Lecture 5 Assignment  Due: Tuesday morning

Problem 5.1: Transformation of y-velocity  This is Spacetime Physics L-7

A particle moves with uniform speed $v'_y = \frac{\Delta y'}{\Delta t'}$ along the $y'$-axis of the rocket frame. The rocket frame ($S'$) is moving in the $x$ direction with speed $v_{rel}$. Transform $\Delta y'$ and $\Delta t'$ using the Lorentz transformation equations. Show that the $x$ and $y$ components of the velocity, as observed in the $S$ frame are,

\[ v_x = v_{rel} \]
\[ v_y = v'_y \sqrt{1 - \frac{v_{rel}^2}{c^2}} \]

Problem 5.2: Tilted meter stick  This is Spacetime Physics L-10

[You will need the answer to the previous problem for this one.]

A meter stick lying parallel to the $x$-axis moves in the $y$-direction in the laboratory frame with speed $v_y$ as shown in the figure. The rocket frame moves at speed $v_{rel}$ in the $x$ direction.

a) In the rocket frame the stick is tilted upward in the positive $x'$-direction as shown in the figure. Explain why this is, first without using any equations.

b) Let the center of the meter stick pass the point $x = y = x' = y' = 0$ at time $t = t' = 0$. Calculate the angle $\phi'$ at which the meter stick is inclined to the $x'$-axis as observed in the rocket frame.

Discussion: Where and when does the right end of the meter stick cross the $x$-axis as observed in the laboratory frame? Where and when does this event of right-end crossing occur as measured in the rocket frame? What is the direction and magnitude of the velocity of the meter stick in the rocket frame (Previous exercise)? Therefore where is the right end of the meter stick at $t' = 0$, when the center is at the origin? Thus....

Problem 5.3: Superluminal expansion of quasar 3C273  (in groups)[OPTIONAL]

This is Spacetime Physics 3-16

To reinstating the factors of $c$, the following statements should be changed.

a): “$\Delta t(1 - v^2 \cos \theta)$” should be replaced with “$\Delta t(1 - \frac{v^2}{c^2} \cos \theta)$”

c): “$\frac{v \sin \theta}{1 - v \cos \theta}$” should be replaced with “$\frac{v \sin \theta}{1 - \frac{v}{c} \cos \theta}$”

e): “$\frac{v}{(1 - v^2)^{\frac{3}{2}}}$” should be replaced with “$\frac{v}{(1 - \frac{v}{c}^2)^{\frac{3}{2}}}$”. Also “$\cos \theta_{max} = v$” should be “$\cos \theta_{max} = \frac{v}{c}$”

figure: “$v \Delta t \sin \theta$” should be replaced with “$\frac{v}{c} \Delta t \sin \theta$”

Problem 5.4: Contraction or rotation?  This is Spacetime Physics 3-17

A cube at rest in the rocket frame has an edge of length 1 meter in that frame. In the laboratory frame the cube is Lorentz contracted in the direction of motion. Determine this Lorentz contraction, for example, from locations of four clocks at rest and synchronized in the laboratory frame lattice with which the four corners of the cube $E,F,G,H$ coincide when all four clocks read the same time. This latticework measurement eliminates time lags in the travel of light from different corners of the cube.

Now for a different observing procedure! Stand in the lab frame and look at the cube with one eye as the cube passes overhead. What one sees at any time is light that enters the eye at that time, even if it left the different corners of the cube at different times. Hence, what ones sees visually may not be the same as what one observes with the latticework of clocks. If the cube is viewed from the bottom then the distance $GO$ is equal to the distance $HO$, so the light that leaves $G$ and $H$ simultaneously will arrive at $O$ simultaneously. Hence, when one sees the cube to be overhead one will see the Lorentz contraction of the bottom edge.
a) Light from $E$ that arrives at $O$ simultaneously with light from $G$ will have to leave $E$ earlier than light from $G$ left $G$. How much earlier? How far has the cube moved in this time? What is the value of the distance $x$ in the right top figure?

b) Suppose the eye interprets the projection in the figures as a rotation of a cube that is not Lorentz contracted. Find an expression for the angle of apparent rotation $\phi$ of this uncontracted cube. Interpret this expression for the limiting two cases of cube speed in the lab frame: $v \rightarrow 0$ and $v \rightarrow c$.

c) **Discussion questions:** Is the word “really” an appropriate word in the following quotations?

1. An observer using the rocket latticework of clocks says, “The stationary cube is really neither rotated nor contracted.”
2. Someone riding in the rocket who looks at the stationary cube agrees, “The cube is really neither rotated nor contracted.”
3. An observer using the laboratory latticework of clocks says, “The passing cube is really Lorentz contracted but not rotated.”
4. Someone standing in the lab frame looking at the passing cube says, “The cube is really rotated but not Lorentz contracted.”

What can one rightfully say – in a sentence or two – to make each observer think it reasonable that the other observers should come to different conclusions?

**Problem 5.5: Does a relativistic submarine float or sink?**

This is a less well known paradox than the pole and barn paradox, and has a more subtle resolution.

Consider a submarine that has a neutral buoyancy with respect to water it is in when it is at rest. For simplicity, we take the sea it is in to have zero viscosity and constant density. Then consider the submarine moving through the fluid at some relativistic speed and as always, consider from two frames of reference.

Here is the paradox: From the fluid’s reference frame, where the fluid is at rest, the density of the fluid is the same as when the submarine is at rest. However, due to length contraction, the submarine is shorter, the volume is smaller, and the mass density of the submarine is now greater. Thus, the submarine sinks in this frame of reference. From the submarine’s frame of reference, the density of the submarine is the same but the water is length contracted and thus the density of the water is greater. In this case the submarine floats up! These are mutually exclusive results and cannot both be true. Is relativity wrong? How do you resolve this?

Some caveats: First, this problem involves gravity and thus should properly be treated by general relativity. However, we don’t know enough about GR to resolve this, we will use special relativity only. To help see the resolution, place this submarine in a sea that has a flat floor and sea surface in the water’s frame. [Hint: Think of the sea floor and do Spacetime Physics L-10 (and maybe L-11, 12 as well).]

**Readings out of “Incomprehensible”**

For what we’ve covered sections 3.1 through 3.3. For alternative derivation of LTs, 3.4. For vectors and the Minkowski metric: sections 1.13, 1.14, 1.15, 3.5. For geometry (manifolds, metrics) read chapter 4. (Advanced 4-vectors read 5.1, 5.2.)
Problem 5.6: Polar and Spherical Coordinates I

In order to build up our background knowledge for later in the course we will slowly introduce different coordinate systems. The end goal is to understand spherical coordinates, something rarely covered prior to university. So to begin we will cover polar coordinates.

Polar coordinates are an alternative form for specifying points in the two-dimensional Euclidean plane. As Cartesian coordinates require two parameters \((x, y)\) to specify any point in the plane, so do polar coordinates, labeled as \(r\) and \(\phi\).¹

The radial coordinate, \(r\), is the straight-line distance from the origin to the point in question (which we’ll call \(P\) for now). The angle \(\phi\) is measured from the \(x\)-axis to the straight line defining \(r\).

Converting between Cartesian and Polar coords.

Find the mathematical equation that gives \(x\) in terms of \(r\) and \(\phi\) and \(y\) in terms of \(r\) and \(\phi\).

\[x = \quad y = \]

Find the mathematical equation that gives \(r\) in terms of \(x\) and \(y\) and \(\phi\) in terms of \(x\) and \(y\).

\[r = \quad \phi = \]

Introducing Spherical coordinates

To take this up one dimension we need to introduce another parameter to our polar coordinates to label all points in three-dimensional Euclidean space (labeled in Cartesian coordinates by \(x, y, \) and \(z\)). The three parameters will be labeled as \(r, \theta, \phi\). \(r\) and \(\phi\) will have the same definition as in polar coordinates - \(r\) is the straight-line distance from the origin to the point in question \((P)\), and \(\phi\) will be the angle from the \(x\)-axis to the projection of the point \(P\) onto the \(x\)-\(y\) plane. Lastly, the coordinate \(\theta\) is the angle from the \(z\)-axis to the line defining \(r\).²

Now that we have the coordinates defined, give the relation between the Cartesian coordinate system and the spherical coordinate system. Find the mathematical equation that gives \(x, y, \) and \(z\) in terms of \(r, \theta, \) and \(\phi\).

\[x = \quad y = \quad z = \]

Spherical coordinates are convenient when dealing with objects that have spherical symmetry, like stars and static black holes. We will be using these coordinates quite a bit next week.

¹These coordinates are more traditionally labeled \(r\) and \(\theta\), however we want to be able to easily extrapolate the results here to three-dimensions (to spherical coordinates).

²Notational convention is one of the things that all have to struggle with when doing advanced research. Here, we have defined the three coordinates \(r, \theta, \phi\), which is the common physics convention. However, if you study math (or learn spherical coordinates from a math book) the convention is often the same but with the roles of \(\theta\) and \(\phi\) switched. Other authors will use \(\rho\) instead of \(r\) and other, less common, notations are sometimes used. A skill to develop is to quickly determine the convention so you can make sense of the math.