like this:

(1.1) $$Y = cX + bM + const + \varepsilon$$

(1.2) $$M = aX + const + \varepsilon$$

Where ε are the errors in the regression, or the residuals.

* If M mediates between X and Y, that means that some significant part of the apparent effect of X on Y is explained by M. Mediation analyses most often crop up in experimental research, but we use them in observational studies also, especially when we have panel data (i.e. longitudinal data) because panel data help us put the variables in temporal order. Temporal order is important here. The three variables should be in this order: X->M->Y for the mediation to make sense. If M comes between X and Y temporally, and the mediation effect is significant, then we have some evidence toward a causal claim about how X causes Y through M. Causality is hard to pin down in observational studies however because there are always variables that are unmeasured and alternative causes that cannot be ruled out, and we do not have the advantage of randomization in observational studies. Nonetheless, temporal order X->M->Y and a significant mediation effect provide some real causal leverage even in observational studies.

Testing the significance of the mediation effect with the Baron-Kenney product method:

* The mediation coefficient known as the Baron-Kenney (1986) product method, is ab, that is a (from equation 1.2) multiplied by b (from equation 1.1). The standard error of ab is adapted from Sobel

(1982) as $SE_{ab} = \sqrt{\left(b^2 se_a^2 + a^2 se_b^2 + se_a^2 se_b^2\right)}$ where se_a is the standard error of a from equation 1.2 and se_b is the standard error of b from equation 1.1. The mediation test of ab divided by this standard error, evaluated as Normally distributed is known as the Sobel test, $Z = \frac{ab}{SE_{ab}}$

* If ab is significantly different from zero, then the simplest version of the total effect of X on Y is $ab + c$, and the proportion of the total effect of X on Y that is explained by the mediation effect is $\frac{ab}{(ab+c)}$, that is the mediation part divided by the whole.

* When we run mediation tests in Stata we will use the Stata function `sureg` to estimate both eq 1.1 and eq 1.2 together, and then we will use the function `nlcom` to generate the standard error of ab, and thereby to run a Sobel test on whether the mediation is statistically significant.

*a,b, and c are generally going to be in different units. That is OK! It can sometimes make sense to standardize predictors (subtract the mean and divide by SD, yielding a new variable with mean 0 and SD=1) so that a, b, and c are all in standardized units, but often that is not a good idea (for instance if you have dummy variables that are awkward to standardize).

Including more controls

* The simple mediation system easily incorporates additional control variables $X_2, X_3 ... X_n$ (note, we are adding the additional controls to both models)

(1.3) $$Y = cX + bM + \beta_2 X_2 + \beta_3 X_3 + ... + \beta_n X_n + const + \varepsilon$$

(1.4) $$M = aX + \delta_2 X_2 + \delta_3 X_3 + ... + \delta_n X_n + const + \varepsilon$$

* The extra controls give us more ability to know that the mediation relationship is robust to alternative explanations. Otherwise everything else is the same as above: the product method still gives ab as the mediation effect and the Sobel standard error is a reasonable first estimate for SE_{ab}.

Some issues with the Baron-Kenney approach:

* There is a lot of research on mediation tests and a lot of scholarship since Baron and Kenney (1986) and Sobel (1982). See Hayes (2018) for a good introduction to advances in this literature.

* One problem with the Baron-Kenney approach is that even if a and b have a nice Normal or t-distribution, their product ab is not guaranteed to be normally distributed. So what to do? Some of the modern mediation packages use the bootstrap to generate a standard error for ab, while others take the known t or Normal distributions of a and of b, sample repeatedly from the known distributions of a and of b and multiply them together and generate a standard error of ab that way. This is known as Monte Carlo simulation. The Stata package `medeff` (a user-generated package you can download) generates the standard error of the mediation test through simulation. But note: if the sample size is large enough, the Sobel test is likely to be very close to the simulated or bootstrap standard errors. For convenience and the simplicity that the Normal assumption provides, we will rely on the Sobel test.

* If eq 1.1 and 1.2 are not linear probability models with OLS estimation (i.e. if 1.1 and 1.2 are not in the form of regression we are most familiar with), then the Baron-Kenney product method may not be the right method for estimating the strength of the mediation effect. See the Stata user-written packages `medeff`, `paramed`, and `med4way` which generate mediation tests for logistic and other binary models for Y and or for M under some more advanced assumptions. Hicks and Tingley (2011) are the authors of the Stata package `medeff`. Tingley and several other authors wrote the R package `mediate`, which is more up-to-date and more flexible than their `medeff` package for Stata. Hayes (2018) describes his package PROCESS for mediation analysis with SAS or SPSS. Stata has good procedures for estimating systems of simultaneous equations, including nonlinear equations, see Stata commands `sem` and `gsem`.

<u>A worked example:</u> Do having gay friends at least partly explain why people with BA degrees are so much more supportive of marriage equality in the US?

Data: General Social Survey, 2006-2008-2010 panel data. GSS asked about gay friends in 2006, asked about marriage equality every year in this period.

Outcome: subject support for marriage equality (original GSS variable name marhomo). Question: "Do you agree or disagree: Homosexual Couples should have the right to marry one another" [5 categories, recoded by me so that it goes from strongly disagree to strongly agree]

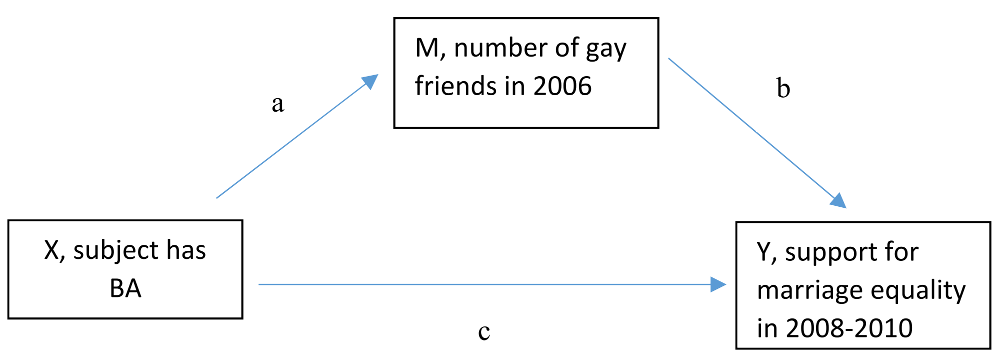
Mediator: subject reported number of gay friends in 2006 (original GSS variable name acqgay). Question: "I'm going to ask you some questions about all the people that you are acquainted with, meaning that you know their name and would stop and talk at least for a moment if you ran into the person on the street or in a shopping mall. Some of these questions may seem unusual but they are an important way to help us understand more about social networks in America. Please answer the questions as best you can. How many are you pretty certain are gay men or women?" [options 0,1,2-5, 6-10, <10; recoded by me as a continuous variable]

Original predictor: College degree. The college degree is associated with stronger support for marriage equality, and also turns out to be associated with having gay friends.

Substance: On the subject of what predicts support for gay rights in general, and marriage equality in particular, see Rosenfeld (2007) and Baunach (2011; 2012). On the subject of how having gay friends influenced attitudes toward gay rights, see Rosenfeld (2017; 2021) and DellaPosta (2018).

Further notes: To retrieve the GSS data, including the panel datasets, go to https://gss.norc.org/. To do search for variables and to see what years they are available, go to http://sda.berkeley.edu/sdaweb/analysis?dataset=gss18 and search for keywords, and then tabulate that variable by year. The GSS sample size is modest every other year or so (approximately 3K) and even when questions are fielded they are not always fielded to the full 3K participants. So sample size can be quite small.

The picture:



```
. sureg (full: marhomo_approve i.educ_recode gay_friends_2006 ) (med: gay_friends_2006
i.educ_recode) if year>=2008

Seemingly unrelated regression
------------------------------------------------------------------------------
Equation             Obs   Params         RMSE  "R-squared"      chi2   P>chi2
------------------------------------------------------------------------------
full                 418        4     1.357438      0.1767      89.72   0.0000
med                  418        3     3.701416      0.0460      20.14   0.0002
------------------------------------------------------------------------------

----------------------------------------------------------------------------------
                 | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-----------------+----------------------------------------------------------------
full             |
     educ_recode |
             HS  |   .1443438   .2326073     0.62   0.535    -.3115582    .6002458
   some college  |   .5007837    .223772     2.24   0.025     .0621986    .9393688
(THIS is c) BA+  |   .9399669   .2295153     4.10   0.000     .4901251    1.389809
                 |
(This below is b)
gay_friends_2006 |   .1228393   .0179376     6.85   0.000     .0876823    .1579964
           _cons |   1.925588   .1885561    10.21   0.000     1.556024    2.295151
-----------------+----------------------------------------------------------------
med              |
     educ_recode |
             HS  |   1.721154   .6286543     2.74   0.006     .4890141    2.953294
   some college  |   1.999303    .602287     3.32   0.001     .8188423    3.179764
(This is a) BA+  |   2.720844   .6115215     4.45   0.000     1.522284    3.919404
                 |
           _cons |   .6057692   .5132941     1.18   0.238    -.4002687    1.611807
----------------------------------------------------------------------------------


* Now we generate the coefficient and SE and Z score for ab

. nlcom [med]4.educ_recode*[full]gay_friends_2006

       _nl_1: [med]4.educ_recode*[full]gay_friends_2006

------------------------------------------------------------------------------
             | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       _nl_1 |   .3342267   .0895813     3.73   0.000     .1586505    .5098028
------------------------------------------------------------------------------

* The interaction term is significant.

* Figure out the mediated effect as a fraction of the total: ab/(ab+c)

. display 0.33422/(0.33422+0.93997)
.26229997

*That is version 1, doing it by hand.
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```
* Note: medeff does not allow the newer factor variables with i.educ_recode, so I had
to create the dummy variables myself for education first with the xi command.
```

```
. medeff (regress gay_friends_2006 _Ieduc_reco_2 _Ieduc_reco_3 _Ieduc_reco_4 )
(regress marhomo_approve gay_friends_2006 _Ieduc_reco_2 _Ieduc_reco_3 _Ieduc_reco_4 )
if year>=2008, mediate( gay_friends_2006 ) treat(_Ieduc_reco_4 ) sims(500)

Using 0 and 1 as treatment values

      Source |       SS           df       MS      Number of obs   =       418
-------------+----------------------------------   F(3, 414)       =      6.65
       Model |  275.947891         3  91.9826302   Prob > F        =    0.0002
    Residual |  5726.80151       414  13.8328539   R-squared       =    0.0460
-------------+----------------------------------   Adj R-squared   =    0.0391
       Total |   6002.7494       417  14.3950825   Root MSE        =    3.7193

------------------------------------------------------------------------------
gay_frie~2006 | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
_Ieduc_reco_4 |   2.720844   .6144686     4.43   0.000     1.512976    3.928711
_Ieduc_reco_2 |   1.721154    .631684     2.72   0.007      .479446    2.962862
_Ieduc_reco_3 |   1.999303   .6051896     3.30   0.001     .8096755    3.188931
        _cons |   .6057692   .5157678     1.17   0.241     -.408081    1.619619
------------------------------------------------------------------------------

      Source |       SS           df       MS      Number of obs   =       418
-------------+----------------------------------   F(4, 413)       =     22.16
       Model |  165.327691         4  41.3319226   Prob > F        =    0.0000
    Residual |  770.222549       413  1.86494564   R-squared       =    0.1767
-------------+----------------------------------   Adj R-squared   =    0.1687
       Total |  935.550239       417  2.24352575   Root MSE        =    1.3656

----------------------------------------------------------------------------------
 marhomo_approve | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-----------------+----------------------------------------------------------------
   _Ieduc_reco_4 |   .9399669   .2309005     4.07   0.000     .4860802    1.393854
gay_friends_2006 |   .1228393   .0180458     6.81   0.000     .0873662    .1583125
   _Ieduc_reco_2 |   .1443438   .2340111     0.62   0.538    -.3156576    .6043453
   _Ieduc_reco_3 |   .5007837   .2251225     2.22   0.027     .0582549    .9433125
           _cons |   1.925588   .1896941    10.15   0.000     1.552701    2.298474
----------------------------------------------------------------------------------
The number of observations in the data is less than the number of simulations.
Expanding the data to the number of simulations
--------------------------------------------------------------------------------------
        Effect                  |  Mean          [95% Conf. Interval]
--------------------------------+-----------------------------------------------------
        ACME                    |   .337374      .1793395      .5158513
        Direct Effect           |  .9412827      .4139323      1.393659
        Total Effect            |  1.278657      .7274903      1.757601
        % of Tot Eff mediated   |  .2603454      .1919515      .4637505
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```

* Very similar results to the Sobel test.

References:

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