The EPG coefficients are stored in a single matrix. The unbalanced parts of many generic sequences can be modeled as small-tip angle approximations. We will formulate sequence design problems as constrained least-squares problems. The purpose of this research is to use EPG to model MR signal progression. These algorithms are greedy and do not yield an optimal result. The output at time $S$ (approximating $\Delta\alpha_x \Delta\alpha_y \approx 0$) is determined recursively:

$$Q(S) = \left( \prod_{s=0}^{S-1} A_s \right) Q_0 + \sum_{s=S}^{S-1} \left( \prod_{s'=s+1}^{S-1} A_{s'} \right) B_s \left( \prod_{s'=0}^{s-1} A_{s'} \right) Q_0 \Delta\alpha_s .$$

The signal at time $S$ is the progression of the initial condition summed with the contributions from the progressions of each recovery component:

$$Q(S) = \left( \prod_{s=0}^{S-1} A_s \right) \hat{Q}_0 \left( \prod_{s=1}^{S-1} A_{s} \right) B_s \left( \prod_{s'=0}^{s-1} A_{s'} \right) Q_0 + \sum_{s=S}^{S-1} \left( \prod_{s'=s+1}^{S-1} A_{s'} \right) B_s \left( \prod_{s'=0}^{s-1} A_{s'} \right) d_s .$$

This can be reformulated as

$$\begin{align*}
\text{minimize} & \quad ||H_S \Delta\alpha - b||_2^2 \\
\text{subject to} & \quad ||\Delta\alpha||_{\infty} \leq \theta
\end{align*}$$

where $\theta$ is a constraint requiring that $\Delta\alpha$ be small. If specific states at many times are desired, then the problem can become

$$\begin{align*}
\text{minimize} & \quad ||H_x \Delta\alpha - b_x||_2^2 + ||H_y \Delta\alpha - b_y||_2^2 + \cdots + ||H_S \Delta\alpha - b_S||_2^2 \\
\text{subject to} & \quad ||\Delta\alpha||_{\infty} \leq \theta
\end{align*}$$

Example: Smoothing the Signal of FSE

This signal is retained through several more echoes when tip-angles are altered than when compared to a constant tip-angle pulse.

Conclusion

We have developed a formalism that allows for sequence design in a non-greedy fashion when making the small-tip angle approximation.