The Design of School Choice Systems in NYC and Boston: Game-Theoretic Issues

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Outline of today's class

- NYC Schools: design of a centralized high school allocation procedure (implemented in 2003-04, for students entering Sept. '04)
- Boston Schools: redesign of a school allocation procedure (implemented for students entering K, 6, and 9 in Sept. 2006)
- New game theory problems and results
  - Generic indifferences (non-strict preferences)
  - Complete and incomplete information/ ex post versus ex ante evaluation of welfare/ restrictions on domains of preferences

Some Papers:

Abdulkadiroglu, Atila, and Tayfun Sönmez, "School Choice: A Mechanism Design Approach", *American Economic Review,* 93-3: 729-747, June 2003.

Abdulkadiroglu, Atila , Parag A. Pathak, and Alvin E. Roth, "The New York City High School Match," *American Economic Review*, Papers and Proceedings, 95,2, May, 2005, 364-367.

Abdulkadiroglu, Atila , Parag A. Pathak, and Alvin E. Roth, "The New York City High School Match," *American Economic Review, Papers and Proceedings*, 95,2, May, 2005, 364-367.

Abdulkadiroglu, Atila, Parag A. Pathak, Alvin E. Roth, and Tayfun Sönmez, "Changing the Boston School Choice Mechanism," January, 2006.

Erdil, Aytek and Haluk Ergin, What's the Matter with Tie-breaking? Improving Efficiency in School Choice, *American Economic Review*, 98(3), June 2008, 669-689.

Abdulkadiroglu, Atila , Parag A. Pathak, and Alvin E. Roth, "Strategy-proofness versus Efficiency in Matching with Indifferences: Redesigning the NYC High School Match,'', *American Economic Review*, 99(5) December 2009, 1954-1978.

Featherstone, Clayton and Muriel Niederle, "Manipulation in School Choice Mechanisms", October 2008.

Market design for school choice

- Thickness
  - In both NYC and Boston, the market for public school places was already quite thick.
- Congestion
  - In NYC, congestion was the most visible problem of the old system, which let to problems of safe participation (and thickness)
  - In Boston there was already a centralized mechanism in place
- Safety
  - In NYC, there were both participation problems and incentive problems about revealing preferences.
  - In Boston, the big problem was about revealing preferences

Matching students to schools—overcoming **congestion** in New York City

- Old NYC high school choice system
  - Decentralized application and admission
  - ***congested***: left 30,000 kids each year to be administratively assigned (while about 17,000 got multiple offers)
    - Waiting lists run by mail
    - Gaming by high schools; withholding of capacity
- The new mechanism is a centralized clearinghouse that produces stable matches.
  - We now have enough data to begin to say something about how it is working.

Old NYC High School Match

(Abdulkadiroglu, Pathak, Roth 2005)

Overview: *Congestion*

- Over 90,000 students enter high school each year in NYC
- Each was invited to submit list of up to 5 choices
- Each student's choice list distributed to high schools on list, who independently make offers
  - Gaming by high schools—withholding of capacity—only recently recentralized school system.
  - Gaming by students: first choice is important
- Only approx. 40% of students receive initial offers, the rest put on waiting lists—3 rounds to move waiting lists…
- Approx. 30,000 students assigned to schools not on their choice list.

Issues in old (2002) system

- Schools see rank orders

  Some schools take students' rankings into account & consider only those that rank their school first

From 2001

http://nymag.com/urban/articles/schools01/

- **"How hard is it to get in?** Preference is given to students who live in District 3. **Only students who list Beacon as their first choice are considered for admission**. Last year, 1,300 kids applied for 150 spots in the ninth grade.
- **"Only students who list Townsend Harris as their first choice** and who meet the cutoff and have an exceptionally high grade-point average **are considered**. Students living anywhere in New York City may apply.
- **Young Women's Leadership School**: Students who want to be considered for admission **must list the school as their first choice**.
- Open to any student living in Brooklyn; students living in a specified zone around the school have priority. **Applicants must list Murrow as their first choice to be considered**.
- Applicants may list **Midwood as their first or second choice** to be considered.

Making it **safe to reveal preferences**

- Redesign of the Boston Public Schools choice mechanism
  - The old centralized assignment system tried to give as many people as possible their first choice: this made it unsafe to reveal true preferences.
    - Some parents acted on these strategic incentives, others did not (and suffered).
  - Replace the existing mechanism in 2006 (for entry into grades K, 1, 6, 9) with a clearinghouse that lets parents safely list their true preferences
.

Issues in old (2002) system

Students need to strategize. The 2002-03 *Directory of the NYC Public High Schools* : “determine what your competition is for a seat in this program”

- **Principals concealed capacities**

  Deputy Chancellor (NYT 11/19/04):

  “Before you might have had a situation where a school was going to take 100 new children for 9th grade, they might have declared only 40 seats and then placed the other 60 children outside the process.”

(think “blocking pairs”)

Issues in old (2002) system

1. "5" choices
   - 52% of kids rank five choices→ constraint binding
   - Congestion, nevertheless (Roth and Xing, 1997): Not enough offers and acceptances could be made to clear the market
   - Only about 50,000 out of 90,000 received offers initially.
   - About 30,000 assigned outside of their choice
2. Multiple offers—are they good for some kids?
   - about 17,000 received multiple offers
   - Students may need time to make up their mind, especially if we want to keep desirable students from going to private school
   - Only 4% don't take first offer in 02-03 at the cost of over 30,000 kids not getting any offer

NYC School System (in 2002)

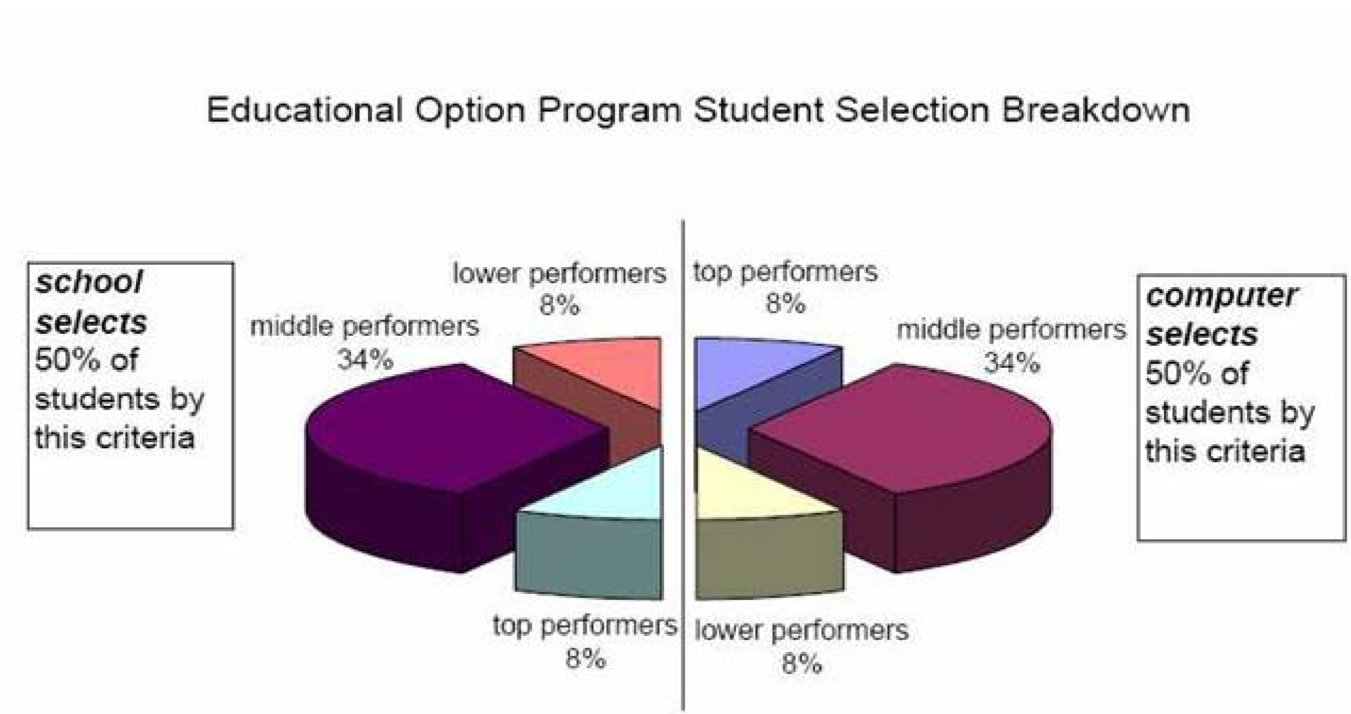
	# of Programs
Unscreened (no preferences)	**86**
Screened & Auditioned	**188**
Specialized HS	**6**
Educational Option (no preferences for half seats)	**252**

- **In Brooklyn, Bronx, Manhattan, Staten Island, and Queens**
- **Unscreened capacity largest**
- **Roughly 25,000 kids take Specialized High School Test**

NYC School System

Ed-Opt Schools – based on city or state standardized reading test score grade 7

(preferences for only half the seats)



Are NYC Schools a *two-sided* market?

Two facts:

1. Schools conceal capacities

   i.e. principals act on instabilities

2. Principals of different EdOpt schools have different preferences, some preferring higher scores, some preferring better attendance records

Recall our (too) simple basic model

- PLAYERS: Schools = $\{f_1,..., f_n\}$ Students = $\{w_1,..., w_p\}$
  \# positions $q_1,...,q_n$

- PREFERENCES (complete and transitive):
  $P(f_i) = w_3, w_2, ... f_i ...$ $[w_3 \; P(f_i) \; w_2]$ (not all strict)
  $P(w_j) = f_2, f_4, ... w_j ...$

- An OUTCOME of the game is a *MATCHING*:
  μ: $F \cup W \rightarrow F \cup W$
  such that $\mu(f) = w$ iff $\mu(w) = f$, and for all f and w $|\mu(f)| \leq q_f$, and either $\mu(w)$ is in F or $\mu(w) = w$.

- A matching μ is *BLOCKED BY AN INDIVIDUAL* k if k prefers being single to being matched with $\mu(k)$ $[kP(k)\,\mu(k)]$
- A matching μ is *BLOCKED BY A PAIR OF AGENTS* (f,w) if they each prefer each other to μ:
  [w P(f) w' for some w' in $\mu(f)$ or w P(f) f if $|\mu(f)| < q_f$] and f P(w) $\mu(w)$]

- A matching μ is *STABLE* if it isn't blocked by any individual or pair of agents.

Basic Deferred Acceptance (Gale and Shapley 1962)

- Step 0.0: students and schools **privately** submit preferences
- Step 0.1: arbitrarily break all ties in preferences
- Step 1: Each student "proposes" to her first choice. Each school tentatively assigns its seats to its proposers one at a time in their priority order. Any remaining proposers are rejected.

  …
- Step k: Each student who was rejected in the previous step proposes to her next choice if one remains. Each school considers the students it has been holding together with its new proposers and tentatively assigns its seats to these students one at a time in priority order. Any remaining proposers are rejected.
- The algorithm terminates when no student proposal is rejected, and each student is assigned her final tentative assignment.

Theorems (for the simple model)

1. The outcome that results from the student proposing deferred acceptance algorithm is stable, and (when preferences are strict) student optimal among the set of stable matchings (Gale and Shapley, 1962)
2. The student proposing outcome is weakly Pareto optimal for students (Roth, 1982)
3. The SPDAA makes it a dominant strategy for students to state their true preferences. (Dubins and Friedman 1981, Roth, 1982, 1985)
4. There is no mechanism that makes it a dominant strategy for schools to state their true preferences. (Roth, 1982)
5. When the market is large, it becomes unlikely that schools can profitably misrepresent their preferences. (Immorlica and Mahdian, 2005, Kojima and Pathak, 2009)

The New (Multi-Round) Deferred Acceptance Algorithm in NYC

- We advised, *sometimes convinced*, the NYC DOE
- Software and the online application process has been developed by a software consulting company
- The new design adapted to the regulations and customs of NYC schools

Some (Imperfectly Resolved) Design issues

(It’s important to choose your fights:)

Strategic Risks for Students

- Tradition: Top 2% students are automatically admitted to EdOpt programs of their choice if they rank them as their first choice

  → Strategic risk to the decisions of top 2% students

Partial incentive compatibility for top 2%-ers

- **Proposition**: In the student-proposing deferred acceptance mechanism where a student can rank at most k schools, if a student is guaranteed a placement at a school only if she ranks it first, then she can do no better than
  - either ranking that program as her first choice, and submit the rest of her preferences according to her true preference ordering, or
  - submitting her preferences by selecting at most k schools among the set of schools she prefers to being unassigned and ranking them according to her true preference ordering.

still…tough choices

- **Sent:** Friday, June 24, 2005 6:58 AM
  … Briefly, my daughter Eliana xxx, who just graduated from IS 98 in Brooklyn (with honors -- I'm a proud parent) **exercised her "top 2% option" during the High School Admissions process and selected Leon M. Goldstein HS in Brooklyn. Her real choice -- which she loved above all the many schools we visited -- was Beacon High School in Manhattan**. As you know, Beacon has a selection process and we were advised by everyone consulted, in and outside the Department of Education, not to take a chance and to absolutely exercise the 2% option and not risk losing *any* choice. Eliana has been restless and losing sleep every since. Mr. Dorosin, she is a steady 94% Arista student, she just was awarded the medal for "Media Communications" at graduation and we learned she scored a "perfect" 830 on the reading exam. Needless to say, it is distressing that she may not have the opportunity to go to the High School of her choice because of an error in our judgement.
  We have since learned that she was indeed ranked by Beacon for acceptance so this would not be an obstacle. I ask that you please grant this request for a deserving student. Thank you very much.

Redesign: 12 choice constraint

- DOE thought this would be sufficient, we encouraged more

New Process: Average Number of Rankings Each Round

Round	**Ranking** 1	2	3	4	5	6	7	8	9	10	11	12
Round 1	91,286	84,554	79,646	73,398	66,724	59,911	53,466	47,939	42,684	37,897	31,934	22,629
	100%	93%	87%	80%	73%	66%	59%	53%	47%	42%	35%	25%
Round 2	87,810	81,234	76,470	70,529	64,224	57,803	51,684	46,293	41,071	35,940	29,211	18,323
	100%	93%	87%	80%	73%	66%	59%	53%	47%	41%	33%	21%
Round 3	8,672	8,139	7,671	7,025	6,310	5,668	5,032	4,568	4,187	3,882	3,562	3,194
	100%	94%	88%	81%	73%	65%	58%	53%	48%	45%	41%	37%

3,476 Specialized High Schools Students
91,286 Total students

Partial incentive compatibility for constrained choosers

- **Proposition** (Haeringer and Klijn, Lemma 8.1.): In the student-proposing deferred acceptance mechanism where a student may only rank k schools,
  - if a student prefers fewer than k schools, then she can do no better than submitting her true rank order list,
  - if a student prefers more than k schools, then she can do no better than employing a strategy which selects k schools among the set of schools she prefers to being unassigned and ranking them according to her true preference ordering.

Multiple Rounds

- Historical/legal constraints: difficult to change specialized high school process/cannot force a student who gets an offer from a specialized high school to take it
  - Round 1: run algorithm with all kids in round 1, not just specialized students; only inform specialized students
    - Unstable if a specialized kid does not get a spot at a non-specialized high school when considered at round 1, but could get that spot in round 2
  - May not a big problem if students with specialized high schools offers are ranked high in all schools' preferences, and/or if most students prefer to go to a specialized school
    - In old system, ~70% of kids with an offer from a specialized program took it, 10% of kids went to private school and 14% kids went to either their first or second choice from the other schools.
      - Potential instabilities among these 14% will not be large if they are also considered highly desirable by the non-specialized schools they apply to.
      - …(however, we do observe several hundred children who decline a specialized school for their not-top-choice mainstream school…)

Multiple Rounds

- Need to assign unmatched kids; unlike medical labor markets everyone must go to school
  - → Round 3
    - “No time” for high schools to re-rank students in round 3, so no new high school preferences expressed
      - Another place where random preferences are used for some screened schools.

Lotteries: Equity and perception

How should we rank students in schools that do not have preferences over students?

- For unscreened schools and in round 3
- A single lottery that applies to each school?
- Or a different lottery for every such school?

- A single lottery avoids instabilities that are due to randomness (Abdulkadiroglu & Sonmez, 2003)

Lotteries, cont.:
Explaining and defending

NYC DOE argued that a more equitable approach would be to draw a new random order for each school:

Here are some of the emails we got on the subject:

- "I believe that the equitable approach is for a child to have a new chance... This might result in both students getting their second choices, the fact is that each child had a chance. If we use only one random number, and I had the bad luck to be the last student in line this would be repeated 12 times and I never get a chance. I do not know how we could explain that to a student and parent."

- "When I answered questions about this at training sessions, (It did come up!) people reacted that the only fair approach was to do multiple runs."

Lottery, cont.

- Ran simulations. These simulations showed that the efficiency loss due to multiple draws was considerable; and increases with correlation in students' preferences.
- We pushed hard on this one, but it looked like the decision was going to go against us. But we did get the NYC DOE to agree to run the algorithm both ways and compare the results on the submitted preference lists.
- They agreed, and eventually decided on a **single rank order** after seeing welfare gains on the submitted preferences

Tie-breaking in Student-Proposing Deferred Acceptance in the First Round 2003-04

Choice	Number Ranking	Single Tie-Breaking (250 draws)	Multiple Tie-Breaking (250 draws)
1	5,797 (6.7%)	21,038 (24.82%)	19,783 (23.34%)
2	4,315 (5.0%)	10,686 (12.61%)	10,831 (12.78%)
3	5,643 (6.6%)	8,031 (9.48%)	8,525 (10.06%)
4	6,158 (7.2%)	6,238 (7.36%)	6,633 (7.83%)
5	6,354 (7.4%)	4,857 (5.73%)	5,108 (6.03%)
6	6,068 (7.1%)	3,586 (4.23%)	3,861 (4.56%)
7	5,215 (6.1%)	2,721 (3.21%)	2,935 (3.46%)
8	4,971 (5.8%)	2,030 (2.40%)	2,141 (2.53%)
9	4,505 (5.2%)	1,550 (1.83%)	1,617 (1.91%)
10	5,736 (6.7%)	1,232 (1.45%)	1,253 (1.48%)
11	9,048 (10.5%)	1,016 (1.20%)	894 (1.05%)
12	22,239 (25.8%)	810 (0.96%)	372 (0.44%)
unassigned	-	20,952 (24.72%)	20,795 (24.54%)

No stochastic dominance

First Year of Operation

- Over 70,000 students were matched to one of their choice schools
  - an increase of more than 20,000 students compared to the previous year match
- An additional 7,600 students matched to a school of their choice in the third round
- 3,000 students did not receive any school they chose
  - 30,000 did not receive a choice school in the previous year

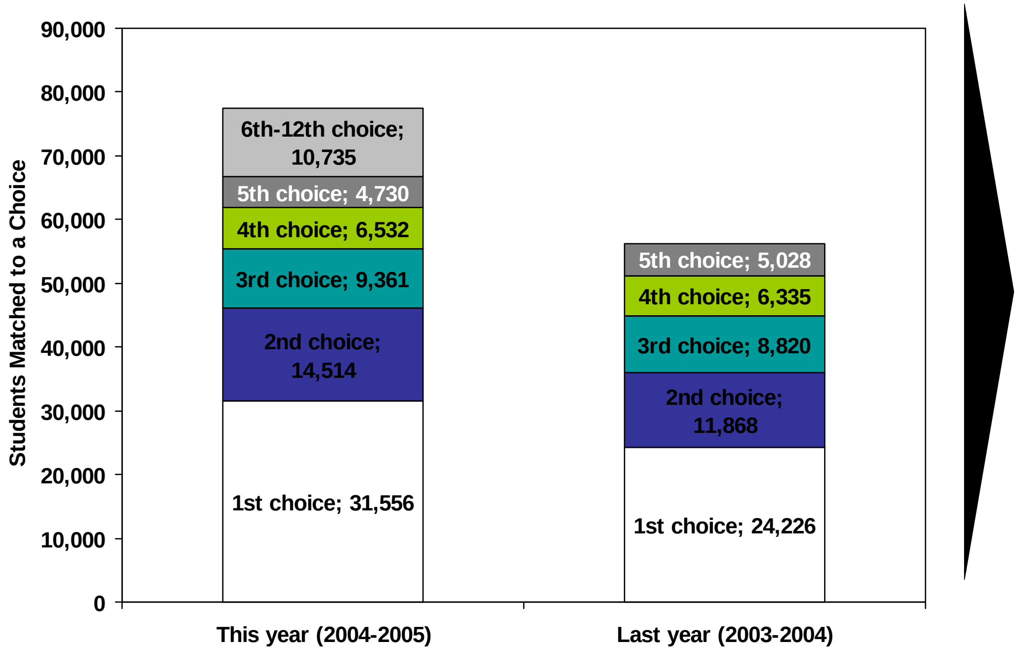
First year, cont

Much of the success is due to

- relieving congestion
  - Allowing many offers and acceptances to be made, instead of only 3
  - giving each student a single offer rather than multiple offers to some students
- allowing students to rank 12 instead of 5 choices
- But more than that is going on…

First year results:
More students get top choices
(this is a chart prepared by NYCDOE, comparing academic years 04-05 and 03-04)

Number of students matched at the end of Round II



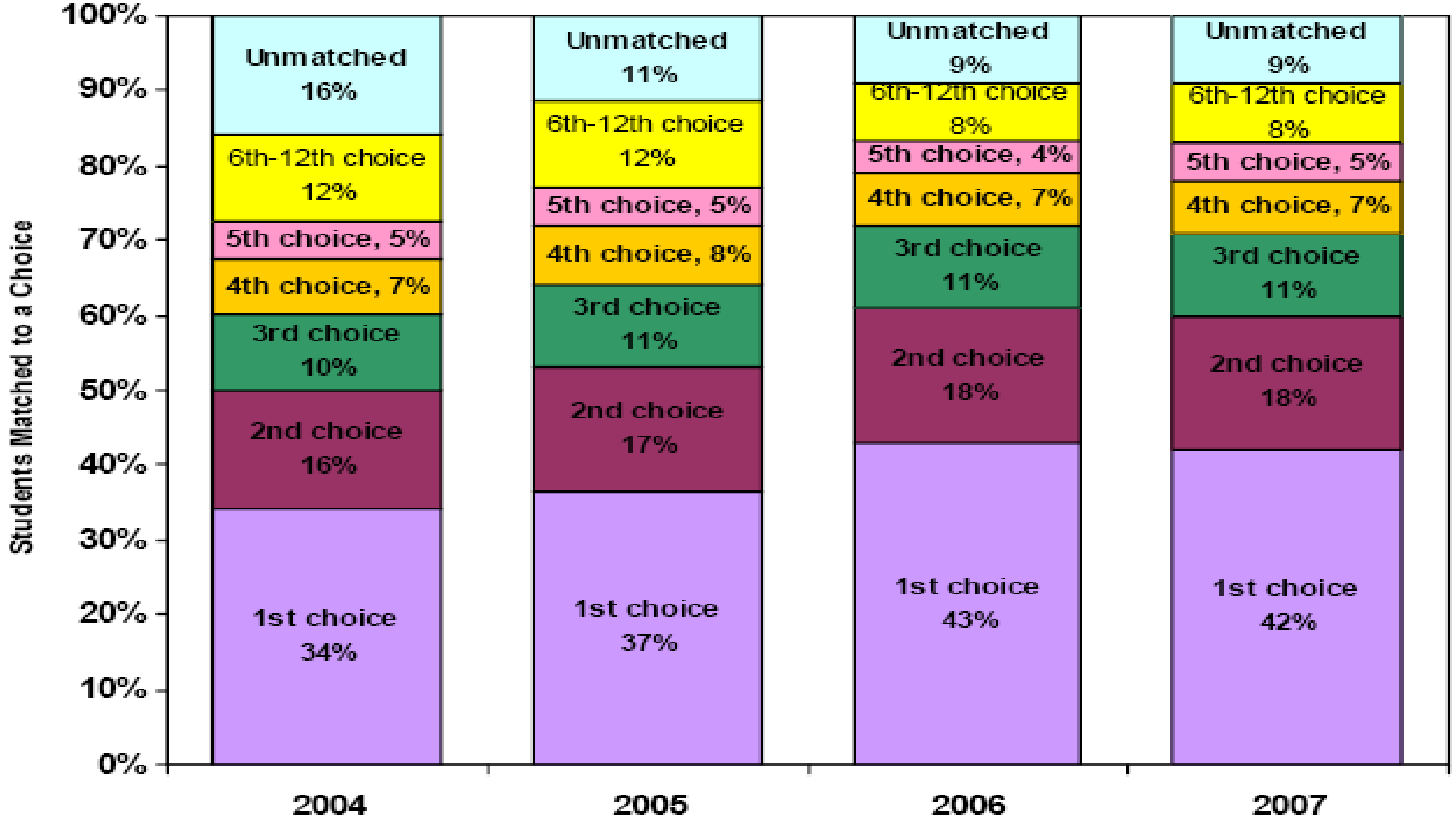
- **21,000** more students matched to a school of their choice
- **7,000** more students receiving their first choice
- **10,000** more students receiving one of their top 5 choices

The results show continued improvement from year to year

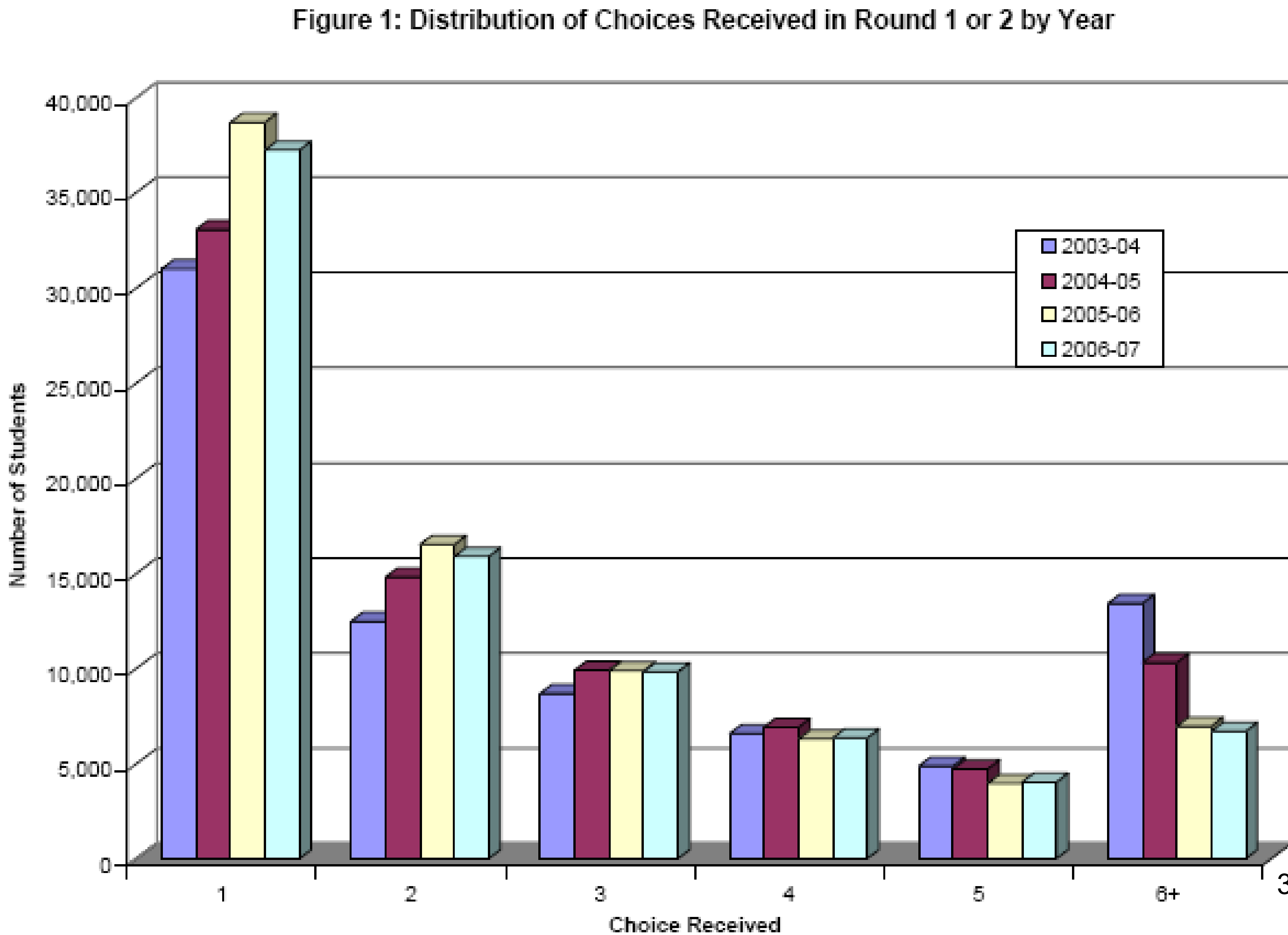
- Even though no further changes have been made in the algorithm…

First 4 years: March 23, 2007
Results at end of Round 2

(Schools have learned to change their reporting of capacities)



What happened in NYC after the algorithm was introduced in 2003-04?



What is going on?

- It appears that schools are no longer withholding capacity.
- Some high schools (even top high schools like Townsend Harris) have learned to rank substantially more than their capacity, because many of their admitted students go elsewhere (e.g. admissions to Townsend Harris provides good leverage for bargaining over financial aid with private schools).
- This allows more students to be accepted to their top choice, second choice, etc. during the formal match process.

Immediate Issue: Appeals

- Just over 5,100 students appealed in the first year
- Around 2,600 appeals were granted
- About 300 of the appeals were from students who received their first choice
- Designing an efficient appeals process—top trading cycles?
  - A dry run in year 2 showed that many students could be granted appeals without modifying school capacities.
    - One 40-student cycle…
- In 2006-08 TTC was used
  - One 26 student cycle

NYC--summary

- **Waiting lists** are a congested allocation mechanism—congestion leads to instabilities and strategic play.
- NYC high schools—only recently re-centralized—are active players in the system.
- Information about the mechanism is part of the mechanism.
  - Information dissemination within and about the mechanism is part of the design
- New mechanisms can have both immediate and gradual effects.
- Appeals may be a big deal
  - when the preferences are those of 13 and 14 year olds
  - When a nontrivial percentage of assigned places aren't taken up because of withdrawals from the public school system (moves, and private schools)
- Open question:
  - How best to design appeals, in light of changing preferences of 13 year olds, mobile school population, but to continue to give good incentives in the main match?

Changing the **Boston** school match: A system with incentive problems (Abdulkadiroglu, Pathak, Roth and Sonmez)

- Students have priorities at schools set by central school system
- Students entering grades K, 6, and 9 submit (strict) preferences over schools.
- In priority order, everyone who can be assigned to his first choice is. Then 2nd choices, etc.
  - Priorities: sibling, walk zone, random tie-breaker
  - There are lots of people in each priority class (non-strict preferences)
- Unlike the case of NYC, in Boston, there weren't apparent problems with the system.

Incentives

- First choices are important: if you don't get your first choice, you might drop far down list (and your priority status may be lost: all 2nd choices are lower priority than all 1st...).
- Gaming of preferences?—the vast majority are assigned to their first choice
- Chen and Sonmez (2005): experimental evidence on preference manipulation under Boston mechanism (see also Featherstone and Niederle 2008)

Advice from the West Zone Parent's Group: Introductory meeting minutes, 10/27/03

"One school choice strategy is to find a school you like that is undersubscribed and put it as a top choice, OR, find a school that you like that is popular and put it as a first choice and find a school that is less popular for a ''safe'' second choice."

Formalizing what the WZPG knows

- **Definition**: A school is overdemanded if the number of students who rank that school as their first choice is greater than the number of seats at the school.
- **Proposition**: No one who lists an overdemanded school as a second choice will be assigned to it by the Boston mechanism, and listing an overdemanded school as a second choice can only reduce the probability of receiving schools ranked lower.

But not everyone knows

- Of the 15,135 students on whom we concentrate our analysis, 19% (2910) listed two overdemanded schools as their top two choices, and about 27% (782) of these ended up unassigned.

Costs of incentive problems

- Many preferences are “gamed,” and hence we don’t have the information needed to produce efficient allocations (and don’t know how many are really getting their first choice, etc.)
  - There are real costs to strategic behavior borne by parents—e.g. West Zone Parents group
  - BPS can’t do effective planning for changes.
- Those who don’t play strategically get hurt.

Design issues for Boston Schools

- Is the market one-sided or two?
  - Unlike NYC, no gaming by schools (Boston school system has been centralized for a long time)
  - Are priorities intended to facilitate parent choice, or do they represent something important to the school system?
  - If one sided, “stable” matches wouldn’t be Pareto optimal: e.g. it would be Pareto improving to allow students to trade priorities—top trading cycles.
    - Other Pareto improvements may be possible (Kesten).
    - “Pareto” optimality involves decisions about who are the players…

Recommendations for BPS

- Switch to a strategy-proof mechanism.
- *We suggested two* choices:
  - Student Proposing Deferred Acceptance Algorithm (as in NYC)
    - Would produce “stable” assignments—no student is not assigned to a school he/she prefers unless that school is full to capacity with higher priority students
  - Top Trading Cycles
    - Would produce a Pareto efficient match.

Student Proposing Deferred Acceptance

- **Stable**: no student who loses a seat to a lower priority student and receives a less-preferred assignment
- **Incentives**: makes truthful representation a dominant strategy for each student
- **Efficiency**: selects the stable matching that is preferred to any other stable matching by all students—no “justified envy” (when preferences are strict)

Top Trading Cycles (TTC)

- If welfare considerations apply only to students, tension between stability and Pareto efficiency
- Might be possible to assign students to schools they prefer by allowing them to trade their priority at one school with a student who has priority at a school they prefer
- Students trade their priorities via **Top Trading Cycles** algorithm
- Theorems:
  - makes truthful representation a dominant strategy for each student
  - Pareto efficient

A too simple 1-sided model: House allocation

- Shapley & Scarf [1974] housing market model: n agents each endowed with an indivisible good, a “house”.
- Each agent has preferences over all the houses and there is no money, trade is feasible only in houses.
- Gale’s top trading cycles (TTC) algorithm: Each agent points to her most preferred house (and each house points to its owner). There is at least one cycle in the resulting directed graph (a cycle may consist of an agent pointing to her own house.) In each such cycle, the corresponding trades are carried out and these agents are removed from the market together with their assignments.
- The process continues (with each agent pointing to her most preferred house that remains on the market) until no agents and houses remain.

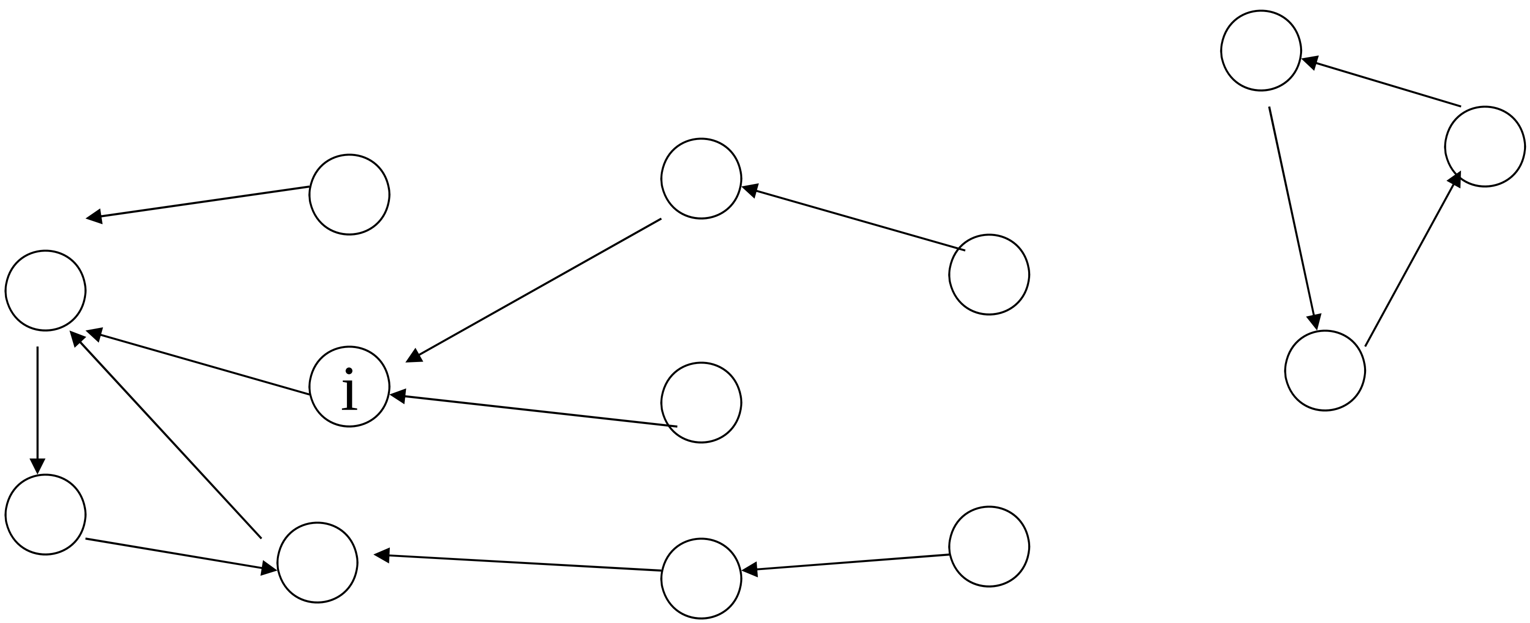
Theorem (Shapley and Scarf): the allocation x produced by the top trading cycle algorithm is in the core (no set of agents can all do better than to participate)

- When preferences are strict, Gale's TTC algorithm yields the *unique* allocation in the core (Roth and Postlewaite 1977).

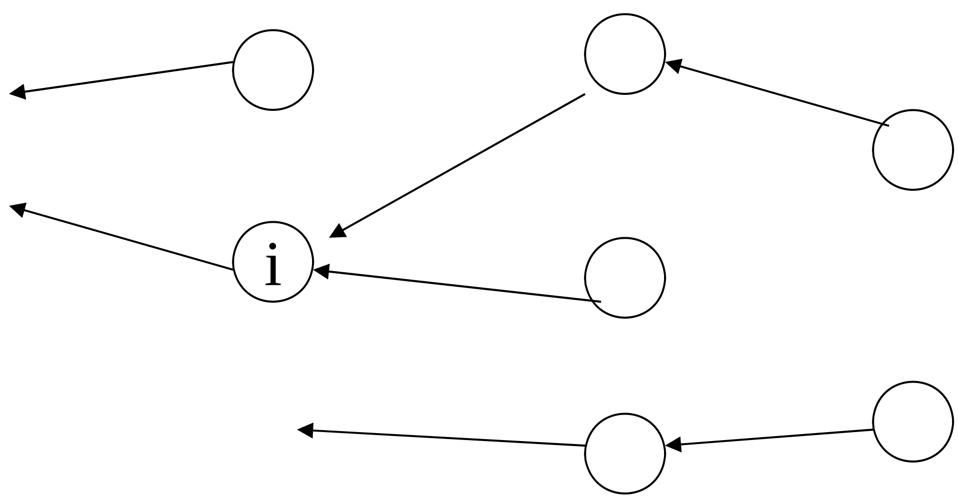
Theorem (Roth ’82): if the top trading cycle procedure is used, it is a dominant strategy for every agent to state his true preferences.

- The *idea* of the proof is simple, but it takes some work to make precise.
- When the preferences of the players are given by the vector P, let $N_t(P)$ be the set of players still in the market at stage t of the top trading cycle procedure.
- A *chain* in a set N_t is a list of agents/houses a_1, a_2, …a_k such that a_i’s first choice in the set N_t is a_{i+1}. (A cycle is a chain such that $a_k=a_1$.)
- At any stage t, the graph of people pointing to their first choice consists of cycles and chains (with the ‘head’ of every chain pointing to a cycle…).

Cycles and chains



The cycles leave the system (regardless of where i points), but i's choice set (the chains pointing to i) remains, and can only grow



Top Trading Cycles

- Step 1: Assign counters for each school to track how many seats remain available. Each student points to her favorite school and each school points to the student with the highest priority. There must be at least one cycle. (A cycle is an ordered list of distinct schools and students (student 1 - school 1 - student 2 - ... - student k - school k) with student 1 pointing to school 1, school 1 to student 2, ..., student k to school k, and school k pointing to student 1.) Each student is part of at most one cycle. Every student in a cycle is assigned a seat at the school she points to and is removed. The counter of each school is reduced by one and if it reaches zero, the school is removed.

- Step k: Each remaining student points to her favorite school among the remaining schools and each remaining school points to the student with highest priority among the remaining students. There is at least one cycle. Every student in a cycle is assigned a seat at the school she points to and is removed. The counter of each school in a cycle is reduced by one and if it reaches zero, the school is removed.

- The procedure terminates when each student is assigned a seat (or all submitted choices are considered).

The choice? Boston School Committee

- "Would anyone mind if two students who each preferred the schools in the other student's walk zone were to trade their priorities and enroll in those schools?"
- YES: transportation costs, externalities when parents walk child to school, lawsuits when a child is excluded from a school while another with lower priority is admitted
  - DAA
- NO: efficiency of allocation is paramount
  - TTC

Explaining and defending

- In the final weeks before a decision was made, our BPS colleagues told us that their main concern was their ability to explain and defend the choice of (which) new algorithm to the public and to Boston politicians.
- We came up with some simpler descriptions of TTC in this process
  - Lines in front of schools in priority order

Explaining and Defending: DA "FAQ"

Q: Why didn't my child get assigned to his first choice, school X?
A: School X was filled with students who applied to it and who had a higher priority.

Q: Why did my child, who ranked school X first, not get assigned there, when some other child who ranked school X second did?

A: The other child had a higher priority at school X than your child did, and school X became that other child's first choice when the school that he preferred became full. (Remember that this assignment procedure allows all children to rank schools in their true order of preference, without risk that this will give them a worse assignment than they might otherwise get.)

TTC "FAQ"

Q: Why didn't my child get assigned to his first choice, school X?
A: School X was filled before your child's priority (to be admitted to school X or to trade with someone who had priority at school X) was reached.

Q: Why did a child with lower priority at school X than my child get admitted to school X when my child did not?

A: Your child was not admitted to school X because there were more children with higher priority than yours than the school could accommodate. One of these children traded his priority with the child who had lower priority at school X.

The recommendation to the School Committee: School Superintendent Payzant Memorandum on 5/25/05 states:

"The most compelling argument for moving to a new algorithm is to enable families to list their true choices of schools without jeopardizing their chances of being assigned to any school by doing so."

"The system will be more fair since those who cannot strategize will not be penalized."

Fairness rationale for strategy-proof mechanisms

Further benefits of a strategy proof mechanism

"A resulting benefit for the system is that this alternative algorithm would provide the district with more credible data about school choices, or parent "demand" for particular schools. Using the current assignment algorithm, we cannot make assumptions about where families truly wish to enroll based on the choices they make, knowing many of those choices are strategic rather than reflective of actual preference."

BPS's Recommendation: Deferred Acceptance

- The Gale-Shapley *Deferred Acceptance Algorithm* will best serve Boston families, as a centralized procedure by which seats are assigned to students based on both student preferences and their sibling, walk zone and random number priorities.
- Students will receive their highest choice among their school choices for which they have high enough priority to be assigned. The final assignment has the property that a student is not assigned to a school that he would prefer **only** if every student who is assigned to that school has a higher priority at that school.
- Regardless of what other students do, this assignment procedure allows all students to rank schools in their true order of preference, without risk that this will give them a worse assignment than they might otherwise get.

Why not top trading cycles?

"Another algorithm we have considered, Top Trading Cycles, presents the opportunity for the priority for one student at a given school to be "traded" for the priority of a student at another school, assuming each student has listed the other's school as a higher choice than the one to which he/she would have been assigned. **There may be advantages to this approach, particularly if two lesser choices can be "traded" for two higher choices. It may be argued, however, that certain priorities -- e.g., sibling priority -- apply only to students for particular schools and should not be traded away.**

Moreover, Top Trading Cycles is less transparent-- and therefore more difficult to explain to parents -- because of the trading feature executed by the algorithm, which may perpetuate the need or perceived need to "game the system."

The Vote

- The Boston School Committee decided to adopt a deferred acceptance algorithm
- It was implemented for use starting January 2006, for assignment of students to schools in September, 2006.

Boston: summary remarks

- Transparency is a virtue in a mechanism
  - Both when it is used and for it to be adopted
  - New mechanisms have to be explained and defended
- Strategy proofness can be understood in terms of fairness/equal access
- Efficient allocation based on personal preferences requires the preferences to be known

Atila Abdulkadiroglu, Atila, Parag A. Pathak, Alvin E. Roth, and Tayfun Sonmez, "Changing the Boston School Choice Mechanism: Strategy-proofness as Equal Access" working paper, May 2006.

New questions raised by school choice

- How to do tie breaking?
- Tradeoffs between Pareto optimality, stability, strategy proofness—what are the ‘costs’ of each?
- Evaluating welfare from different points in time
- Restricted domains of preferences?

Recent developments

- IIPSC—the Institute for Innovation in Public School Choice (run by Neil Dorosin, former Director of HS Operations for NYCDOE)
- We have developed new school choice systems in Denver and New Orleans
  - New Orleans uses a version of top trading cycles…

Matching with indifferences

- When we were mostly using matching models to think about labor markets, strict preferences didn't seem like too costly an assumption
  - Strict preferences might be generic
- But that isn't the case with school choice
  - We already saw that one of the first NYC design decisions we faced in 2003 was how to randomize to break ties.

New Theoretical Issues

- Erdil, Aytek and Haluk Ergin, What's the Matter with Tie-breaking? Improving Efficiency in School Choice , American Economic Review , 98(3), June 2008, 669-689
- Abdulkadiroglu, Atila , Parag A. Pathak , and Alvin E. Roth, " Strategy-proofness versus Efficiency in Matching with Indifferences: Redesigning the NYC High School Match ," American Economic Review, 99(5) December 2009, 1954-1978.
- Featherstone, Clayton and Muriel Niederle, “EX ANTE EFFICIENCY IN SCHOOL CHOICE MECHANISMS: AN EXPERIMENTAL

Other new issues we won't get to today…

- Pathak, Parag and Tayfun Sönmez "Leveling the Playing Field: Sincere and Strategic Players in the Boston Mechanism" , *American Economic Review,* 98(4), 1636-52, 2008
- Ergin, Haluk and Tayfun Sonmez, Games of School Choice under the Boston Mechanism " , *Journal of Public Economics* , 90: 215-237, January 2006.
- Kesten, Onur On Two Kinds of Manipulation for School Choice Problems March, 2011, forthcoming in *Economic Theory*.
- Kesten, Onur, "School Choice with Consent," *Quarterly Journal of Economics* 125(3), August, 2010; 1297-1348.
- Abdulkadiroglu, Atila, Yeon-Koo Che, and Yosuke Yasuda, " Resolving Conflicting Preferences in School Choice: The "Boston Mechanism" Reconsidered" American Economic Review, February, 2011, 101(1): 399–410.

Matching with indifferences

I: a finite set of students (individuals) with (strict) preferences P_i over school places.

S: a finite set of schools with responsive weak preferences/priorities R_s over students (i.e. can include indifferences: P_s ($\succ_s$) is the asymmetric part of R_s).

As before:

$q = (q_s)_{s \epsilon S}$: a vector of quotas ($q_s \geq 1$, integer).

A matching is a correspondence μ: I U S → S U I satisfying:

(i) For all i є I : μ(i) є S U {i}

(ii) For all s є S : $|\mu(s)| \leq q_s$, and $i \in \mu(s)$ implies μ(i) = s.

We'll mostly concentrate on student welfare and student strategy, and regard R_S as fixed.

Matchings and student welfare

A matching μ is ***individually rational*** if it matches every x ∈ I ∪ S with agent(s) that is(are) acceptable for x.

A matching μ is ***blocked*** by (i, s) if $sP_i\mu(i)$, and either [$|\mu(s)| < q_s$ and $i \succ_s s$] or [$i \succ_s i'$ for some $i' \in \mu(s)$]. μ is stable if μ is individually rational and not blocked by any student-school pair (i, s).

A matching μ ***dominates*** matching if $\mu(i)R_i(i)$ for all i ∈ I, and $\mu(i)P_i(i)$ for some i ∈ I. (Weak Pareto domination for students.)

A stable matching μ is a ***student-optimal stable matching*** if it is not dominated by any other stable matching.

"A" not "the": When school preferences aren't strict, there won't generally be a unique optimal stable match for each side, rather there will be a non-empty set of stable matches that are weakly Pareto optimal for agents on

Example: multiple optimal stable matchings

Example 1. (Tie-breaking does not always yield student-optimal stable matchings.) Tie-breaking has important welfare consequences. Suppose that school s_1 is indifferent among students, students i_1, i_2, i_3 and schools s_2 and s_3 have the following strict preferences:

Student Preferences	School Preferences
$s_2 P_{i_1} s_1 P_{i_1} s_3$	$i_1 \sim_{s_1} i_2 \sim_{s_1} i_3$
$s_1 P_{i_2} s_2 P_{i_2} s_3$	$i_2 \succ_{s_2} i_1 \succ_{s_2} i_3$
$s_1 P_{i_3} s_2 P_{i_3} s_3$	$i_3 \succ_{s_3} i_1 \succ_{s_3} i_2$

The stable matchings are the following:

$$\mu_1 = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_2 & s_3 \end{pmatrix}, \ \mu_2 = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_2 & s_1 & s_3 \end{pmatrix}, \ \mu_3 = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_3 & s_2 & s_1 \end{pmatrix}.$$

Note that μ_1, μ_2 and μ_3 are produced by the student proposing deferred acceptance algorithm (DA) when the indifference in s_1's preferences is broken as $i_1 \succ_{s_1} i_3 \succ_{s_1} i_2$, $i_2 \succ_{s_1} i_x \succ_{s_1} i_y$ and $i_3 \succ_{s_1} i_x \succ_{s_1} i_y$, respectively. However, μ_2 dominates μ_1 despite μ_1 being stable. That is, DA need not produce a student-optimal stable matching even if ties at schools are broken the same way.

Weak Pareto optimality generalizes…

- Proposition 1. If μ is a student-optimal stable matching, there is no individually rational matching ν (stable or not) such that $\nu(i) P_i \mu(i)$ for all $i \in I$.

- (terminology: a student optimal stable matching is *weakly* Pareto optimal because it can't be *strictly* Pareto dominated, but the outcome of student proposing deferred acceptance algorithm might not be *strongly* Pareto optimal, i.e. might not be student optimal, because it can be *weakly* Pareto dominated)

Tie breaking

- A tie-breaker is a bijection $r: I \to N$, that breaks ties at school s by associating R_s with a strict preference relation P_s :

$$iP_s j \Leftrightarrow [(i \succ_s j) \text{ or } (i \sim_s j \text{ and } r(i) < r(j))].$$

Basic Deferred Acceptance (Gale and Shapley 1962)

- Step 0: arbitrarily break all ties in preferences
- Step 1: Each student "proposes" to her first choice. Each school tentatively assigns its seats to its proposers one at a time in their priority order. Any remaining proposers are rejected.

  …
- Step k: Each student who was rejected in the previous step proposes to her next choice if one remains. Each school considers the students it has been holding together with its new proposers and tentatively assigns its seats to these students one at a time in priority order. Any remaining proposers are rejected.
- The algorithm terminates when no student proposal is rejected, and each student is assigned her final tentative assignment.

Deferred acceptance algorithm with tie breaking: DA^{τ}

- A single tie breaking rule uses the same tie-breaker $r_s = r$ at each school, while a multiple tie breaking rule may use a different tie breaker r_s at each school s.
- For a particular set of tie breakers $\tau=(r_s)_{s \in S}$, let the mechanism DA^{τ} be the student-proposing deferred acceptance algorithm acting on the preferences (P_I,P_S), where P_s is obtained from R_s by breaking ties using r_s, for each school s.

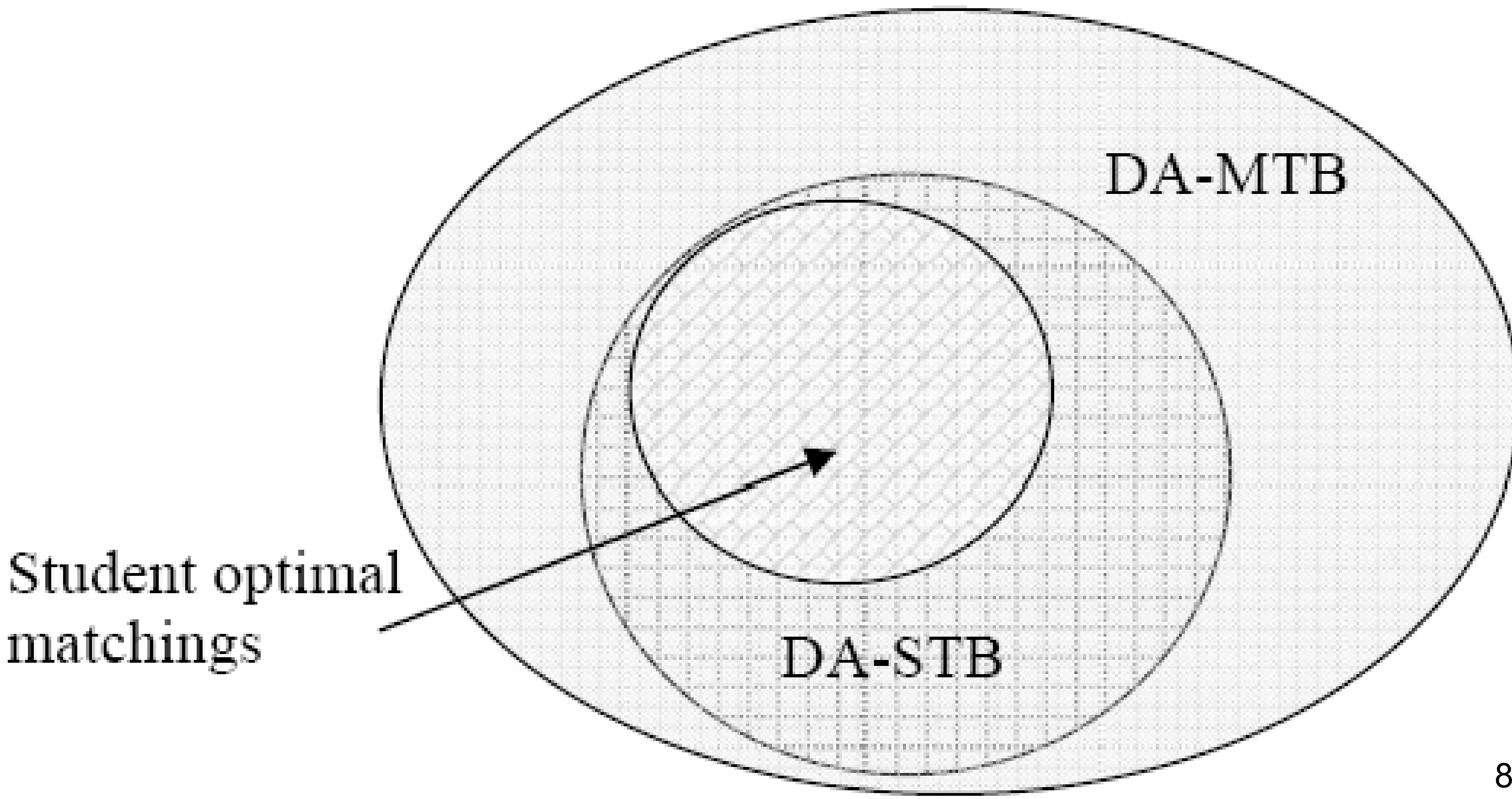
Single and Multiple tie breaking

- The dominant strategy incentive compatibility of the student-proposing deferred acceptance mechanism for every student implies that DA^τ is strategy-proof for any τ.
- But the outcome of DA^τ may not be a student optimal stable matching.
  - We already saw this is true even for single tie breaking.

Single versus multiple tie breaking
NYC Grade 8 applicants in 2006-07
(250 random draws: simulation standard errors in parentheses)

Choice	Deferred Acceptance Single Tie-Breaking DA-STB (1)	Deferred Acceptance Multiple Tie-Breaking DA-MTB (2)
1	32,105.3 (62.2)	29,849.9 (67.7)
2	14,296.0 (53.2)	14,562.3 (59.0)
3	9,279.4 (47.4)	9,859.7 (52.5)
4	6,112.8 (43.5)	6,653.3 (47.5)
5	3,988.2 (34.4)	4,386.8 (39.4)
6	2,628.8 (29.6)	2,910.1 (33.5)
7	1,732.7 (26.0)	1,919.1 (28.0)
8	1,099.1 (23.3)	1,212.2 (26.8)
9	761.9 (17.8)	817.1 (21.7)
10	526.4 (15.4)	548.4 (19.4)
11	348.0 (13.2)	353.2 (12.8)
12	236.0 (10.9)	229.3 (10.5)
unassigned	5,613.4 (26.5)	5,426.7 (21.4)

Proposition: For any (P_I,R_S), any matching that can be produced by deferred acceptance with multiple tie breaking, but not by deferred acceptance with single tie breaking is not a student-optimal stable matching.



Dominating stable matchings

- **Lemma**: Suppose μ is a stable matching, and ν is some matching (stable or not) that dominates μ. Then the same set of students are matched in both ν and μ

Proof

- If there exists a student who is assigned under μ and unassigned under v, then $v(i)=iP_i\mu(i)$, which implies that μ is not individually rational, a contradiction. So every i assigned under μ is also assigned under v.
- Therefore $|v(S)|\geq|\mu(S)|$. If $|v(S)|>|\mu(S)|$ then there exists some $s\in S$ and $i\in I$ such that $|v(s)|>|\mu(s)|$ and $v(i)=s\neq\mu(i)$. This implies there is a vacancy at s under μ and i is acceptable for s. Furthermore, $sP_i\mu(i)$ since v dominates μ. These together imply that μ is not stable, a contradiction. So $|v(S)|=|\mu(S)|$.
- Then the same set of students are matched in

Stable Improvement Cycles (Erdil and Ergin, 08)

Fix a stable matching μ w.r.t. given preferences P and priorities R.

Student i ***desires*** s if $sP_i\mu(i)$.

Let B_s = the set of highest R_s-priority students among those who desire school s.

Definition: A ***stable improvement cycle C*** consists of distinct students $i_1, \ldots, i_n = i_0$ ($n \geq 2$) such that

(i) $\mu(i_k) \in S$ (each student in the cycle is assigned to a school),

(ii) i_k desires $\mu(i_{k+1})$, and

(iii) $i_k \in B\mu(i_{k+1})$, for any) $k = 0, \ldots, n-1$.

Given a stable improvement cycle define a new matching μ' by:

$\mu'(j) = \mu(j)$ if j is not one of $\{i_1, \ldots, i_n\}$

$\mu'(j) = \mu(i_{k+1})$ if $j = i_k$

Proposition: μ' is stable and it (weakly) Pareto dominates μ.

Improving on DA^T

- **Theorem** (Erdil and Ergin, 2008): Fix P and R, and let μ be a stable matching. If μ is Pareto dominated by another stable matching , then μ admits a stable improvement cycle.

- **Algorithm** for finding a student optimal matching: start with a stable matching. Find and implement a stable improvement cycle, as long as one exists.

Outline of proof

Fix P and R. Suppose μ is a stable matching Pareto dominated by another stable matching ν.

Simplifying assumption: Each school has one seat.

1. I' := $\{i \in I \mid \nu(i) P_i \mu(i)\} = \{i \in I \mid \nu(i) \neq \mu(i)\}$.
2. All students in I' are matched to a school at ν.
3. S' := ν(I')=μ(I').

Hence, I [S] can be partitioned into two subsets I' and I\I' [S' and S \ S'] such that

- Those in I \ I' [S \ S'] have the same match under μ and ν.
- The matches of those in I' [S'] have been "shuffled" among themselves to obtain ν from μ.

4. For all s є S':

I'$_s$:= (i є I'|i desires s at μ, and no j є I' desires s at μ and j P_s i) is nonempty;.

5. Construct a directed graph on S':

- For each s є S', arbitrarily choose and fix i_s є I'$_s$.
- i_s є B_s: i.e., i_s desires s at μ, and there is no j є I who desires s at μ and j P_s i. (from stability of ν)
- For all s, t є S', let t →s if t = μ(i_s).

6. The directed graph has a cycle of n ≥ 2 distinct schools: $s_1 \rightarrow s_2 \rightarrow \cdot \cdot \rightarrow s_n \rightarrow s_1$

7. The students $i_{s1}, i_{s2}, \ldots, i_{sn}$ constitute a stable improvement cycle at μ

How much room is there to improve on deferred acceptance?

- Are there costs to Pareto improvements in welfare?

Strategy-proof mechanisms

A ***direct mechanism*** φ is a function that maps every (P_I ,R_S) to a matching.

For $x \in I \cup S$, let $\varphi_x(P_I ;R_S)$ denote the set of agents that are matched to x by φ.

A mechanism φ is ***dominant strategy incentive compatible*** (DSIC) for $i \in I$ if for every (P_I ,R_S) and every P'_i ,

$$\varphi_i(P_I ;R_S) R_i\, \varphi_i(P'_i , P_{-i};R_S).$$

A mechanism will be called strategy-proof if it is DSIC for all **students**.

Pareto improvement and strategy proofness

Fix R_S.

We say that a mechanism φ ***dominates*** ψ if

for all P_I : $\varphi_i(P_I ;R_S)R_i\ \psi_i(P_I ;R_S)$ for all $i \in I$, and

for some P_I : $\varphi_i(P_I ;R_S)P_i\ \psi_i(P_I ;R_S)$ for some $i \in I$.

Theorem (Abdulkadiroglu, Pathak, Roth): For any tie breaking rule τ, there is no

Proof

- Suppose that there exists a strategy-proof mechanism ϕ and tie-breaking rule τ such that ϕ dominates DA^{τ}. There exists a profile P_I such that

$$\phi_i(P_I;R_S)R_i\ DA^{\tau}(P_I;R_S) \text{ for all } i\in I, \text{ and}$$

$$\phi_i(P_I;R_S)P_i\ DA^{\tau}(P_I;R_S) \text{ for some } i\in I.$$

Let $s_i=DA_i^{\tau}(P_I;R_S)$ and $s'_i=\phi i(P_I;R_S)$ be i's assignment under $DA^{\tau}(P_I;R_S)$ and $\phi(P_I;R_S)$, respectively, where $s'_iP_is_i$.

…continued

- Consider profile $P_I' = (P_i', P_{-i})$, where P_i' ranks s'_i as the only acceptable school. Since DA^T is strategy-proof, $s_i = DA_i^T(P_I; R_S) R_i DA_i^T(P_I'; R_S)$, and since $DA_i^T(P_I'; R_S)$ is either s'_i or i, we conclude that $DA_i^T(P_I'; R_S) = i$. Then the Lemma implies $\phi_i(P_I'; R_S) = i$.
- Now let $(P_I'; R_S)$ be the actual preferences. In this case, i could state P_i and be matched to $\phi_i(P_I; R_S) = s'_i$, which under P_i' she prefers to $\phi(P_I'; R_S) = i$.
- So ϕ is not strategy-proof.

Let’s look at some data

- We can’t tell what preferences would have been submitted with a different (non strategy-proof) mechanism, but we can ask, given the preferences that were submitted, how big an apparent welfare loss there might be due to not producing a student optimal stable matching.

Inefficiency in the NYC match (cost of strategy-proofness)

Table 1— Tie-breaking for Grade 8 Applicants in NYC in 2006-07

Choice	Deferred Acceptance Single Tie-Breaking DA-STB (1)	Deferred Acceptance Multiple Tie-Breaking DA-MTB (2)	Student-Optimal Stable Matching (3)	Improvement from DA-STB to Student-Optimal	Number of Students (4)
1	32,105.3 (62.2)	29,849.9 (67.7)	32,701.5 (58.4)	+1	633.2 (32.1)
2	14,296.0 (53.2)	14,562.3 (59.0)	14,382.6 (50.9)	+2	338.6 (22.0)
3	9,279.4 (47.4)	9,859.7 (52.5)	9,208.6 (46.0)	+3	198.3 (15.5)
4	6,112.8 (43.5)	6,653.3 (47.5)	5,999.8 (41.4)	+4	125.6 (11.0)
5	3,988.2 (34.4)	4,386.8 (39.4)	3,883.4 (33.8)	+5	79.4 (8.9)
6	2,628.8 (29.6)	2,910.1 (33.5)	2,519.5 (28.4)	+6	51.7 (6.9)
7	1,732.7 (26.0)	1,919.1 (28.0)	1,654.6 (24.1)	+7	26.9 (5.1)
8	1,099.1 (23.3)	1,212.2 (26.8)	1,034.8 (22.1)	+8	17.0 (4.1)
9	761.9 (17.8)	817.1 (21.7)	716.7 (17.4)	+9	10.2 (3.1)
10	526.4 (15.4)	548.4 (19.4)	485.6 (15.1)	+10	4.7 (2.0)
11	348.0 (13.2)	353.2 (12.8)	316.3 (12.3)	+11	2.0 (1.1)
12	236.0 (10.9)	229.3 (10.5)	211.2 (10.4)		
unassigned	5,613.4 (26.5)	5,426.7 (21.4)	5,613.4 (26.5)	Total:	1,487.5

Cost of stability in NYC

Table 2— Welfare Consequences of Stability for Grade 8 Applicants in in 2006-07

Choice	Student-Optimal Stable Matching (1)	Efficient Matching (2)	Improvement from Student-Optimal Stable Matching	Number (3)	k	Count of Students with k Blocking Pairs (4)
1	32,701.5 (58.4)	34,707.8 (50.5)	+1	1,819.7 (41.3)	1	22,287.5 (205.1)
2	14,382.6 (50.9)	14,511.4 (51.1)	+2	1,012.8 (26.4)	2	6,707.8 (117.9)
3	9,208.6 (46.0)	8,894.4 (41.2)	+3	592.0 (19.5)	3	2,991.0 (79.6)
4	5,999.8 (41.4)	5,582.1 (40.3)	+4	369.6 (16.0)	4	1,485.4 (56.5)
5	3,883.4 (33.8)	3,492.7 (31.4)	+5	212.5 (12.0)	5	716.6 (32.5)
6	2,519.5 (28.4)	2,222.9 (24.3)	+6	132.1 (9.1)	6	364.6 (22.9)
7	1,654.6 (24.1)	1,430.3 (22.4)	+7	77.0 (7.1)	7	183.1 (13.6)
8	1,034.8 (22.1)	860.5 (20.0)	+8	43.0 (5.6)	8	85.6 (10.9)
9	716.7 (17.4)	592.6 (16.0)	+9	26.3 (4.5)	9	44.7 (6.4)
10	485.6 (15.1)	395.6 (13.7)	+10	11.6 (2.8)	10	22.6 (4.9)
11	316.3 (12.3)	255.0 (10.8)	+11	4.8 (2.0)	11	9.9 (3.0)
12	211.2 (10.4)	169.2 (9.3)			12	3.2 (1.6)
unassigned	5,613.4 (26.5)	5,613.4 (26.5)	Total:	4,296.6		34,898.8

Comparison with Boston

Table 3— Tie-breaking for Elementary School Applicants in Boston in 2006-07

Choice	Deferred Acceptance Single Tie-Breaking DA-STB (1)	Deferred Acceptance Multiple Tie-Breaking DA-MTB (2)	Student-Optimal Stable Matching (3)	Improvement from DA-STB to Student-Optimal	Number of Students (4)
1	2,251.8 (8.4)	2,157.3 (13.4)	2,256.6 (8.2)	+1	4.6 (2.6)
2	309.8 (10.3)	355.5 (12.0)	307.4 (10.0)	+2	1.2 (1.1)
3	154.9 (7.9)	189.3 (10.1)	154.0 (7.7)	+3	0.5 (0.7)
4	59.7 (5.5)	76.1 (7.0)	58.7 (5.5)	+4	0.3 (0.5)
5	27.4 (4.5)	34.1 (4.8)	27.0 (4.4)	+5	0.0 (0.1)
6	4.9 (1.9)	6.0 (2.5)	4.9 (1.9)	+6	0.0 (0.1)
7	2.6 (1.4)	2.8 (1.6)	2.5 (1.4)	+7	0.0 (0.1)
8	1.9 (1.2)	0.9 (0.9)	1.9 (1.2)	+8	0.0 (0.1)
9	1.2 (1.1)	0.4 (0.6)	1.2 (1.0)	+9	0.0 (0.0)
10	0.3 (0.6)	0.1 (0.2)	0.3 (0.5)		
unassigned	112.4 (4.6)	104.6 (4.5)	112.4 (4.6)	Total:	6.5

Open questions

- (Equilibrium) misrepresentation in stable improvement cycles? (Can potential gains be realized?)
  - It appears there will be an incentive to raise popular schools in your preferences, since they become tradeable endowments…
- Restricted domains of preference?
  - Manipulation will be easier on some domains than others, and potential welfare gains greater on some domains than others.

CAN WE MAKE SCHOOL CHOICE MORE EFFICIENT? AN EXAMPLE EDUARDO M. AZEVEDO AND JACOB D. LESHNO (2011)

School Priorities		Student Preferences			
Arts	*Science*	a	z	b_1	b_2
a	b_1, b_2	*Science*	*Science*	*Arts*	*Arts*
b_1, b_2, z	a, z	*Arts*	ϕ	ϕ	ϕ
		ϕ			

School S has two seats, and school A has a single seat ($q_A = 1$, $q_S = 2$). We assume that all agents are expected utility maximizers. We normalize the utility functions by setting the utility of an assignment to the most preferred school to be 1, and the utility from being unmatched to 0. We assume that $u_{b_i}(Science) > -\frac{1}{2}$.

The DA-STB assignment depends on the tie breaking only to determine which b_i student gets rejected from 1. The resulting (random) assignment is:

DA-STB assignment:
(truthful)

$$\begin{array}{ll} a \rightarrow & S \\ z \rightarrow & S \\ b_i \rightarrow & \frac{1}{2}A,\ \frac{1}{2}\phi \end{array}$$

This assignment is Pareto efficient, and truth-telling is an equilibrium under DA-STB.

Consider the *equilibrium* of (any) SOSM in which everyone reports truthfully except the two bi who both (mis)report A>S>ϕ (notice that S is popular and the bi's have priority there…)

- The outcome of the DA-STB for this profile is:
  - a: ½ S, ½ A
  - z: ϕ
  - bi: ¼ A, ¾ S
- SOSM: stable improvement cycles would allow a to trade A for S with a bi
  - a: S,
  - z: ϕ
  - bi: ½ A, ½ S
- None of the students do better under this equilibrium, and some do strictly worse.

AZEVEDO AND LESHNO

Proposition 3.1. *Consider any mechanism that is Pareto efficient with respect to reported preferences, and Pareto dominates DA-STB. In the economy above, this mechanism has a unique equilibrium assignment, which is Pareto dominated by the DA-STB assignment, and is unstable with respect to the true preferences.*

Ex post versus ex ante evaluation?

- E.g. Boston mechanism in uncorrelated environment, where you don't have to pay the cost for lack of strategy proofness…Featherstone and Niederle 2008

- Recall that DA is strategy-proof (DSIC) while the Boston mechanism is not.
- (The following slides are adapted from F&N's)

Example; correlated preferences (likely the general case…)

- 3 schools are commonly ranked by students as follows.

School	Best	Second	Third	No school
Seats	2	1	1	–
Payoff	100	67	25	0

- Two types of students: Top and Average.
  - Top always has priority over Average.
  - Within group, ties are broken by a lottery.
- 3 Tops and 2 Averages
- DA Outcome:

  Top: 2 get Best, 1 gets Second

  Average: 1 gets Third, 1 is unassigned
- What is the equilibrium under Boston?

Boston mechanism in the correlated environment—complex eq. strategies

School	Best	Second	Third	No school
Seats	2	1	1	–
Payoff	100	67	25	0

Boston:

- Strategies:

  Top: (Best, Third, . . .) (skipping the middle)
  Average: (Second, ...) (skipping the top)

- Outcome:

  Top: 2 get Best, 1 gets Third
  Average: 1 gets Second, 1 is unassigned

Uncorrelated preferences: (a conceptually illuminating simple environment)

- 2 schools, one for Art, one for Science, each with one seat
- 3 students, each iid a Scientist with p=1/2 and Artist with p=1/2. Artists prefer the art school, scientists the science school.
- The (single) tie breaking lottery is equiprobable over all orderings of the three students.

Consider a student after he knows his own type, and before he knows the types of the others. Then (because the environment is uncorrelated) his type gives him no information about the popularity of each school. So, **under the Boston mechanism**, **truthtelling is an equilibrium**. (Note that for some utilities this wouldn't be true e.g. of the school-proposing DA, even in this environment.)

Boston can stochastically dominate DA in an uncorrelated environment

Example: 3 students, 2 schools each with one seat

- DA:

Lottery rank	First choice	Second choice	No school
1	1	0	0
2 s	$1/2$ 1:a	$1/2$ 1:s	0
3	0	0	1
Average	$1/2$	$1/6$	$1/3$

- Boston:

Lottery rank	First choice	Second choice	No school
1	1	0	0
2 s	$1/2$ 1:a	$1/4$ 1:s; 3:s	$1/4$ 1:s; 3:a
3	$1/4$	0	$3/4$
Average	$1/2 + 1/12$	$1/6 - 1/12$	$1/3$

Things to note

- The uncorrelated environment let's us look at Boston and DA in a way that we aren't likely to see them in naturally occurring school choice.
- In this environment, there's no incentive not to state preferences truthfully in the Boston mechanism, even though it isn't a dominant strategy. (So on this restricted domain, there's no corresponding benefit to compensate for the cost of strategyproofness.)
- Boston stochastically dominates DA, even though it doesn't dominate it ex-post (ex post the two mechanisms just redistribute who is unassigned)

Recap: New questions raised by school choice

- How to do tie breaking?
- Tradeoffs between Pareto optimality, stability, strategy proofness—what are the ‘costs’ of each?
- Evaluating welfare from different points in time
- Restricted domains of preferences?