Judgment Error in Lottery Play: When the Hot-Hand Meets the Gambler's Fallacy*

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Abstract

We demonstrate that lottery markets can exhibit the "hot-hand" phenomenon, in which past winning numbers tend to have a greater share of the betting proportion in future draws, even though past and future events are independent. This result is surprising, since Clotfelter and Cook (1993) and Terrell (1994) have instead documented the presence of an opposite effect, the "gambler's fallacy" in the US lottery market. The current literature also suggests that the gambler's fallacy prevails when random numbers are generated by mechanical devices, such as in lottery games (e.g., Ayton and Fisher (2004), Burns and Corpus (2004), Caruso et al. (2010)). We use two datasets from lotteries to show that both the gambler's fallacy and the hot-hand fallacy can exist in different types of lottery games. We then run experimental studies that mimic lottery games with one, two, or three winning numbers. Our results show that the number of winning prizes impacts behavior. In particular, whereas a single-prize game leads to a strong presence of the gambler's fallacy, we observe a significant increase in hot-hand behavior in multiple-prize games with two or three winning numbers.

Keywords: Perception of Randomness, Hot-Hand Fallacy, Gambler's Fallacy, Lottery Game

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1 Introduction

People are known to rely on heuristics or simple mental models to interpret random events (Tversky and Kahneman, 1974, Oskarsson et al., 2009). Although such heuristics ensure fast decisions that generate reasonably good outcomes under most circumstances, they can trigger systematic deviations from certain notions of rationality. Among others, the gambler's and hot-hand fallacies are cognitive biases that have been commonly observed in the perception of randomness and they lead humans to misjudge probabilities.

The gambler's fallacy is an erroneous belief in negative correlations of independent outcomes generated by a random process. Accordingly, when tossing a fair coin, people erroneously believe that tail is due after a sequence of heads. The gambler's fallacy has long been observed in the field, particularly in lottery games (e.g., Clotfelter and Cook (1993), Terrell (1994), Sundali and Croson (2006)). The hot-hand fallacy—the counterpart to the gambler's fallacy—is the fallacious belief that a person who has experienced a success in one random event is more likely to succeed in subsequent attempts. This fallacy was first described by Gilovich et al. (1985) in basketball games, in which basketball players and fans believe that a player who has made a successful streak of shoots will continue to score as if having "hot-hands." Later, Camerer (1989) showed that betting markets for basketball games exhibit a small hot-hand bias. Although the two fallacies seem to contradict each other, the current literature shows that they may coexist in the same individual and/or appear within the same setting. For example, Croson and Sundali (2005) and Sundali and Croson (2006) found that casino players exhibited the gambler's fallacy when they bet on numbers, but the hot-hand fallacy regarding their own luck of themselves. Guryan and Kearney (2008) examined the sales data for lottery outlets that had sold winning jackpot tickets, and showed that those stores experienced a significant increase in game-specific ticket sales, exhibiting a "lucky store" effect.

The current literature posits two main explanations for the coexistence of the two seemingly opposite phenomena. One line of research suggests that the two phenomena both arise from the representativeness heuristics or belief in the law of small numbers (e.g., Tversky and Kahneman (1971), Rabin (2002), Asparouhova et al. (2009), Rabin and Vayanos (2010)). These works identify the importance of the length of streaks in influencing the perception of randomness: The gambler's fallacy prevails in short streaks, but as the streaks lengthen, beliefs in the hot-hand fallacy dominate. In the other line of research, observations that the gambler's fallacy is more prevalent in lottery games, and the hot-hand fallacy more prevalent in games that require skill, are often attributed to the source of the random number generator: We tend to observe the gambler's fallacy if the random generator is a mechanical device and the hot-hand fallacy if it is a human being (e.g., Ayton and Fisher (2004), Burns and Corpus (2004),Caruso et al. (2010)).

This paper provides empirical evidence using two sets of naturally occurring data to demonstrate that both the gambler's and hot-hand fallacies can appear in lottery games. We collected field data from two fixed-odds lottery games. The first is a 3-digit (3D) lottery game played daily in China, the scheme of which resembles the Pick 3 lottery game reported by Clotfelter and Cook (1993); the other game is a 4-digit (4D) lottery game played in Southeast Asia that picks 23 winning numbers at a time, three times per week. Consistent with the current literature, we observe the gambler's fallacy in the 3D lottery game. Surprisingly, however, the 4D game exhibits a hot-hand fallacy, for which the current literature cannot provide an explanation. First, long streaks are unlikely to be observed in lottery games, and therefore, theories building on streaks in sequences of random events cannot be applied directly to these environments. Second, because winning numbers for both the 3D and the 4D games are machine-generated, theories that focus on the source of randomness would predict the gambler's fallacy in both games.

Our paper instead proposes a novel explanation for the field observations and reveals the role of the lottery game design in shaping the perception of randomness. In particular, we show that the number of winning prizes has an impact on the occurrence of the two fallacies: Single-prize games can lead to the gambler's fallacy while multiple-prize games may result in the hot-hand fallacy under appropriate conditions. In addition to the number of winning prizes, the two lottery games differ in other aspects, such as frequency of draw and sample population. We, therefore, test our conjecture in an experimental study, which allows us to isolate the effect of the number of prizes and to control for all other differences.

In the experiment, participants are asked to guess the outcome of simulated lottery games that only differ with respect to the number of prizes. This design contributes to experimental literature on the gambler's fallacy, which mainly focuses on binary choice sets and single prize. We expand the choice set beyond the binary case and examine how behavior changes when multiple prizes are present. Our main results show that whereas the gambler's fallacy still prevails in the single-prize game, the fallacy immediately disappears when the number of prizes increases to two or three. At the individual level, we observe more hot-hand behavior with multiple prizes than with a single prize. We infer from our observations that game design, in the form of the number of prizes drawn, can systematically influence a player's betting behavior.

We further analyze the data to unravel this puzzle: Why do multiple-prize games lead to more hot-hand behavior? We observe a strong gambler's fallacy in the first few rounds across all experimental conditions, which suggests that participants start out with a predisposition for the gambler's fallacy. This observation is consistent with cognitive evidence on the prevalence of the gambler's fallacy with binary choice sets (see Rabin 2002, Section 2, for a detailed literature review). We also observe strong evidence that participants underestimate the probability of repetition in winning numbers in multiple-prize games, in which the correct answer for the true probability of repetition is less obvious than in single-prize games. This form of misjudgment in probability is also well documented in Tversky (1974). We argue that when participants are eventually confronted with unexpected repetitions, they abandon the initial belief of negative autocorrelation (gambler's fallacy), and some may tend to adopt a new belief of positive autocorrelation (hot-hand fallacy), which is more consistent with their subjective game experience. This interpretation is compatible with our observation that the gambler's fallacy quickly disappears after playing several rounds of multiple-prize games, whereas it persists in the single-prize game.

This interpretation can also explain the differences in behavior for our field data between 3D and 4D lottery games. The probabilities are calculated for two different games, one with one in 1,000 numbers winning (3D numbers game), and another with 23 in 10,000 numbers winning (4D numbers game). The second game, with 23 winners, clearly has higher chance of seeing a winning number again in the next draw. If the players observe that a winning number itself (or some related numbers) has a higher chance of appearing again, how will that affect their choice of lottery numbers to bet on? The empirical data suggest that the gambler's fallacy prevails in the first case (1 prize in 1,000 numbers), whereas the hot-hand prevails in the latter (23 prizes in 10,000 numbers).

In summary, we provide empirical and experimental evidence that behavior can be altered through a careful lottery-system design. These results have important practical implications for risk management and decision making under uncertainty, because they suggest that the perception

	1,000 numbers Pick 1 winning number	10,000 numbers Pick 23 winning numbers
Probability of winning	$\frac{1}{1000} = 0.001$	$\frac{23}{10000} = 0.0023$
Probability of some winning number being drawn in the next draw	$\frac{1}{1000}$ $= 0.001$	$1 - (1 - \frac{23}{10000})^{23}$ ≈ 0.05

Table 1: The probability of winning and seeing at least one repetition in subsequent draws in 3D and 4D

of randomness can be manipulated, and hence behavior can be nudged with the appropriate design. For instance, several countries have attempted to influence commuters' behavior by offering incentive schemes in which commuters earn credit for each journey taken (with triple credit for off-peak journeys) and to enter in weekly cash lotteries. The success of these schemes hinges on an insight from behavioral economics: On average, people are risk-seeking when the stakes are small. Hence, a 1-in-1000 chance to win $100 is more attractive than a cash award of $0.10. For such low probability events, our results suggest that we can do better if we can induce the belief that certain choices made by players are more likely to win. This is done through designing lottery games to allow the hot-hand fallacy to dominate. Increasing the number of prizes is enough to make what is actually a 1-in-1,000 chance of winning appear to be a much more attractive bet to commuters.

2 Literature Review

Both the gambler's and hot-hand fallacies have long been observed in both laboratory experiments and the field. Among many others, Clotfelter and Cook (1993) observed that lottery players in 3D numbers games in the United States are subject to the gambler's fallacy. Figure 1 shows the percentiles of betting ratios (betting volume index over average index on a particular day) on different days after the numbers have been drawn as winners. Once a 3D number is drawn as the winner, subsequent betting ratios drop immediately and then gradually pick up. The immediate drop after the winning number is drawn provides strong evidence of the gambler's fallacy at work. In a different context, Camerer (1989) showed that betting markets for basketball games exhibit a small hot-hand bias. Through examining a set of panel data, Green and Zwiebel (2017) found strong hot-hand fallacy among baseball players in Major League Baseball. Suetens et al. (2016) found that people in Lotto games usually avoid numbers that have recently been drawn, exhibiting the gambler's fallacy, but tend to bet more on winning numbers in streaks (that have been drawn several times in a row), suggesting the presence of the hot-hand fallacy.

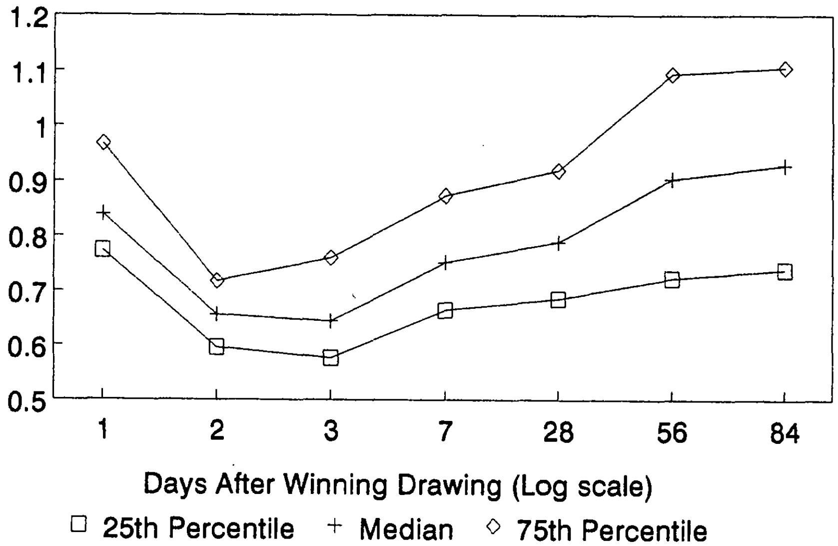

Figure 1: Betting Ratios on Previous Winning Numbers in 3-Digit Numbers Games (Clotfelter and Cook, 1993)

The gambler's and hot-hand fallacies may coexist in the same individual and appear within the same setting. One line of research posits that they both arise from representativeness bias or belief in the law of small numbers. Tversky and Kahneman (1971) coined this term and examined its connection with the gambler's fallacy. Rabin (2002) and Rabin and Vayanos (2010) built theoretical models to model the law of small numbers that directly leads to the gambler's fallacy. Asparouhova et al. (2009) ran a set of lab experiments on binary choice games, and the results support the work of Rabin (2002). Kendall (2010) used a discount factor to refine the model presented in Rabin (2002) and proposed a simple but effective model to explain the two fallacies.

These studies built the decision biases (gambler's fallacy) directly into their model and showed that other decision biases may emerge as the streaks of winning lengthen. However, as streaks are unlikely in lottery games, these models cannot be directly applied in these environments. Our paper instead examines the role of lottery game design in shaping the perception of randomness. We show that due to game design, either the gambler's or the hot-hand fallacy can arise under appropriate conditions, and we can thus manipulate the behaviors through careful system design.

Another stream of research argues that the type of random-event generator—whether an inanimate device or a human being—determines the occurrence of the gambler's or hot-hand fallacy (Ayton and Fisher, 2004, Burns and Corpus, 2004). If the random process is believed to be generated by a mechanical device, people expect negative recency; if human beings generate the sequences, people expect positive recency. For example, the experimental study of Caruso et al. (2010) demonstrated that human subjects tend to predict the continuation of a streak of outcomes when the agents that generate the streak are perceived to be intentional. In contrast, when the streak is generated by some kind of mechanical device (as in lottery games), people believe that the streak will end; that is, the gambler's fallacy dominates. These results are reinforced by field observations made in casinos and lottery stores (Sundali and Croson, 2006, Guryan and Kearney, 2008). Green et al. (2010) developed several ingenious arguments to demonstrate that experimental cues can alter how subjects learn about the generating process (using optimal Bayesian updating with erroneous beliefs) and thus substantially affect subjects' choices, without altering the underlying outcome probabilities. Our work suggests that these explanations are incomplete; our empirical evidence shows that either the gambler's or the hot-hand fallacy can occur even if the random process is generated by mechanical devices.

In addition, more evidence on cognitive biases also suggests that the gambler's fallacy occurs in situations in which information is experienced sequentially over time (Baron and Leider (2010)). For instance, Militana et al. (2010) found that the bias was stronger when research subjects had longer time intervals between choices. On the other hand, the gambler's fallacy decreased when people's inaccurate assessments of the probability of an outcome imposed costs (Terrell (1994)).

Navarrete and Santamari (2012) showed, in two lab experiments, that the incidence of the gambler's fallacy was reduced when the size of the *choice set* was increased. Our paper has a different focus: We investigate how the number of prizes—the size of the *outcome set*—influences player's betting behavior while fixing the size of choice set. Our results, consequently, are different: We observe more hot-hand behavior in multiple-prize games compared to single-prize games.

3 Field Evidence

In this section, we use field data from two fixed-odds lottery games to demonstrate that both the gambler's fallacy and the hot-hand fallacy can exist in lottery games. The first is a 3-digit (3D) lottery game played in China, the scheme of which resembles the "Pick 3" lottery game reported in Clotfelter and Cook (1993). It draws a single prize-winning number each day, seven times per week. The other game, a 4-digit (4D) lottery game played in Southeast Asia, picks 23 winning numbers at a time, three times per week. In what follows, we describe the two games in detail and provide evidence that the gambler's fallacy prevails in the 3D game, while the hot-hand fallacy is the dominant behavior in the 4D game.

3.1 3D Numbers Game

In the 3D numbers game, a single winning number from 000 to 999 is randomly drawn with equal probability at 8:30 p.m. each day. Lottery tickets for the day's draw are sold on the same day until 8:00 p.m. Each wager costs 2 RMB. There are three types of bets: Straight, Box 3, and Box 6. In the Straight bet, players bet on a specific 3D number and receive 1,000 RMB per wager if the number wins. The Box 3 bet allows players to make one wager on the permutation of any 3D number with two identical digits, and in the Box 6 bet, on the permutation of any 3D number with three unique digits. For example, a player who places a 2 RMB wager on 223 and chooses the Box 3 bet type actually bets 2/3 RMB on each of the 3 numbers 223, 322, and 232. The payout per wager[1] and odds of winning are shown in Table 2.

Table 2: Prize Structure and Winning Odds in 3D Numbers Game

Bet Type	Match to Win	Payout per Wager	Odds
Straight	Match the exact order	1,000	1 in 1,000
Box 3	Match any order (2 identical digits)	320	1 in 333
Box 6	Match any order (3 unique digits)	160	1 in 167

Testing for the presence of the gambler's fallacy usually requires a full dataset that includes

[1] There are minor payout variations by areas: In Beijing, the game pays 980 per wager for the Straight bet, and in Shanghai and Tibet, the payout is 333 for the Box 3 bet and 166 for the Box 6 bet.

the sales of every number in every draw. Such a dataset is not publicly available for the lottery of interest. Nevertheless, we were able to obtain a rich enough dataset from the official website of the game operator[2], which consists of (i) the winning number drawn; (ii) the number of winning wagers (of the three types), and (iii) total sales for each draw. We extract this information on 4,466 draws on a daily basis over a 150-month period, from May 8, 2005, to October 30, 2017.

In this dataset, 10 numbers have never been drawn as prize winners, 59 numbers appeared only once, and 931 numbers have been drawn at least twice. We define our dependent variable, "payout rate", as the amount paid to bettors relative to total sales for a given draw. The higher the payout rate of a winning number the more popular this number was among bettors relative to the market size of the draw. The overall mean payout rate is 49.2% and the median is 42.7%. The lowest payout rate of 7.3% occurred on September 18, 2010, when "945" was drawn as the winning number; the highest payout rate was 530.3%, when "353 " was drawn as the winning number on July 18, 2005. A Welch's two sample *t-test* does not detect any significant difference in mean payout rates between numbers that have been drawn only once (with mean $m = 0.526$) and numbers that have been drawn at least twice ($m = 0.488$, $p\text{-}value = 0.329$).

We seek to estimate the change in subsequent sales of a 3D number after it has been drawn as the winning number. As we only have sales information for winning numbers, we thus construct a subsample of 3,476 draws, in which the winning numbers have been drawn previously (repeated) at least once within the period the full dataset covers. To measure the influence of past wins on future ticket sales, we define the "Delay" of a winning number as the number of draws that elapsed since last time it was drawn. We then bin the draws according to the Delay with intervals of a week. Each bin contains around 30 data points (draws). Figure 2a shows the 25th percentile, median, and 75th percentile of the payout rates in each bin for the first eight weeks. The first data point represents the same statistics for the full dataset. We observe a clear pattern of declining payout rates with a one-week Delay, which then eventually bounce back to their initial levels. Bettors display a tendency to avoid past winning numbers with short delays, exhibiting strong evidence for the gambler's fallacy. A more detailed account on the data underlying Figure 2a is provided in Table 8 in Appendix A.

Our latter interpretation of the data is corroborated by a series of paired *t-tests*. For each number,

[2] `http://www.zhcw.com/3d/kaijiangshuju/index.shtml`.

we calculate its average payout rate for all draws that repeat within x weeks (early repetition) and that repeat in more than x weeks (late repetition). We compare average payout rates between "early" and "late" repetitions at the intra-number level. We then test the null hypothesis of equal payout rates between early and late repetitions for varying values of x, with $x = 3, 5, 7, 9, 11$. All tests unanimously show that the mean payout rate for early repetitions is significantly lower than that for late repetitions, and it is worth noting that all *p-values* are in the order of magnitude of $e - 05$ (see detailed results in Table 9 in Appendix A). However, one drawback of our approach is that we must exclude numbers that lack either early or late repetition. As a next step, we will therefore demonstrate the existence of the gambler's fallacy in 3D games via rigorous regression analysis that takes into account a broader set of winning numbers.

Figure 2: Betting on Previous Winning Numbers in 3D and 4D

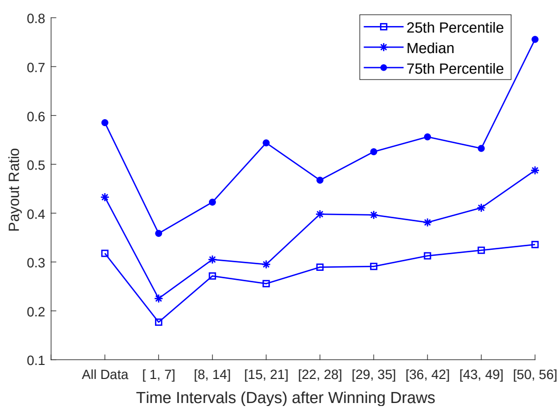

Figure 2a: 3D Game

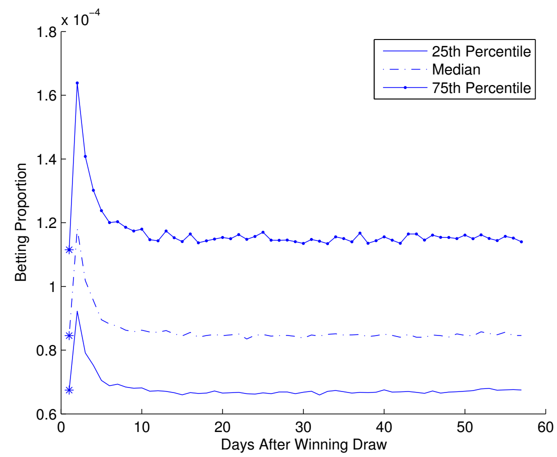

Figure 2b: 4D Game

For the regression analysis, we consider again all numbers that were drawn at least twice, this time including the first draw so that we can estimate the payout rates at 0 for the horizontal axis. We use the demeaned payout rate denoted as DR as the dependent variable. To obtain demeaned payout rates, we first calculate the average payout rate for each number in all draws of the sample, then subtract this number-specific average from the observed payout rate of the winning number in each draw. Demeaning mimics a number fixed-effects approach, and allows us to control for number-specific effects (e.g., superstitions, lucky numbers, etc.) [3]. For each number in the sample, we calculate the average payout ratio of this number and subtract this number-specific average from the actual observed payout ratios for this number. By construction, the mean of the demeaned payout ratio is zero. We use the Delay (denoted as T) as the independent variable. We characterize the relationship between DR and $log_{10}(T+1)$ as a higher-degree polynomial. Polynomials are functionally flexible, and their curvatures can capture the type of effects we were expecting to observe in the data: a diminishing effect over time of past wins on current payout rates. We use an ordinary least squares approach with robust standard errors to estimate the effect of $log_{10}(T+1)$ on DR. We start with the quadratic function and add successively higher-degree polynomial terms, as long as they turn out to be significant. Our final model for the 3D game is a 6th degree polynomial, and takes the following form:

$$DR_i = \beta_0 + \sum_{k=1}^{6} \beta_k * (\log_{10}(T_i + 1))^k + \alpha_i, \tag{1}$$

in which DR_i is the demeaned payout ratio of the $i-th$ draw in the subsample; β_0 is the fixed intercept, β_k is the coefficient of the k^{th} order polynomial function, and $\alpha_i N(0, \sigma_\alpha^2)$ represents unobserved random effects. Estimated model coefficients are shown in Table 3. All estimated parameters are significant with small *p-values.*

To visualize our estimation results, we plot the predicted values and their 95% confidence interval in Figure 3a. We observe a strong gambler's fallacy effect, in which the demeaned payout rate significantly drops after a winning draw, gradually picks up, and goes back to its intial level after around 60 draws. At a delay of one day, payout rates are reduced by almost 30 percentage points. Even at a delay of 10 days, payout rates are down by 10 percentage points.

3.2 4D Numbers Game

In the 4D lottery game, players bet on numbers 0000 to 9,999. Different from the 3D game, in which sales only occur on the draw day, 4D sales for each draw start one week before and close at 6 p.m. on the draw day. There are three draws every week, on Wednesday, Saturday, and Sunday.

[3]Since winning numbers are drawn identically and independently, demeaning and a fixed-effects approach should theoretically yield the same estimated effect of past wins on future betting behavior. We decided not to follow the fixed effects approach, as this would have required us to estimate an additional 990 parameters in a model with 4,407 total observations.

Table 3: OLS Estimated Regression Coefficients on Payout Ratio in 3D & 4D Numbers Games. Standard Errors are Robust and Provided in Parentheses. Significance Coding: *** - 1%, ** - 5%, * - 10%.

	3D	4D
$\log_{10}(T)$	-1.817***	0.233***
	(0.375)	(0.078)
$\big(\log_{10}(T)\big)^2$	3.683***	-0.286***
	(0.954)	(0.100)
$\big(\log_{10}(T)\big)^3$	-2.969***	0.086***
	(0.942)	(0.032)
$\big(\log_{10}(T)\big)^4$	1.181***	
	(0.446)	
$\big(\log_{10}(T)\big)^5$	-0.231**	
	(0.101)	
$\big(\log_{10}(T)\big)^6$	0.018**	
	(0.009)	
Intercept	0.014*	-0.005
	(0.009)	(0.006)
Observations	4,407	1,103
F-value	7.03	3.06
degrees of freedom F-value	(6, 4400)	(3, 1099)
Robust std. err.	Y	Y

Figure 3: Estimated Average Payout Rate and 95% CI of Previous Winning Numbers in 3D and 4D after a Winning Draw

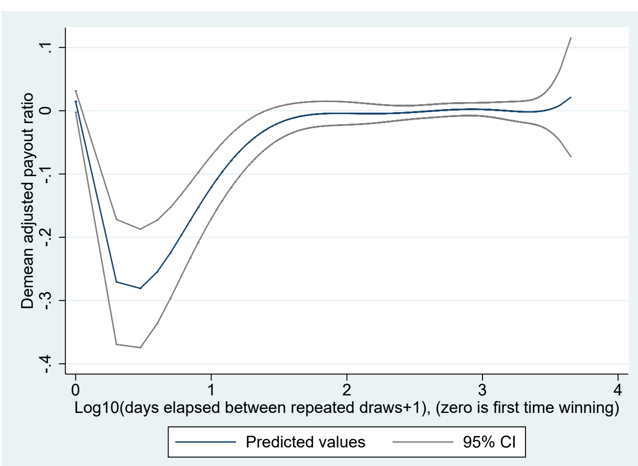

Figure 3a: 3D Game

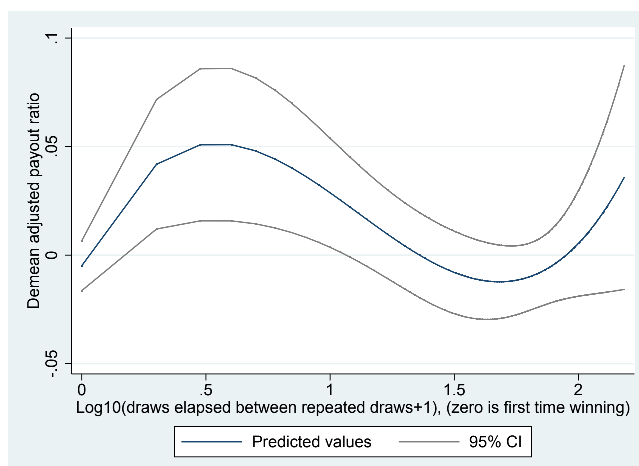

Figure 3b: 4D Game

The minimum cost of a bet is 1 local currency. At each draw, 23 numbers are picked as winning numbers. The 23 winning numbers are generated with replacement by rolling four boxes such that each contains 10 balls. There are 5 prize categories. Table 4 shows the payout for each prize category and winning odds.

Table 4: Prize Table and Winning Odds in 4D Numbers Games

Prize Category	Payout per Wager	Odds
1st Prize	2,000	1 in 10,000
2nd Prize	1,000	1 in 10,000
3rd Prize	500	1 in 10,000
10 Starter	250	1 in 1,000
10 Consolation	60	1 in 1,000

We investigate the effect of a winning draw on subsequent betting behavior for the winning numbers. We aim to demonstrate that a simple change in the game design from drawing one winning ball in 3D to drawing 23 winning balls in 4D substantially impacts betting behavior. While we have established the gambler's fallacy as the prevalent behavior in the 3D game, we will establish that hot-hand behavior is prevalent in the 4D game.

We obtained data from a game operator in the Southeast Asia region, with information on 156 draws over a 1-year period. The dataset consists of sales volume for each of the 10,000 4D numbers and the 23 winning numbers in each draw. Due to the presence of different prize categories in the 4D numbers game, we use the *betting proportion*, defined as the sales volume on a number divided by total sales in a particular draw, as our measure for the popularity of a number.[4] We examine all winning numbers from draw 1 to draw 100 and calculate their betting proportions respectively, on the day they are drawn and the subsequent 56 draws. Sales increase by about 40% immediately following a winning draw (cf. Figure 2b), suggesting that the hot-hand fallacy prevails at the aggregate level. Next, we will demonstrate the existence of the hot-hand fallacy in 4D via

[4]The change in the dependent variable from payout rates in 3D to betting proportions in 4D is innocuous. The payout rate in the 3D is a simple linear function of betting proportions. All of our 3D results hold true if we replace payout rates with betting proportions.

rigorous regression analysis.

To facilitate a fair comparison with the 3D game, we also construct a subsample of 1,103 draws, consisting of winning numbers that have been drawn at least twice in our dataset. We denote DR as the demeaned value of the betting proportion and T as the *number of draws* elapsed since this winning number last won. We characterize the relationship between DR and $log_{10}(T+1)$ as a higher-degree polynomial. The final relationship after adding significant higher-order polynomial terms is a 3rd-order polynomial and takes the following form:

$$DR_i = \beta_0 + \sum_{k=1}^{3} \beta_k * (\log_{10}(T_i + 1))^k + \alpha_i, \quad (2)$$

Corresponding regression results are reported in Table 3. All of the coefficients of the polynomials are significant, and the signs are opposite to those of corresponding results in 3D. To visualize our estimated effects, we plot the predicted values and their 95% confidence interval in Figure 3b. We observe an increase in the demeaned betting proportion for short Delays, which is a clear manifestation of bettors actively seeking past winning numbers. This effect gradually decreases as the Delay becomes longer, with betting proportions reaching their initial levels at around 25 winning draws. This suggests the existence of the hot-hand fallacy in the 4D numbers games.

In summary, we observe that lottery players' betting behaviors are significantly affected by the gambler's fallacy in the 3D game and the hot-hand fallacy in the 4D game. We conjecture that a plausible explanation is that the game design—in particular, the size of outcome sets—leads to the different observations in the two fixed-odds lottery games. In the 4D game there are 23 winning numbers in each draw, and therefore repetition in winning numbers across draws occur more frequently and similarities across draws can be easily observed. This fosters the bettors' belief that the same or similar numbers have a higher chance of being drawn again, leading them to seek past winning numbers. On the other hand, consecutive wins or extremely short delays in repetitions are rather rare events in the 3D game. From the bettors' perspective, it is therefore the norm not to observe recurring past winning numbers. This, in turn, can lead bettors to falsely infer that past wins have a lower chance of being drawn again, which motivates them to avoid past winning numbers.

Nevertheless, we cannot rule out other factors that might be the driving force behind our observations. For example, cultural differences might exist between the two populations. The two games also differ in the number of winning prizes, number of digits in the outcome set, and the frequency with which draws are staged. Due to the limitation of the data, we cannot exclude the effects that be due to these structural difference. We therefore decided to examine the effect of how the game design influences betting behavior via a controlled experimental study. Our goal is to make the causal statement that a ceteris paribus change in the size of the winning numbers set increases bettors' tendency to seek past winning numbers, thereby confirming our conjecture about the mechanism underlying our empirical observations. In the following section, we report on the details of our experiment.

4 Experimental Studies

Our experimental study investigates how game design—in particular, the size of winning numbers—affects gambler's and hot-hand fallacy behaviors via a simulated lottery game. We have developed an online simulated lottery game that allows us to induce ceteris paribus changes in the number of winning prizes drawn. We believe this to be the ideal environment to investigate the mechanisms of human probabilistic predictions. The game is widespread in the population, requires comparatively little skill, and each draw from the lottery machine "follows a known, truly random process with a fixed probability for each number" (Suetens et al. (2016)). In our game, a computer randomly draws winning number(s) from an urn that contains 10 numbered balls from 0 to 9. Each number is represented by exactly one ball in the urn, and each ball has an equal chance of being drawn.[5].

4.1 Method

Participants

We recruited 210 participants via the crowd-sourcing platform Prolific for our online experiment[6].

[5]Our choice for 10 balls was motivated by practical concerns. Our aim was to create an environment in which winning is less likely than losing, but still occurs frequently enough to make incentives salient. At the same time we aimed for enough flexibility to create different experiences with regard to how frequently past winning number reoccur. 10 had the additional advantage that it allowed us to create a natural and homogeneous outcome set with regard to the number of digits present.

[6]Prolific is a UK-based crowd-sourcing platform that connects researchers with participants for their studies.

Participants were screened based on English as first language to avoid any linguistic ambiguity for the instructional aspects of the experiment. Our participants were either UK or US based, with 89% and 8%, respectively. Female participants comprised 58.6% of our sample, 12% were students, and the average age was 36.5 years. Self-reported median household income was between $35,000 and $49,999, and 66% of the participants reported that they played lottery or similar betting games less frequently than on a monthly basis, with 35% stating that they never played such games. On average, it took participants 8 minutes to complete the experiment. They received a flat completion fee of £0.80 (approx. $1.05). The best-guessing participant within a treatment was awarded a bonus payment of £15 ($19.80). The expected remuneration per participant amounted to 1.5 times the recommended remuneration set by Prolific.

Design, apparatus, and procedure

Participants received an invitation link to the simulated lottery game via Prolific. The lottery game itself was administered via Qualtrics, an online survey tool, which recorded participants' self-paced responses. Screenshots of the decision screens as well as transcripts of the online instructions are provided in Appendix B. In total, there were 45 rounds with one draw of winning number(s) in each round. The first 5 rounds constituted test rounds and gave participants the opportunity to familiarize themselves with the presentation style. In each of the remaining 40 rounds, participants were asked to guess one number from 0 to 9. If the guess matched either of the winning numbers drawn, the participant earned 1 point. Points were accumulated over the 40 guessing rounds, and the final score determined the winner of the prize. We mimicked the design of lottery websites as observed in 3D and 4D games. In particular, participants always saw the winning number(s) in the last draw on the screen and they could click a button to reveal the results of the 5 most recent draws.

Participants were randomly assigned to 3 experimental conditions, which differed only with respect to how many winning numbers were drawn in each round. In treatment $T = 1$, 75 participants played the lottery game with one winning number drawn. In treatments $T = 2$ and $T = 3$, 63 and 72 participants played lottery games with 2 and 3 winning numbers drawn, respectively. We implemented a stratified randomization procedure, which explains the minimal differences in terms

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In contrast to alternative platforms like Amazon's MTurk, it is specifically geared toward the needs of researchers; participating in research studies is the only activity platform members can engage in. The platform has been shown to produce reliable, high-quality data (cf. Peer et al. (2017)).

of number of participants across games.

4.2 Results

For our analysis of the 40 guessing rounds, we follow the same empirical methods as Suetens et al. (2016) and Wang et al. (2016), who analyze naturally occurring data with the same information structure as ours. We run probit regressions to estimate the effect of past winning draws on current betting behavior. Independent observations are taken at the participant-period-number level. Our dependent variable takes the value 1 if a participant bets in a given period on a given number, and 0 otherwise. Independent variables include a dummy for the most recent history (period $t-1$) and a count variable for number of wins in earlier histories (periods $t-5$ to $t-2$). For brevity of exposition, we only report estimation results for these two variables. However, all estimated models include fixed effects (FE) at the round and number level. We also control for subject-specific "hotness" of lucky numbers. For this, we include a variable that measures the share of past bets a number has received in the past. To control for other subject-specific effects, we either include subject FE or a set of demographic controls in terms of gender, age, income, and gambling habits. Standard errors are clustered at the participant level. Table 5 reports the average marginal effects of our regressions.

Table 5: Average Marginal Effects of Probit Regression on Historical Wins (Aggregate Level)

	T=1		T=2		T=3	
Won period t-1=1	-0.043***	-0.044***	0.004	0.008	-0.011	-0.009
	(0.007)	(0.006)	(0.014)	(0.014)	(0.009)	(0.009)
Wins in t-2 to t-5	-0.021***	-0.017***	-0.002	0.002	-0.005	-0.003
	(0.005)	(0.005)	(0.004)	(0.003)	(0.003)	(0.003)
Number of participants	75	75	63	63	72	72
Subject FE	Y	N	Y	N	Y	N
Period FE	Y	Y	Y	Y	Y	Y
Number FE	Y	Y	Y	Y	Y	Y
Demographic controls	N	Y	N	Y	N	Y
Clustered std. err.	Y	Y	Y	Y	Y	Y
Obs	30000	30000	25200	25200	28800	28800

We focus our discussion of results on the model with subject FE, since differences across estimation models are minimal. We find that if a number has won the most recent draw (period $t-1$), the probability of participants' betting on this particular number is reduced by 4.3 percentage points in $T=1$. This effect lasts beyond the most recent draw. For every win in the periods $t-2$

to $t-5$, the betting proportion on that number is reduced by 2.1 percentage points in $T=1$. On the aggregate level, we observe a strong gambler's fallacy effect in $T=1$. However, aggregate betting behavior is significantly impacted by a change in the number of winning balls drawn. In treatments $T=2$ and $T=3$, we do not detect any significant manifestation of the gambler's fallacy in aggregate betting behavior.

Next, we analyze participants' betting behavior conditional on streak length, defined as the number of consecutive wins. As discussed by Rabin (2002) and Rabin and Vayanos (2010), streak length plays a prominent role in misperceptions of positive or negative autocorrelation in i.i.d. processes. We run probit regressions to estimate the impact of streak length on the betting proportion of a number, following the same empirical strategy as we did in Table 5. Table 6 reports the corresponding average marginal effects. In $T=1$, we find the gambler's fallacy to emerge in short streaks (with streak lengths one and two), but then betting behavior switches and turns into hot-hand fallacy behavior when the streak length is four. It is worth noting that this pattern of behavior is compatible with the theoretical predictions derived by Asparouhova et al. (2009). In $T=2$, we find only hot-hand behavior with longer streaks. This has already emerged when streak length is three and persists when streak length is four. In $T=3$, there is no significant impact of streak length on betting behavior.

Table 6: Average Marginal Effects of Probit Regression on Streak Length (Aggregate Level)

	T=1		T=2		T=3	
Length streak=1	-0.051***	-0.053***	-0.004	0.000	-0.014	-0.012
	(0.007)	(0.006)	(0.013)	(0.013)	(0.009)	(0.009)
Length streak=2	-0.054***	-0.044**	0.013	0.018	-0.002	-0.000
	(0.018)	(0.019)	(0.020)	(0.021)	(0.012)	(0.012)
Length streak=3	0.035	0.051	0.094**	0.100***	-0.004	-0.005
	(0.047)	(0.047)	(0.038)	(0.038)	(0.017)	(0.016)
Length streak=4	0.166**	0.182***	0.169**	0.153**	0.022	0.013
	(0.065)	(0.064)	(0.070)	(0.065)	(0.029)	(0.028)
Number of participants	75	75	63	63	72	72
Subject FE	Y	N	Y	N	Y	N
Period FE	Y	Y	Y	Y	Y	Y
Number FE	Y	Y	Y	Y	Y	Y
Demographic controls	N	Y	N	Y	N	Y
Clustered st. err.	Y	Y	Y	Y	Y	Y
Obs	30000	30000	25200	25200	28800	28800

So far, our analysis has revealed a strong tendency in aggregate betting behavior to avoid past winning numbers in $T=1$, which then disappears in $T=2$ and $T=3$. To demonstrate that this pattern is the result of an increased inclination to bet on past winning numbers in $T = 2$ and $T = 3$, we next focus our analysis on individual-level data. If a participant's betting strategy follows an i.i.d. process, we would expect him or her to bet on a number involved in an ongoing streak in 4 out of 40 rounds in $T = 1$. Analogously, we would expect her to continue an ongoing streak in 8 and 12 out of 40 rounds in $T = 2$ and $T = 3$, respectively. We bin participants based on the number of times they continue streaks and plot the bin counts in Figure 4. For example, in $T = 1$, 17 participants never bet on a number involved in a streak, regardless of streak length, during the experiment. In comparison to $T = 2$ and $T = 3$, the inclination to follow streaks is skewed to the left relative to i.i.d. expectations.

Figure 4: The Gambler's and Hot-Hand Fallacies at the Individual Level

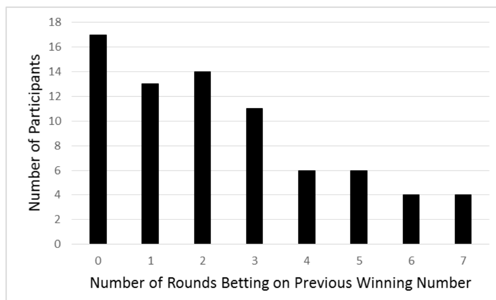
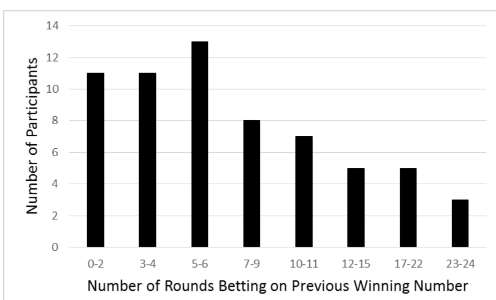


Figure 4a: Individual Behavior with $T = 1$



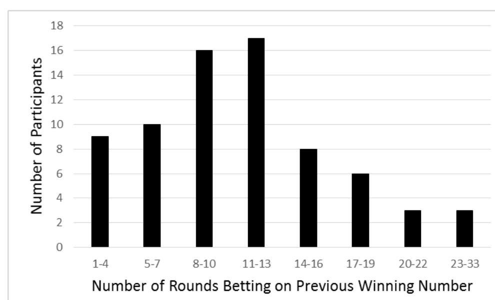


Figure 4b: Individual Behavior with $T = 2$ Figure 4c: Individual Behavior with $T = 3$

We can now classify participants as seeking past numbers if they disproportionately often continue streaks in comparison to the aforementioned treatment-specific expectations. We translate these expectations into binomial success probabilities, in which a success is defined as continuing a streak. A participant is then classified as a hot-hand player if her frequency to follow streaks is below the threshold of 10% (5%), assuming a binomial distribution in continuing a streak. Under a significance level of 10%, we observe 5.3%, 20.6%, and 16.7% hot-hand players in treatment $T = 1$, $T = 2$, and $T = 3$, respectively. When measured at a significance level of 5%, there are 0%, 17.5%, and 12.5% hot-hand palyers in the three treatments, respectively. Table 7 reports the corresponding *p-values* of one-sided Fisher-Boschloo tests on the proportions of hot-hand and non-hot-hand players. Even after accounting for family-wise error rates via Holm-Bonferroni, we find evidence that hot-hand behavior significantly increases from $T = 1$ to $T = 2$ and 3, but no evidence for differences in hot-hand behavior between $T = 2$ and $T = 3$.

Table 7: *p-values* of One-Sided Exact Fisher-Boschloo Tests

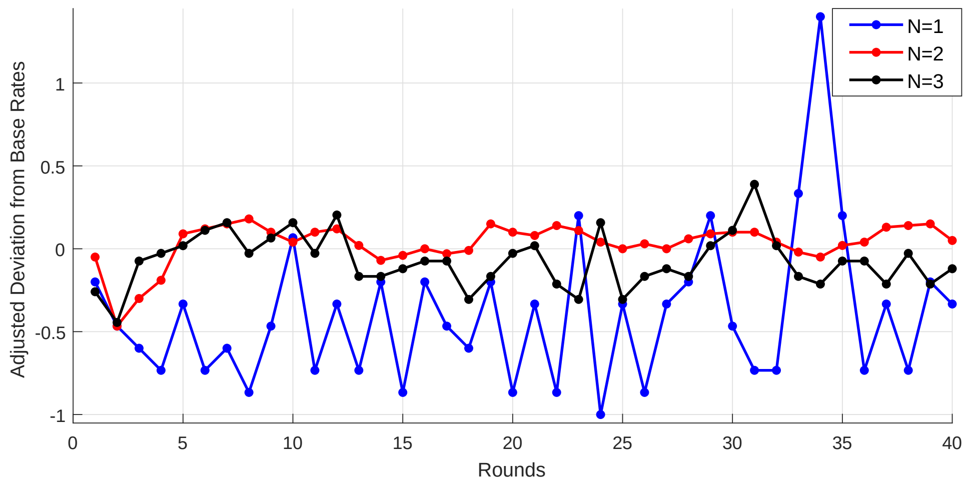
Threshold at 10%				**Threshold at 5%**			
	vs	$T=2$	$T=3$		vs	$T=2$	$T=3$
$T=1$		0.004	0.017	$T=1$		< 0.001	< 0.001
$T=2$			0.746	$T=1$			0.818

4.3 Additional Analyses

Why do participants exhibit more hot-hand behavior when we increase the number of balls drawn in a simulated lottery game? We present here a set of observations with regard to betting behavior that will motivate our answer to this question. For sake of brevity, we only state our main finding and refer interested readers to Appendix C for details.

First, we explore behavioral change over the course of 40 rounds. We calculate the percentage of participants who bet on previous winners in each round over the course of the experiment, and then the percentage deviation from the treatment-specific base rates (10%, 20%, and 30%, respectively, for $T = 1, 2,$ and 3). Figure 5 plots the adjusted deviation from the base rates, in which negative (positive) deviations indicate the gambler's fallacy (hot-hand fallacy). At the beginning of the experiment, deviations from the base rates are all negative, suggesting that participants in all treatments show a predisposition for the gambler's fallacy. However, this tendency only persists in $T = 1$, but not in $T = 2$ and $T = 3$. Participants in multiple-prize conditions alter their behavior as the game proceeds; evidence of a game-design induced change in beliefs about the underlying random process.

Figure 5: Behavioral Change in Multiple-Prize Games



Second, we recorded each individual's decision time, i.e., the time it took a participant to enter his or her guess for the winning number. It is well established that decision time is a reliable indicator of task difficulty and task automaticity, with quicker decision time representing easier, more automatic decisions (Achtziger and Alós-Ferrer (2014), Alós-Ferrer et al. (2016)). We ran panel regressions with individual random-effects to explore the intra-subject relationship between decision time and hot-hand behavior (continuing existing streaks); see detailed regression results in Appendix C. We find that hot-hand behavior is associated with slower decision times in $T = 1$. This suggests that following past winning numbers in $T = 1$ is a more difficult decision compared to not betting on past numbers. In $T = 2$ and $T = 3$, we observe a strong reversal in our results, in the sense that betting on past winning numbers is a quicker decision than not betting on past winning numbers. These results suggest the existence of an underlying factor that triggers a change in participants' behavior and their perception of randomness when more balls are being drawn.

Our results suggest that participants start out with the disposition to avoid past winning numbers, possibly because they wrongly believe that past winning numbers have a lower chance of being redrawn. To lend further credence to this interpretation, we conducted a small-scale replication study with 160 participants randomly assigned to the conditions $T = 1$ and $T = 3$. In replicating our main findings, we added one new question to the design. We implemented an incentivized belief-elicitation question about the probability of redrawing past winning numbers.

We observe strong evidence that participants underestimate the probability of repetition in winning numbers in $T = 3$, in which the correct answer for the true probability of repetition is less obvious than is the case for the single-prize game $T = 1$. In fact, there is no significant difference in average beliefs between $T = 1$ and $T = 3$.

4.4 Discussion

We examined the role of game design—specifically, multiple outcomes in a random process—on the emergence of the gambler's and the hot-hand fallacies in a lottery game prediction task. Participants were asked to guess the outcome of simulated lottery games with one, two, or three winning numbers. Results show that whereas the gambler's fallacy still prevailed in the single-prize game, it immediately disappeared when the number of outcomes was increased to two and three. On the individual level, we observe significantly more hot-hand behavior in multiple-prize games compared to single-prize games. We infer from our observations that the lottery system design, in the form of the number of prizes drawn, systematically influence participants' perceptions of randomness, and thereby their betting behavior. These findings offer a new insight on the prevalence of the gambler's fallacy and the hot-hand fallacy for which the current literature does not account for.

One plausible explanation for our observations is that participants strongly underestimate the likelihood of experiencing streaks or repeated outcomes in multiple-prize lottery games. The famous birthday paradox is a classical illustration of these types of common misjudgments in probabilities of repetition among multiple outcomes. In our experiment, the probability of observing at least one repetition in two consecutive draws are, respectively, 10%, 37.8%, and 70.8%, with $T = 1$, $T = 2$, and $T = 3$. Repeated outcomes or streaks are perceived to be a rare and remarkable event in independent and identically distributed draws (Sun and Wang, 2010). When asked to generate a sequence, people tend to avoid repetition (Falk and Konold, 1997). In our experiment, participants underestimate the likelihood of observing repetition in the multiple-prize games. When they are confronted with more repetitions than expected during the game, they prefer to judge the underlying mechanism as "different from what they believed"—namely, if they start with a predisposition toward the gambler's fallacy, they may switch to an opposite behavior, the hot-hand fallacy, as their prior beliefs are challenged. We indeed observe this switch from the gambler's to the hot-hand fallacy in multiple-prize games.

We conclude this section with some final remarks on the validity of our experimental set-up. First, our results with single-outcome games replicate/support some relevant findings in the literature. Based on Rabin's (2002) model, Asparouhova et al. (2009) predicted that the gambler's fallacy prevails when streaks are short, and the hot-hand fallacy emerges when streaks become longer. To the best of our knowledge, our experiment is the first to observe this pattern with a clear switching point from the gambler's fallacy to the hot-hand fallacy. Second, our experiment also replicates empirical findings in Suetens et al. (2016) based on real Lotto data from Denmark; we applied the authors' approach to test for the presence of the gambler's fallacy. In line with their results, we find a reduction in bets on past winning numbers, which is consistent with the gambler's fallacy. They find a magnitude of between 1.6% and 3.8%s, depending on the sample under investigation, while we find a reduction of 4.3% in the treatment in which one ball is drawn. Finally, our findings are also consistent with observations in Wang et al. (2016), who found that participants avoid numbers that were drawn only once over the last six draws, while those numbers that were drawn three or four times are relatively popular.

5 Conclusion

In this paper, we used data from two different lottery games to demonstrate the impact of game design in shaping erroneous beliefs: the gambler's and hot-hand fallacies. Field data showed that when there are multiple prizes in a lottery game, players tend to favor previous winning numbers, suggesting that they believe the same outcome will repeat in the future. We then run a set of online experimental studies and found that behavior changed under different game designs; in particular, whereas a single-prize game lead to the gambler's fallacy, consistent with the current literature, multiple-prize games lead to more hot-hand behavior.

The notion that humans can be manipulated to believe in the hot-hand fallacy through appropriate game design has important ramifications in various fields. First, it provides a behavioral explanation for the "medium-prizes puzzle" (Haruvy et al., 2001)—why lottery-game operators typically offer prize distributions of a few large prizes and a large number of medium ones. This is surprising, especially if we assume that gamblers are typically risk-seeking. There are two common explanations for this observation. One relies on prospect theory: A large number of medium prizes reduces the probability of losing from near certainty to some smaller probability. Another explanation follows the line of adaptive learning, in that human behavior is best captured by simple adaptive learning models, and actions that did better in the past will tend to be adopted more frequently compared to actions that did worse. Thus the presence of medium prizes slows down the agent's inclination to gamble less. Our paper provides another explanation: A large number of medium prizes can induce more players to believe in the hot-hand fallacy. This reduces players' inclination to quit the game, and also increase their desire to bet on those numbers they believe to have a higher probability of winning.

Second, it provides guidelines for designing lottery games that will induce desirable behavior. A recent trend is for governments to encourage good civic behavior through the use of lottery games. Richard Thaler has written about the merits of this approach.[7] For instance, New Taipei City in Taiwan recently initiated a lottery as an inducement for dog owners (and other citizens) to clean up after their pets in order to win gold ingots worth as much as $2,000. In addition, the Singapore government is experimenting with an incentive scheme in which commuters earn credit for each journey taken (with triple credit for off-peak journeys) for a chance to win cash prizes in weekly lotteries. Our study highlights two features that will render these lottery games more effective for influencing behavior: (1) a sufficiently large number of medium prizes (to induce more hot-hand believers) should be inducled, and (2) some mechanism that allows players to bet on numbers they believe to have a higher probability of winning—and thus increase their incentive to participate in lottery games—is desirable. One way to do this is to use personalized numbers (instead of random numbers) that players can easily relate to. The Dutch government uses this principle very effectively; one of its state lotteries is based on postal codes. The idea is to make use of the near-miss effect, and also to exploit the lucky-store effect that has been shown to exist in various lottery games. The chances of winning may be falsely believed to be higher if the postal code has been drawn before (or is a near miss) in the previous draws.

[7] "Making Good Citizenship Fun," *New York Times*, February 13, 2012, p A25.

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A Empirical Evidence

Table 8: Payout Rate Statistics of Winning Numbers in 3D Game

Numbers Repeating in	Data Point	Mean	Stdev	25th Percentile	Median	75th Percentile
All data	4466	0.49	0.27	0.32	0.43	0.59
1 week	27	0.34	0.36	0.18	0.23	0.36
1 to 2 weeks	27	0.35	0.12	0.27	0.31	0.42
2 to 3 weeks	22	0.40	0.24	0.26	0.30	0.54
3 to 4 weeks	31	0.42	0.20	0.29	0.40	0.47
4 to 5 weeks	28	0.47	0.26	0.29	0.40	0.53
5 to 6 weeks	34	0.46	0.23	0.31	0.38	0.56
6 to 7 weeks	31	0.43	0.16	0.32	0.41	0.53
7 to 8 weeks	30	0.58	0.32	0.34	0.49	0.76
8 to 9 weeks	37	0.45	0.21	0.33	0.38	0.52
9 to 10 weeks	29	0.45	0.20	0.31	0.39	0.52
More than 10 weeks	3180	0.49	0.24	0.33	0.44	0.59
No Past Win	990	0.50	0.34	0.30	0.42	0.59

B Experimental Design

General introduction to study for all experimental conditions

Thank you for your interest in this study. In this study, you are invited to play a simulated lottery game and your task is to guess which numbers a lottery-computer will draw. On top of your fixed earnings of 0.80, you can earn a bonus payment of 10 for this study which will be awarded to every 50th respondent. More details are provided during the study. Please read all questions carefully and follow the on-screen instructions. Completing the survey will take about 9 minutes.

Table 9: Payout Rate Statistics of Winning Numbers in 3D Game

x	sample size	*t-stat.*	*p-value*	95% confidence interval
3	70	-5.07	3.20e-06	[-0.219, -0.095]
5	125	-4.23	4.55e-05	[-0.147 -0.053]
7	176	-4.80	3.48e-06	[-0.120, -0.050]
9	231	-4.13	5.08e-05	[-0.100, -0.035]
11	264	-4.34	2.05e-05	[-0.094, -0.035]

If you close your browser before completing the survey, your data will be lost and you will have to start anew. Make sure that javascript is enabled in your browser. The survey works best on desktops and tablets, but might be inconvenient to complete on smartphones. Participant IDs and completion codes are automatically transferred. You can open this study in a new window.

Introduction to the simulated lottery game with $T = 1$

Welcome to the Simulated Lottery Game! Today, you are invited to play a simulated lottery game. There will be 40 rounds in total. In each round, a lottery computer will randomly draw 1 numbered ball (0 - 9) from an urn. Each ball in the urn has an equal chance of being drawn. Your task is to guess, at the beginning of each round, which number will be drawn by the computer. If you guess it correctly, you earn 1 point. Roughly 70 respondents will play this game and the one with the highest points score will earn a bonus payment of 15. It is therefore important that you read on-screen instructions carefully. Please click on NEXT to proceed. The computer will randomly draw 1 ball from an urn of 10 balls. The balls are numbered consecutively from 0 to 9. That is, each number is represented by exactly one ball (one ball with 0, one ball with 1, ..., one ball with 9). Guess one number from 0 to 9. If your number matches the number drawn, you earn 1 point. This game is repeated for 40 rounds.

Introduction to the simulated lottery game with $T = 2$

Welcome to the Simulated Lottery Game! Today, you are invited to play a simulated lottery game. There will be 40 rounds in total. In each round, a lottery computer will randomly draw 2 numbered balls (0 - 9) from an urn. Each ball in the urn has an equal chance of being drawn.

Your task is to guess, at the beginning of each round, which number will be drawn by the computer. If you guess it correctly, you earn 1 point. Roughly 70 respondents will play this game and the one with the highest points score will earn a bonus payment of 15. It is therefore important that you read on-screen instructions carefully. Please click on NEXT to proceed. The computer will randomly draw 2 balls from an urn of 10 balls. The balls are numbered consecutively from 0 to 9. That is, each number is represented by exactly one ball (one ball with 0, one ball with 1, ..., one ball with 9). Guess one number from 0 to 9. If your number matches one of the numbers drawn, you earn 1 point. This game is repeated for 40 rounds.

Introduction to the simulated lottery game with $T = 3$

Welcome to the Simulated Lottery Game! Today, you are invited to play a simulated lottery game. There will be 40 rounds in total. In each round, a lottery computer will randomly draw 3 numbered balls (0 - 9) from an urn. Each ball in the urn has an equal chance of being drawn. Your task is to guess, at the beginning of each round, which number will be drawn by the computer. If you guess it correctly, you earn 1 point. Roughly 70 respondents will play this game and the one with the highest points score will earn a bonus payment of 15. It is therefore important that you read on-screen instructions carefully. Please click on NEXT to proceed. The computer will randomly draw 3 balls from an urn of 10 balls. The balls are numbered consecutively from 0 to 9. That is, each number is represented by exactly one ball (one ball with 0, one ball with 1, ..., one ball with 9). Guess one number from 0 to 9. If your number matches one of the numbers drawn, you earn 1 point. This game is repeated for 40 rounds. **Figure 6 shows two screen shots from the experiment.**

C Regression Results with Decision Time

We followed the empirical strategy in Achtziger and Alós-Ferrer (2014) and Alós-Ferrer et al. (2016) and adopted a GLS panel approach with random effects at the individual to analyze the impact of various variables on decision times. Our dependent variable in all models is the log of Decision Time measured from the onset of presentation of the decision screen until submission of a guess for a winning number. Independent observations are taken at the individual-guess-period level. The estimation results are reported in Table 10. We include dummies for our multiple-prize

Figure 6: Screenshots from the experiment

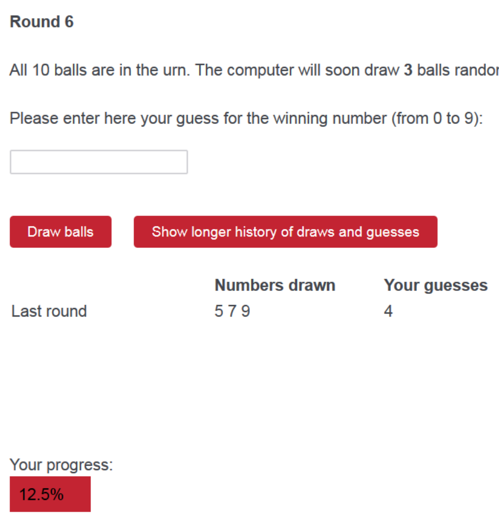

Figure 6a: Treatment $T = 1$

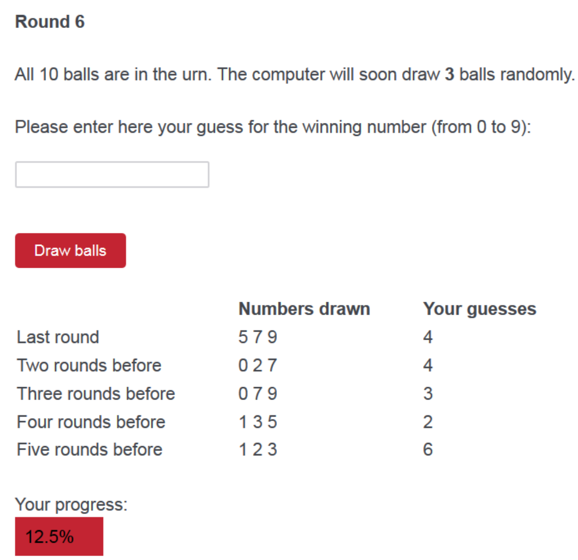

Figure 6b: Treatment $T = 3$

treatments $T = 2$ and $T = 3$ and a dummy for whether the guess of an individual continues an ongoing streak (Hot Hand). As reported in the main text, hot-hand behavior was slower than non-hot-hand behavior in $T = 1$, whereas hot-hand behavior was quicker in than non-hot-hand behavior in multiple-prize games. We include additional control variables to validate decisions times and to control for possible confounding factors. Participants guessed faster in later periods of the experiment, evidence of learning and task-experience effects. Decisions were slower when the history button was clicked. Naturally, it is time consuming to click the button and process the additional information displayed from clicking the button. We also observe that older participants were slower than younger ones, and that women and participants associating themselves to a Trans gender were slower than men. The lotto playing habits of participants had no influence on the speed of decisions.

Table 10: GLS panel regression with random effects at the individual level and cluster robust standard errors (reported in parentheses). Significance coding: *** - 1%, ** - 5%, * - 10%.

Dependent Var Log(Decision Time)	Model (1)	Model (2)	Model (3)
Hot Hand	0.127***	0.083*	0.083*
	(0.048)	(0.043)	(0.043)
T=2	0.160***	0.091	0.069
	(0.062)	(0.061)	(0.056)
T=3	0.061	-0.001	-0.018
	(0.068)	(0.060)	(0.059)
Hot Hand × T=2	−0.201***	−0.183***	−0.181***
	(0.058)	(0.050)	(0.050)
Hot Hand × T=3	−0.162***	−0.125***	−0.125***
	(0.052)	(0.047)	(0.047)
Period	−0.017***	−0.014***	−0.014***
	(0.001)	(0.001)	(0.001)
Hotness Number		−0.602***	−0.595***
		(0.044)	(0.044)
Click History Button		1.073***	1.075***
		(0.052)	(0.052)
Age			0.012***
			(0.002)
Female			0.101**
			(0.048)
Trans*			0.527***
			(0.094)
Playing Lotto Monthly			0.095
			(0.070)
Playing Lotto < Monthly			0.022
			(0.060)
Playing Lotto Never			-0.027
Income			-0.009
			(0.010)
Constant	1.579***	1.613***	1.164***
	(0.052)	(0.047)	(0.104)
Number of participants	210	210	210
Obs	8400	8400	8400
Clustered std. err.	Y	Y	Y
Overall R^2	0.098	0.264	0.313
Wald χ^2 p	0.000	0.000	0.000