Choice-Screen Auctions

Michael Ostrovsky* Stanford University and NBER ostrovsky@stanford.edu

June 22, 2023

Abstract

Choice-screen auctions have been recently deployed in 31 European countries, allowing consumers to choose their preferred search engine on Google's Android platform instead of being automatically defaulted to Google's own search engine. I show that a seemingly minor detail in the design of these auctions—whether they are conducted on a "per appearance" or a "per install" basis—plays a major role in the mix and characteristics of auction winners and, consequently, in their expected market share. Furthermore, per install auctions distort search engines' incentives. Empirical evidence from Android choice-screen auctions conducted in 2020 is consistent with my theoretical results.

^{*}I am grateful to John Asker, Anirudha Balasubramanian, Jeremy Bulow, Jacques Crémer, Antoine Dubus, Megan Gray, Ginger Jin, Aurélien Mähl, Suraj Malladi, Preston McAfee, Katie McInnis, David Salant, Tim Schumacher, Andy Skrzypacz, Hal Varian, Frank Yang, anonymous referees, and the Editor, Jeff Ely, for helpful comments and suggestions.

1 Introduction

Optimal regulation of digital platforms is one of the thorniest issues in competition policy. A particularly challenging dimension for regulation is the fact that dominant platforms are often active in multiple distinct businesses and may leverage their position in one area into gaining an advantage in another. The net effect of such leverage on consumer welfare is often ambiguous and hard to determine. On one hand, a dominant platform's expertise and technological complementarities may make the adjacent product genuinely superior to the alternatives. On the other hand, such leverage may make it harder for other firms to successfully compete, even if their products, on their own, would be preferred by some consumers to that of the platform.

These linkages across product lines have led regulators to sometimes propose extreme measures to regulate large digital platforms, all the way to breaking them up and prohibiting them from entering certain lines of business. Notable examples in the U.S. include the Microsoft case of the late 1990s, in which the initial court decision was to break up the company, and the recently concluded congressional investigation into the business practices of Amazon, Apple, Facebook, and Google, which proposes "structural separations and line of business restrictions" as a solution for "restoring competition in the digital economy." Regulators in the European Union and other parts of the world have often reached similar conclusions. Of course, the breakup of a company is a very heavy-handed solution, difficult to implement, rife with potential unintended consequences, and, unsurprisingly, adamantly opposed by the digital platforms.

In light of these problems, platforms and regulators have, in some cases, adopted a more "lightweight" alternative as a compromise solution: choice screens. The logic of a choice screen is straightforward: instead of having the consumer use the dominant platform's product automatically and by default, the platform agrees to present the consumer with a menu of choices. This menu includes the platform's own product as one of the options, but also includes several competing products as alternatives. Consumers can then choose whichever products they prefer, leveling the playing field between the dominant platform and its competitors.

Choice screens for Web browsers on the Windows platform were first proposed by Microsoft in 1999 as a remedy in its negotiations with the U.S. Department of Justice.⁴ They were not adopted

¹United States v. Microsoft Corp., 97 F. Supp. 2d 59 (D.D.C. 2000), https://law.justia.com/cases/federal/district-courts/FSupp2/97/59/2339529/.

² "To address this underlying conflict of interest, Subcommittee staff recommends that Congress consider legislation that draws on two mainstay tools of the antimonopoly toolkit: structural separation and line of business restrictions. Structural separations prohibit a dominant intermediary from operating in markets that place the intermediary in competition with the firms dependent on its infrastructure. Line of business restrictions, meanwhile, generally limit the markets in which a dominant firm can engage." (Section VI.A.I, https://judiciary.house.gov/uploadedfiles/competition_in_digital_markets.pdf.)

³To give just one recent example (out of many available ones): "A government effort to break up Facebook Inc. from Instagram and WhatsApp would defy established law, cost billions of dollars and harm consumers, according to a paper company lawyers have prepared in the wake of rising antitrust legal threats. [...] In the paper, Facebook says unwinding the deals would be nearly impossible to achieve, forcing the company to spend billions of dollars maintaining separate systems, weakening security and harming users' experience." (https://www.wsj.com/articles/facebook-says-government-breakup-of-instagram-whatsapp-would-be-complete-nonstarter-11601803800.)

⁴https://www.wired.com/2000/11/microsoft-7/.

at that time, but were subsequently accepted as a compromise solution between the European Commission and Microsoft in 2009, and were displayed to users in Europe from 2010 until 2014.⁵ (Due to a technical error, the choice screen was not displayed on one of the versions of Windows from May 2011 to July 2012, affecting approximately 15 million users. Microsoft admitted its responsibility for this error and was subsequently fined €561 million.⁶) In 2017, Google reached a settlement with the competition authority in Russia to display choice screens for the default search engine on the Android platform there.⁷ A similar agreement was reached between Google and the European Commission following a €4.3 billion fine imposed on the company by the Commission in 2018,⁸ and Google began displaying choice screens for both default search engines and web browsers to Android users in Europe in 2019.⁹ An analogous solution is now being considered by the Australian Competition & Consumer Commission.¹⁰

Choice-screen menus can be an effective and powerful tool. For instance, following the 2010 introduction of browser choice-screen menus on the Windows platform in Europe, the number of downloads of Opera Software's web browser more than doubled. Discussing Google's introduction of choice-screen menus on Android, the European Commissioner for Competition, Margrethe

First, the tying of the Google Search app. As a result, Google has ensured that its Google Search app is pre-installed on practically all Android devices sold in the EEA. Search apps represent an important entry point for search queries on mobile devices. The Commission has found this tying conduct to be illegal as of 2011, which is the date Google became dominant in the market for app stores for the Android mobile operating system.

Second, the tying of the Google Chrome browser. As a result, Google has ensured that its mobile browser is pre-installed on practically all Android devices sold in the EEA. Browsers also represent an important entry point for search queries on mobile devices and Google Search is the default search engine on Google Chrome. The Commission found this tying conduct to be illegal as of 2012, which is the date from which Google has included the Chrome browser in its app bundle." (https://ec.europa.eu/commission/presscorner/detail/en/IP_18_4581.)

⁵ "Under the commitments approved by the Commission, Microsoft will make available for five years in the European Economic Area [...] a "Choice Screen" enabling users of Windows XP, Windows Vista and Windows 7 to choose which web browser(s) they want to install in addition to, or instead of, Microsoft's browser Internet Explorer.

The Commission's preliminary view was that competition was distorted by Microsoft tying Internet Explorer to Windows. This was because it offered Microsoft an artificial distribution advantage not related to the merits of its product on more than 90 per cent of personal computers. Furthermore, the Commission's preliminary view was that this tying hindered innovation in the market and created artificial incentives for software developers and content providers to design their products or web sites primarily for Internet Explorer.

The approved commitments address these concerns. PC users, by means of the Choice Screen, will have an effective and unbiased choice between Internet Explorer and competing web browsers. This should ensure competition on the merits and allow consumers to benefit from technical developments and innovation both on the web browser market and on related markets, such as web-based applications." (https://ec.europa.eu/commission/presscorner/detail/en/IP_09_1941.)

⁶https://ec.europa.eu/commission/presscorner/detail/en/IP_13_196.

⁷https://www.reuters.com/article/us-alphabet-google-russia-idUSKBN17J11C, https://yandex.com/blog/yacompany-com/choosing-yandex-search-on-android.

⁸ "The Commission decision has concluded that Google has engaged in two instances of illegal tying:

⁹https://www.blog.google/around-the-globe/google-europe/presenting-search-app-and-browser-options-android-users-europe/.

¹⁰ "The ACCC is seeking submissions from consumers and industry participants about choice screens, which give users a choice of internet search services on mobiles and tablets, rather than a pre-selected search service, and about the supply of web browsers in Australia." (https://www.accc.gov.au/media-release/feedback-sought-on-choice-and-competition-in-internet-search-and-web-browsers.)

¹¹https://press.opera.com/2010/03/18/opera-more-than-doubles-download-numbers-in-europe-after-choice-screen-introduction/.

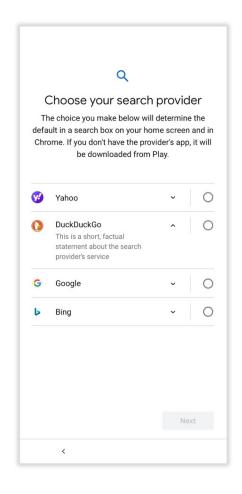


Figure 1: Android Choice Screen

Vestager, stated, "We've seen in the past that a choice screen can be an effective way to promote user choice." However, from the point of a company that owns the platform, the initial implementations of choice-screen menus suffered from one serious shortcoming: zero revenue. This may be a particularly salient issue in the case of search engines. First, being chosen by a consumer is extremely valuable to a search engine due to the advertising revenues it expects to receive when the consumer uses it. Second, the dominant company itself may be making large payments to another platform to have consumers use its search engine there. In this case, it is logical for the company to argue that it should be allowed to charge others for the right to have their products be shown on its platform's choice screens—and a natural way to do so is via an auction.

¹²https://ec.europa.eu/commission/presscorner/detail/en/STATEMENT_19_1774.

¹³While the companies do not directly disclose these numbers, analysts estimate that Google is paying Apple on the order of \$8-\$12 billion per year to have Google be the default search engine on Apple's Safari browser (https://www.npr.org/2020/10/22/926290942/google-paid-apple-billions-to-dominate-search-on-iphones-justice-department-says). It is worth noting that this agreement itself has also been a subject of recent regulatory scrutiny (https://www.justice.gov/opa/press-release/file/1328941/download). Similarly, Google is paying an estimated \$400-\$450 million per year to Mozilla to be the default search engine on the Firefox browser (https://www.zdnet.com/article/sources-mozilla-extends-its-google-search-deal/).

That is the decision that Google announced in August 2019,¹⁴ and the first "choice-screen auctions" took place in early 2020 (see Figure 1 for an illustration of an Android choice screen), with the subsequent auctions run on a quarterly basis. The basic rules of Google's choice-screen auctions are very simple.

In each country auction, search providers will state the price that they are willing to pay each time a user selects them from the choice screen in the given country. The three highest bidders will appear in the choice screen for that country. The provider that is selected by the user will pay the amount of the fourth-highest bid.¹⁵

In the same document, Google explains why it chose to auction off slots in the choice screen this way:

Q: Why does Google use an auction to determine the search providers that appear in the choice screen?

A: An auction is a fair and objective method to determine which search providers are included in the choice screen. It allows search providers to decide what value they place on appearing in the choice screen and to bid accordingly.

The auction revenues help us to continue to invest in developing and maintaining the Android platform.

In this paper, I show that a seemingly minor detail of the implementation of choice-screen auctions plays a major role in their outcomes—and thus in the overall effectiveness of the antitrust remedy. Specifically, while the answer in the Q&A section of the document states that an auction "allows search providers to decide what value they place on appearing in the choice screen and to bid accordingly," the auction, as implemented, charges these providers not for appearing in the choice screen but for being chosen by a user.

While the difference may seem to be just a matter of language, it is not. To see the intuition for the difference, consider a version of the auction with just one available spot and two bidders. Bidder A gets revenue \$10 from each user who installs its search engine, and if it is shown as an option in the choice screen, then the probability that a user will choose it is 10%. Bidder B gets revenue \$20 from each user who installs its search engine, but the probability that a user will choose it (if it is shown as an option in the choice screen) is only 1%. The value that bidder A has for appearing on the screen is therefore \$1, and the value that bidder B has for appearing on the screen is \$0.20. Thus, if the auction is conducted on the "per appearance" basis, then bidder A will win, will pay \$0.20 per appearance, and will have its search engine chosen by users 10% of the time, while the dominant platform's own search engine will be chosen 90% of the time. If, instead, the auction is conducted as implemented, with bidding and payment on the "per install" basis, then bidder B will win and will pay \$10 every time its search engine is chosen (corresponding to \$0.10

 $^{^{14} {\}tt https://www.blog.google/around-the-globe/google-europe/update-android-search-providers-europe/.}$

 $^{^{15}}$ https://web.archive.org/web/20200501202513/https://www.android.com/choicescreen/.

per appearance). The winner's search engine will be chosen only 1% of the time, and the dominant platform's one will be chosen the remaining 99% of the time. Thus, relative to the per appearance auction, the per install auction results in a lower likelihood that an alternative search engine will be chosen by the user (making it correspondingly more attractive to the dominant platform) and gives advantage to search engines that generate higher revenue per user vs. those that are more popular but generate less revenue on a per-user basis. In Section 2, I show that these conclusions hold more generally, in a basic model in which alternative search engines differ on two dimensions: revenue per user (i.e., how much revenue the search engine generates, on average, when a user chooses to install it) and popularity (i.e., the likelihood that the search engine will be chosen if it is shown to the user in the choice screen). Moreover, I show that the difference is exacerbated by competition. As the number of alternative search engines grows, under the per appearance auction, the expected popularity of the winner also grows, so the probability that the dominant platform's own search engine is chosen decreases. By contrast, under the per install auction, these measures are not affected by the number of bidders.

In the example above and in the model of Section 2, a search engine's popularity and revenue per user (RPU) are fixed. In practice, a search engine has some ability to trade them off against each other. For instance, a search engine may choose to show more intrusive ads, increasing its revenue per user but decreasing its popularity. Conversely, a search engine may donate some of its proceeds to charity or implement very strict privacy rules, lowering its revenue per user but increasing the probability that a user will choose it. I introduce this possibility in Section 3 and show that the two auction formats result in very different incentives to the search engines regarding this tradeoff. Under the per appearance auction, each bidder chooses the same point on the popularity-RPU frontier as it would choose if it were the only bidder (and were thus guaranteed the spot on the choice screen). In particular, this implies that just as in the model of Section 2, as the number of competitors increases, the expected popularity of the winner also increases, and the expected probability that the dominant platform's search engine is chosen goes down. By contrast, under the per install auction, each bidder has a strong incentive to distort the choice toward higher revenue per each user who chooses the product, at the expense of lowering the probability of actually being chosen. This distortion grows stronger as the number of bidders grows. In the limit, as that number approaches infinity, the distortion results in a "race to the bottom," with all bidders pushing to the extreme point on the popularity-RPU frontier: the highest possible RPU and the lowest possible popularity. As a result, the expected popularity of the winner goes in the opposite direction vs. that in the case of the per appearance auction, minimizing the probability that an alternative search engine will be chosen.

In Section 4, I present empirical evidence using publicly available data from the four sets of choice-screen auctions (for the periods of March–June 2020, July–September 2020, October–December 2020, and January–March 2021) conducted by Google in 2020 in 31 European countries, as well as various statistics publicly available on Google Play Store on the numbers of downloads and ratings of various search engine apps. The evidence is consistent with my theoretical conclusions.

In particular, to mention just one data point from the section, the search engine that was most successful in these auctions, winning a slot in every country and in every period, has been installed only approximately 100,000 times worldwide, has only 74 user reviews in Google's Play Store, and has one of the lowest ratings among the search engines participating in choice-screen auctions. For the sake of comparison, Google's own search app has been installed more than 5 billion times, while alternative search engines Bing (produced by Microsoft) and DuckDuckGo (produced independently and focused on user privacy) have been installed more than 10 million times. (These numbers include all installs, including those that come from choice screens and those that do not.)

Subsequent to the circulation of the initial draft of this paper, Google has announced the decision to discard the use of choice-screen auctions and instead determine the list of providers shown in the choice screen by their overall popularity. ¹⁶ In Section 5, I conclude by discussing this decision and other issues related to the design of choice-screen auctions.

1.1 Related Literature

Two recent surveys by Crémer et al. (2019) and Scott Morton et al. (2019) provide extensive discussions of the challenges of regulating digital platforms, potential remedies, and other related issues. Salinger (2020) reviews the economics of self-preferencing. On the specific issue of choice screens (without auctioning off the slots), Economides and Lianos (2011) provide a discussion of the 2009 Microsoft–EU agreement regarding the Windows platform and the Internet Explorer web browser.

On the issue of search engine monetization, see Edelman et al. (2007) and Varian (2007). These papers also contain discussions of search engines adjusting advertisers' "per click" bids by their estimated probabilities of being clicked, essentially transforming those "per click" auctions into "per appearance" ones. See also Varian (2008) for a discussion of reasons why search engines adjust "per click" bids in ad auctions by the click-through rates, instead of ranking the ads solely by "per click" bids. This distinction is conceptually similar to the difference between the "per install" and "per appearance" formats in choice-screen auctions that I discuss in the current paper. There are, however, several important differences. First, there is a matter of degree. In sponsored search auctions, the "per click" format, while imperfect, has worked successfully at the ad auction pioneer GoTo (subsequently renamed Overture and then acquired by Yahoo) for almost a decade, from 1998 until 2007. By contrast, the failure of the "per install" choice-screen auction format was so dramatic that the entire auction was scrapped after barely a year, despite initially getting the blessing of both Google and the European Commission. Second, the auctioneer's incentives are very different in the sponsored search setting vs. the choice-screen one: in the latter, the auctioneer places a high value on the outcome in which the winning bidder's listing is not chosen by the end

¹⁷See also the discussion of practical implementation of per appearance auctions, and the connections to sponsored search auctions, in Section 5.

¹⁸https://www.nytimes.com/2007/02/05/technology/05yahoo.html.

user, which is not the case in the former setting. Finally, in the choice-screen auction setting, I explore the important issue of search engines' endogenous choice of popularity vs. revenue per user, and show how the per install auction distorts this decision (with the distortion growing larger as the number of bidders grows), while the per appearance auction does not.

The "non-distortionary" property of the per appearance auction is not a coincidence. In the choice-screen setting, the per appearance auction can be viewed as an implementation of the Vickrey-Clarke-Groves mechanism (Vickrey, 1961; Clarke, 1971; Groves, 1973), with the right to be shown on the choice screen being the object auctioned off. VCG is known to preserve various incentives (e.g., pre-auction investment or information acquisition), even in cases in which other auction formats may not (Rogerson, 1992; Bergemann and Välimäki, 2002; Arozamena and Cantillon, 2004; Hatfield et al., 2014, 2018). As I show in Section 3.1, the per appearance auction in the choice-screen setting likewise has this non-distortionary property.

2 Basic Model: Exogenous Popularity and Revenue-per-User

A platform is auctioning off the right to be shown on the choice screen. There is one slot available, next to the platform's own product.¹⁹ There are n = 2 bidders, $i \in \{1, 2\}$. Each bidder i has an exogenously determined popularity q_i and if its product is chosen by a user, then bidder i receives revenue r_i from that. Variables q_i and r_i are private information of bidder i. Variables q_1 , q_2 , r_1 , and r_2 are independently and identically distributed, and each is drawn from the uniform distribution on [0, 1].²⁰

If a product of popularity q is shown to a user, then it is chosen with probability q. With probability 1-q, the platform's own product is chosen instead, in which case the platform gets benefit $\pi > 1$.²¹

Under the "per appearance" auction, each bidder submits a bid for the right to be shown to users. The bidder with the highest bid wins, is shown on the choice screen next to the platform's own product, and pays the amount equal to the bid of the second-highest bidder.

Under the "per install" auction, each bidder submits a bid. The bidder with the highest bid wins, is shown on the choice screen next to the platform's own product, and pays the amount equal to the bid of the second-highest bidder if the user chooses its product.

Note that both auction formats are incentive-compatible: it is a dominant strategy for each bidder to submit its valuation truthfully. That is, under the "per install" auction, each bidder i will bid r_i , while under the "per appearance" auction, each bidder i will bid $q_i r_i$.²²

To characterize the distribution of outcomes in the per appearance auction, we need to perform

¹⁹In Google's Android choice-screen auctions, there are three slots next to the platform's own listing. I consider the case of only one alternative slot for simplicity; this assumption does not qualitatively change my conclusions.

²⁰This assumption is made for the ease of exposition. In Appendix A.1, I extend the results of the current section to the case of more general distributions.

²¹I.e., the platform's most preferred outcome is to have its own product chosen by a user; after all, if that was not the case, there would be no need for the antitrust remedy.

²²I will ignore other equilibria of these auctions.

some calculations.

First, observe that the unconditional distribution of each bid $b_i = q_i r_i$ is given by the CDF $G(x) = x - x \ln x$ and the corresponding density function $g(x) = -\ln x$ (for $x \in (0,1]$).²³ Thus, for a bidder with type (q,r), the probability of winning the auction is $(qr) - (qr) \ln(qr)$. For the population of bidders with type q and with types r distributed uniformly on [0,1], the probability of winning is therefore $\int_0^1 ((qr) - (qr) \ln(qr)) dr = q \int_0^1 (r - r \ln q - r \ln r) dr = q \left(\frac{1}{2} - \frac{1}{2} \ln q + \frac{1}{4}\right) = \frac{3}{4}q - \frac{1}{2}q \ln q$.²⁴ We can now calculate the expected popularity of the winner of the auction, which is equal to $2 \int_0^1 q \left(\frac{3}{4}q - \frac{1}{2}q \ln q\right) dq = \int_0^1 \frac{3}{2}q^2 dq - \int_0^1 q^2 \ln q dq = \frac{1}{2} + \frac{1}{9} = \frac{11}{18}$.²⁵

The expected payment made by the winner of the auction is equal to $E[\min\{q_1r_1,q_2r_2\}]$. Given the distribution $G(\cdot)$ of each q_ir_i derived above, the distribution of $\min\{q_1r_1,q_2r_2\}$ is given by $G(x)^2 + 2G(x)(1 - G(x))$, with the corresponding density $2g(x) - 2g(x)G(x) = -2\ln x(1 - x + x \ln x)$. Thus, $E[\min\{q_1r_1,q_2r_2\}] = -2\int_0^1 (x \ln x(1 - x + x \ln x)) dx = \frac{7}{54}$, and the expected payoff of the platform is $\frac{7}{54} + \frac{7}{18}\pi$.

Characterizing the outcomes of the per install auction is straightforward. Under this format, the expected popularity of the winner is 1/2—it is independent of the bids in the auction and plays no role in determining the winner. This is lower than the expected popularity of the winner in the per appearance auction $(\frac{11}{18})$, and thus the probability that an alternative product will be chosen is reduced and the probability that the dominant platform's product is chosen is increased.

The expected per install payment made by the winner is 1/3 (and is independent of its popularity), and the expected payoff of the platform in the per install auction is therefore $1/6 + 1/2\pi$, which is higher than its expected payoff in the per appearance auction $(\frac{7}{54} + \frac{7}{18}\pi)$.

The contrast between the outcomes of the per appearance and per install auctions becomes even more striking if the number of bidders, n, becomes large; i.e., the number of potential alternative products grows. It is immediate that as n grows, the expected popularity of the winner of the per appearance auction converges to one: the highest possible popularity. By contrast, the expected popularity of the winner of the per install auction remains unchanged, at one half; under that format, the increase in the competition has no impact on the popularity of the winner, and thus on the probability that an alternative product will be chosen. Of course, if the platform's payoff from having a user choose its own product is higher than the revenue that an alternative product generates, this outcome is preferred by the platform, just as it was in the case of n = 2.

For $x \in [0, 1]$, the probability that $q_i r_i$ is less than or equal to x is equal to $G(x) = x + \int_x^1 \frac{x}{q} dq = x + x(\ln 1 - \ln x) = x - x \ln x$.

²⁴This calculation uses the fact that $\int x \ln x = \frac{1}{2}x^2 \ln x - \frac{x^2}{4} + c$. In the next step, we will also use the fact that $\int x^2 \ln x = \frac{1}{3}x^3 \ln x - \frac{x^3}{9} + c$.

²⁵The logic behind this formula is that by symmetry, (the expected popularity of the winner of the auction) is equal to (the expected popularity of the winner of the auction conditional on that winner being bidder 1). The latter expression is equal to $\frac{\int_0^1 q_1 Prob(\text{bidder 1 is the winner of the auction}|q_1)dq_1}{Prob(\text{bidder 1 is the winner of the auction})}$, whose denominator is equal to $\frac{1}{2}$.

3 Extension: Endogenous Popularity and Revenue-per-User

It is clear from the results of Section 2 that the choice between per install and per appearance auction formats is a first-order issue. However, that only tells a part of the story. In this section, I consider the incentives of bidders to choose between making their product more attractive to users (at the expense of lower per-user revenue) vs. moving in the opposite direction. This is an important issue for the case of default search engines on a platform. Search engines may be able to increase their popularity by reducing the intrusiveness of ads, enhancing privacy protections, or donating their advertising revenue to charitable causes. These measures, while making the search engine more attractive to users, reduce the revenue it makes from each one of them. In this section, I show that the issues discussed in Section 2 are exacerbated once these incentives are taken into account; in some cases, dramatically so.

I maintain most of the assumptions of Section 2 and make only one change. Instead of assuming that each bidder i has an exogenously given popularity q_i and revenue-per-user r_i , I assume that each bidder has an exogenously given type t_i drawn independently from the uniform distribution F(t) on [0,1] (with the corresponding density function f(t)). The bidder can then select its popularity q_i from $[0,t_i]$ and its revenue-per-user is then equal to $r_i = t_i - q_i$. After making this decision, the bidder submits its auction bid as before.

Bidding decisions are still straightforward under both rules: it is optimal to bid truthfully in both per appearance and per install auctions. However, each bidder now needs to decide, given its type t_i , how much of that type to allocate to popularity q_i and how much to allocate to revenue-per-user. I work out the resulting equilibria in the next two subsections.

3.1 Per Appearance Auction

Consider a per appearance auction with n bidders, and suppose bidder i has type $t_i \in [0,1]$. Fix other bidders' strategies, let G(x) denote the distribution of the first-order statistic of those bidders' bids, and let P(x) denote the expected payment that bidder i would make, conditional on winning the auction, if it submitted bid x (note that G(x) and P(x) are purely functions of the other bidders' strategies and x). Bidder i has two decisions to make: popularity q_i and bid b_i . Its payoff as a function of these two decisions is given by

$$\Pi(q_i, b_i) = G(b_i) \times (q_i(t_i - q_i) - P(b_i)).$$

It is immediate that bidder i's optimal choice of popularity is to set

$$q_i = \frac{t_i}{2}. (1)$$

This is an optimal strategy regardless of what other bidders' strategies are (or how many of those bidders there are). Thus, the strategy profile in which each bidder sets $q_i = t_i/2$ and then bids

²⁶In Appendix A.2, I extend the results of the current section to the case of more general distributions.

 $(t_i/2)^2$ per appearance constitutes an equilibrium.

3.2 Per Install Auction

Equilibrium characterization in the case of the per install auction requires a more involved argument. Consider a symmetric equilibrium of the per install auction with n bidders, and suppose equilibrium strategies are given by functions q(t) and b(t), with the first one denoting the popularity chosen by a bidder with type t and the second one denoting its bid. We know that in equilibrium, function b(t) will be truthful (given the choice of popularity q(t) and the corresponding revenue-per-user t - q(t)); however, just as in the case of the per appearance auction, it is more convenient to not yet impose that restriction on function b(t).

Take a bidder of type $t_i \in (0,1)$ and a real number Δ_q such that $q(t_i) + \Delta_q \in (0,t_i)$. Let $\Pi(\Delta_q; t_i)$ denote the expected payoff of bidder i whose type is t_i if it chooses popularity $q(t_i) + \Delta_q$ but bids $b(t_i)$, given that other bidders are bidding according to strategies q(t) and b(t). We then have

$$\Pi(\Delta_q; t_i) = F^{n-1}(t_i) \times (q(t_i) + \Delta_q) \times \left(t_i - (q(t_i) + \Delta_q) - E\left[b(\max_{j \neq i} \{t_j\}) | \max_{j \neq i} \{t_j\} \le t_i \right] \right).$$

Because we started with an equilibrium profile of strategies, the partial derivative of $\Pi(\Delta_q; t_i)$ with respect to Δ_q has to be equal to zero when evaluated at $(0; t_i)$. This implies the following equation for $q(t_i)$:

$$q(t_i) = \frac{t_i - E[b(\max_{j \neq i} \{t_j\}) | \max_{j \neq i} \{t_j\} \le t_i]}{2}.$$
 (2)

Note that even without fully characterizing the equilibrium, from the comparison of equations (1) and (2) it is immediate that equilibrium popularity chosen by each type $t_i > 0$ will be strictly lower under the per install auction than under the per appearance auction (assuming, of course, that the equilibrium of the per install auction actually exists, which we will show below).

Recall that by incentive compatibility, we have $b(t_j) = t_j - q(t_j)$. We can then rewrite the expectation in equation (2) as

$$E\left[b(\max_{j\neq i}\{t_j\})|\max_{j\neq i}\{t_j\} \le t_i\right] = \frac{\int_0^{t_i}(s-q(s))dF^{n-1}(s)}{F^{n-1}(t_i)}$$
$$= \frac{(n-1)\int_0^{t_i}(s-q(s))f(s)F^{n-2}(s)ds}{F^{n-1}(t_i)},$$

and subsequently rewrite equation (2) as

$$2q(t_i)F^{n-1}(t_i) = t_iF^{n-1}(t_i) - (n-1)\int_0^{t_i} (s - q(s))f(s)F^{n-2}(s)ds.$$
(3)

The next step is to take a derivative of both sides of equation (3) with respect to t_i , which gives

$$\begin{aligned} 2q'(t_i)F^{n-1}(t_i) + 2(n-1)q(t_i)f(t_i)F^{n-2}(t_i) &= F^{n-1}(t_i) + (n-1)t_if(t_i)F^{n-2}(t_i) \\ &- (n-1)t_if(t_i)F^{n-2}(t_i) + (n-1)q(t_i)f(t_i)F^{n-2}(t_i), \end{aligned}$$

which simplifies to

$$2q'(t_i) + (n-1)q(t_i)\frac{f(t_i)}{F(t_i)} = 1.$$
(4)

Equation (4) is a first-order linear differential equation, with the initial condition q(0) = 0. In our case, $F(t_i) = t_i$ and $f(t_i) = 1$, and so the equation becomes

$$2q'(t_i) + (n-1)q(t_i)\frac{1}{t_i} = 1,$$

with the solution

$$q(t_i) = \frac{t_i}{n+1}.$$

3.3 Comparison

The comparison between the two formats is immediate. Even with just two bidders, n = 2, a bidder of each type t_i chooses a much lower popularity under the per install auction than under the per appearance auction: $\frac{t_i}{3}$ vs. $\frac{t_i}{2}$. Of course, this lower popularity on a per type basis immediately translates into a correspondingly lower overall probability that a product alternative to the dominant platform's one will be picked by users from the choice screen.

As the number of bidders grows, the expected popularity of the winner of the per appearance auction also grows: with n bidders, it is equal to $\frac{n}{2(n+1)}$, which is increasing in n.²⁷ By contrast, the expected popularity of the winner of the per install auction moves in the *opposite* direction, falling as the number of bidders grows: with n bidders, it is equal to $\frac{n}{(n+1)^2}$, which is decreasing in n.²⁸

In the limit, as $n \to \infty$, the difference becomes particularly dramatic. Under the per appearance auction format, the popularity of the winning bidder converges to $\frac{1}{2}$. By contrast, under the per install format, as $n \to \infty$, the popularity of the winner converges to zero; therefore, the share of installs that goes to the dominant platform's own product converges to 100%, completely undoing the choice-screen auction's raison d'être.

²⁷It is helpful to recall that with n independent draws from the uniform distribution, the expected value of their maximum is $\frac{n}{n+1}$.

²⁸The derivative of function $f(x) = \frac{x}{(x+1)^2}$ is equal to $f'(x) = \frac{(x+1)^2 - 2(x+1)x}{(x+1)^4} = \frac{1-x}{(x+1)^3}$, which is less than zero for x > 1.

4 Evidence from Android Choice-Screen Auctions

I now turn to the empirical evidence on the outcomes of Google's choice-screen auctions for default search engines on the Android platform conducted in 2020. These auctions took place in January 2020 (for the period from March to June), June 2020 (for the period from July to September), September 2020 (for the period from October to December), and December 2020 (for the period from January to March 2021). For each time period, 31 independent auctions were conducted (one per country). These auctions were conducted on a per install basis, with those submitting top three bids being shown on the choice screen and paying the fourth-highest bid every time a user chose one of them from the choice screen. In the event of a tie, more than three bidders could win, with the ties broken randomly on a per device basis.

Historical data on the winners of these auctions are publicly available on the Internet Archive at https://web.archive.org/web/*/https://www.android.com/choicescreen-winners/. These results, on a country-by-country basis, are summarized in Table 1. In each search engine—country cell in the table, I list the periods in which that search engine won a slot on the choice-screen menu in that country. Countries are sorted by population, while search engines are sorted by the average population they won across the four time periods. E.g., the search engine DuckDuckGo won a slot in every country in the first period (total population 519.4 million) and also in the second one (same total population), but only four countries in period 3 (with the total population 11.4 million) and one country in period 4 (population 11.6 million). The average population won by DuckDuckGo across the four time periods is thus (519.4 + 519.4 + 11.4 + 11.6)/4 = 265.5 million, as listed in the last row of Table 1. Sorting the search engines by the total number of auctions won (next-to-last row in the table), i.e., giving equal weight to countries of different sizes, would result in an almost identical ordering.

Table 2 lists the search engines by the average population they won across the four periods, includes again the total number of auctions they won, and adds data collected from the Android Play Store (https://play.google.com/store/apps) on the popularity and quality of these search engines.²⁹ To be eligible to participate in a choice-screen auction, a "search provider must have an app that is available for free in Google Play" and if a search engine is chosen by a user from the choice screen, "[that app] will be downloaded from Play" in addition to the search engine being set as the default in the Chrome browser on the user's device (https://www.android.com/choicescreen/). Of course, users can install these apps even without the choice screen (as they do, e.g., outside of Europe); moreover, the numbers listed on Play Store count the number of installs worldwide, not just in Europe. Thus, while the numbers of downloads resulting from choice-screen auctions are not publicly disclosed by Google or the European Commission, the install numbers from Play Store listed in Table 2 provide upper bounds on those numbers.³⁰ These install numbers, of course, also

²⁹I collected the data on March 1, 2021—exactly one year after the first day of the first choice-screen auction period. The screenshots of Play Store pages from which these data were taken are available upon request.

³⁰Note that Play Store does not disclose the exact number of installs but instead shows a range, which is what Table 2 reports.

Table 1: Winners of Android Choice-Screen Auctions (by country)

Country	Population	Info.com	PrivacyWall	DuckDuckGo	Bing	GMX	Yandex	Qwant	Seznam	Givero	Ecosia
Germany	83,990,646	1, 2, 3, 4	2, 3, 4	1, 2	3, 4	1					
United Kingdom	67,999,326	1, 2, 3, 4	2, 3, 4	1, 2	1, 3, 4						
France	65,297,182	1, 2, 3, 4	2, 3, 4	1, 2	3, 4			1			
Italy	60,457,546	1, 2, 3, 4	2, 3, 4	1, 2	3, 4			1			
Spain	46,789,532	1, 2, 3, 4	2, 3, 4	1, 2	3, 4			1			
Poland	37,847,219	1, 2, 3, 4	3, 4	1, 2		2, 3, 4	1				
Romania	19,196,044	1, 2, 3, 4	1, 3, 4	1, 2		2, 3, 4	1, 3, 4				
Netherlands	17,142,323	1, 2, 3, 4	2, 3, 4	1, 2	3, 4	1					
Belgium	11,608,284	1, 2, 3, 4	2, 3, 4	1, 2, 4	3			1			
Czech Republic	10,717,516	1, 2, 3, 4	3, 4	1, 2					1, 2, 3, 4		
Greece	10,415,204	1, 2, 3, 4	1, 3, 4	1, 2		2, 3, 4	1, 3, 4	1			
Portugal	10,190,296	1, 2, 3, 4	1, 4	1, 2		2, 3, 4	1, 3	1			
Sweden	10,114,623	1, 2, 3, 4	1, 2, 3, 4	1, 2	3, 4						
Hungary	9,657,366	1, 2, 3, 4	1, 3, 4	1, 2		2, 3, 4	1, 3, 4				
Austria	9,032,162	1, 2, 3, 4	2, 3, 4	1, 2	3	1, 4					
Bulgaria	6,938,828	1, 2, 3, 4	1, 3, 4	1, 2, 3		2, 3, 4	1, 3, 4				
Denmark	5,795,666	1, 2, 3, 4	1, 2, 3, 4	1, 2	3, 4		1	1		1	
Finland	5,543,674	1, 2, 3, 4	2, 3, 4	1, 2	3, 4		1				
Slovakia	5,461,816	1, 2, 3, 4	4	1, 2		3			1, 2, 3, 4		
Norway	5,428,345	1, 2, 3, 4	1, 2, 3, 4	1, 2	3, 4						
Ireland	4,960,177	1, 2, 3, 4	1, 2, 3, 4	1, 2	3, 4						
Croatia	4,099,199	1, 2, 3, 4	1, 4	1, 2, 3		2, 3, 4	1, 4				
Lithuania	2,710,479	1, 2, 3, 4	3, 4	1, 2		2, 3, 4	1				
Slovenia	2,079,635	1, 2, 3, 4	1	1, 2		2, 3, 4	1				3, 4
Latvia	1,881,006	1, 2, 3, 4	3, 4	1, 2		2, 4	1, 3, 4				
Estonia	1,328,929	1, 2, 3, 4		1, 2		2, 3, 4	1, 3, 4				
Republic of Cyprus	1,190,962	1, 2, 3, 4	1	1, 2		2, 3, 4	1, 3, 4				
Luxembourg	629,798	1, 2, 3, 4		1, 2		3, 4		1, 2, 3, 4			
Malta	514,564	1, 2, 3, 4	1, 3, 4	1, 2		2, 3, 4	1				
Iceland	341,834	1, 2, 3, 4	1, 4	1, 2, 3		2, 3, 4	1				
Liechtenstein	38,150	1, 2, 3, 4	4	1, 2, 3		2, 3, 4		1			
# Auctions won		124	76	67	25	51	32	12	8	1	2
Average pop. (M)	519.4	519.4	373.3	265.5	208.9	112.3	58.8	53.3	16.2	1.4	1.0

Table 2: Popularity and Ratings of Android Search Engine Apps

Search Engine	Av. Pop.	# Auct. Won	# Installs	Rating	# Reviews	
info.com	519.4	124	100,000+	4.1	74	
PrivacyWall	373.3	76	100,000+	4.2	378	
DuckDuckGo	265.5	67	10,000,000+	4.8	1,014,301	
Bing	208.9	25	10,000,000+	4.4	187,314	
GMX	112.3	51	10,000+	4.5	19	
Yandex	58.8	32	100,000,000+	4.5	1,042,638	
Qwant	53.3	12	1,000,000+	3.8	10,968	
Seznam	16.2	8	1,000,000+	4.3	67,137	
Givero	1.4	1	100+	n/a	n/a	
Ecosia	1.0	2	5,000,000+	4.6	125,723	

provide a measure of the overall popularity of these search engines by including the installs that were made independently of choice-screen auctions. 31

Table 2 also includes the ratings from Android users on the quality of these apps, as well as the number of user reviews. Play Store requires a minimum number of installs, ratings, and reviews before it starts disclosing this information. One of the winners of choice-screen auctions did not pass this bar and, thus, does not have average ratings reported by the Play Store platform (indicated as "n/a" in the table).

Consistent with the theoretical results from Section 2, there is no meaningful correlation between the quality and popularity of the search engines participating in these auctions and the average population or the number of auctions they won in them.³² Strikingly, of the top five winners, each of whom has on average won a slot in choice-screen auctions covering a population of more than 100 million, three had on the order of only 100,000 installs or less (from all sources, worldwide, over their lifetimes), and had very small numbers of user reviews (74 for info.com, 378 for PrivacyWall, and 19 for GMX—compared with, e.g., one million reviews for DuckDuckGo or 187 thousand reviews for Bing). The two largest winners, info.com and PrivacyWall, had some of the lowest user ratings among the search engines available (4.1 and 4.2, respectively, vs., e.g., 4.8 for DuckDuckGo or 4.6 for Ecosia—the two most highly rated search engines). These two biggest winners of choice-screen auctions also do not show up in the aggregate market share statistics for search engines on mobile platforms in Europe, despite winning very large populations in choice-screen auctions for an entire year (519 million people in the case of info.com and 373 million in the case of PrivacyWall).³³

³¹One caveat on this measure is that the search engines' popularity may vary by geography. For example, the majority of users of the Yandex search engine are from Russia and countries of the former Soviet Union.

³²Ideal data to check this correlation would include the actual choice probabilities from choice screens displayed to the users and the actual bids submitted by the search engines to the auctioneer. Unfortunately, these data are not available; they are proprietary and confidential. Instead, I use publicly available proxies for these measures: I use the average population and the number of auctions won by these various search engines as a proxy for their average value per user and the numbers of installs and reviews as a measure of popularity.

³³Data on aggregate market share statistics for search engines on mobile platforms in Europe are available at https://gs.statcounter.com/search-engine-market-share/mobile/europe. In February 2021, the following

Regarding the theoretical results of Section 3 on endogenous quality choice, I do not have direct evidence on the impact of the incentives from the auction on the actual design choices that search engines make to trade off revenue they make per each user vs. their popularity and quality. The time period of only a year may also be insufficient for these design changes to manifest themselves. However, these incentives are clearly understood (and lamented) by at least some of the participants in these auctions.

The most highly rated search engine, DuckDuckGo, won a slot in every country in periods 1 and 2, but then by period 3 only won slots in four countries covering in total only 2% of the available population, and then won a slot in only one country in period 4. As explained by DuckDuckGo, "Despite DuckDuckGo being robustly profitable since 2014, we have been priced out of this auction because we choose to not maximize our profits by exploiting our users. In practical terms, this means our commitment to privacy and a cleaner search experience translates into less money per search. This means we must bid less relative to other, profit-maximizing companies." ³⁴

The second most highly rated search engine is Ecosia, which uses its profits to plant trees around the world (https://info.ecosia.org/). Despite its overall popularity with users (more than 5,000,000 installs and more than 100,000 user reviews), it only won coverage in the last two periods and in only one country (covering less than 0.4% of overall available population). Ecosia explained its decision not to participate in the initial auction (for the first time period) as follows: "Ecosia is a not-for-profit search engine. Taking part in Google's auction would force us to spend our income on an unnecessary bidding war with other (profit-oriented) search engines. We'd rather use it to plant trees on our endangered planet." 35

5 Concluding remarks

On June 8, 2021, Google announced the decision to discontinue the use of choice-screen auctions, saying, "Following further feedback from the Commission, we are now making some final changes to the Choice Screen including making participation free for eligible search providers." ^{36,37} While Google and the European Commission did not further elaborate on the details of the Commission's feedback and reasoning, the makeup of the winners of choice-screen auctions, and the fact that some virtually unknown and unpopular search engines have won most of the positions in them, likely played an important role in them.

search engines (of those participating in choice-screen auctions) had nonnegligible market shares: Yandex (1.34%), Bing (0.51%), DuckDuckGo (0.38%), Ecosia (0.22%), Seznam (0.07%), and Qwant (0.04%).

³⁴https://spreadprivacy.com/search-preference-menu-duckduckgo-elimination/.

³⁵https://blog.ecosia.org/google-auction-choice-screen/.

 $^{^{36} \}mathrm{https://blog.google/around-the-globe/google-europe/changes-android-choice-screen-europe/.}$

³⁷The specifics of how Google will list the search engines on the choice screen starting September 2021 are as follows: (i) "The five most popular eligible general search services in each country according to StatCounter (including Google) will be displayed at the top, ordered randomly each time the choice screen is shown." and (ii) "Up to seven remaining eligible general search services will be shown below the initial five services, similarly ordered randomly. In the event there are more than seven remaining general search services in a given country, the seven services to be shown on the choice screen in that country will be selected at random each time the choice screen is displayed." (https://www.android.com/choicescreen/.)

The results of my paper, however, show that abandoning the auction is not the only possible solution. The issues described in Section 4 arise not from the usage of auctions per se, but rather from the specific implementation of these auctions. Replacing the "per install" bidding rule in choice-screen auctions with a "per appearance" one might be sufficient to alleviate the shortcomings of Google's implementation of choice-screen auctions, or at least meaningfully reduce their impact.

A "per appearance" auction can be implemented in practice in a number of different ways. Search engines can bid directly per appearance, specifying how much they are willing to pay every time they are shown on the choice screen, and paying the highest losing bid every time they are shown. To help them make bidding decisions, the platform could provide to each search engine historical data on its "conversion probability," i.e., the likelihood that the search engine will be chosen by a user conditional on being shown on the choice screen. Alternatively, the auction could still ask bidders to report their values "per install," but then rank bidders by the product of those reported values and estimated conversion probabilities and charge each winning bidder the lowest bid they could have reported and still won the auction, similarly to how many search engines auction off ad slots (Edelman et al., 2007; Varian, 2007). The platform can estimate these conversion probabilities by allocating a small fraction of choice-screen impressions to random selections of all eligible search engines, and then using the data from this fraction of traffic to estimate the probabilities. In summary, while deploying per appearance choice-screen auctions in one of the above forms would require some additional work on the part of the auctioneer, they are sufficiently similar to the existing formats in other domains that their implementation would not be too problematic.

While per appearance choice-screen auctions may provide a simple, transparent, and potentially effective tool in the regulatory toolbox, it is important to note that this solution is not necessarily optimal. To judge the optimality of this (or any other) solution, one would need, at a minimum, to explicitly specify the regulator's objective function. Moreover, even holding the overall auction format fixed, there are parameters of its implementation that will have an impact on its overall effectiveness and thus need to be chosen appropriately. E.g., one would need to decide how many options to show on the choice screen, ³⁸ how often to show the choice screen to the users, and how often to conduct these auctions. And going beyond just the design of a particular choice-screen auction, there is a question of how common they should be. E.g., many apps from Apple and Google come pre-installed on iOS and Android platforms, respectively, and in principle, each one could have a choice screen to give a level playing field to the competitors. But having to go through a choice screen for every single one of them may lead to a poor experience for end users, substantially reducing consumer utility, which gives rise to the question of how to determine the circumstances and cases to which choice-screen auctions should be applied, without overwhelming the end users. I leave the analysis of these questions to future research.

³⁸As discussed above, along with abandoning auctions in the choice-screen solution, Google and the European Commission have also increased the number of search engines displayed in the menu. See also https://www.auction-technologies.net/images/Papers/competitive-edge-the-google-auction-that-may-turn-into-a-trap.pdf for a discussion of whether the number of slots should be increased.

Choice-screen auctions can be a powerful tool for leveling the playing field in the now widespread settings in which a dominant platform offers a product that is competing with several others. More generally, the tools of market design can be useful for constructing elegant and effective regulatory solutions for the increasingly complex and interconnected digital economy. Having said that, when deploying such solutions, it is important to analyze the equilibrium properties of the resulting systems on a detailed level. Seemingly minor details can have major effects on the outcomes. And the case of Android Choice-Screen Auctions serves as a cautionary tale: getting these details wrong can undermine the entire mechanism.

A Model of Choice-Screen Auctions with General Distributions of Bidder Types

In Sections 2 and 3, I restrict attention to uniform distributions of bidder types. In this Appendix, I consider analogous settings with more general distributions.

A.1 Exogenous Popularity and Revenue-per-User

First, consider the setting of Section 2, with the following change. Instead of assuming that variables q_1 , q_2 , r_1 , and r_2 are all distributed uniformly on [0,1], I now make the following set of assumptions:

- Variables q_1 and q_2 are drawn from a continuous distribution F with full support on [0,1].
- Variables r_1 and r_2 are drawn from a continuous distribution G with full support on [0,1].
- Variables $q_1, q_2, r_1,$ and r_2 are independent.

Let $\overline{q} = E[F]$. Just as in the uniform case, it is immediate that in the per install auction, the expected popularity of the winner is equal to \overline{q} . Let us now show that in the per appearance auction, the expected popularity of the winner (which we denote q^A) is strictly greater than \overline{q} .

For each $q \in [0, 1]$, let W(q) be the probability that $qr_1 > q_2r_2$, when q_2 is drawn from F while r_1 and r_2 are drawn from G, independently of each other and of q_2 . As observed in footnote 25 in the main body of the paper, the expected popularity of the winner of the auction is equal to the expected popularity of the winner of the auction conditional on that winner being bidder 1 (simply by symmetry between the bidders). That latter quantity, in turn, is equal to the ratio

$$\frac{\int_0^1 q_1 Prob(\text{bidder 1 is the winner of the auction}|q_1) dF(q_1)}{Prob(\text{bidder 1 is the winner of the auction})}$$

whose denominator (again simply by symmetry) is equal to $\frac{1}{2}$.

We thus have the following expression for q^A :

$$q^A = 2 \int_0^1 qW(q)dF(q).$$

Note that both q and W(q) are nonnegative, strictly increasing functions of q, and that $\int_0^1 q dF(q) = \overline{q}$ and $\int_0^1 W(q) dF(q) = \frac{1}{2}$. We thus have

$$q^A = 2 \int_0^1 qW(q)dF(q) > 2 \left(\int_0^1 qdF(q) \right) \left(\int_0^1 W(q)dF(q) \right) = 2 \cdot \overline{q} \cdot \frac{1}{2} = \overline{q},$$

concluding the proof.

Next, let us show that that in the per appearance auction, the expected payoff of the platform is strictly lower than that in the per install auction. Consider two draws q_1 and q_2 from distribution

F and two draws r_{α} and r_{β} from distribution G. Let $\Pi^{A}(q_{1}, q_{2}, r_{\alpha}, r_{\beta})$ be the payoff to the platform from the per appearance auction when bidder 1's popularity is equal to q_{1} and her revenue-per-user is equal to r_{α} , while bidder 2's popularity is equal to q_{2} and his revenue-per-user is equal to r_{β} . Define $\Pi^{I}(q_{1}, q_{2}, r_{\alpha}, r_{\beta})$ analogously for the per install auction. We need to show that

$$E_{q_1,q_2,r_{\alpha},r_{\beta}} \left[\Pi^{I}(q_1,q_2,r_{\alpha},r_{\beta}) - \Pi^{A}(q_1,q_2,r_{\alpha},r_{\beta}) \right] > 0.$$
 (5)

Note that by symmetry,

$$E_{q_1,q_2,r_{\alpha},r_{\beta}} \left[\Pi^I(q_1,q_2,r_{\alpha},r_{\beta}) - \Pi^A(q_1,q_2,r_{\alpha},r_{\beta}) \right] = E_{q_1,q_2,r_{\alpha},r_{\beta}} \left[\Pi^I(q_1,q_2,r_{\beta},r_{\alpha}) - \Pi^A(q_1,q_2,r_{\beta},r_{\alpha}) \right],$$

where we swapped r_{α} and r_{β} under Π^{I} and Π^{A} , and so equation (5) is equivalent to

$$E_{q_1,q_2,r_{\alpha},r_{\beta}} \left[\Pi^I(q_1,q_2,r_{\alpha},r_{\beta}) + \Pi^I(q_1,q_2,r_{\beta},r_{\alpha}) - \Pi^A(q_1,q_2,r_{\alpha},r_{\beta}) - \Pi^A(q_1,q_2,r_{\beta},r_{\alpha}) \right] > 0. \quad (6)$$

To prove equation (6), it is sufficient to show that for any q_1 , q_2 , r_{α} , and r_{β} such that $r_{\alpha} \neq r_{\beta}$, $q_1 r_{\alpha} \neq q_2 r_{\beta}$, and $q_2 r_{\alpha} \neq q_1 r_{\beta}$ (so that ties do not occur),

$$\Pi^{I}(q_{1}, q_{2}, r_{\alpha}, r_{\beta}) + \Pi^{I}(q_{1}, q_{2}, r_{\beta}, r_{\alpha}) - \Pi^{A}(q_{1}, q_{2}, r_{\alpha}, r_{\beta}) - \Pi^{A}(q_{1}, q_{2}, r_{\beta}, r_{\alpha}) \ge 0, \tag{7}$$

with the inequality being strict for a positive mass of tuples $(q_1, q_2, r_\alpha, r_\beta)$.

Without loss of generality, assume that $r_{\alpha} > r_{\beta}$, and recall that we denote by $\pi > 1$ the payoff of the platform when its own search engine is chosen from the choice screen. Then $\Pi^{I}(q_{1}, q_{2}, r_{\alpha}, r_{\beta}) = (1 - q_{1})\pi + q_{1}r_{\beta}$ and $\Pi^{I}(q_{1}, q_{2}, r_{\beta}, r_{\alpha}) = (1 - q_{2})\pi + q_{2}r_{\beta}$. Next, without loss of generality, suppose that $q_{1} \geq q_{2}$. Then $q_{1}r_{\alpha} > q_{2}r_{\beta}$, and $\Pi^{A}(q_{1}, q_{2}, r_{\alpha}, r_{\beta}) = (1 - q_{1})\pi + q_{2}r_{\beta}$. Finally, we need to consider two cases: $q_{2}r_{\alpha} > q_{1}r_{\beta}$ and $q_{2}r_{\alpha} < q_{1}r_{\beta}$.

Case 1: $q_2 r_{\alpha} > q_1 r_{\beta}$. In this case, $\Pi^A(q_1, q_2, r_{\beta}, r_{\alpha}) = (1 - q_2)\pi + q_1 r_{\beta}$, and so $\Pi^I(q_1, q_2, r_{\alpha}, r_{\beta}) + \Pi^I(q_1, q_2, r_{\beta}, r_{\alpha}) - \Pi^A(q_1, q_2, r_{\alpha}, r_{\beta}) - \Pi^A(q_1, q_2, r_{\beta}, r_{\alpha}) = (1 - q_1)\pi + q_1 r_{\beta} + (1 - q_2)\pi + q_2 r_{\beta} - (1 - q_1)\pi - q_2 r_{\beta} - (1 - q_2)\pi - q_1 r_{\beta} = 0$.

Case 2: $q_2r_{\alpha} < q_1r_{\beta}$. In this case, $\Pi^A(q_1, q_2, r_{\beta}, r_{\alpha}) = (1 - q_1)\pi + q_2r_{\alpha}$, and so $\Pi^I(q_1, q_2, r_{\alpha}, r_{\beta}) + \Pi^I(q_1, q_2, r_{\beta}, r_{\alpha}) - \Pi^A(q_1, q_2, r_{\alpha}, r_{\beta}) - \Pi^A(q_1, q_2, r_{\beta}, r_{\alpha}) = (1 - q_1)\pi + q_1r_{\beta} + (1 - q_2)\pi + q_2r_{\beta} - (1 - q_1)\pi - q_2r_{\beta} - (1 - q_1)\pi - q_2r_{\alpha} = (q_1 - q_2)\pi + (q_1r_{\beta} - q_2r_{\alpha})$. By our assumptions, $(q_1 - q_2)\pi \geq 0$ and $q_1r_{\beta} - q_2r_{\alpha} > 0$, and so in this case, the expression in equation (7) is strictly positive.

Note that Case 2 happens with positive probability, and thus the expression in equation (6) is also strictly positive, concluding the proof.

For the limit case of the number of bidders n approaching infinity (last paragraph of Section 2), the conclusions and the arguments are identical to those for the uniform case, because for those arguments the only characteristics of the distributions that matter are the upper bounds of their supports.

³⁹For concreteness, whenever there are ties, one can assume that they are broken randomly with equal probabilities (though the specific tie-breaking rule will not matter, since ties happen with probability zero).

Finally, the proof above also shows that for the case with two bidders, the search engine's expected payoff from the per install auction exceeds that from the per appearance auction even if its own payoff π from getting a user is less than 1, as long as $\pi \geq 0$. For the limit case of large n, the comparison does rely on the assumption $\pi > 1$.

A.2 Endogenous Popularity and Revenue-per-User

I now consider an extension of the model in Section 3, in which instead of assuming that distribution F of types t is uniform on [0,1], I make the following set of assumptions:

- Types t_i are drawn independently from distribution F(t) with full support on [0,1].
- Distribution F(t) has a continuous positive density function f(t) on [0,1].

The analysis of the per appearance auction (Section 3.1) remains unchanged.

The analysis of the per install auction also remains unchanged up to the differential equation (4). Thus, in particular, as in Section 3.2, we can immediately conclude that if the per install auction has a symmetric equilibrium characterized by some function q(t), the equilibrium popularity chosen by each type $t_i > 0$ will be strictly lower under the per install auction than under the per appearance auction.

Next, dropping index i in equation (4) for convenience, we have

$$2q'(t) + (n-1)q(t)\frac{f(t)}{F(t)} = 1.$$

To solve this differential equation, first multiply both sides by $\frac{F(t)^{\frac{n-1}{2}}}{2}$ to get

$$q'(t)F(t)^{\frac{n-1}{2}} + \frac{n-1}{2}q(t)f(t)F(t)^{\frac{n-3}{2}} = \frac{F(t)^{\frac{n-1}{2}}}{2}.$$

This equation, in turn, can be rewritten as

$$\left(q(t)F(t)^{\frac{n-1}{2}}\right)' = \frac{F(t)^{\frac{n-1}{2}}}{2},$$

and thus

$$q(t)F(t)^{\frac{n-1}{2}} = \int_0^t \frac{F(x)^{\frac{n-1}{2}}}{2} dx + C$$

for some constant C.

Since q(0) = 0 and F(0) = 0, it has to be that C = 0, and so

$$q(t) = \frac{\int_0^t F(x)^{\frac{n-1}{2}} dx}{2F(t)^{\frac{n-1}{2}}}.$$

E.g., for the case of the uniform distribution considered in Section 3, we get $q(t) = \left(\frac{2}{n+1}t^{\frac{n+1}{2}}\right)/\left(2t^{\frac{n-1}{2}}\right) = \frac{1}{n+1}\left(\frac{2}{n+1}t^{\frac{n+1}{2}}\right)$

 $\frac{t}{n+1}$. Note also that the search engine's bid, equal to its revenue-per-user, is increasing in its type:

$$r(t) = t - q(t) = t - \frac{\int_0^t F(x)^{\frac{n-1}{2}} dx}{2F(t)^{\frac{n-1}{2}}},$$

and so

$$r'(t) = 1 - \frac{2F(t)^{n-1} - (n-1)f(t)F(t)^{\frac{n-3}{2}} \int_0^t F(x)^{\frac{n-1}{2}} dx}{4F(t)^{n-1}} \ge \frac{1}{2} > 0.$$

Finally, since F(x) is a strictly increasing function, for any t>0 (including the highest possible type t=1), we have $\lim_{n\to\infty}\frac{\int_0^t F(x)^{\frac{n-1}{2}}dx}{2F(t)^{\frac{n-1}{2}}}=0$, and so just like in the uniform case, as the number of bidders becomes large, the popularity of the winner of the per install auction converges to zero.

References

- Arozamena, L. and E. Cantillon (2004). Investment incentives in procurement auctions. *Review of Economic Studies* 71(1), 1–18.
- Bergemann, D. and J. Välimäki (2002). Information acquisition and efficient mechanism design. *Econometrica* 70(3), 1007–1033.
- Clarke, E. H. (1971). Multipart pricing of public goods. Public Choice 11(1), 17–33.
- Crémer, J., Y.-A. de Montjoye, and H. Schweitzer (2019). Competition policy for the digital era. Report for the European Commission.
- Economides, N. and J. Lianos (2011). A critical appraisal of remedies in the EU Microsoft cases. Columbia Business Law Review 2010(2), 346–420.
- Edelman, B., M. Ostrovsky, and M. Schwarz (2007). Internet advertising and the generalized second-price auction: Selling billions of dollars worth of keywords. *American Economic Review* 97(1), 242–259.
- Groves, T. (1973). Incentives in teams. Econometrica 41(4), 617–631.
- Hatfield, J. W., F. Kojima, and S. D. Kominers (2014). Investment incentives in labor market matching. *American Economic Review Papers & Proceedings* 104(5), 436–41.
- Hatfield, J. W., F. Kojima, and S. D. Kominers (2018). Strategy-proofness, investment efficiency, and marginal returns: An equivalence. *Becker Friedman Institute for Research in Economics Working Paper*.
- Rogerson, W. P. (1992). Contractual solutions to the hold-up problem. Review of Economic Studies 59(4), 777–793.
- Salinger, M. A. (2020). Self-preferencing. The Global Antitrust Institute Report on the Digital Economy 10, 329–368.
- Scott Morton, F., P. Bouvier, A. Ezrachi, B. Jullien, R. Katz, G. Kimmelman, A. D. Melamed, and J. Morgenstern (2019). Committee for the study of digital platforms: Market structure and antitrust subcommittee report. Stigler Center for the Study of the Economy and the State, University of Chicago Booth School of Business.
- Varian, H. R. (2007). Position auctions. International Journal of Industrial Organization 25(6), 1163–1178.
- Varian, H. R. (2008). Quality scores and ad auctions. https://googleblog.blogspot.com/2008/10/quality-scores-and-ad-auctions.html.
- Vickrey, W. (1961). Counterspeculation, auctions, and competitive sealed tenders. *Journal of Finance* 16(1), 8–37.