Fast Parallel Detection of Strongly Connected Components (SCC) in Small-World Graphs

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PPL Retreat – January 25, 2014
Outline

- SCC Background and Motivation
- Shortcomings of Existing Algorithm and Our Solutions
- Experimental Results
Twitter: 215 million monthly active users
500 million Tweets per day

Source: Eric Fischer via Flickr and Twitter IPO Filing
Example: Twitter SCCs

@katyperry

@justinbieber

@BarackObama

@DalaiLama

@user1
In a directed graph, an SCC is a maximally connected subgraph with a path in both directions between any two nodes.
Datasets contain millions to billions of nodes \((n)\) and billions of edges \((m)\)

Fastest sequential SCC algorithms require \(O(n + m)\) work

\(\rightarrow\) SCC on large graphs is time-consuming

Solution: PARALLELIZE!
Existing Algorithms

- **Optimal sequential algorithm**
  - Tarjan’s Algorithm [Tarjan, SIAM 1972]
  - Cannot be parallelized effectively due to depth-first search (DFS)

- **Forward-Backward-Trim parallel algorithm**
  - Recursive application of reachability
    [Fleischer et al., IPDPS 2000]
  - Trim of trivial SCCs
    [McLendon et al., Parallel & Dist. Computing 2005]
Node $a$ is **reachable** from node $b$ if there is a path from $b$ to $a$.
FW-BW-Trim Algorithm: Reachability

- Four partitions
  - $FW_G(i) \cap BW_G(i)$ [SCC]
  - $FW_G(i) \setminus BW_G(i)$
  - $BW_G(i) \setminus FW_G(i)$
  - $V \setminus (FW_G(i) \cup BW_G(i))$

- Additional SCCs must be completely contained within one of the three additional partitions
FW-BW-Trim Algorithm: Reachable Set Recursion

- Recursively apply the algorithm to each of the three partitions created besides the pivot’s SCC
- Utilizes task parallelism
FW-BW-Trim Algorithm: Trimming

- Can identify trivial SCCs (size 1) by looking only at the number of neighbors
  - If the node has in-degree=0 or out-degree=0, it is a size 1 SCC
- Repeat iteratively
- Implement in parallel on disconnected nodes
FW-BW-Trim Algorithm

Apply iterative Trim step

Choose ANY node in the graph

Calculate forward & backward sets

New SCC is intersection of FW & BW sets

Algorithm 1: FW-BW-Trim(G, SCC)

In-Out: G: a graph (a subgraph of the original input graph)
In-Out: SCC: a collection of node sets; each set corresponds to an SCC of the original graph

Trim(G, SCC)

if |Nodes(G)| = 0 then return;

u ← pick any node in G

FW ← Forward-Reach(G, u)

BW ← Backward-Reach(G, u)

S ← FW ∩ BW

SCC ← SCC ∪ {S}

begin in parallel

FW-BW-Trim(FW \ S, SCC')

FW-BW-Trim(BW \ S, SCC')

FW-BW-Trim(G \ (FW ∪ BW), SCC')

Recursively apply algorithm to each partition
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Real-World Graphs and the Small-World Property

- Social networks, web graphs, citation networks

- Relevant properties
  - Small-world property (small diameter)
  - Giant SCC size $O(N)$
  - Skewed SCC size distribution
    - Small SCCs are more frequent than large SCCs
Example Small-World Graph: LiveJournal

- $N = 4,848,571; M = 68,993,773$
- Estimated diameter $= 18$
- Largest SCC size $= 3,828,682$ (79% of all nodes)
Shortcomings of the FW-BW-Trim Algorithm

- High probability that we initially pick a pivot node in the giant SCC
- Giant SCC is likely identified at the beginning by a single thread
- Other threads idle because no other tasks yet

→ Workload imbalance
→ Insufficient parallelism
Our Algorithm Extensions
Method 1: Two-Phase Parallelization

- Adds **data parallelism**
  - All threads work on the same partition of the graph to find reachable sets
- Implement with parallel breadth-first search (BFS)
Method 1: Two-Phase Parallelization

**FW-BW-Trim(G):**
- // Data parallel
- Trim(G)
- // Task parallel
- Recur-FWBW(G)

**Method1(G):**
- // Data parallel
- Trim(G)
- Par-FWBW(G)
- Trim(G)
- // Task parallel
- Recur-FWBW(G)
Shortcomings of Method 1

- Insufficient tasks in the task parallel recursive FW-BW step

Mostly disconnected SCCs

The Giant SCC
Method 2: Weakly Connected Components (WCC)

Now each WCC is a separate parallel task

→ Significantly increases parallelism in recursive FWBW step
Method 2: Weakly Connected Components (WCC)

- In a directed graph, a WCC is a maximally connected subgraph with a path in one direction between any two nodes.
Method 2: Trim2

- Parallel detection of a subset of size 2 SCCs
  - Tight loop between nodes A and B
  - No other outgoing (or incoming) edges from A and B

- Apply only once rather than iteratively
  - Higher computational cost than Trim

- Reduces execution time of WCC step by up to 50%
Method 2: WCC + Trim2

Method1(G):
// Data parallel
Trim(G)
Par-FWBW(G)
Trim(G)

// Task parallel
Recur-FWBW(G)

Method2(G):
// Data parallel
Trim(G)
Par-FWBW(G)
Trim'(G) \{ Trim2(G)
Par-WCC(G) \}

// Task parallel
Recur-FWBW(G)
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Experimental Datasets

- Online social networks
  - Flickr
  - Friendster*
  - Twitter
  - Orkut*

- Web link networks
  - LiveJournal
  - Baidu
  - Wikipedia

- Citation
  - US Patents

- Non small-world
  - CA-road*

*the original graph is undirected; we randomly assign a direction for each edge with 50% probability for each direction
Experimental Setup

- Commodity server
  - 2 Intel Xeon E5-2660 (2.20GHz) CPUs
  - Total of 16 cores and 32 hardware threads
  - Total of 20 MB of last-level cache and 256 GB of main memory

- OpenMP threading library
Algorithm Recap

FW-BW-Trim(G):
// Data parallel
Trim(G)

// Task parallel
Recur-FWBW(G)

Method1(G):
// Data parallel
Trim(G)
Par-FWBW(G)
Trim(G)

// Task parallel
Recur-FWBW(G)

Method2(G):
// Data parallel
Trim(G)
Par-FWBW(G)
Trim'(G)
Par-WCC(G)

// Task parallel
Recur-FWBW(G)
Parallel Speedup Results vs. Tarjan’s Alg.
Parallel Speedup Results

- LiveJournal
- Flickr
- Baidu
- Wikipedia
- Friendster
- Twitter
- Orkut
- Patents
- CA-road
Method 2 = Method 1
Results: Friendster
Method 2 = Method 1
Results: Friendster
Method 2 > Method 1
Results: LiveJournal

![Graph showing speedup vs. number of threads for FW-BW-Trim, Method 1, and Method 2. The graph indicates that Method 2 outperforms Method 1 as the number of threads increases.]
Method 2 > Method 1
Results: LiveJournal
Tarjan > Methods 1&2
Results: CA-road

- FW-BW-Trim
- Method 1
- Method 2
Tarjan > Methods 1&2
Results: CA-road
Conclusions

- We extend the FW-BW-Trim parallel SCC detection algorithm by taking advantage of small-world graph properties

- Result: Significant parallel speedup on small-world graphs
  - Speedup from 5x to 29.4x
  - Mean speedup 14x
Questions?

Thank you

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