ECONOMETRICA

JOURNAL OF THE ECONOMETRIC SOCIETY

An International Society for the Advancement of Economic Theory in its Relation to Statistics and Mathematics

http://www.econometricsociety.org/

Econometrica, Vol. 78, No. 4 (July, 2010), 1435–1452

CAN RELAXATION OF BELIEFS RATIONALIZE THE WINNER'S CURSE?: AN EXPERIMENTAL STUDY

ASEN IVANOV
Virginia Commonwealth University, Richmond, VA 23284, U.S.A.

DAN LEVIN
The Ohio State University, Columbus, OH 43210, U.S.A.

MURIEL NIEDERLE
Stanford University, Stanford, CA 94305-6072, U.S.A.

Econometrica, Vol. 78, No. 4 (July, 2010), 1435–1452

NOTES AND COMMENTS

CAN RELAXATION OF BELIEFS RATIONALIZE THE WINNER'S CURSE?: AN EXPERIMENTAL STUDY

BY ASEN IVANOV, DAN LEVIN, AND MURIEL NIEDERLE[1]

We use a second-price common-value auction, called the *maximal game*, to experimentally study whether the winner's curse (WC) can be explained by models which retain best-response behavior but allow for inconsistent beliefs. We compare behavior in a regular version of the maximal game, where the WC can be explained by inconsistent beliefs, to behavior in versions where such explanations are less plausible. We find little evidence of differences in behavior. Overall, our study casts a serious doubt on theories that posit the WC is driven by beliefs.

KEYWORDS: Common-value auctions, winner's curse, beliefs, cursed equilibrium, level-k model.

1. INTRODUCTION

A WELL DOCUMENTED PHENOMENON in common-value auctions is the winner's curse (WC)—a systematic overbidding relative to Bayesian Nash equilibrium (BNE) which results in massive losses in laboratory experiments.[2] Two recent papers, Eyster and Rabin (2005) and Crawford and Iriberri (2007), rationalize the WC within theories that retain the BNE assumption that players best respond to beliefs (hence, we refer to these theories as belief-based), but relax the requirement of consistency of beliefs. Eyster and Rabin introduced the concept of cursed equilibrium (CE) in which players' beliefs do not fully take into account the connection between others' types and bids. Crawford and Iriberry used the level-k model which was introduced by Stahl and Wilson (1995) and Nagel (1995). In this model, level-0 (L_0) players bid in some prespecified way and level-k (L_k) players ($k = 1, 2, \ldots$) best respond to a belief that others are L_{k-1}.[3]

In response to Eyster and Rabin (2005) and Crawford and Iriberri (2007), we investigate experimentally whether the WC in common-value auctions is indeed driven by beliefs.[4] We use a second-price common-value auction, called the maximal (or maximum) game, that was first introduced in Bulow and Klem-

[1]We would like to thank David Harless and Oleg Korenok for useful discussions. We would also like to thank a co-editor and three anonymous referees for their comments and suggestions. An extended working-paper version is available at http://www.people.vcu.edu/~aivanov/.

[2]See Bazerman and Samuelson (1983), Kagel and Levin (1986), Kagel, Harstad, and Levin (1987), Dyer, Kagel, and Levin (1989), Lind and Plott (1991), and the papers surveyed in Kagel (1995, Section II) and Kagel and Levin (2002).

[3]CE and the level-k model can be applied to environments other than common-value auctions.

[4]Our study applies to any belief-based explanation of the WC. This includes, for example, analogy-based expectation equilibrium (Jehiel (2005), Jehiel and Koessler (2008)). We focus on CE and the level-k model because they are the two most prominent belief-based explanations.

 DOI: 10.3982/ECTA8112

perer (2002) and Campbell and Levin (2006). This game has the special property of being two-step dominance-solvable and our experimental design exploits this property. We focus on initial periods of play as this seems like a natural starting point for evaluating belief-based theories.

The paper most closely related to ours is Charness and Levin (2009). This study finds that the WC is alive and well in an individual-choice variant of the "acquiring a company" game, that is, in an environment where the WC cannot be rationalized by inconsistent beliefs about other players' behavior.

Three concerns arise in interpreting the results in Charness and Levin (2009). First, one cannot reasonably expect CE or the level-k model to explain every aspect of behavior.[5] Thus, even if Charness and Levin's setup rules out belief-based explanations, we should still expect some anomalies. The key question is, "Is the WC more pronounced in environments where it can be rationalized by belief-based explanations than in environments where such explanations are less plausible?" If the answer is "yes," the difference could be attributed to the level-k model or CE, whereas a negative answer casts doubt on the validity of such models. However, Charness and Levin (2009) cannot answer this question because it does not include a regular acquiring a company game against human opponents, that is, an environment of the former type. In contrast, our paper studies and compares behavior in both types of environments.

Second, the acquiring a company game represents a lemons market (see Akerlof (1970)) and is not a common-value auction. Although both types of environments admit a WC, they are quite different and it is not obvious that Charness and Levin's conclusions readily extend to common-value auctions.[6]

Finally, Charness and Levin (2009) studied behavior in individual-choice settings. However, it is possible that subjects employ very different cognitive mechanisms in interactions with other players; such interactions may trigger all sorts of thought processes about others' reasoning, beliefs, and intentions. Thus, the conclusions from Charness and Levin (2009) do not necessarily extend to games against human opponents. In our study, subjects play against other people.[7]

Another related study is Costa-Gomes and Weizsäcker (2008), which finds a systematic inconsistency between chosen actions and stated beliefs in normal-form games. This study differs from ours in two important ways. First, it concerns an environment which is very different from common-value auctions. Second, it is based on eliciting subjects' beliefs. In addition, the study cannot distinguish between two possible interpretations: (i) that subjects do not best

[5]For example, they do not explain bidding above values in private-value second-price auctions.

[6]It is plausible that the WC in both types of environments is driven by the same forces. However, given that these are quite different environments, this cannot be taken for granted.

[7]In one of our environments, each subject plays against the computer which, however, mimics the strategy of a person (actually, the subject's own past strategy).

respond to beliefs when choosing actions and (ii) that subjects form different beliefs when choosing actions and stating beliefs.[8]

We proceed as follows. In Section 2, we describe the maximal game and derive the relevant theoretical predictions. In Section 3, we describe our experimental design and in Section 4 we examine the experimental data. Section 5 concludes.

2. THEORETICAL CONSIDERATIONS

We begin by describing the maximal game. There are n bidders, each of whom privately observes a signal X_i that is independent and identically distributed (i.i.d.) from a cumulative distribution function $F(\cdot)$ on $[0, 10]$. Let $X^{\max} = \max(\{X_i\}_{i=1}^n)$ be the highest of the n signals, and let x_i and $x^{\max}$ denote particular realizations of X_i and $X^{\max}$, respectively. Given $(x_1, \dots, x_n)$, the ex post common value to the bidders is $v(x_1, \dots, x_n) = x^{\max}$. Bidders bid in a sealed-bid second-price auction where the highest bidder wins, earns the common value, $x^{\max}$, and pays the second highest bid. In case of a tie, each tying bidder gets the object with equal probability.

We will say that, given signal x_i, a player bidding b underbids, bids her signal, overbids, or bids above 10 if $b < x_i$, $b = x_i$, $x_i < b \leq 10$, or $b > 10$, respectively. We now state our first result.

PROPOSITION 1: *$b(x_i) = x_i$ is the unique bid function remaining after two rounds of iterated deletion of weakly dominated bid functions.*[9] *In the first round, all bid functions $b_i(\cdot)$ with $b_i(x_i) < x_i$ or $b_i(x_i) > 10$ for some x_i are deleted. In the second round, all bid functions $b_i(\cdot)$ with $x_i < b_i(x_i) \leq 10$ for some x_i are deleted.*

The proof is given in the Appendix A. Here, we give the intuition. It is obvious that bidding above 10 is weakly dominated. Underbidding is also weakly dominated since, under the second-price rule, one could lose the auction at a price below one's signal even though the value of the object is greater than or equal to one's signal. Given that no one underbids, $b_i(x_i) > x_i$ is weakly dominated for any x_i, because, in case the highest bid among others is between x_i and $b_i(x_i)$, i makes nonpositive (and possibly negative) profits.

That bidding one's signal is a BNE follows directly from Proposition 1. In fact, we can say more than that (the proof is given in Appendix A):

[8]Another related study is Pevnitskaya (2008). This study investigates whether deviations from the risk-neutral BNE in first-price private-value auctions are caused by inconsistent beliefs, risk aversion, or probability misperception. All components seem to be at work.

[9]A bid function is weakly dominated if, for some signal, it prescribes a weakly dominated bid.

PROPOSITION 2: *The bid function* $b(x_i) = x_i$ *is the unique symmetric BNE* (*including mixed strategies*).[10]

We now show that overbidding can arise within the level-k model and CE. First, let us consider the level-k model. In this model, level-0 (L_0) players bid in some prespecified way and level-k (L_k) players ($k = 1, 2, \ldots$) best respond to a belief that others are L_{k-1}. For auction settings, Crawford and Iriberry (2007) considered a version of L_0, called *random* L_0 (RL_0), which, regardless of its signal, bids uniformly over all bids between the minimal and maximal value of the object. RL_k ($k \geq 1$) best responds to RL_{k-1}. The next proposition shows that RL_1 can overbid.[11]

PROPOSITION 3: *The bid function of* RL_1 *is* $b^{RL_1}(x_i) = E(X^{\max}|X_i = x_i) \geq x_i$. *If* $F(x_i) < 1$, *the inequality is strict*.[12]

The proof is given in Appendix A. It hinges on the fact that, because RL_0's bid is uninformative about its signal, RL_1 cannot draw any inference about $X^{\max}$ from winning the auction.[13]

Let us turn to CE. In a χ-CE ($\chi \in [0, 1]$), players best respond to a belief that each other player j, with probability χ, chooses a bid that is type-independent and is distributed according to the ex ante distribution of j's bids and, with probability $1 - \chi$, chooses a bid according to j's actual type-dependent bid function. Thus, χ captures players' level of "cursedness": if $\chi = 0$, we have a standard BNE, and if $\chi = 1$, players are fully cursed and draw no inferences about other players' types. Based on Proposition 5 in Eyster and Rabin (2005), we can state the following proposition.

PROPOSITION 4: *Assuming* X_i *has a strictly positive probability density function* (*p.d.f.*),[14] *the following bid function constitutes a symmetric* χ*-CE*: $b^{\mathrm{CE}}(x_i) = (1-\chi)x_i + \chi E(X^{\max}|X_i = x_i) \geq x_i$. *If* $\chi > 0$ *and* $F(x_i) < 1$, *the inequality is strict*.[15]

[10] In our experiment, matching of subjects is anonymous and there is no feedback, so it seems implausible that subjects should coordinate on an asymmetric BNE. For more on asymmetric equilibria, see our working paper at http://www.people.vcu.edu/~aivanov/

[11] Crawford and Iriberry (2007) also considered a version of the level-k model based on a so-called *truthful* L_0 (TL_0). In our settings, the behavior of TL_k and RL_{k+1} coincides for $k \geq 0$.

[12] If signals have the discrete uniform distribution on the set $\{0, 1, 2, \ldots, 10\}$ and there are two bidders (this is relevant for our experiment), then $b^{RL_1}(x_i) = E(X^{\max}|X_i = x_i) = (x_i^2 + x_i + 110)/22$.

[13] The behavior of RL_k for $k \geq 2$ is not uniquely determined. The point, however, is that a RL_1 can rationalize overbidding.

[14] Although the assumption of a strictly positive p.d.f. is not satisfied for the discrete distribution in our experiment, we suspect that the proposition nevertheless holds.

[15] If signals have the discrete uniform distribution on the set $\{0, 1, 2, \ldots, 10\}$ and there are two bidders, then $b^{\mathrm{CE}}(x_i) = (1-\chi)x_i + \chi E(X^{\max}|X_i = x_i) = (1-\chi)x_i + \chi(x_i^2 + x_i + 110)/22$.

3. EXPERIMENTAL DESIGN

3.1. *Treatments and Procedures*

The experiment consists of the *Baseline*, *ShowBidFn*, and *MinBid* treatments. The *Baseline* treatment consists of two parts. In part I, subjects play the maximal game for 11 periods. In each period, subjects are randomly and anonymously rematched in separate two-player auctions. Each subject's signals for the 11 auctions are drawn with equal probability and without replacement from the set $\{0, 1, 2, \ldots, 10\}$.[16] Signals are independent across subjects. Subjects can bid anything between 0 and 1,000,000 experimental currency units (ECU). To minimize the effect of learning, we provide no feedback whatsoever during the experiment. This also ensures that, in any auction, each bidder's prior over the other bidder's signal is the discrete uniform distribution on $\{0, 1, 2, \ldots, 10\}$.

Part II is similar to part I. The only difference is that each subject i bids against the computer rather than against another subject. The computer, which "receives" a uniformly distributed signal, mimics i's behavior from part I by using the same bid function that i used in part I. For example, if the computer receives signal y, it makes the same bid that i made in part I when she received signal y. Effectively, in part II each subject is playing against herself from part I (and knows that this is the case).[17]

The *ShowBidFn* treatment is identical to the *Baseline* treatment except that in part II we explicitly show subjects their bid functions from part I. The *MinBid* treatment is identical to the *Baseline* treatment except that subjects are explicitly not allowed to underbid.

We conducted three sessions of the *Baseline* (62 subjects), two sessions of the *ShowBidFn* (46 subjects), and one session of the *MinBid* treatment (26 subjects). Subjects were students at The Ohio State University (OSU) who were enrolled in undergraduate economics classes. The sessions were held at the Experimental Economics Lab at OSU and lasted around 45 minutes. At the start of each session, the experimenter read the instructions for part I aloud as subjects read along. After that, subjects did a practice quiz. Experimenters walked around checking subjects' quizzes, answering questions, and explaining mistakes. After part I of the relevant treatment, the instructions for part II were read. After part II, subjects were paid. Subjects' earnings consisted of a \$5 show-up fee, plus 10 ECU starting balances, plus their cumulative earnings from the 22 auctions,[18] converted at a rate of \$0.50 per ECU. Average earnings

[16]Our design for part I ensures that each subject receives each signal from the set $\{0, 1, 2, \ldots, 10\}$ exactly once. In effect, we are eliciting subjects' bid functions. This simplifies the design of part II.

[17]Note that although in part II a subject bids against the computer, the bidding strategy of the opponent is that of a person. The fact that this person is herself from part I should only make the cognitive processes of the opponent all the more salient.

[18]In case a subject incurred losses which could not be covered by the 10 ECU starting balances, she was paid just her \$5 show-up fee.

were \$18.53/\$18.03/\$15.53 in the *Baseline*/*ShowBidFn*/*MinBid* treatment. The instructions for the *Baseline* treatment are given in the Supplemental Material (Ivanov, Levin, and Niederle (2010)).[19] The experiment was programmed and conducted with the software z-Tree (Fischbacher (2007)).

3.2. *Possible Implications for Belief-Based Theories*

In part I of the *Baseline* and *ShowBidFn* treatments, underbidding and bidding above 10 are weakly dominated and can hardly be explained by any belief-based theory. The most interesting behavior is overbidding because it leads to a WC (as long as others are also appropriately overbidding) and because it could potentially be explained by belief-based theories. Notice that, to explain overbidding, both the level-k model and CE require that beliefs place a positive weight on underbidding, that is, on weakly dominated bids. Although not implausible, this requirement puts some strain on belief-based explanations of overbidding.

However, the real test of belief-based theories comes from part II of each treatment and part I of the *MinBid* treatment. In particular, we argue below that if behavior is driven by beliefs, we should observe a reduction in overbidding (i) in part II of each treatment relative to part I and (ii) in part I of the *MinBid* treatment relative to part I of the *Baseline* and *ShowBidFn* treatments. The absence of any such reduction would cast a serious doubt on belief-based theories.

Our argument is based on the assumption that if behavior is driven by beliefs, these beliefs are at least consistent with the objectively known features of the environment. That is, we assume that a subject's belief in part II is consistent with the fact that the computer uses her own bid function from part I[20] and that a subject's belief in the *MinBid* treatment is consistent with the fact that the opponent cannot underbid. Later, we will consider alternative interpretations of belief-based theories under which beliefs can be at odds with the objectively known features of the environment.

Consider a subject i who overbids (for all signals) in part I of one of the three treatments.[21] From Proposition 1, it follows that bidding her signal is a best response in part II. Although underbidding may not be a best response, it is at least a response in the right direction.[22] If i continues to overbid but

[19]The instructions in the other two treatments are very similar and are available upon request.

[20]In the *Baseline* treatment, this assumption entails that subjects are able to recall their bidding behavior from part I (which was just a few minutes ago) or perhaps, at least, whether they tended to underbid, bid their signal, overbid, or bid above 10. In the *ShowBidFn* treatment, subjects do not need to recall anything because they are explicitly shown their bid functions.

[21]To be precise, overbidding is not possible for signal 10: a subject can underbid (except in the *MinBid* treatment), bid her signal, or bid above 10. Therefore, the correct statement is "a subject i who overbids for all signals 0–9 and bids above 10 for signal 10."

[22]Of course, underbidding is not possible in the *MinBid* treatment.

corrects her overbidding downward, this may or may not be a best response,[23] but again it is a response in the right direction. On the other hand, if i continues overbidding without a downward correction or even starts bidding above 10 in part II, she is clearly not best responding to her behavior from part I. The bottom line is that if i's behavior is driven by beliefs, we should observe a downward correction of bids in part II relative to part I.

In part I of the *MinBid* treatment, anything other than bidding one's signal is weakly dominated. Thus, if behavior is driven by beliefs, we would expect a reduction in the frequency and (average) magnitude of overbidding relative to part I of the *Baseline* and *ShowBidFn* treatments.[24]

Let us turn to three interpretations of belief-based theories under which beliefs can be at odds with the objectively known features of the environment. The first interpretation is that subjects are using some simple rule of thumb which leads them to behave "as if" they were best responding to beliefs. For example, a player using a rule like "bid based on the expected value conditional on my signal and ignore everything else" would behave just like a fully cursed or a RL_1 player. Because subjects do not deliberately form beliefs, the beliefs describing their behavior could be at odds with objectively known features of the environment. Thus, subjects in any of our environments could behave as if they had cursed or RL_1 beliefs.

Note that this interpretation requires that the rule of thumb be rigid across environments. For example, a subject using the above rule of thumb needs to ignore the opponent's bidding strategy just as much in part II as in part I, even though in part II it is her own past bidding strategy (which is even explicitly shown to her in the *ShowBidFn* treatment).

The second and third interpretations pertain to CE. According to the second interpretation, cursed players do not fully think through the connection between others' types and bids. As a result, they come up with cursed beliefs to which, however, they best respond by appropriately conditioning the expected value of the object on winning the auction. Under this interpretation, CE would explain behavior in our experiment only if players equally fail to realize the connection between others' types and bids when bidding against (i) other people whose bids are unrestricted, (ii) their own bidding strategy (even when it is shown to them), and (iii) other people who are explicitly not allowed to underbid.

[23]For overbidding in part II to be a best response, i would need to shift her bid function in part II, $b_i^{\mathrm{II}}(\cdot)$, downward in a way that, for all signals x_i, none of the bids she made in part I lies in $(x_i, b_i^{\mathrm{II}}(x_i)]$. Otherwise, there is a positive probability that she wins the auction and loses money.

[24]In the *MinBid* treatment, a subject's available bids depend on her type so that CE is not formally defined. Nevertheless, our point remains valid: if subjects' behavior is driven by beliefs (whether these beliefs are appropriately redefined cursed beliefs or other beliefs), we should observe a reduction in overbidding.

According to the third interpretation, players are aware of others' type-contingent strategies but underappreciate the information content of winning the auction. Under this interpretation, CE could explain overbidding in any of the environments of our experiment.[25] The problem with this interpretation is that rather than being about inconsistent beliefs, it is about a failure to properly update the expected value of the object conditional on winning. Such a failure is not at all part of the formal definition of CE according to which players perfectly update given their (albeit cursed) beliefs. Although theoretically awkward, this interpretation could have validity if CE can accurately capture behavior which is actually driven by improper updating. For example, in the special case of bidders who have correct beliefs but completely fail to condition on winning, fully CE perfectly describes behavior. However, it is unclear to what extent CE can accurately capture the behavior of bidders who update, albeit incompletely. Thus, it is unclear to what extent this interpretation is generally valid.[26]

4. RESULTS

We start by studying and comparing behavior in parts I and II within each treatment. After that, we compare the part I behavior of the *Baseline* and *ShowBidFn* treatments with that of the *MinBid* treatment.

4.1. *Behavior in Part I and Part II*

We start by placing each bid b, given signal x, in one of the following categories: (i) $b < x - 0.25$, (ii) $x - 0.25 \leq b \leq x + 0.25$, (iii) $x + 0.25 < b \leq 10$, and (iv) $b > 10$.[27] That is, we count all bids within 0.25 ECU of one's signal as if they were precisely equal to the signal.[28] Based on this, we classify subjects (separately for each part of each treatment) in the following way: *Underbidders/Signal Bidders/Overbidders/Above-10 Bidders* are those who make at least 6 (out of 11) bids in category (i)/(ii)/(iii)/(iv); subjects who fall in none of these four classes are classified as *Indeterminate*.[29]

We start the analysis with the *Baseline* treatment. Table I shows how many subjects were in each class in part I (last column) and part II (last row). The

[25]The same holds for Charness and Levin's variant of the acquiring a company game.

[26]To shed light on the issue, one would formally have to define an equilibrium concept in which players have correct beliefs but incompletely update their beliefs (conditional on winning).

[27]Actually, for signal $x = 10$, a bid needs to be above 10.25 to fall into category (iv); a bid $9.75 \leq b \leq 10.25$ falls into category (ii). We ignore this into our notation.

[28]Counting only bids which are precisely equal to the signal in category (ii) (and adjusting the other categories appropriately) does not change any of our results.

[29]Using 7 or 8 (instead of 6) class-consistent decisions as the cutoff for a player to be assigned to a class does not affect the analysis much (apart from increasing the number of *Indeterminate* subjects).

TABLE I

SUBJECT CLASSIFICATION IN PARTS I AND II OF THE *BASELINE* TREATMENT

Part I / II	Underbidders	Signal Bidders	Overbidders	Above-10 Bidders	Indeterminate	
Underbidders	2	0	2	1	0	**5**
Signal Bidders	0	5	3	1	0	**9**
Overbidders	1	5	14	1	4	**25**
Above-10 Bidders	2	1	1	6	0	**10**
Indeterminate	2	2	3	5	1	**13**
	7	**13**	**23**	**14**	**5**	

table also shows how subjects switched between classes from part I to part II. For example, the entry in the first row and third column shows that 2 subjects who were *Underbidders* in part I became *Overbidders* in part II. Based on the table, we can make the following statements:

RESULT 1:
(a) In part I, a large percentage of subjects make a weakly dominated bid ($b < x - 0.25$ or $b > 10$) in at least 6 (out of 11) auctions (30.7%).[30]
(b) In part I, *Overbidders* are the largest class (40.3%).
(c) Only a minority of *Overbidders* from part I become *Signal Bidders* or *Underbidders* in part II (24%).
(d) The majority of *Overbidders* from part I remain *Overbidders* in part II (56%).

For a large proportion of subjects, behavior can hardly be explained by belief-based theories (point (a)). However, the largest proportion of subjects in part I are *Overbidders*. These subjects' behavior is potentially driven by beliefs. However, only a minority of them (best) respond in part II by becoming *Signal Bidders* or *Underbidders*. The key question is whether those who remain *Overbidders* in part II are best responding in part II or are at least responding in the right direction by correcting their bids downward.[31]

For subjects who are *Overbidders* in parts I and II, we find that only 23% of bids in part II are best responses to part I behavior. These subjects are foregoing, on average, 5.62 ECU (median is 4.07 ECU) in expected profits by not behaving optimally in part II. Figure 1 plots, for each signal, the median bid in part I (circles) and part II (stars).[32] Based on the figure, we see no downward correction of bids in part II. We can state the following result:

[30]This percentage includes all *Underbidders* and *Overbidders*, as well as 4 *Indeterminate* subjects.
[31]The one *Overbidder* from part I who becomes an *Above-10 Bidder* in part II is clearly not (best) responding to her behavior from part I.
[32]We plot median, rather than average, bids because averages are distorted by bids above 10.

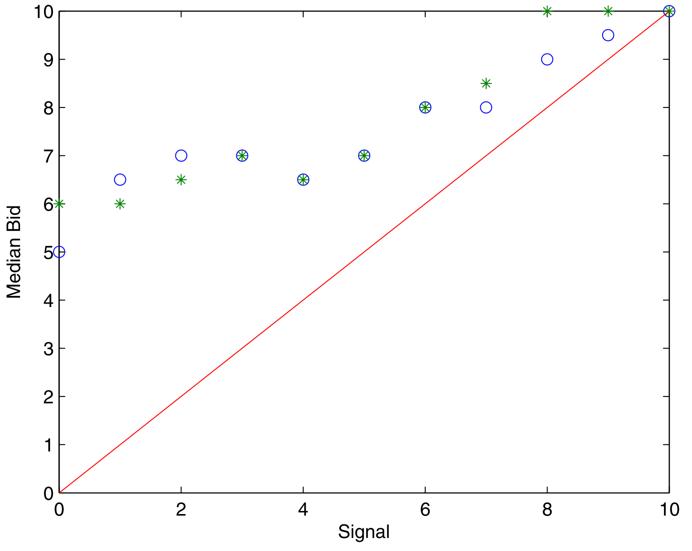

FIGURE 1.—Median bids in parts I (circles) and II (stars) for subjects who are *Overbidders* in parts I and II of the *Baseline* treatment.

RESULT 2: For subjects who are *Overbidders* in parts I and II, we make the following observations:

(a) In part II, they forego substantial expected profits.

(b) In part II, there is no evidence of a downward correction of bids.

Result 1 extends to the *ShowBidFn* and *MinBid* treatments; see Tables II and III in Appendix B which are the analogs of Table I.[33] Result 2 also extends to the *ShowBidFn* and *MinBid* treatments. In the *ShowBidFn*/*MinBid* treatment, for subjects who are *Overbidders* in parts I and II, 15%/19% of bids in part II are best responses to part I behavior; these subjects are foregoing, on average, 6.60 ECU[34]/5.40 ECU (median is 7.19 ECU/3.84 ECU) in expected profits in part II. For analogs of Figure 1, see Figures 4 and 5 in Appendix B.

4.2. Baseline *and* ShowBidFn *versus* MinBid

If behavior is driven by beliefs, we would expect a reduction in the frequency and (average) magnitude of overbidding in part I of the *MinBid* treatment rel-

[33]In the *ShowBidFn*/*MinBid* treatment, the percentage of subjects who make a weakly dominated bid in at least 6 (out of 11) auctions is 28.3%/7.8%. In the *MinBid* treatment, the percentage is smaller largely because subjects cannot underbid.

[34]This average excludes one subject who bid very high both in parts I and II so that she incurred huge losses in part II.

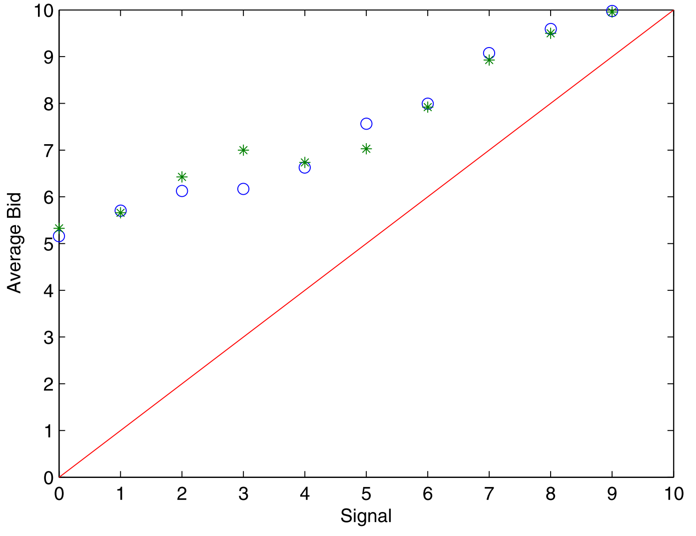

FIGURE 2.—Average bids in part I of *Baseline* and *ShowBidFn* (circles) and *MinBid* (stars) (based on bids of form $x + 0.25 < b \leq 10$).

ative to part I of the *Baseline* and *ShowBidFn* treatments. The frequency of overbidding is 42.8% in the *Baseline* and *ShowBidFn* treatments[35] and 60.5% in the *MinBid* treatment. Overbidding is probably more frequent in the *MinBid* treatment because underbidding is impossible, so all bids are distributed in three, rather than four, categories. Given this, the frequencies of overbidding seem quite comparable.

What about the magnitude of overbidding? Figure 2 shows, for each signal, the average bid of the form $x + 0.25 < b \leq 10$ in part I of the *Baseline* and *ShowBidFn* treatments (circles) and in part I of the *MinBid* treatment (stars). Average bids are astonishingly close. We can now state our final result:

RESULT 3: Relative to part I of the *Baseline* and *ShowBidFn* treatments, we find no evidence in part I of the *MinBid* treatment of (a) a lower frequency of bids of the form $x + 0.25 < b \leq 10$ or (b) a reduction in the average size of bids of the form $x + 0.25 < b \leq 10$.

[35]We pool the data from part I of the *Baseline* and *ShowBidFn* treatments because part I is the same in both treatments.

5. CONCLUDING REMARKS

We investigate experimentally whether belief-based theories can explain the WC in common-value auctions in initial periods of play.[36] The main idea of our approach is to compare behavior in an environment where overbidding can be rationalized by belief-based theories with behavior in environments where belief-based explanations are less plausible. We observe no reduction in overbidding in the latter environments. We conclude that, our results cast serious doubt on belief-based explanations of the WC in initial periods of play unless one is willing to accept one of the following statements:

(i) Subjects use a rule of thumb which leads them to behave as if they were best responding to beliefs and which is fixed across the environments in our study.

(ii) Subjects equally fail to realize the connection between others' types and bids in all environments in our study.

(iii) CE, contrary to its formal definition, can be interpreted as being about improper updating rather than about inconsistent beliefs.

APPENDIX A: PROOFS

PROOF OF PROPOSITION 1: *First round of deletion of weakly dominated bid functions*. It is obvious that bidding above 10 is weakly dominated. Under the second-price rule, for any x_i, any bid strictly below x_i is also weakly dominated (by bidding x_i) since one could lose the auction at a price below x_i even though $x^{\max} \geq x_i$. Therefore, we can delete all bid functions, such that $b_i(x_i) < x_i$ or $b_i(x_i) > 10$ for some x_i.

Second round of deletion of weakly dominated strategies. Suppose that bidder i with signal x_i considers bidding $b^+ > x_i$. In the event that bidding x_i wins, bidding b^+ rather than x_i does not matter. In the event that bidding b^+ does not win, bidding b^+ rather than x_i also does not matter.

Now consider the third possible event: that bidding x_i does not win but bidding b^+ does. Then bidder i pays the highest bid among the other $n-1$ players, $\widehat{b}$, where $\widehat{b} \geq x^{\max}$. The inequality holds because $\widehat{b} \geq x_i$ (otherwise x_i would have won) and because none of the other bidders ever underbid (by the first round of deletion of weakly dominated bid functions). But then i would make nonpositive profits by bidding b^+, whereas she would make zero profits by bidding x_i. Moreover, if $\widehat{b}$ is strictly above $x_{\max}$, then b^+ makes strictly negative

[36]Our study, and particularly behavior in part II of our treatments, may have implications for theories of learning, such as fictitious play (Brown (1951) and Robinson (1951)), in which players best respond to others' past actions. (We thank an anonymous referee for this point.) We do not emphasize this point because learning models are usually about multiple repetitions. Therefore, even if players in part II in our experiment do not best respond to their own past behavior, with experience, they may very well learn to best respond to others' past behavior.

profit. Therefore, b^+ is weakly dominated and we can delete all bid functions such that $b_i(x_i) > x_i$ for some x_i. *Q.E.D.*

PROOF OF PROPOSITION 2[37]: A strategy for player i is a probability measure H on $[0, 10] \times [0, \infty)$ with marginal cumulative distribution function (c.d.f.) on the first coordinate equal to $F(\cdot)$. A pure strategy is a bid function $b : [0, 10] \mapsto [0, \infty)$ such that $H(\{x, b(x)\}_{x\in[0,10]}) = 1$. That $b(x) = x$ is a BNE follows directly from Proposition 1. Here, we prove uniqueness among all symmetric BNE.[38]

Assume that H is a symmetric BNE. Let $L = \{(x, b)|x \in [0, 10], b < x\}$ and $U = \{(x, b)|x \in [0, 10], b > x\}$. That is, L and U are the sets in $[0, 10] \times [0, 10]$ strictly below and strictly above the 45° line, respectively. We need to show that $H(L \cup U) = 0$ or, equivalently, that $H(L) = 0$ and $H(U) = 0$.

First, assume $H(L) > 0$. Let $s_k(\cdot)$ be the step function, defined by $s_k(x) = \frac{10}{k}\,\mathrm{int}(\frac{kx}{10})$, where $\mathrm{int}(\cdot)$ gives the integer part of a real number ($s_3(\cdot)$ is depicted in the left graph in Figure 3). Let $A_k = \{(x, b)|b \leq s_k(x)\} \cap L$, that is, A_k is the area in L below the $s_k(\cdot)$ function. Note that $k' < k''$ implies $A_{2^{k'}} \subset A_{2^{k''}}$ and that $L = \bigcup_{k\geq 1} A_{2^k}$. Therefore, $H(L) = \lim_{k\to\infty} H(A_{2^k}) > 0$.[39] Therefore,

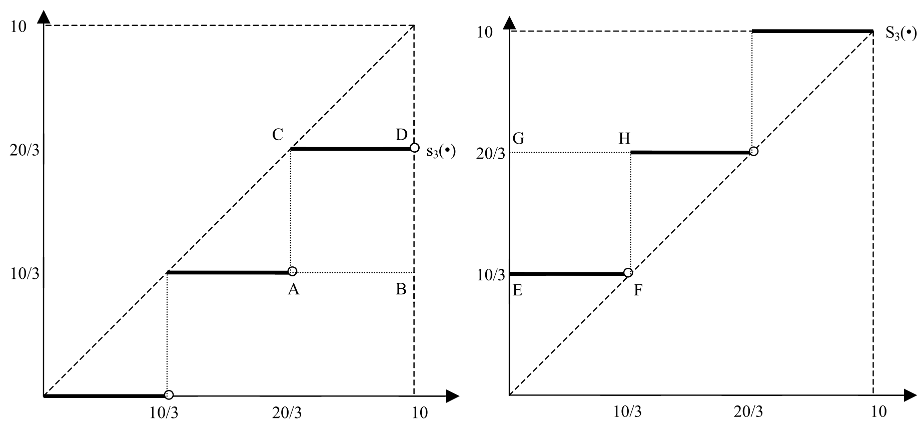

FIGURE 3.—$s_3(\cdot)$ and $S_3(\cdot)$.

[37]Under standard assumptions on $F(\cdot)$, we could simply invoke Proposition 1 in Pesendorfer and Swinkels (1997) so that no proof would be necessary. However, these assumptions do not hold in the case of the discrete distribution in our experiment.

[38]Of course, any bid function which differs from $b(x) = x$ only on a set of measure zero will also be a symmetric BNE.

[39]To see this, let $B_2 = A_2$ and $B_l = A_l/A_{l-1}$ for $l \geq 3$. Then $H(L) = H(\bigcup_{l\geq 2} A_l) = H(\bigcup_{l\geq 2} B_l) = \sum_{l\geq 2} H(B_l) = \lim_{k\to\infty} \sum_{l=2}^{k} H(B_l) = \lim_{k\to\infty} H(A_k) = \lim_{k\to\infty} H(A_{2^k})$. The third and fifth equalities follow from the (countable) additivity of probability measures.

for some $\overline{k}$, $H(A_{2^{\overline{k}}}) > 0$. Because $A_{2^{\overline{k}}}$ consists of finitely many rectangles like $ABCD$ in Figure 3 ($ABCD$ includes its boundaries, except for point D), it follows that at least one of these rectangles has positive measure. Assume, without loss of generality, $H(ABCD) > 0$.

We will show that, for a positive measure (with respect to (w.r.t.) H) of points $(x, b) \in ABCD$, bidding b given signal x is strictly worse than bidding x because there is a positive probability that one will lose the auction to a bid strictly below x. Let $g(\widetilde{b}) = H(\{(x, b)|(x, b) \in ABCD, b \leq \widetilde{b}\})$. Note that $g(\cdot)$ is a nondecreasing function and that $g(\underline{b}) \geq 0$ and $g(\overline{b}) > g(\underline{b})$, where $\underline{b} = \min(\{b|(x, b) \in ABCD\})$ and $\overline{b} = \max(\{b|(x, b) \in ABCD\})$.

If $g(\underline{b}) > 0$, then $\{(x, b)|(x, b) \in ABCD, b = \underline{b}\}$ has positive measure. For any point (x, b) in this set, bidding $\underline{b}$ given signal x is strictly worse than bidding x since there is a positive probability of a tie at $\underline{b}$.

Assume $g(\underline{b}) = 0$. If $g(\cdot)$ is continuous, choose $b^* \in (\underline{b}, \overline{b})$, such that $0 < g(b^*) < g(\overline{b})$.[40] Then $\{(x, b)|(x, b) \in ABCD, b \leq b^*\}$ and $\{(x, b)|(x, b) \in ABCD, b > b^*\}$ each have positive measure. But then for a positive measure of points (x, b) (the points in the former set), bidding b given signal x is strictly worse than bidding x since there is a positive probability of losing the auction to a bid b, such that $b^* < b < x$.

If $g(\cdot)$ is not continuous, then it has a jump point[41] at, say, b^{**}. Therefore, $\{(x, b)|(x, b) \in ABCD, b = b^{**}\}$ has positive measure. For any point (x, b) in this set, bidding b^{**} given signal x is strictly worse than bidding x since there is a positive probability of a tie at b^{**}. This proves that we cannot have $H(L) > 0$.

The proof that we cannot have $H(U) > 0$ is analogous, so only a brief outline is provided. Assume that $H(U) > 0$. Let $S_k(\cdot)$ be the step function, defined by $S_k(x) = s_k(x + \frac{10}{k})$ ($S_3(\cdot)$ is depicted in the right graph in Figure 3). Then we show analogously to the above discussion that a rectangle of the sort $EFGK$ in Figure 3 has positive measure. Then defining $h(\widetilde{b}) = H(\{(x, b)|(x, b) \in EFGH, b \leq \widetilde{b}\})$, we show that for a positive measure (w.r.t. H) of points $(x, b) \in EFGH$, bidding b given signal x is strictly worse than bidding x because there is a positive probability that one will win the auction at a price strictly above $x^{\max}$. *Q.E.D.*

PROOF OF PROPOSITION 3: Let $\widehat{B}$ denote the highest bid among the $n - 1$ subjects other than i. Given $X_i = x_i$, subject i chooses her bid, b, to maximize[42]

$$\begin{aligned} &E(\text{payoff}|X_i = x_i) \\ &\quad = \text{prob}(\widehat{B} < b|X_i = x_i)E(X^{\max} - \widehat{B}|X_i = x_i, \widehat{B} < b) \end{aligned}$$

[40] This can clearly be done by the intermediate value theorem.

[41] Any nondecreasing function is either continuous or has countably many jump points.

[42] Ties are ignored because they occur with zero probability.

$$= \text{prob}(\widehat{B} < b)[E(X^{\max}|X_i = x_i) - E(\widehat{B}|\widehat{B} < b)]$$
$$= \frac{b^{n-1}}{10^{n-1}}\left[E(X^{\max}|X_i = x_i) - \frac{n-1}{n}b\right].$$

The second equality follows, because (i) others' bids (and $\widehat{B}$ in particular) are not informative about $X^{\max}$ and (ii) X_i is not informative about others' bids (and about $\widehat{B}$ in particular). The third equality uses facts about the distribution and expectation of the first-order statistic of $n-1$ i.i.d. random variables which have the uniform distribution on $[0, 10]$. From the last expression, it is straightforward to verify that the unique optimal bid equals $E(X^{\max}| X_i = x_i)$. *Q.E.D.*

APPENDIX B: FIGURES AND TABLES

TABLE II

SUBJECT CLASSIFICATION IN PARTS I AND II OF THE *SHOWBIDFN* TREATMENT

Part I / II	Underbidders	Signal Bidders	Overbidders	Above-10 Bidders	Indeterminate	
Underbidders	3	0	0	0	1	**4**
Signal Bidders	0	5	1	0	1	**7**
Overbidders	4	0	10	2	2	**18**
Above-10 Bidders	0	1	0	4	0	**5**
Indeterminate	3	1	5	1	2	**12**
	10	**7**	**16**	**7**	**6**	

TABLE III

SUBJECT CLASSIFICATION IN PARTS I AND II OF THE *MINBID* TREATMENT

Part I / II	Underbidders	Signal Bidders	Overbidders	Above-10 Bidders	Indeterminate	
Underbidders	0	0	0	0	0	**0**
Signal Bidders	0	3	0	1	0	**4**
Overbidders	0	3	14	2	0	**19**
Above-10 Bidders	0	1	1	0	0	**2**
Indeterminate	0	1	0	0	0	**1**
	0	**8**	**15**	**3**	**0**	

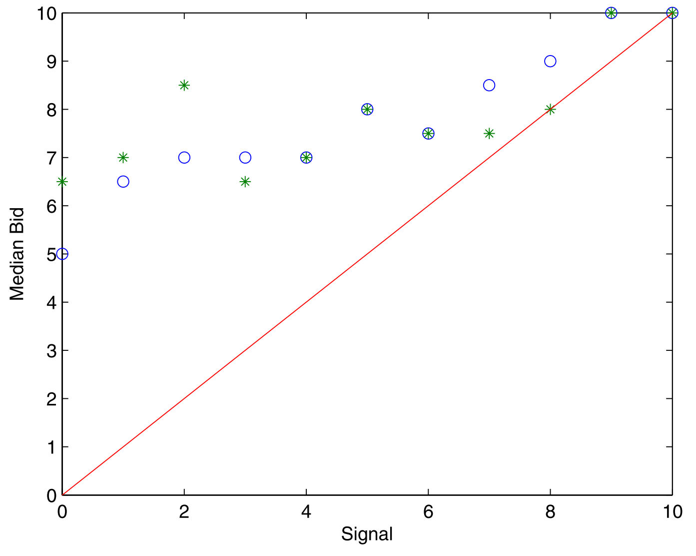

FIGURE 4.—Median bids in parts I (circles) and II (stars) for subjects who are *Overbidders* in parts I and II of the *ShowBidFn* treatment.

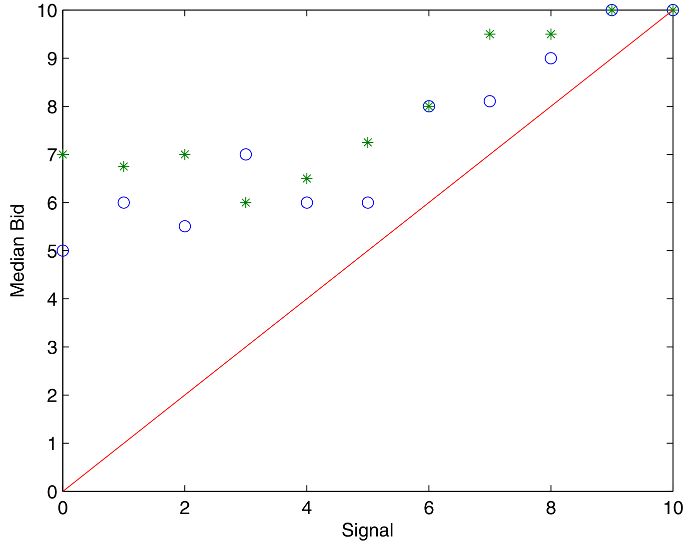

FIGURE 5.—Median bids in parts I (circles) and II (stars) for subjects who are *Overbidders* in parts I and II of the *MinBid* treatment.

REFERENCES

AKERLOF, G. (1970): "The Market for Lemons: Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, 84, 488–500. [1436]

BAZERMAN, M. H., AND W. F. SAMUELSON (1983): "I Won the Auction But Don't Want the Prize," *Journal of Conflict Resolution*, 27, 618–634. [1435]

BROWN, G. (1951): "Iterative Solution of Games by Fictitious Play," in *Activity Analysis of Production and Allocation*, ed. by T. C. Koopmans. New York: Wiley. [1446]

BULOW, J., AND P. KLEMPERER (2002): "Prices and the Winner's Curse," *RAND Journal of Economics*, 33, 1–21. [1435,1436]

CAMPBELL, C. M., AND D. LEVIN (2006): "When and why not to Auction," *Economic Theory*, 27, 583–596. [1436]

CHARNESS, G., AND D. LEVIN (2009): "The Origin of the Winner's Curse: A Laboratory Study," *American Economics Journal: Microeconomics*, 1, 207–236. [1436]

COSTA-GOMES, M., AND G. WEIZSÄCKER (2008): "Stated Beliefs and Play in Normal-Form Games," *Review of Economic Studies*, 75, 729–762. [1436]

CRAWFORD, V. P., AND N. IRIBERRI (2007): "Level-*k* Auctions: Can a Non-Equilibrium Model of Strategic Thinking Explain the Winner's Curse and Overbidding in Private-Value Auctions?" *Econometrica*, 75, 1721–1770. [1435,1438]

DYER, D., J. H. KAGEL, AND D. LEVIN (1989): "A Comparison of Naive and Experienced Bidders in Common Value Offer Auctions: A Laboratory Analysis," *Economic Journal*, 99, 108–115. [1435]

EYSTER, E., AND M. RABIN (2005): "Cursed Equilibrium," *Econometrica*, 73, 1623–1672. [1435, 1438]

FISCHBACHER, U. (2007): "z-Tree: Zurich Toolbox for Ready-Made Economic Experiments," *Experimental Economics*, 10, 171–178. [1440]

IVANOV, A., D. LEVIN, AND M. NIEDERLE (2010): "Supplement to 'Can Relaxation of Beliefs Rationalize the Winner's Curse?: An Experimental Study'," *Econometrica Supplemental Material*, 78, http://www.econometricsociety.org/ecta/Supmat/8112_instructions to experimental subjects.pdf. [1440]

JEHIEL, P. (2005): "Analogy-Based Expectation Equilibrium," *Journal of Economic Theory*, 123, 81–104. [1435]

JEHIEL, P., AND F. KOESSLER (2008): "Revisiting Games of Incomplete Information With Analogy-Based Expectations," *Games and Economic Behavior*, 62, 533–557. [1435]

KAGEL, J. H. (1995): "Auctions: A Survey of Experimental Research," in *The Handbook of Experimental Economics*, ed. by J. H. Kagel and A. E. Roth. Princeton, NJ: Princeton University Press. [1435]

KAGEL, J. H., AND D. LEVIN (1986): "The Winner's Curse and Public Information in Common Value Auctions," *American Economic Review*, 76, 894–920. [1435]

——— (2002): *Common Value Auctions and the Winner's Curse*. Princeton, NJ: Princeton University Press. [1435]

KAGEL, J., R. HARSTAD, AND D. LEVIN (1987): "Information Impact and Allocation Rules in Auctions With Affiliated Private Valuations: An Experimental Study," *Econometrica*, 55, 1275–1304. [1435]

LIND, B., AND C. R. PLOTT (1991): "The Winner's Curse: Experiments With Buyers and With Sellers," *American Economic Review*, 81, 335–346. [1435]

NAGEL, R. (1995): "Unraveling in Guessing Games: An Experimental Study," *American Economic Review*, 85, 1313–1326. [1435]

PESENDORFER, W., AND J. M. SWINKELS (1997): "The Loser's Curse and Information Aggregation in Common Value Auctions," *Econometrica*, 65, 1247–1281. [1447]

PEVNITSKAYA, S. (2008): "Effect of Beliefs and Risk Preferences on Bidding in Auctions," Working Paper, Florida State University. [1437]

ROBINSON, J. (1951): "An Iterative Method of Solving a Game," *Annals of Mathematics*, 54, 296–301. [1446]

STAHL, D., AND P. WILSON (1995): "On Players' Models of Other Players: Theory and Experimental Evidence," *Games and Economic Behavior*, 10, 218–254. [1435]

Virginia Commonwealth University, 301 West Main Street, Room B3149, Richmond, VA 23284, U.S.A.; aivanov@vcu.edu,

The Ohio State University, 1945 North High Street, Arps Hall 410, Columbus, OH 43210, U.S.A.; levin.36@osu.edu,

and

Stanford University, 579 Serra Mall, Stanford, CA 94305-6072, U.S.A.; niederle@stanford.edu.

Manuscript received September, 2008; final revision received December, 2009.