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Adoption of standards under uncertainty

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The presence of noise in compliance times may have a critical impact on the selection of new technological standards. A technically superior standard is not necessarily viable because an arbitrarily small amount of noise may render coordination on that standard impossible. We introduce the concept of a firm's "support ratio," defined as a function that depends only on characteristics of that firm. We show that for sufficiently patient firms, the viability of a standard does not depend on the distribution of noise in compliance times. The criterion for the viability of a standard is that the sum of support ratios of all firms be smaller than one.

1. Introduction

■ When a firm invests in supply chain management (SCM) software, it hopes that by the time the system is deployed, its supply chain partners will have already deployed their SCM software. In a perfect world, all partners would pick a single date for their systems to go online, and it would be individually optimal for each firm to follow that schedule. In reality, however, even trains do not always come on time, and it is impossible to perfectly anticipate the exact time of SCM deployment—some firms will necessarily be later than others. When an individual firm schedules SCM software deployment, it does not take into account the expected negative externality of postponing its investment. Thus, deployment may happen inefficiently late or, if externalities are sufficiently large, may never happen.

This problem arises whenever firms schedule complementary projects or investments. For another example, consider Bluetooth. Bluetooth is a technological standard that enables "wireless links between mobile computers, mobile phones, portable handheld devices, and connectivity to the Internet."[1] To take advantage of this technology, a firm needs to install an additional chip in its hardware and write software integrating the chip with the rest of the system. This will be profitable only if by the time the design and manufacture of these products are complete, there are other Bluetooth-enabled devices from other firms to communicate with.

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[1] www.bluetooth.com.

In this article we study such synchronization issues, when firms want to make complementary business decisions that need advance planning. Specifically, we talk about the adoption of standards, but the insights are applicable in a broader context, from investments in complementary technologies to large-scale real estate development. We show that adding a stochastic component (noise) to adoption times may have a critical impact on the viability of a standard. Perhaps surprisingly, in a wide range of cases, a universally desired standard may not be viable in the sense that any amount of noise may render coordination on that standard impossible. As a result, given several alternatives, market participants may forgo a technically superior (Pareto-dominant) standard in favor of an inferior one, if the former is not viable. Proposition 1 shows that for sufficiently patient players, the viability of a standard depends on firms' per-period "waiting costs" and "post-adoption benefits," where firm i's per-period waiting cost c_i is the loss it suffers each period after complying with the standard while some other firms have not yet complied, and its post-adoption benefit d_i is the profit it gets from the standard each period after all firms have complied. More specifically, we define the firm's "support ratio" as $s_i = c_i/c_i + d_i$. The criterion for the viability of a standard is that the sum of support ratios of all firms must be smaller than one.

Waiting costs may take many forms. For instance, consider two businessmen planning to undertake complementary investments: agent one plans to open a restaurant, and agent two plans to open a store nearby. The restaurant and the store are complementary investments, because the restaurant draws customers to the store and vice versa.[2] The restaurant loses c_1 per period until the store opens for business. Similarly, the store would lose c_2 per period if it opened before the restaurant. After both the restaurant and the store are open, they make per-period profits of d_1 and d_2, respectively. Waiting cost c_i is the rent and other costs of keeping the business open, minus the income that the business generates before the other firm completes its complementary investment. Similarly, a customer looking for a printer and a laptop will be willing to pay more for a laptop with an infrared (IR) port if a printer with an infrared port is also available. Of course, infrared transmitters are not free, and so making a laptop compatible with the infrared communication standard is costly. If a firm incorporates the IR transmitter before printers become available, the price premium that it can charge for IR-compatible products may be smaller than necessary to cover the extra cost. In this case, the per-period loss of firm i when it is the only adopter of the new standard is c_i. Of course, the nature of cost c_i depends on a setting and may take many other forms: obsolescence of software applications developed for a new platform when the release of that platform is delayed, inventory costs when one component of a complex product is manufactured earlier than others, time value of money, and so on.

Let us sketch a two-player example that illustrates the model considered herein. For simplicity, we assume that there are only two possible options: the status quo and the new standard, which Pareto dominates the status quo. Both firms simultaneously choose target dates for compliance with the new standard. The actual compliance time is uncertain: it is equal to the target time plus noise.[3] As soon as a firm is compliant with the standard, it incurs a per-period cost of supporting the standard, c. Complying with the new standard starts paying off only after the standard is adopted by both firms. When (and if) this happens, each firm starts receiving a stream of net benefits at the rate d.

Without noise in compliance times, the game has a continuum of pure-strategy equilibria: any adoption time is an equilibrium as long as both players choose that time to comply with the

[2] Gould, Pashigian, and Prendergast (2005) find that businesses located in a shopping mall impose large positive externalities on each other. In the case of shopping malls, the externalities are partiality internalized by means of contracts between the mall operator and the businesses located in the mall, but in many other settings such contracts are not feasible.

[3] This modelling assumption was inspired by our experiences of working for two large technology companies. The time required to incorporate a new feature into a product was always uncertain, and implementation schedules were often revised. Similarly, when professors or students are promised a new research building or a dormitory, it is rarely possible to predict precisely when the construction will be finished. Of course, in many situations the amount of noise in compliance times may be small. That is why it is important to note that our results hold even for an arbitrarily small amount of noise.

standard.[4] Proposition 1 shows that this multiplicity of equilibria is a knife-edge result. If there is noise in adoption times, *at most* two equilibria survive. There is always a trivial equilibrium where neither player ever adopts the new standard. The equilibrium where the new standard is adopted may or may not exist. Proposition 1 also establishes a necessary condition for the viability of a standard. This condition becomes necessary and sufficient as the players' discount rate converges to one. Coordination on the new standard is impossible if the per-period cost of waiting is greater than the per-period net benefit that a standard yields after adoption by both players. In other words, a standard is not viable if its support ratio, $c/c + d$, is greater than one-half. This is true for any distribution of noise.

Let us sketch the intuition behind this result. First, observe that the best outcome for both players is simultaneous adoption. From the *ex post* perspective, the first player to comply "wishes" he had targeted a slightly later compliance time, since that would have saved him c per period. Similarly, the last player to comply wishes he had targeted a slightly earlier compliance time, because then he would have earned additional benefit d per period (which is equal to c for a "borderline viable" standard). Thus, roughly speaking, a standard is not viable if the benefit to the second adopter from decreasing his target compliance time is smaller than the cost to the first adopter from lowering his target compliance time. In this case, first-order conditions imply that each player's best response is to try to be last with probability greater than one-half; consequently, the equilibrium where the new standard is adopted disappears.

The model of the standard adoption process presented in Section 3 is highly stylized: players get benefits only after everyone complies. It highlights the dramatic effects that an arbitrarily small amount of uncertainty has on the equilibrium standard selection. In a deterministic world, only the net benefits of a standard matter; in this case the support ratio is irrelevant. If, however, there is any amount of noise in compliance times, support ratios become important. These effects do not go away in a more sophisticated model where firms choose among several competing standards. Also, small amounts of noise continue to have a large impact on equilibrium selection in models where network externalities gradually increase in the number of adopters (See Section 6).[5]

A standard does not have to fail even if the sum of support ratios is greater than one. Such a standard must fail unless there are institutions that facilitate coordination. Therefore, one should expect that if firms could not coordinate on a universally desired standard in a decentralized market, they would attempt to build institutions to overcome the inefficiency. This argument tells us when coordination in a decentralized market can be achieved, and when institutions facilitating coordination must emerge. Mechanisms for overcoming coordination failures range from contractual arrangements to cross-ownership among firms. Some of these mechanisms are discussed in Section 7.

2. Related literature

■ The idea that adding noise to the model may reduce the set of equilibria has a long history in economics. Recently, it figures prominently in the work on global games, first introduced in Carlsson and van Damme (1993). In global games, agents receive noisy signals about true economic fundamentals. This captures the lack of common knowledge about the true state of the economy (see Morris and Shin (2003) for the most recent survey of the global games literature).[6]

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[4] We are assuming that the new standard is Pareto dominant if, when adopted, it yields a positive net benefit for all players.

[5] It is worth mentioning that adding noise to other parameters of the model, such as discount rates, compliance costs, and adoption benefits, does not pin down the equilibrium of the model. That is why a model of standard adoption should capture the uncertainty about compliance times and could neglect the uncertainty about other parameters of the model.

[6] Some equilibrium refinements, e.g., trembling-hand perfection, are also based on the idea of perturbing a game. However, there is a significant difference between the logic behind equilibrium refinements and global games. Both this article and the global games literature attempt to consider games that capture some features of the underlying economic

The strand of the global games literature closest to our results is the work on synchronization games with asynchronous clocks. This work was preceded by an article by Halpern and Moses (1990), who show that asynchronous clocks may prevent synchronization because statements about timing never become common knowledge. Abreu and Brunnermeier (2003) show that a bubble may persist despite the presence of rational arbitrageurs who learn about the existence of the bubble at different times; essentially the difficulty in coordinating an attack on an asset is due to arbitrageurs' clocks not being synchronized. Morris (1995) considers a synchronization problem faced by agents who decide when to start working. Each worker knows the time on his watch, but watches are not perfectly synchronized. Morris shows that if clocks are not perfectly synchronized, coordination may not be achieved.

The setting of Morris's article is similar to ours—in both models agents gain once everybody participates, but "early arrival" is costly. However, Morris's model is a global game, and the inability to coordinate is due to agents having private information and thus lacking common knowledge about timing. In contrast, in our model there is no issue of clock synchronization, our agents have no private information, and the common-knowledge assumption is maintained. The difficulty in coordination is due to the inability to exactly control compliance times. Thus, our model is not a global game. Nevertheless, our results share some of the remarkable features often encountered in global games, namely (1) without noise there is a continuum of equilibria, and adding noise to the model pins the equilibrium down; (2) there exists an equilibrium robust to noise.[7] Blume (2003) and Frankel, Morris, and Pauzner (2003) explore the role of noise in equilibrium selection in certain classes of potential games. In a web Appendix (available at www.rje.org/main/sup-mat.html), we show that the models considered in the present article are also potential games, and we discuss the connection between our models and potential global games.

A coordination game similar in spirit to the present article and Morris (1995) is considered in Anderson, Goeree, and Holt (2001). In their article, agents simultaneously choose effort levels, and the payoff of an agent is equal to the lowest effort level minus the individual cost of effort chosen by the agent. Without noise, this game has a continuum of equilibria. Anderson, Goeree, and Holt consider a logit equilibrium, which essentially assumes a particular distribution of noise, and show its uniqueness. In contrast, Morris (1995) and the present article make no assumptions about the distribution of noise.

Basu and Weibull (2003) also study synchronization, in the context of social norms. In their model, an individual may choose to be "punctual" or "tardy," and punctuality may be just one of several equilibria, rather than a society's innate trait.

Our results show that in a wide range of cases, a technically superior (Pareto-dominant) standard may not be viable in the sense that any amount of noise may render coordination on that standard impossible. The failure of a useful standard to get adopted is a common result in the standards literature. There are many possible reasons why this may happen or why an inferior standard may prevail. They include ownership/sponsorship of standards, current technical superiority and acceptance versus future/long-term superiority, and incompleteness of information (Katz and Shapiro, 1985, 1986, 1994; Farrell and Saloner, 1985; Besen and Farrell, 1994; Liebowitz and Margolis, 1994).

This is the opposite of the conclusion of Farrell and Saloner (1985, p. 71) that if players make adoption decisions sequentially, "a somewhat surprising result emerges: if all firms would benefit from change [to a new standard] then all *will* change." Farrell and Saloner point out that in most cases players make adoption decisions simultaneously. In that case their model has multiple equilibria. However, they show that it is an equilibrium for players to switch to the Pareto-dominant standard. This result hinges on the assumption that the adoption of a new standard by a firm is an instantaneous process, and thus there are no costs of imperfect coordination of adoption times.

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reality that may play an important role in the selection of equilibrium. Unlike equilibrium refinements, we do not seek to improve the equilibrium concept—we seek to improve the model.

[7] Loosely speaking, robustness to noise means that the same equilibria are pinned down by a small amount of noise, regardless of the exact form of the noise.

The result of Proposition 1 of our article implies that in a simultaneous-move game, the Pareto-efficient equilibrium considered in Farrell and Saloner may disappear if any amount of uncertainty is present. Thus, the predictions of sequential- and simultaneous-move models are very different: the adoption of the Pareto-dominant standard is the unique equilibrium of the sequential-move game. In contrast, in a simultaneous-move game, the adoption of the Pareto-dominant standard may be impossible if any amount of uncertainty is present. In Section 5 we reconcile the difference by adding a dynamic aspect to the game—we make the assumption that once a firm complies with a standard, others observe that and can begin to comply as well. Under this assumption, for a small average compliance time (i.e., as the expected compliance time goes to zero), the sequential-move game of Farrell and Saloner is a valid approximation, and the Pareto-efficient outcome is an equilibrium. For a large average compliance time (i.e., as the expected compliance time goes to infinity), the simultaneous-move model considered herein is a valid approximation.

Therefore, average compliance time (or, equivalently, average time-to-build) plays an important role in the adoption of standards. Pacheco-de-Almeida and Zemsky (2003) find that time-to-build also has a significant impact on investment timing and the tradeoff between flexibility and commitment, firm heterogeneity, and the evolution of prices under demand uncertainty.

Finally, Farrell and Saloner (1986) present a model related to ours, where agents can switch from an old standard to a new one. Each agent faces occasional switching opportunities, arriving randomly in an independent Poisson process. For some values of the costs and benefits of the standards (with or without other agents complying with them), agents do not switch to the new standard even if they unanimously favor the switch, because each prefers others to switch first. This effect, however, is driven by the technological infeasibility of the new standard in the absence of transfers between agents—the agent who switches first is in expectation worse off than he would be if nobody switched. In our setup, in contrast, the effect is driven by strategic considerations and the inability of agents to commit to their compliance times. If noise is small (i.e., the rate of the Poisson process is high), then in the setup of Farrell and Saloner (1986), technological frictions vanish and the efficient standard gets adopted, whereas in our setup, commitment problems remain and the result is unchanged.

3. The model

■ We start with a simple model where complying with a standard is only profitable if all other firms comply as well. This simple model is sufficient to illustrate the importance of noise in the process of standard adoption and creation. In Section 6 we will consider a more general model of network externalities. Each firm can select the target time by which it expects to become compliant with a new standard. The key assumption of our model is that the actual compliance time is uncertain: it is equal to the target compliance time plus a disturbance term. The random disturbances are uncorrelated across firms, and thus perfect coordination is impossible: some firms are bound to comply earlier than others. While a firm is waiting for others to comply, it bears waiting costs. It only gets benefits after everyone (or, in a more general model, a sufficient number of other firms) complies.

More formally, suppose there are N firms that consider adopting a new standard. Each firm can choose a target compliance time $\mu_i \geq m_i \geq 0$ at which it plans to comply with the standard (m_i is an exogenous constraint—for each firm there is some minimum time required to comply), or a firm can choose not to comply at all, which we denote by *out*. All firms select their target times simultaneously. If the firm decides not to comply, its payoff is zero. Otherwise, its actual compliance time t_i is equal to μ_i plus a random disturbance ε_i drawn from a continuous-probability distribution F_i with support on $[0, \overline{\varepsilon_i}]$, independent of other firms' disturbances. As soon as a firm complies with the new standard, it has to pay a cost of supporting it of $c_i > 0$ per period. When (and if) all firms adopt the standard, firm i starts getting a flow of net benefits $d_i \geq 0$. The adoption time, i.e., the time when all firms comply, is denoted by $t_* = \max_i \{t_i\}$ (if one of the agents never adopts, we say that $t_* = \infty$). For simplicity, we assume that c_i and d_i do not change over time. The firm's payoff is a discounted flow of costs and benefits from the new standard:

$\Pi_i = E[\int_0^\infty \beta^t \pi_i(t)dt]$, where $\pi_i(t)$ is the net benefit accrued at time t,

$$\pi_i(t) = \begin{cases} 0 & \text{for } t \leq t_i \\ -c_i & \text{for } t_i < t \leq t_* \\ d_i & \text{for } t_* < t. \end{cases}$$

Assume that the discount factor, β, is strictly less than one. We will refer to the game described above as $\Gamma(\beta)$.

To analyze the equilibria of $\Gamma(\beta)$, we construct an approximation with no time discounting. To be able to do that, we renormalize payoffs, and for each i we subtract the net benefit after universal adoption, d_i, from firm i's instantaneous payoff in every period, i.e.,

$$\pi_i^1(t) = \begin{cases} -d_i & \text{for } t \leq t_i \\ -(c_i + d_i) & \text{for } t_i < t \leq t_* \\ 0 & \text{for } t_* < t. \end{cases}$$

More precisely, we define $\Gamma(1)$ as follows. Action space and probability distributions of disturbances are the same as before, but payoffs are different. If any player chooses *out*, every player's payoff is $-\infty$. If all players choose to comply, the payoff of player i is given by the expected value of $-c_i(t_* - t_i) - d_i t_*$, where $t_* = \max_i\{t_i\}$, and vector $\boldsymbol{t}$ is equal to vector $\boldsymbol{\mu}$ plus random vector $\boldsymbol{\epsilon}$ of disturbances drawn from the distribution $F_1 \times \cdots \times F_N$. We also define the support ratio of firm i, $s_i = c_i/(c_i + d_i)$.

4. The viability of a standard

■ The following proposition characterizes the equilibrium set of $\Gamma(\beta)$ as $\beta \to 1$. It gives a criterion for the viability of a standard, i.e., a necessary and sufficient condition for the existence of equilibrium where the standard is adopted provided that players are sufficiently patient. If the sum of support ratios of all players is less than one, the standard is viable. This condition *does not depend* on the distribution of noise—it depends only on the firms' support ratios. Also, as β increases, equilibrium target compliance times decrease, i.e., as players become more patient, they adopt earlier.

Proposition 1. If $\sum_{i=1}^N s_i < 1$, then

(i) there exists $\beta_0 < 1$ such that for any $\beta_0 < \beta < 1$, game $\Gamma(\beta)$ has exactly two equilibria—one in which all players choose to adopt, and one in which all players choose not to adopt,[8]

(ii) $\lim_{\beta \to 1} \boldsymbol{\mu}^*(\beta) = \boldsymbol{\mu}^*(1)$, where $\boldsymbol{\mu}^*(\cdot)$ denotes the vector of target compliance times in the equilibrium where the standard is adopted, and

(iii) for any $\beta_0 < \beta_1 \leq \beta_2 \leq 1$, $\boldsymbol{\mu}^*(\beta_1) \geq \boldsymbol{\mu}^*(\beta_2)$.

If $\sum_{i=1}^N s_i \geq 1$, then

(iv) for any $\beta < 1$, game $\Gamma(\beta)$ has only one equilibrium, and in that equilibrium all players choose not to adopt.

Proof. We prove this proposition in two steps. Step 1 characterizes the equilibria of the game with no time discounting, $\Gamma(1)$—this is done in Lemma 3. Step 2 describes the behavior of equilibria of $\Gamma(\beta)$ as $\beta \to 1$.

Step 1. First, we prove two auxiliary results.

[8] Note that there are no mixed equilibria.

Lemma 1. Suppose players have distributions of disturbances $\{F_i\}$. Take any strictly positive numbers $\{p_i\}$ such that $\sum p_i = 1$. Then there exists a vector of target times such that each player i adopts last with probability p_i.

Proof. See Appendix A.

Lemma 2. Suppose players have distributions of disturbances $\{F_i\}$. Take any strictly positive numbers $\{p_i\}$ such that $\sum p_i \leq 1$. Take any numbers $\{m_i\}$. Then there exists a vector of target times, $\boldsymbol{\mu}$, such that (i) for all i, $\mu_i \geq m_i$, (ii) each player i adopts last with probability greater than or equal to p_i, and (iii) if $\mu_i > m_i$, player i adopts last with probability exactly equal to p_i. If $\sum p_i < 1$, such vector μ is unique.

Proof. See Appendix A.

Now we can state the necessary and sufficient condition for the existence of an equilibrium of $\Gamma(1)$ where firms choose to adopt. Such an equilibrium exists if and only if the sum of the probabilities with which players want to be last is less than or equal to one.

Lemma 3. Game $\Gamma(1)$ has a Nash equilibrium with guaranteed adoption if and only if

$$\sum_{i=1}^{N} s_i \leq 1, \tag{1}$$

and when the above inequality is strict, such an equilibrium is unique. In all other equilibria, every player's expected payoff is $-\infty$.[9]

Proof. First, notice that if in an equilibrium at least one player plays *out* with a positive probability, all players have expected payoffs equal to $-\infty$. Therefore, in all equilibria with finite payoffs (if they exist), players have to mix among target times and never play *out*.

Suppose player i takes the distribution of adoption times of other players as given. Then, in his personal optimum, he will choose his adoption time μ_i in such a way that either $\mu_i = m_i$ and the probability of him being last is $q_i \geq s_i$, or $\mu_i > m_i$ and the probability of him being last is $q_i = s_i$. (To see that, suppose that $\mu_i > m_i$ and the probability of him being last is $q_i > c_i/(c_i+d_i)$. If instead he plans to adopt slightly earlier, at $\mu_i - \varepsilon$, in expectation he gains $q_i d_i \varepsilon + O(\varepsilon^2)$ (when he is the last one to adopt) and loses $(1 - q_i)c_i\varepsilon + O(\varepsilon^2)$ (when he is not). For μ_i to be optimal, it has to be the case that $q_i d_i - (1 - q_i)c_i = 0 \Longrightarrow q_i = c_i/(c_i + d_i)$. Similar arguments apply to the case $q_i < c_i/(c_i + d_i)$.)

If the sum of these "desired" probabilities s_i is greater than one, then, since each player wants to adopt last with at least his "desired" probability, no $\boldsymbol{\mu}$ can satisfy these conditions. When $\sum s_i \leq 1$, by Lemma 2, such $\boldsymbol{\mu}$ exists[10] and is an equilibrium: for each player i, by construction, μ_i is the best target compliance time, and playing *out* cannot be a better action, because it would give everyone, including player i, the payoff of $-\infty$.

When $\sum s_i < 1$, uniqueness also follows directly from Lemma 2.

Step 2. The proof that when $\sum s_i < 1$, the equilibrium with compliance of $\Gamma(\beta)$ converges to that of $\Gamma(1)$ as $\beta \to 1$ and the proof of statement (iv) are rather technical, and we present them in Appendix A. This completes the proof of Proposition 1. *Q.E.D.*

Note that it is clear from the proof that if inequality (1) is strict, then in the equilibrium where the standard is adopted, some firms comply as soon as they can, while for the rest, the probability of being last is equal to the support ratio.

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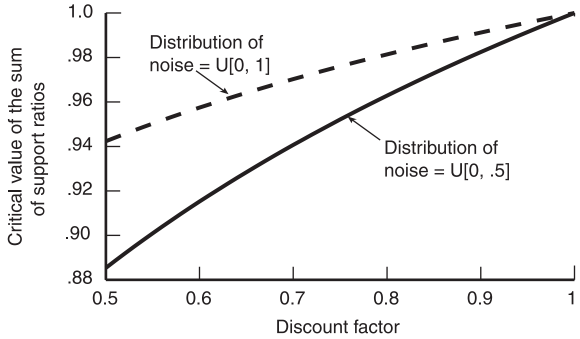
9 In particular, this proposition implies that if benefits are small relative to costs, e.g., $d_i = 0$ for all i, there is no equilibrium with finite payoffs.

10 When $\sum s_i = 1$, there is a continuum of such vectors $\boldsymbol{\mu}$, obtained by adding a constant to all elements of the vector. In this case, take the smallest $\boldsymbol{\mu}$ satisfying the constraints $\mu_i \geq m_i$.

FIGURE 1

CRITICAL VALUE OF THE SUM OF SUPPORT RATIOS AS A FUNCTION OF DISCOUNT FACTOR



It is also important to note that as players become less patient, the standard may fail to get adopted even if the sum of support ratios is less than one and, more generally, the viability of a standard may depend not only on support ratios but also on the distributions of disturbances. Figure 1 illustrates this by showing the critical values of the sum of support ratios for different values of discount factor β and two identical players with uniformly distributed disturbances. The two lines correspond to two different distributions of disturbances: uniform on [0, 1] and uniform on [0, .5]. Our criterion provides a good approximation for patient players. For example, when disturbances are uniform from zero to one year, and the discount factor is .9 per year, a standard can get adopted when the sum of support ratios is around .98 or less. See Appendix B for the derivation of the critical value for the case of two identical players and uniform disturbances.

An interesting question is what happens if we relax the assumption that costs and benefits are constant over time. The results change little if costs are decreasing and benefits are increasing over time—a standard can be adopted if and only if the sum of *limit* support ratios is less than one.

Finally, in many situations firms can control the amount of noise in their compliance times. Van Damme (1987) and van Damme and Weibull (2002) introduce the idea of control costs, whereby players can invest in reducing the amount of noise in their decisions, although completely eliminating noise is prohibitively expensive. Proposition 2 shows that in our setting, endogenizing the amount of noise does not change the criterion for the viability of a standard for sufficiently patient players.

Consider the following modification of game $\Gamma(\beta)$, denoted by $\Upsilon(\beta)$. Instead of choosing just the target compliance time, μ_i, each player now chooses both the target compliance time and the amount of noise, $(\mu_i, \sigma_i) \in [m_i, \infty) \times (0, 1]$. The realized compliance time t_i is equal to $\mu_i + \sigma_i \varepsilon_i$, where, as before, ε_i is the random disturbance drawn from a continuous distribution F_i. Controlling noise is costly. The cost is $r_i(\sigma_i)$, where $r_i(\cdot)$ is a differentiable monotonically decreasing function such that $r_i(1) = 0$ and $\lim_{\sigma_i \to 0} r_i(\sigma_i) = \infty$. As before, if a firm decides not to comply with the standard, its payoff is zero. Otherwise, its payoff is the same as in game $\Gamma(\beta)$, minus the cost of controlling noise $r_i(\sigma_i)$. The benchmark model of Section 3 is equivalent to game $\Upsilon(\beta)$ where all players are forced to pick the maximum possible amount of noise. Support ratios s_i are defined as before.

Proposition 2. If $\sum s_i \geq 1$, then for any $\beta < 1$ game $\Upsilon(\beta)$ has a unique equilibrium: all players choose *out*. If $\sum s_i < 1$, for β sufficiently close to one, game $\Upsilon(\beta)$ has an equilibrium where all players comply.

Intuitively, since it is impossible to eliminate the noise completely, if the sum of support ratios is greater than one, then some firm would have to be the last one to comply with probability greater than its support ratio, which cannot happen in equilibrium. See Appendix A for the formal proof.

5. The observability of compliance times

■ In Section 4 we showed that in a wide range of cases a technically superior standard may not be viable. This is the opposite of the conclusion of Farrell and Saloner (1985) that if players make adoption decisions sequentially, then a Pareto-superior standard gets adopted in equilibrium. In this section we reconcile the difference by adding a dynamic aspect to the game: we make an assumption that once a firm complies with a standard, others observe that and can begin to comply as well. Proposition 3 shows that for small average compliance times (i.e., as the expected compliance time goes to zero) the sequential-move game of Farrell and Saloner (1985) is a valid approximation, and the Pareto-efficient outcome is the only equilibrium. However, for large average compliance times (i.e., as the expected compliance time goes to infinity), the simultaneous-move model of Section 3 is a valid approximation (Proposition 4), and therefore a Pareto-optimal standard may be impossible to implement.

Assume that at each time t a firm can initiate the compliance process if it has not already done so. Once initiated, the process takes an uncertain amount of time. Let T_i denote the expected amount of time it takes firm i to comply; the actual compliance time (if the firm initiated the process at time t) is thus $t + T_i + \varepsilon_i$, where ε_i is a random deviation. We assume that distributions of random deviations are bounded for all players and independent of each other. Define $T_{\min} = \min_i \{T_i + \underline{\varepsilon_i}\}$ and $T_{\max} = \max_i \{T_i + \overline{\varepsilon_i}\}$, where $\underline{\varepsilon_i}$ and $\overline{\varepsilon_i}$ are the lower and the upper bounds of stochastic deviations ε_i of firm i. In other words, $T_{\min}$ is the shortest amount of time it takes any firm to comply, and $T_{\max}$ is the longest amount of time it takes any firm to comply once it has initiated the compliance process. Once a firm has initiated the process, it cannot reverse it or influence the time it is going to take.

Each firm observes when others comply, i.e., at time t each firm knows who has complied prior to time t and when they did it. However, it does not know who has *initiated* the compliance process. We also assume that the discount rate $\beta < 1$ is held constant. Then the following propositions hold.

Proposition 3. If the new standard is strictly Pareto optimal, then as $T_{\max} \to 0$, equilibrium payoffs of the players approach the payoffs they would obtain if each firm immediately decided to comply with the standard.[11]

Proof. See Appendix A.

Now consider a family of games with observable compliance times where all players' expected initiation-to-compliance times T_i are increased by the same $x \geq 0$, while holding the distributions of disturbances the same. Notice that without observable compliance times all these games are identical, up to multiplying all players' payoffs by β^x, and thus we can without ambiguity talk about the corresponding simultaneous-move game without observable compliance times.

Proposition 4. As $x \to \infty$, the game with observable compliance times has an equilibrium where the standard is adopted if and only if there exists an equilibrium of the corresponding simultaneous-move game where the standard is adopted.

Proof. See Appendix A.

6. Network externalities

■ Up to this point we have assumed a very specific form of network externalities. We now show how our results can be extended to network externalities of a general form. We continue

[11] Notice that $T_{\max} \to 0$ implies that for each player, both average compliance time and the amount of noise go to zero.

to assume that there are N players who choose their target compliance times and whose actual compliance times are independent stochastic deviations from their targets. We also assume that all players are identical. Firms bear a per-period cost c after they comply. The per-period net benefit of a firm that has complied with the standard is now $d(k)$ (i.e., the gross benefit is $c+d(k)$), where k is the number of firms that have complied up to that moment, including itself; $k \in \{1, 2, \dots, N\}$. We assume that $d(N) > 0$, i.e., the new standard is profitable if everyone adopts it, and that $d(k)$ is weakly increasing in k.[12]

Then the following result holds.

Proposition 5. For sufficiently patient players, there exists an equilibrium where the new standard is adopted if and only if $(1/N)\sum_k(1/s(k)) > 1$, where $s(k) = c/(c + d(k))$.[13]

Proof. We omit the approximation part of the proof, since it is completely analogous to step 2 of the proof of Proposition 1, and we go directly to the case with no discounting. Notice that since players are identical, in the equilibrium where they comply they have to target the same time. Therefore, for a given firm, its probability of being the kth firm to comply is equal to $(1/N)$ for any k. Therefore, its expected net benefit from delaying its compliance by a small amount of time is proportional to $c - (1/N)\sum_k(c+d(k))$, which is negative if and only if $(1/N)\sum_k(1/s(k)) > 1$.

We have to be careful with the case $(1/N)\sum_k(1/s(k)) = 1$. Then in the game with no discounting, players do not want to deviate if all target the same compliance times. In the presence of any nontrivial discounting, however, a player's higher benefits are discounted at a higher rate, since, on average, they happen when he complies later, and therefore he would be strictly better off deviating by a small amount. *Q.E.D.*

The proposition states that a standard can be adopted in equilibrium if and only if the average of inverse support ratios $((1/N)\sum_k(1/s(k)))$ is greater than one. From symmetry, it follows that in equilibrium each player has the same probability of complying first, last, or anything in between. Thus, the condition simply states that in expectation, the flow of benefits at the time of compliance is greater than the flow of costs of maintaining a standard. If that were false, a player would prefer to comply later.

The following corollary of Proposition 5 reflects the fact that the free-rider problem does not become more severe if the number of players in the standard adoption game is increased.

Corollary 1. Consider two games with different numbers of players but identical costs c and identical network externalities $d(k)$. If a standard can be adopted in an equilibrium of a game that has N players, then it can be adopted in a game that has more than N players.

Throughout the article we refer to players as firms, because the results of Section 4 are relevant for standard adoption games where the number of participants is small and each participant may be pivotal (players are a handful of corporations). The results of the present section are also relevant for standards that can only succeed if adopted by millions of consumers or other small players, even though there is no longer any uncertainty about the share of players who have complied at any given moment. The following corollary makes this claim formal. Assume that there is a continuum of identical players. Let $D(\alpha)$ denote the per-period net benefit to a player who is compliant with the new standard at the time when *share* α of the population of players is compliant; as before, the support ratio is $S(\alpha) = c/[c + D(\alpha)]$.

Corollary 2. For sufficiently patient players, there exists an equilibrium where the new standard is adopted if and only if $\int_0^1[1/S(\alpha)]d\alpha > 1$.

[12] This assumption says that externalities from technology adoption are positive, and that investments by different firms are strategic complements. Fudenberg and Tirole (1985, 1986) study the timing of technology adoption in a setting where players impose negative externalities on each other. Bliss and Nalebuff (1984) and Gradstein (1992) study the dynamics of the private provision of public goods. In that setting, externalities from individual donations are positive, but donations by one player make others less willing to contribute.

[13] Of course, $s(k)$ can be greater than one, since $d(k)$ is the *net* benefit and can be negative.

If players are not identical, and externalities are gradually increasing, the situation becomes more complicated. In particular, the viability of a standard may depend on the distribution of noise; see the web Appendix for details and for parallels with the literature on potential global games. However, as the following proposition shows, for a fixed distribution of noise, comparative statics are still very simple—a standard will be adopted earlier if players become more patient or if adoption benefits increase.

Let d_{ij} denote the per-period payoff of player i after he has complied, given that $(j-1)$ other players have complied as well, and let β_i denote the discount factor of player i.

Proposition 6. Suppose vector μ is a vector of equilibrium compliance times. Consider another game with gradually increasing network externalities, with parameters $\{\{d'_{ij}\}, \{\beta'_i\}\}$. If for any i and j, $d'_{ij} \geq d_{ij}$ and $\beta'_i \geq \beta_i$, then this game has a vector of equilibrium compliance times, μ', such that for any i, $\mu'_i \leq \mu_i$.[14]

Proof. See Appendix A.

7. Overcoming coordination failure

■ Just as the presence of adverse selection does not necessarily imply that markets collapse, synchronization problems do not necessarily imply that standards will not be adopted. Rather, institutions may arise to overcome these failures, and understanding the problems helps us better understand the institutions.

The most straightforward way to overcome synchronization failure is to write enforceable contracts, specifying penalties for late compliance. Ostrovsky and Schwarz (2006) characterize the socially optimal target compliance times and present incentive mechanisms that would induce players to target these times in equilibrium. In practice, however, compliance times may be noncontractible. Partially enforceable contracts may go a long way toward overcoming synchronization failure, and figuring out what kinds of enforceability are sufficient is an interesting area for future research.

Another way to achieve coordination is to "discretize" time and thus eliminate the possibility of being "slightly late." Perhaps unintentionally, annual industry trade shows may accomplish that. A trade show provides wide exposure to new products, and missing one may result in a year of lost profits from the new standard. Sometimes, discretization is natural: Christmas comes only once a year, and that is when many consumer goods manufacturers sell most of their merchandise. A videogame producer cannot afford to be slightly late with the new release for a game console.

It may also be easier to adopt a standard if the compliance process is gradual, with a firm's costs and benefits increasing as its degree of compliance, e.g., the number of compliant products, goes up. Alternatively, it may help if the intermediate stages of the firm's compliance process are observable to outsiders—for example, a beta version of a software package may serve as such a signal. Also, when representatives of interested parties work together on specifying and improving a standard, constant communication allows the tracking of other firms' progress and thus helps alleviate synchronization issues. This is, in fact, how Internet standards are developed by the World Wide Web Consortium (www.w3.org). The consortium includes more than 500 entities: all major software firms, many universities, publishers, and even the Library of Congress. When a need for a new standard is identified (e.g., MathML, a way of displaying mathematical equations on web pages, or XML Query, a way of efficiently exchanging data between web pages and databases), a working group of interested parties' engineers is created to develop the standard. These engineers communicate with each other as they work out technical specifications and documentation, and also work with their firms' developers on implementing the standards. Thus, by the time a version of the standard is finalized and publicly released, there is already a critical mass of adopters.

Assuming costs and benefits add up when firms merge, the support ratio of a merged firm is lower than the sum of the support ratios of its components. Thus, mergers reduce the sum of the

[14] It can also be shown that, generically, the vector of equilibrium compliance times is unique.

support ratios of market participants and help make standards viable. Once the standard is adopted, we may see spinoffs. This prediction sounds far-fetched, but this is in fact what happens quite often in large-scale real estate projects, when a single developer builds up a piece of land and then sells or leases the parts. Celebration, Florida (www.celebrationfl.com) and Santana Row in San Jose, California (www.santanarow.com) are just two recent examples of towns built by a single developer from scratch. Residential and commercial property in these areas was subsequently sold or leased.

8. Concluding remarks

■ Our results imply some interesting corollaries. First, they say that a Pareto-improving standard is not necessarily viable. The following quotation from the Court's Findings of Fact in the *U.S.* v. *Microsoft* case (U.S. District Court for the District of Columbia, 2000) gives a very similar argument:

41. In deciding whether to develop an application for a new operating system, an [Independent Software Vendor's] first consideration is the number of users it expects the operating system to attract. Out of this focus arises a collective-action problem: Each ISV realizes that the new operating system could attract a significant number of users if enough ISVs developed applications for it; but few ISVs want to sink resources into developing for the system until it becomes established. Since everyone is waiting for everyone else to bear the risk of early adoption, the new operating system has difficulty attracting enough applications to generate a positive feedback loop.[15]

Another setting where our results apply is the creation of standards by various industry groups. We can view this process as a two-stage game. First, an industry consortium develops and recommends a single standard out of a large universe of technically feasible standards. Then each player decides if and when to adopt a standard recommendation. The subgame is modelled as the standard adoption game considered earlier. The objective of the consortium is to select a Pareto-improving standard that maximizes the total payoff of the industry participants.[16] This objective implies that the consortium will always choose to recommend a viable standard, whenever a viable standard is available. Thus, the equilibrium recommendation of the consortium may be Pareto dominated by some technologically feasible standard.

A disclaimer is in order. It is not our contention that noise in adoption terms determines the outcome of a battle among competing standards. However, looking at support ratios may offer an insight into competition among standards: for a standard to survive and be backed by some coalition of players, its support ratios must be sufficiently low.

Appendix A

■ Proofs of Lemmas 1 and 2 and Propositions 1 (step 2), 2, 3, 4, and 6 follow.

Proof of Lemma 1. Let $f(\boldsymbol{\mu}) = \sum |q_i(\boldsymbol{\mu}) - p_i|$, where $q_i(\boldsymbol{\mu})$ is the probability that player i complies last given that the players' vector of target compliance times is $\boldsymbol{\mu}$. Take $\boldsymbol{\mu}^*$, which minimizes f. Suppose $f(\boldsymbol{\mu}^*) > 0$. Then there exists i such that $q_i^* = q_i(\boldsymbol{\mu}^*) > p_i$. Reduce μ_i^* slightly (call the new vector $\boldsymbol{\mu}'$) so that the new $q_i' = q_i(\boldsymbol{\mu}')$ is between p_i and q_i^*. $f(\boldsymbol{\mu}') = |q_i' - p_i| + \sum_{j \neq i} |q_j' - p_j| = f(\boldsymbol{\mu}^*) - |q_i^* - q_i'| + \sum_{j \neq i}(|q_j' - p_j| - |q_j^* - p_j|)$. Notice that for all $j \neq i$, $q_j' \geq q_j^*$ and for at least one j, $p_j \geq q_j'$. Hence, $f(\boldsymbol{\mu}') < f(\boldsymbol{\mu}^*) - |q_i^* - q_i'| + \sum_{j \neq i} |q_j' - q_j^*| = f(\boldsymbol{\mu}^*)$: contradiction.

Notice that we assumed that the minimizing $\boldsymbol{\mu}^*$ exists. To show this, note first that we can restrict $\boldsymbol{\mu}$ to, say, a set of vectors in which $\mu_1 = 0$, because adding the same number to all μ_i leaves $f(\boldsymbol{\mu})$ unchanged. Within that set, we can restrict $\boldsymbol{\mu}$ to the set of vectors such that $\max_i\{|\mu_i|\} \leq \sum_j \overline{\varepsilon_j}$. The resulting set is compact, function f is continuous, and therefore $\boldsymbol{\mu}^*$ exists. *Q.E.D.*

Proof of Lemma 2. Consider set $T = \{\boldsymbol{\mu} \mid \forall\, i, \mu_i \geq m_i, q_i(\boldsymbol{\mu}) \geq p_i\}$, where $q_i(\boldsymbol{\mu})$ is the probability that player i complies last given that players choose target times $\boldsymbol{\mu}$. Take $\boldsymbol{\mu}^* \in T$, which minimizes $\sum_i \mu_i$ in T. Then $\boldsymbol{\mu}^*$ satisfies

[15] The document goes on to say that "the vendor of a new operating system cannot effectively solve this problem by paying the necessary number of ISVs to write for its operating system, because the cost of doing so would dwarf the expected return." We disagree with this claim—in our opinion, the reason for the operating system vendor's inability to pay the ISVs has to do with complications inherent in writing and enforcing the necessary contracts.

[16] Nothing would change if the objective function of the consortium were to maximize some objective function that is increasing in the profit of each player.

the conditions of the lemma. Indeed, for all i, $q_i \geq p_i$, and we only need to show that for all i, $\mu_i > m_i$ implies $q_i = p_i$. Suppose that is not so. Take i such that $q_i > p_i$ and $\mu_i > m_i$. We can slightly decrease μ_i so that it is still greater than m_i and q_i is still greater than p_i, i.e., the modified $\boldsymbol{\mu}$ is still in T. But we decreased $\sum_i \mu_i$: contradiction.

Of course, it is necessary to prove that such minimizing $\boldsymbol{\mu}^*$ exists. To show that, first notice that set T is not empty: according to Lemma 1, there exists $\boldsymbol{\mu}^{**}$ such that $q_1(\boldsymbol{\mu}^{**}) = 1 - p_2 - \cdots - p_N$ and $q_i(\boldsymbol{\mu}^{**}) = p_i$ for $i > 1$. Second, notice that we can search for $\boldsymbol{\mu}^*$ in the intersection of sets T and $\{\boldsymbol{\mu} \mid \mu_i \geq m_i, \sum \mu_i \leq \sum \mu_i^{**}\}$. The latter set is compact and the former is closed, so their intersection is compact and, since function $\sum \mu_i$ is continuous in $\boldsymbol{\mu}$, there exists $\boldsymbol{\mu}^*$ in that set that minimizes this function.

When $\sum p_i < 1$, such vector has to be unique: if there are two vectors $(\boldsymbol{\mu}^1, \boldsymbol{\mu}^2)$ satisfying the conditions, take player i with the biggest increase in μ_i from $\boldsymbol{\mu}^1$ to $\boldsymbol{\mu}^2$; then both $q_i(\boldsymbol{\mu}^2) > p_i$ and $\mu_i^2 > m_i$: contradiction. *Q.E.D.*

Proof of Proposition 1, step 2. (i) Clearly, the strategy vector where nobody complies is an equilibrium. Let us show that for β sufficiently close to one, there exists exactly one other pure equilibrium, and no mixed ones. The proof is similar to the proof of Lemma 2.

Take a Nash equilibrium in which player i chooses compliance time μ_i with positive probability. For convenience, if player j chooses not to comply, let $t_j = \infty$ and $t_j > t_i$. By the same "marginal delay" reasoning as in Lemma 3, player i's first-order condition for choosing μ_i is

$$c_i \int_{-\infty}^{\infty} \beta^{t_i} \text{Prob}(i \neq last \mid t_i) f_i(t_i - \mu_i) dt_i = d_i \int_{-\infty}^{\infty} \beta^{t_i} \text{Prob}(i = last \mid t_i) f_i(t_i - \mu_i) dt_i$$

if $\mu_i > m_i$ and

$$c_i \int_{-\infty}^{\infty} \beta^{t_i} \text{Prob}(i \neq last \mid t_i) f_i(t_i - \mu_i) dt_i \leq d_i \int_{-\infty}^{\infty} \beta^{t_i} \text{Prob}(i = last \mid t_i) f_i(t_i - \mu_i) dt_i$$

if $\mu_i = m_i$.

By adding $c_i \int_{-\infty}^{\infty} \beta^{t_i} \text{Prob}(i = last \mid t_i) f_i(t_i - \mu_i) dt_i$ to both sides, we get the equivalent first-order condition

$$c_i \int_{-\infty}^{\infty} \beta^{t_i} f_i(t_i - \mu_i) dt_i = (c_i + d_i) \int_{-\infty}^{\infty} \beta^{t_i} \text{Prob}(t_i \geq t_j \forall j \mid t_i) f_i(t_i - \mu_i) dt_i$$

if $\mu_i > m_i$ and

$$c_i \int_{-\infty}^{\infty} \beta^{t_i} f_i(t_i - \mu_i) dt_i \leq (c_i + d_i) \int_{-\infty}^{\infty} \beta^{t_i} \text{Prob}(t_i \geq t_j \forall j \mid t_i) f_i(t_i - \mu_i) dt_i$$

if $\mu_i = m_i$.

Crucially, the ratio of the right-hand side to the left-hand side goes up if μ_i goes up, unless all other players choose not to comply, and so one and only one point on the real line can satisfy this condition. This rules out mixing among compliance times.

Let $\hat{q}_i(\boldsymbol{\mu}, \beta)$ be equal to

$$\frac{\int_{-\infty}^{\infty} \beta^{t_i} \text{Prob}(t_i \geq t_j \forall j \mid t_i) f_i(t_i - \mu_i) dt_i}{\int_{-\infty}^{\infty} \beta^{t_i} f_i(t_i - \mu_i) dt_i}.$$

Consider the set $T(\beta) = \{\boldsymbol{\mu} \mid \forall i \ \mu_i \geq m_i, \hat{q}_i(\boldsymbol{\mu}, \beta) \geq c_i/(c_i + d_i)\}$. It follows from Lemma 2 that there exists $\boldsymbol{\mu}^1 \geq \boldsymbol{m}$ such that $q_i(\boldsymbol{\mu}^1) = \hat{q}_i(\boldsymbol{\mu}^1, 1) = \text{Prob}(t_i \geq t_j \forall j \mid \boldsymbol{\mu}^1) > c_i/(c_i + d_i)$. $\lim_{\beta \to 1} \hat{q}_i(\boldsymbol{\mu}^1, \beta) = q_i(\boldsymbol{\mu}^1)$, and so there exists β_0 such that $\forall \beta > \beta_0$ we have $\hat{q}_i(\boldsymbol{\mu}^1, \beta) > c_i/(c_i + d_i)$. Thus, $T(\beta)$ is nonempty. Take $\boldsymbol{\mu}^* \in T$, which minimizes $\sum_i \mu_i$. It satisfies the first-order condition above and is a Nash equilibrium. Let us show that there are no other equilibria.

First, let us show that there is no mixing between complying and not complying. Suppose player i is indifferent between the two, and his optimal compliance time is μ_i. $\int_t^{\infty} \beta^{\tau} d\tau = (1/\ln\beta)\beta^t$. Thus, $0 = 0 \ln\beta = \ln\beta E[-c_i(\int_{t_i}^{\infty} \beta^{\tau} d\tau) + (c_i + d_i)(\int_{t_{last}}^{\infty} \beta^{\tau} d\tau)] = -c_i E[\beta^{t_i}] + (c_i + d_i) E[\beta^{t_{last}}]$. But $(c_i + d_i) E[\beta^{t_{last}}] = (c_i + d_i) E[\beta^{t_i} \text{Prob}(i = last) + \beta^{t_{last}} \text{Prob}(i \neq last)] > (c_i + d_i) E[\beta^{t_i} \text{Prob}(i = last)] \geq$ (by the first-order condition) $c_i E[\beta^{t_i}]$: contradiction. Therefore, there are no mixed equilibria.

The proof that there cannot be two equilibrium compliance time vectors $\boldsymbol{\mu}^1$, $\boldsymbol{\mu}^2$ is the same as before—if there are, take player i with the biggest increase in μ_i from $\boldsymbol{\mu}^1$ to $\boldsymbol{\mu}^2$; then both $\hat{q}_i(\boldsymbol{\mu}, \beta) > c_i/(c_i + d_i)$ and $\mu_i^2 > m_i$: contradiction.

(ii) Suppose $\boldsymbol{\mu}^*(\beta)$ does not go to $\boldsymbol{\mu}^*(1)$ as β goes to one. Then there exists a subsequence $\{\beta^n\}$ converging to one such that $\boldsymbol{\mu}^*(\beta^n)$ converges to some $\tilde{\boldsymbol{\mu}} \neq \boldsymbol{\mu}^*(1)$ (set of $\boldsymbol{\mu}^*(\beta)$ is bounded as $\beta \to 1$). Then by continuity, $\tilde{\boldsymbol{\mu}}$ satisfies the first-order condition with $\beta = 1$ and is therefore an equilibrium of game $\Gamma(1)$; in this equilibrium, all players' payoffs are finite. But we know that $\Gamma(1)$ has only one equilibrium with finite payoffs, equal to $\boldsymbol{\mu}^*(1)$.

(iii) Take $\beta_1 < \beta_2$, and suppose for some i, $\mu_1 = \mu_i^*(\beta_1) < \mu_2 = \mu_i^*(\beta_2)$. Without loss of generality, assume that

$i = \arg\max_j\{\mu_j^*(\beta_2) - \mu_j^*(\beta_1)\}$. By the first-order condition,

$$(c_i + d_i)\int \beta_1^{t_i}\text{Prob}(t_i = last)f(t_i - \mu_1)dt_i \geq c_i \int \beta_1^{t_i} f(t_i - \mu_1)dt_i.$$

Since $\mu_1 < \mu_2$,

$$(c_i + d_i)\int \beta_1^{t_i}\text{Prob}(t_i = last)f(t_i - \mu_2)dt_i > c_i \int \beta_1^{t_i} f(t_i - \mu_2)dt_i$$

and

$$\int \beta_1^{t_i}((c_i + d_i)\text{Prob}(t_i = last) - c_i)f(t_i - \mu_2)dt_i > 0.$$

Let t_i^* be such that $(c_i + d_i)\text{Prob}(t_i^* = last) - c_i = 0$. The integrand is negative for $t_i < t_i^*$ and positive for $t_i > t_i^*$. $\beta_2 > \beta_1$, and so $(\beta_2/\beta_1)^t$ is an increasing function. Therefore,

$$\int \beta_2^{t_i}((c_i + d_i)\text{Prob}(t_i = last) - c_i)f(t_i - \mu_2)dt_i$$
$$\geq \left(\frac{\beta_2}{\beta_1}\right)^{t_i^*}\int \beta_1^{t_i}((c_i + d_i)\text{Prob}(t_i = last) - c_i)f(t_i - \mu_2)dt_i > 0.$$

But this, together with $\mu_2 > \mu_1 \geq m_i$, is a violation of the first-order condition for an equilibrium.

(iv) To prove the last statement, assume the opposite, and consider an equilibrium with adoption for some $\beta < 1$ and support ratios adding up to one or more. Then there exists some player i whose probability of being last, p_i, is at least as low as his support ratio $s_i = c_i/(c_i + d_i)$. For this player not to be willing to delay his target compliance time by a small ε, it has to be the case that

$$c_i\int_{-\infty}^{\infty} \beta^{t_i} f_i(t_i - \mu_i)dt_i \leq (c_i + d_i)\int_{-\infty}^{\infty} \beta^{t_i}\text{Prob}(i = last \mid t_i)f_i(t_i - \mu_i)dt_i,$$

i.e.,

$$c_i E_{t_i}[\beta^{t_i}] \leq (c_i + d_i)E_{t_i}[\beta^{t_i}\text{Prob}(i = last \mid t_i)].$$

But for $\beta < 1$, $\text{Cov}(\beta^{t_i}, \text{Prob}(i = last \mid t_i)) < 0$, and so $(c_i + d_i)E_{t_i}[\beta^{t_i}\text{Prob}(i = last \mid t_i)] < (c_i + d_i)E_{t_i}[\text{Prob}(i = last \mid t_i)]E_{t_i}[\beta^{t_i}] = (c_i + d_i)\text{Prob}(i = last \mid t_i)E_{t_i}[\beta^{t_i}] = (c_i + d_i)p_i E_{t_i}[\beta^{t_i}] \leq c_i E_{t_i}[\beta^{t_i}]$. *Q.E.D.*

Proof of Proposition 2. The proof of the first statement is straightforward. Consider the distributions of compliance times induced by the firms' (possibly mixed) strategies. At least one firm (say, firm i) has to be the last one to comply with probability less than or equal to its support ratio. Its incentive to marginally increase its target compliance time by the same small amount in all of the strategies it chooses with positive probability is proportional to $E[\beta^{t_i}(c_i - (c_i + d_i)\text{Prob}(t_i = \max\{t_j\}))] = E[\beta^{t_i}]E[(c_i - (c_i + d_i)\text{Prob}(t_i = \max\{t_j\}))] + \text{Cov}(\beta^{t_i}, (c_i - (c_i + d_i)\text{Prob}(t_i = \max\{t_j\})))$. Since firm i's probability of being last is less than or equal to its support ratio, $E[\beta^{t_i}]E[(c_i - (c_i + d_i)\text{Prob}(t_i = \max\{t_j\}))] \geq 0$. In addition, β^{t_i} is a strictly decreasing function of t_i, and $(c_i - (c_i + d_i)\text{Prob}(t_i = \max\{t_j\})))$ is a nonincreasing function, which is strictly decreasing over some intervals that have positive mass. Hence, $\text{Cov}(\beta^{t_i}, (c_i - (c_i + d_i)\text{Prob}(t_i = \max\{t_j\}))) > 0$, and therefore $E[\beta^{t_i}(c_i - (c_i + d_i)\text{Prob}(t_i = \max\{t_j\}))] > 0$, i.e., firm i could improve its expected payoff: contradiction.

Let us now prove the second statement. Suppose the sum of support ratios is less than one. Let $M = \sum_j(m_j + \overline{\varepsilon_j})$ and for each i, $\underline{\sigma_i}$ is the unique solution to the equation $r_i(\underline{\sigma_i}) = 100(c_i + d_i)M$. Consider a modified game $\Upsilon'(\beta)$, where firms cannot play *out*, must choose target compliance times less than or equal to $10M$, and must choose the amount of noise σ_i greater than or equal to $\underline{\sigma_i}$. Also, in $\Upsilon'(\beta)$, firms only get payoff flows until time $20M$; their payoff flows are zero afterwards. Payoffs are continuous, strategy spaces are compact, and so for any β, there exists a Nash equilibrium, possibly in mixed strategies (Glicksberg, 1952; Fudenberg and Tirole, 1991). Let us show that for β sufficiently close to one, this profile of strategies is also an equilibrium of game $\Upsilon(\beta)$.

It is clear that for sufficiently high β, following these strategies is individually better for the firms than playing *out*, and so we only need to show that the firms will not want to change their target compliance times or amounts of noise.

First, let us show that for β sufficiently high, in any equilibrium of game $\Upsilon'(\beta)$, at least one firm assigns a positive probability to target compliance times less than or equal to M. Suppose this is not the case. Then there exists a sequence $\{\beta_k\}$ monotonically converging to one and a corresponding sequence of equilibria of $\Upsilon'(\beta_k)$ where none of the firms could benefit by slightly reducing all target compliance times by the same amount (since $M > m_i$ for any i). That is, in these equilibria, $E[\beta^{t_k}(c_i - (c_i + d_i)\text{Prob}(t_i = \max\{t_j\}))] \geq 0$. Dividing by $(c_i + d_i)$ and summing over all firms we get

$$0 \leq \sum_i E_{t_i}[\beta_k^{t_i}(s_i - \text{Prob}(t_i = \max\{t_j\}))] \tag{A1}$$

$$= \sum_i E_{t_i}[\beta_k^{t_i}]s_i - \sum_i E_{t_i}[\beta_k^{t_i}]E_{t_i}[\text{Prob}(t_i = \max\{t_j\})] - \sum_i \text{Cov}_{t_i}(\beta_k^{t_i}, \text{Prob}(t_i = \max\{t_j\})). \tag{A2}$$

Since target compliance times are restricted to bounded intervals, β_k^t converges to one uniformly as β_k converges to one, and so $\lim_k \sum_i E_{t_i}[\beta_k^{t_i}]s_i = \sum_i s_i$, $\lim_k \sum_i E_{t_i}[\beta_k^{t_i}]E_{t_i}[\text{Prob}(t_i = \max\{t_j\})] = \lim_k \sum_i \text{Prob}(t_i = \max\{t_j\}) = 1$, and $\lim_k \sum_i \text{Cov}_{t_i}(\beta_k^{t_i}, \text{Prob}(t_i = \max\{t_j\})) = 0$. Hence, the limit of expression (A2) as β_k goes to one is equal to $\sum_i s_i - 1 < 0$: contradiction.

Now, notice that in equilibrium, if a firm mixes among different target compliance times and amounts of noise, it cannot target two different compliance times that are more than M apart. That is because at the later target compliance time, the benefit from marginally increasing it, $E_{t_i}[\beta_k^{t_i}(c_i - (c_i + d_i)\text{Prob}(t_i = \max\{t_j\}))$, has to be exactly equal to zero. Since the two compliance times are more than M apart, the highest probability of being last under the earlier target compliance time is lower than the lowest probability of being last under the later time, regardless of the amounts of noise chosen. But then the benefit from marginally increasing the *earlier* target compliance time will be strictly positive: contradiction.

Now take a sufficiently high β, so that in any equilibrium of game $\Upsilon'(\beta)$ at least one firm (say, firm i) assigns a positive probability to target compliance times less than M, and pick any equilibrium, $\boldsymbol{S}$. In this equilibrium, no firm targets a compliance time greater than $3M$, because that would imply that all target compliance times of this firm are greater than $2M$, but this in turn implies that when firm i picks a target compliance time less than M, it has a zero probability of being the last one to comply, which is impossible in equilibrium. But then strategy profile $\boldsymbol{S}$ is an equilibrium of the original game $\Upsilon(\beta)$. Indeed, suppose it is not, and suppose firm j can increase its payoff by switching to another action. Suppose action (μ_j^*, σ_j^*) is a best reply. It has to be the case that (i) $\mu_j^* > 10M$, or (ii) $\mu_j^* \le 10M$ and $\sigma_j^* < \underline{\sigma_j}$, or (iii) $\mu_j^* \le 10M$ and $\sigma_j^* \ge \underline{\sigma_j}$. But (i) is impossible because then firm j would be the last one to comply with probability one, and would be able to increase its payoff by decreasing μ_j^*; (ii) is impossible because setting $\sigma_j^* < \underline{\sigma_j}$ is so expensive that it is strictly better to set $\sigma_j = 1$ and $\mu_j = m_j$ given that other firms target compliance times less than or equal to $3M$. Hence, (iii) is the only remaining option. But notice that in that case, the pair (μ_j^*, σ_j^*) would be a feasible choice under the rules of game Υ', and hence $\boldsymbol{S}$ could not have been an equilibrium. *Q.E.D.*

Proof of Proposition 3. We prove the statement by induction on N, the number of players. For $N = 1$ the statement is obvious. Suppose it holds for $N = k$. Let us show that it also holds for $N = k+1$. Suppose there is a sequence of equilibria for $T_{\max} \to 0$ in which the payoffs of players converge to something other than the payoffs of the Pareto-efficient outcome (i.e., the immediate adoption of the standard). Take any player i whose equilibrium payoff in the limit is strictly less than his payoff under the immediate adoption. If he deviates from his equilibrium strategy, and initiates the compliance process immediately, then others will observe that he has complied at most after $T_{\max}$. After that we are back in the game with $N - 1$ players, which, by the assumption of induction, has equilibria payoffs arbitrarily close to Pareto-optimal ones as $T_{\max}$ goes to zero. But then player i's payoff from deviating is less than the Pareto payoff by at most the costs and forgone profits up to $T_{\max}$ plus the costs and forgone profits while the $(N - 1)$-player subgame takes place. But each of these two components goes to zero as $T_{\max}$ goes to zero, and so player i's payoff from deviating goes to his Pareto payoff—thus any equilibrium payoff has to approach the Pareto payoff as well. *Q.E.D.*

Proof of Proposition 4. Suppose the simultaneous-move game has an equilibrium where the standard is adopted. Let x be such that $\min_i\{T_i + x + \underline{\varepsilon_i}\}$ is greater than $\max_i\{T_i + \overline{\varepsilon_i}\}$. Then the equilibrium with adoption of the simultaneous-move game remains an equilibrium of the game with observable compliance time, since the optimal target compliance times are such that players want to initiate the compliance process before they could have possibly observed other players' compliance.

Conversely, suppose the simultaneous-move game does not have an equilibrium where the standard is adopted, but for arbitrarily large x the game with observable compliance times does. Notice that in an equilibrium with adoption, for a large enough x, each player (say, player 1) has to initiate the compliance process before observing others comply with a positive probability (otherwise the payoff of at least one of the other players is negative, as player 1 always complies too late). Let $\underline{\mu_i}(x)$ denote the earliest target compliance time of player i in an equilibrium with delay x; subtract $\underline{\mu_1}(x)$ from all $\underline{\mu_i}(x)$s to normalize. Now consider the sequence of vectors $\underline{\boldsymbol{\mu}}(x)$ as x goes to infinity. This sequence has to be bounded—otherwise the player with the lowest $\underline{\mu_i}(x)$ would find it profitable to deviate and not comply at all. Thus, it has to converge to some vector $\underline{\boldsymbol{\mu}}$. By assumption, there were no equilibria with compliance in the simultaneous-move game, and thus there is at least one player who would find it strictly profitable to slightly increase his target compliance time if everyone targeted $\underline{\boldsymbol{\mu}}$. But then, for a large enough x, this player would also find it profitable to do that in the observable-compliance game with delay x. *Q.E.D.*

Proof of Proposition 6. Consider the new game, with modified parameters. Let $\pi_i(\boldsymbol{\nu})$ be the expected payoff of firm i given that the firms target the vector of compliance times $\boldsymbol{\nu}$. Let T be the set of target compliance vectors $\boldsymbol{\nu} \ge \boldsymbol{m}$ such that for each i, $\partial\pi_i(\boldsymbol{\nu})/\partial\nu_i \le 0$, i.e., no firm wants to marginally increase its target compliance time.

Let us show that $\boldsymbol{\mu}$—an equilibrium of the old game—is in set T. Let t denote the realized compliance time of firm i and $\phi_j(t)$ the probability that i is the jth firm to comply given that its actual compliance time is t.

$$\frac{\partial\pi_i(\boldsymbol{\nu})}{\partial\nu_i} = -E_t\left[(\beta_i')^t \sum_{j=1}^{N} \phi_j(t) d_{ij}'\right].$$

For all $i, j, d_{ij}' \ge d_{ij}$, so it is sufficient to show that $E[(\beta_i')^t \sum_{j=1}^{N} \phi_j(t) d_{ij}] \ge 0$.

$$E[(\beta_i')^t \sum_{j=1}^{N} \phi_j(t) d_{ij}] = E[(\beta_i')^t d_{i1}] + \sum_{j=2}^{N} E[(\beta_i')^t (d_{ij} - d_{i,j-1})(\phi_j(t) + \cdots + \phi_N(t))]. \tag{A3}$$

If $d_{i1} \geq 0$, this expression is obviously greater than zero, since by assumption, externalities are gradually increasing, i.e., $(d_{ij} - d_{i,j-1}) \geq 0$ for any $j > 1$. (In other words, if there are no waiting costs, and the firm starts getting a flow of net benefits immediately upon compliance, it does not have an incentive to increase the target time.) Let us consider the interesting case $d_{i1} < 0$. Since $\boldsymbol{\mu}$ is an equilibrium of the initial game, we know that $E[(\beta_i)^t d_{i1}] + \sum_{j=2}^{N} E[(\beta_i)^t (d_{ij} - d_{i,j-1})(\phi_j(t) + \cdots + \phi_N(t))] > 0$. Thus, it is enough to show that for each $j \geq 2$,

$$\frac{E[(\beta_i')^t (\phi_j(t) + \cdots + \phi_N(t))]}{E[(\beta_i)^t (\phi_j(t) + \cdots + \phi_N(t))]} \geq \frac{E[(\beta_i')^t]}{E[(\beta_i)^t]}. \tag{A4}$$

Note that for any j, $\Phi_j(t) = (\phi_j(t) + \ldots + \phi_N(t))$, i.e., the probability that at least $j-1$ other firms have complied by time t, is a nondecreasing function of t. Let us also introduce a new probability density function, $g(t) = \beta_i^t f_i(t)/E[\beta_i^t]$. Let $E^*[h]$ denote the expectation of random variable h with respect to probability density g. Then equation (A4) is equivalent to

$$E^*\left[\left(\frac{\beta_i'}{\beta_i}\right)^t \Phi_j(t)\right] \geq E^*\left[\left(\frac{\beta_i'}{\beta_i}\right)^t\right] E^*[\Phi_j(t)],$$

which is true because both $(\beta_i'/\beta_i)^t$ and $\Phi_j(t)$ are nondecreasing functions of t, and therefore their covariance is nonnegative. Hence, $\boldsymbol{\mu} \in T$, and so set T is not empty.

Take a vector $\boldsymbol{\nu}$ in set T that minimizes $\sum \nu_i$ (such a vector exists, because T is closed and is bounded below by the vector of lowest possible compliance times $\boldsymbol{m}$). This vector is a vector of equilibrium compliance times. Indeed, by construction, no firm wants to marginally increase its compliance time. It is easy to check that, therefore, no firm wants to increase its compliance time by any amount. On the other hand, suppose there is a firm that would like to decrease its target compliance time, and whose lower bound (m_i) doesn't bind. Then, if it did decrease its target compliance time, other firms would not want to *increase* theirs (though they might possibly want to decrease them as well), and so this new vector of arrival times would also be in set T. This leads to a contradiction, since this new vector would have a lower sum of target compliance times than vector $\boldsymbol{\nu}$—hence vector $\boldsymbol{\nu}$ is an equilibrium.

We now need to demonstrate that for any i, $\nu_i \leq \mu_i$. Suppose that is not so. Let $\boldsymbol{\eta} = \min\{\boldsymbol{\mu}, \boldsymbol{\nu}\}$, element by element. Then $\boldsymbol{\eta} \in T$, and also $\sum \eta_i < \sum \nu_i$: contradiction. *Q.E.D.*

Appendix B

■ The derivation of the viability criterion for the case of two identical players and uniform disturbances follows.

Suppose there are two identical players $i = \{1, 2\}$ with cost c, net benefit d, discount factor β, and disturbances distributed uniformly on $[0, A]$.

If in equilibrium the firms adopt, they have to target the same compliance time (since they are identical), and the first-order condition that has to hold is therefore

$$c \int_0^A \beta^{t_i} dt_i \leq (c+d) \int_0^A \beta^{t_i} \text{Prob}(t_i = \max\{t_1, t_2\}) dt = (c+d) \int_0^A \beta^{t_i} \frac{t_i}{A} dt_i.$$

Now,

$$\int_0^A \beta^{t_i} dt_i = \frac{1}{\log \beta}\left(\beta^A - 1\right),$$

$$\int_0^A \beta^{t_i} \frac{t_i}{A} dt_i = \frac{1}{A \log \beta}\left(A\beta^A - \frac{1}{\log \beta}\left(\beta^A - 1\right)\right),$$

and so for the standard to be viable, the sum of support ratios, $2c/(c+d)$, has to be less than or equal to

$$\frac{A \log \beta \cdot \beta^A - \left(\beta^A - 1\right)}{A \log \beta \left(\beta^A - 1\right)}.$$

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