

Carpooling and the Economics of Self-Driving Cars

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Three emerging technologies that will transform transportation:

- Self-driving cars
- Frictionless time-dependent tolls
- Convenient, efficient carpooling

What will a market powered by these technologies look like?

How should it be designed and organized?

Outline of the talk

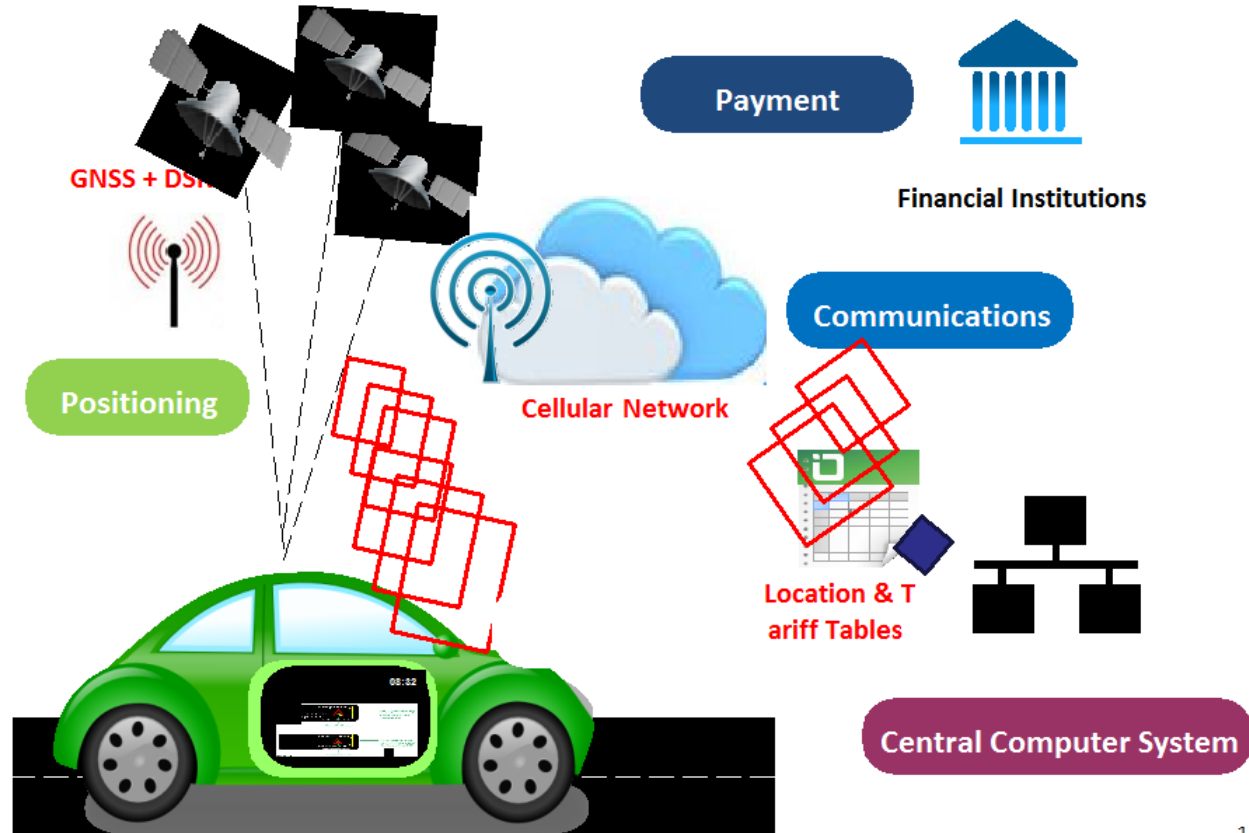
- Emerging technologies and interactions among them
- Main model and results
- Auxiliary result: The benefits of higher utilization

Tolls – current technology



Tolls – coming soon (2020)

Next-Generation ERP System Architecture



Carpooling

Tap into your local Waze community
See friends, neighbors & coworkers going your way

The image shows a smartphone screen displaying the Waze app interface. A map with a blue route is visible. Four carpooling cards are overlaid on the map:

- Elijah**: 5 stars, 18 rides. Price: \$5.00. Note: "Via HOV: Save 8 min" and "5 mutual friends".
- Sophia**: 5 stars, 32 rides. Price: \$4.00. Note: "You've carpoled before" and "General Manager at Orion".
- Brian**: 5 stars, 27 rides. Price: \$6.00. Note: "Chef at Pastelo Patisserie".
- Natalie**: 5 stars, 46 rides. Note: "Software Engineer".

A home button icon is visible in the top right corner of the app interface.

BlaBlaCar

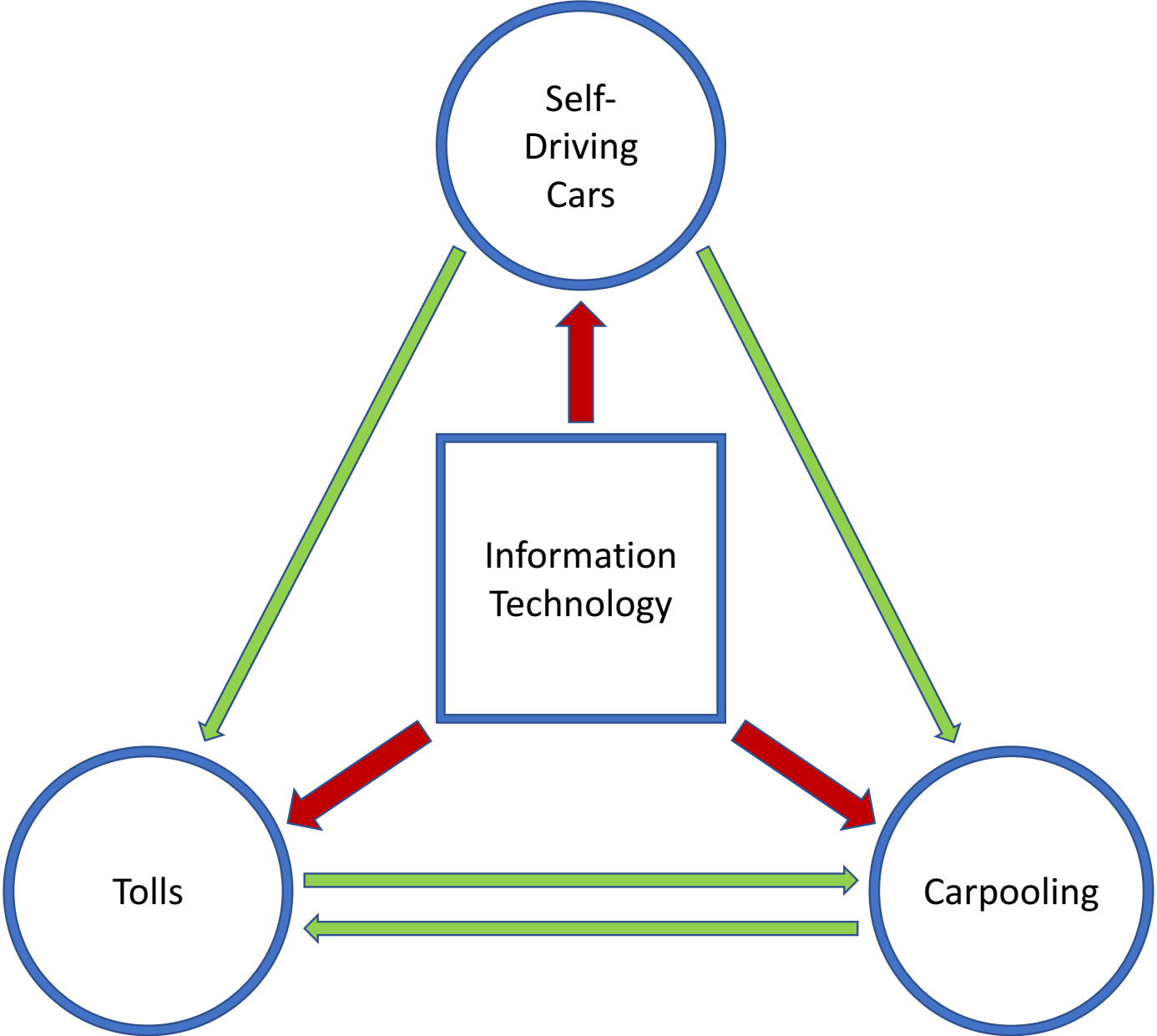
The image shows a woman with long brown hair, wearing a pink sweater, hugging a young child with curly hair. The text "Emma. BlaBla. Mum." is overlaid in white. The BlaBlaCar logo is in the top left corner.

Convenient and enjoyable carpooling

The image shows a smartphone screen displaying the Scoop app interface. The text "Convenient and enjoyable carpooling" is at the top. The app interface includes:

- Navigation icons: Trips, Community, Stream, Account.
- Profile section: "You", "Lea", "Rajesh", "Pick up at 7.", "Get \$8", "Trip details", "Compl".
- Event section: "Evening", "Schedule by 3:30pm".

The Scoop logo is at the bottom left.



Autonomous transportation and carpooling

With self-driving cars, carpooling will blur the line between solo driving and public transportation. Currently, the cost of drivers is a large component of public transit costs \Rightarrow large vehicles, frequent stops, slow and often inconvenient service. Eliminating this cost \Rightarrow smaller vehicles, fewer stops, door-to-door transit.

Carpooling in self-driving cars vs. carpooling now:

- Seamless coordination.
- No loss of flexibility.
- With several people sharing a ride, it does not have to be the case that the same person is the first one getting in the car and the last one getting out of it.

- Detours are much less costly: it costs much less to sit in the car as a passenger and read/sleep/work/watch movies while the car is making a five-minute detour than to drive during that time.
- No need to depend on a potentially unreliable carpool driver.
- Consistency of experience. E.g., no need to worry about being matched with an unsafe driver.
- One of the key challenges in creating a carpooling platform is getting to a critical mass where on-demand carpooling is reliable enough to be practical. With transportation services powered by self-driving cars, this issue is resolved automatically.

Autonomous transportation and road pricing

- Logistical reasons
- Equilibrium effect of self-driving cars on road demand during peak hours (in the absence of road pricing). Disutility of being stuck in traffic goes down \Rightarrow demand goes up \Rightarrow more traffic.

“A lot of people think that once you make cars autonomous that they’ll be able to go faster and that will alleviate congestion and to some degree that will be true. But . . . the amount of driving that will occur will be much greater with shared autonomy and actually, traffic will get far worse.”

– Elon Musk, Tesla CEO (2017)

In this case, intelligent road pricing becomes even more valuable.

Complementarity between carpooling and road pricing

Example based on Vickrey (1969). Consider a congested road: twice as many drivers want to go over the segment over a certain period of time as what the road allows.

- Suppose carpooling becomes very convenient: its disutility (vs. driving solo) is some small $\Delta > 0$. This has no effect on outcomes.
- Suppose there is no carpooling and tolls are set at the socially optimal level. With homogeneous drivers, *none of the drivers are better off*: they simply pay in dollars what they used to pay in time. Also, the cost of suboptimal arrival times is not eliminated.
- Now suppose we both have convenient carpooling *and* set tolls optimally. We now fully relieve congestion, all riders arrive at their most preferred times, and are now much better off than they were before.

Main model and results

Model

- Finite set of riders $m = 1, \dots, M$.
- Finite set of road segments $s = 1, \dots, S$. Each road segment s identifies both a physical road segment and a specific time interval.
- Each road segment s has an integer capacity $q_s > 0$.
- A *trip* is a feasible combination of one or more riders and one or more road segments. There is a finite number T of possible trips $t = 1, \dots, T$.
- Each trip t has a physical cost $c(t)$ associated with it.

- Each rider m has a valuation for every trip t that involves him: $v_m(t)$. Monetary transfers enter m 's utility additively.
- For each m , there exists an “outside option”: a trip that involves only one rider (m) and no road segments, gives m the value of zero, and costs zero.
- An *assignment* is a set of trips A such that each rider is involved in exactly one trip in A .
- Assignment A is *feasible* if for each road segment s , the number of trips in A that include road segment s does not exceed its capacity q_s .
- The *social surplus* of assignment A is equal to the sum of the valuations of all the riders from the trips to which they are assigned in A minus the sum of the costs of the trips in A .

Monetary transfers and outcomes

- A non-negative vector $p \in \mathbb{R}^M$ specifies the price paid by each rider m .
- A non-negative vector $r \in \mathbb{R}^S$ specifies the tolls imposed by a regulator.
- An *outcome* is a triple (A, p, r) that specifies an assignment, the payments made by the riders, and the tolls imposed by the regulator.

An outcome (A, p, r) is

- *feasible* if the corresponding assignment A is feasible,
- *budget-balanced* if the sum of prices paid by the riders for their trips is \geq to the sum of the total physical costs of those trips and the total tolls on the road segments involved in those trips,
- *stable* if it is not possible to organize a trip t that makes all the riders involved in it strictly better off than they were under (A, p, r) ,
- *market-clearing* if for every road segment s that has “excess capacity” in A , the corresponding toll r_s is equal to zero.

Theorem 1:

If an outcome (A, p, r) is feasible, stable, budget-balanced, and market-clearing, then assignment A has the highest possible social surplus among all feasible outcomes.

Proof:

Take a feasible, stable, budget-balanced, and market-clearing outcome (A, p^A, r^A) . Suppose feasible assignment B generates a higher social surplus than assignment A .

Take any trip t in B , and all riders m involved in t . By stability of (A, p^A, r^A) , this coalition cannot benefit from organizing trip t by themselves, given prices p^A and tolls r^A in outcome A . Thus, summing the utilities of all riders involved in trip t :

$$\sum_{m \in t} (v_m(t_m^A) - p_m^A) \geq \left(\sum_{m \in t} v_m(t) \right) - c(t) - r^A(t). \quad (1)$$

Adding up equations (1) across all trips t in assignment B , we get

$$\sum_{m=1}^M v_m(t_m^A) - \sum_{m=1}^M p_m^A \geq \sum_{m=1}^M v_m(t_m^B) - \sum_{t \in B} c(t) - \sum_{s=1}^S r^A(s) k^B(s), \quad (2)$$

where $k^B(s)$ denotes how many trips in assignment B use segment s .

Since the outcome is by assumption budget-balanced, we have

$$\sum_{m=1}^M p_m^A \geq \sum_{t \in A} c(t) + \sum_{s=1}^S r^A(s) k^A(s).$$

Thus, equation (2) implies that

$$\sum_{m=1}^M v_m(t_m^A) - \sum_{t \in A} c(t) - \sum_{s=1}^S r^A(s) k^A(s) \geq \sum_{m=1}^M v_m(t_m^B) - \sum_{t \in B} c(t) - \sum_{s=1}^S r^A(s) k^B(s).$$

The social surplus of A is equal to $\sum_{m=1}^M v_m(t_m^A) - \sum_{t \in A} c(t)$, while the social surplus of B is equal to $\sum_{m=1}^M v_m(t_m^B) - \sum_{t \in B} c(t)$. To show that the former is greater than or equal to the latter, it is now sufficient to observe that

$$\sum_{s=1}^S r^A(s) k^A(s) \geq \sum_{s=1}^S r^A(s) k^B(s),$$

which follows from the market-clearing property of (A, p^A, r^A) . ■

Issue: a feasible, stable, budget-balanced, market-clearing outcome may fail to exist. For example, suppose we have 101 riders and each car fits at most two passengers.

Approximation: allow trips to be “divisible”. Formally, a “quasi-assignment” A is a measure on the set of trips such that overall, each rider is involved in measure 1 of trips. The definitions of an outcome and of feasibility, stability, budget-balancedness, and market-clearing extend naturally.

Theorem 2:

A feasible, stable, budget-balanced, and market-clearing quasi-outcome is guaranteed to exist. Any such quasi-outcome is socially efficient.

Idea of the proof:

Construct an auxiliary economy. Consumers in this economy are new agents called “trip organizers”. There are T types of those agents, one for each trip. There are 2 trip organizers of each type.

The goods are the M riders and S road segments from the original model. Price vector $\rho \in \mathbb{R}^{M+S} \geq 0$ assigns a price to every good in the economy.

Trip organizers can consume bundles of goods $b \in [0, 1]^{M+S}$. Their utility is

$$\tilde{V}_t(b) = \left(\sum_{m \in t} v_m(t) - c(t) \right) \times \min_{g \in t} b(g) - \rho^T b.$$

Competitive equilibrium: set of prices ρ and profile of consumption bundles b such that every consumer is optimizing and markets clear.

Existence of CE: For each ρ , take optimal bundles b , and add them up to get aggregate demand $D \in \mathbb{R}^{M+S}$. Let $Q \in \mathbb{R}^{M+S}$ denote the available supply of all items. Define the tâtonnement price adjustment function $\tau(\rho, D) = \max\{0, \rho + (D - Q)\}$.

Correspondence φ : for each ρ , take all possible D , and compute all possible $\tau(\rho, D)$.

Correspondence φ satisfies all the requirements of Kakutani's fixed-point theorem: $\varphi(\rho)$ is non-empty and convex for all ρ ; and $\varphi(\rho)$ has a closed graph. Thus, correspondence φ has at least one fixed point. Any fixed point ρ^* of φ and a profile of consumer-optimal bundles with aggregate demand D^* such that $\rho^* = \tau(\rho^*, D^*)$ form a competitive equilibrium.

Last step: observe that in every CE, the utility of every consumer (trip organizer) is zero, because by construction, there is “excess supply” of trip organizers.

This observation allows us to go from a CE in the auxiliary market to a feasible, stable, budget-balanced, and market-clearing quasi-outcome:

- For each segment s , toll $r(s)$ is equal to the competitive equilibrium price $\rho^*(s)$.
- For each rider m , the utility $u(m)$ in the original market is set equal to $\rho^*(m)$.
- To construct the quasi-assignment G , take all riders m whose utility $u(m)$ is greater than zero and “read off” the volumes of various trips from these riders.



Additional observations, comments, and results

We made an assumption: “Each road segment s has an integer capacity $q_s > 0$.”

This assumption is:

- Very accurate for highways.
- Less accurate, but still reasonable, for cities.

Upside from making this assumption: when setting tolls, the regulator does not need to estimate marginal costs and benefits of various drivers in various situations; can simply focus on the volume of traffic.

Another Assumption: “Each trip t has a physical cost $c(t)$ associated with it.”

Observation: This assumption makes our model not applicable to the world with professional drivers (taxis, Uber, Lyft, and so on).

However, the model and the main results can be easily adapted to the world of carpooling (with non-professional drivers) without self-driving cars. Each “trip” would need to specify which of the people involved is the driver (with the rest being passengers), and we also need to make some minor modifications to our modeling assumptions.

Why is it OK to make this assumption in our main model with self-driving cars?

Basic model of car utilization

A car costs C and dies after N miles or A years, whichever comes first. The annualized real interest rate is r . A car is driven K miles per year. Fuel, maintenance, and insurance are variable costs, m per mile. Note: car will die at $T = \min\{A, \frac{N}{K}\}$.

Proposition

For $r > 0$, the cost per mile G is equal to $m + \frac{Cr}{K(1-e^{-rT})}$. For $r = 0$, the cost per mile is $G = m + \frac{C}{KT}$.

Proof:

$$C + \int_0^T mKe^{-rt} dt = \int_0^T GKe^{-rt} dt.$$

■

Note that there is no discontinuity at $r = 0$ (because $\lim_{r \rightarrow 0} \frac{r}{1-e^{-rT}} = \frac{1}{T}$).

Corollary

If cars die from usage (rather than old age) and the real interest rate is equal to zero, the increase in annual mileage yields no cost savings.

Numerical example

- Cars die after 200,000 miles or 15 years, whichever happens first
- New cars cost \$30,000, real interest rate is 3% per year
- Gas, maintenance and insurance are variable costs, 30 cents per mile

| | | | | | |
|-------------------|-------|--------|--------|--------|--------|
| Annual mileage | 5,000 | 15,000 | 25,000 | 35,000 | 75,000 |
| Cost per mile (¢) | 80 | 48 | 47 | 46 | 46 |

Implications

- Limited cost reduction of using “Transportation-as-a-service” vs. personal ownership for many people (e.g., those driving 15,000 miles per year or more and living in areas with low parking costs), so personal self-driving cars may easily co-exist with various transportation services. Our framework accommodates this possibility.
- “High utilization” in the form of putting more people in the car (carpooling) is likely to be much more important than “high utilization” in the form of putting more miles on a car. E.g., potential cost savings from doubling car occupancy are an order of magnitude higher than the savings from doubling car usage (i.e., the number of miles a car drives per year).

Thank you!