# LECTURE 16

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## PHYSICS OF ULTRASOUND

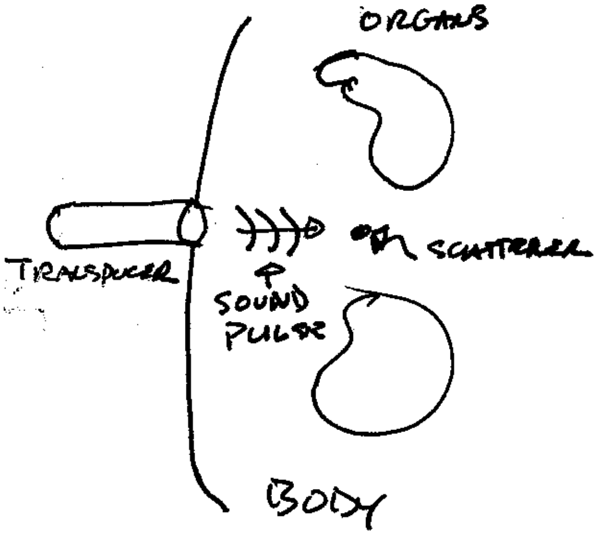
TODAY:
- BASIC IDEA
- PHYSICS OF ULTRASOUND PROPAGATION

WEDNESDAY:
- ULTRASOUND SYSTEMS
- FOCUSING, ARRAYS

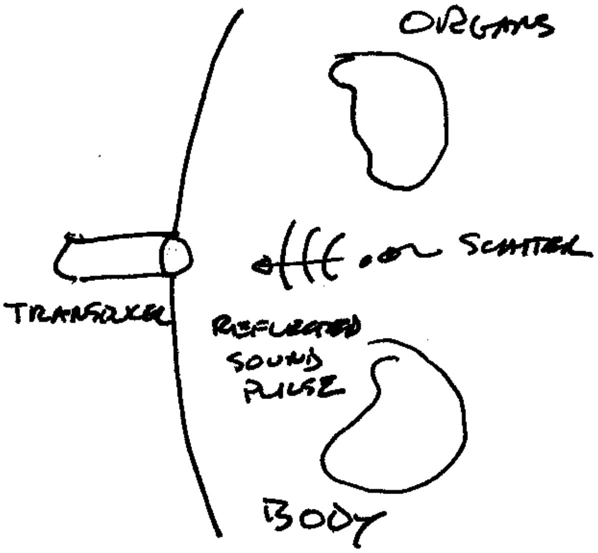
FRIDAY:
- DOPPLER ULTRASOUND
- COLOR FLOW

BOOK
- READ 10.1 – 10.3 IN PRINCE + LINKS

# BASIC ULTRASOUND IDEA

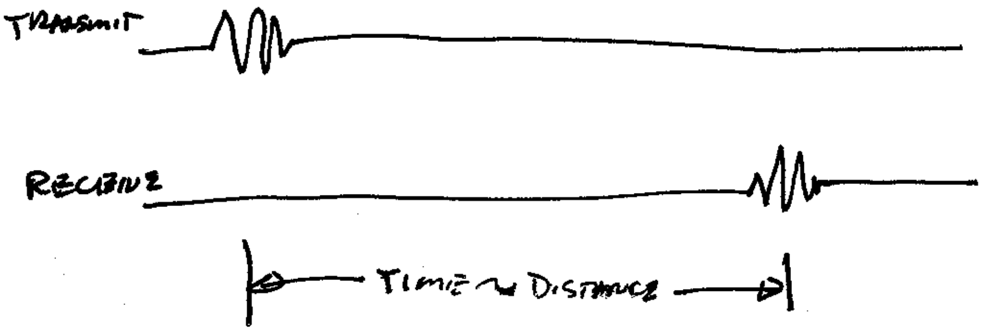


TRANSMIT SOUND PULSE

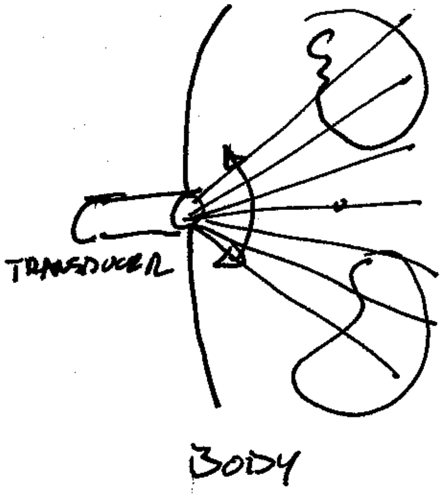


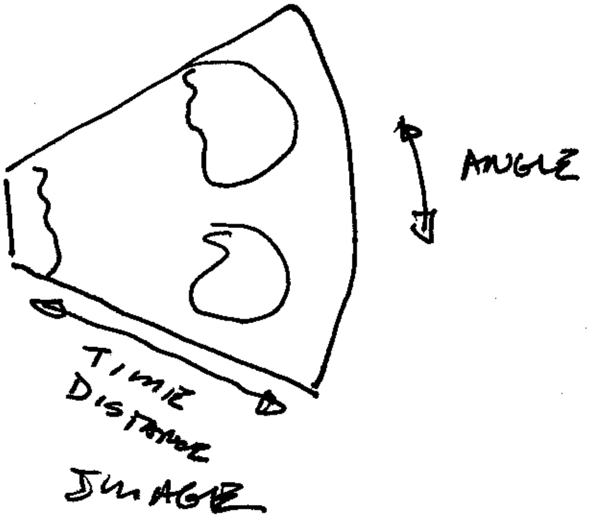
LISTEN FOR REFLECTION

WAVEFORMS:



REPEAT AT DIFFERENT ANGLES



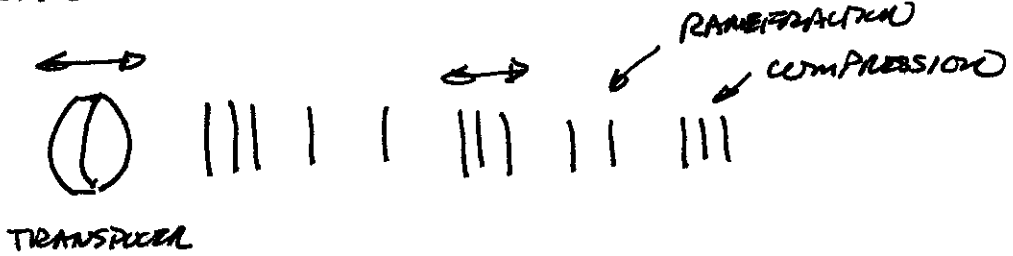


# WHY IS ULTRASOUND INTERESTING?

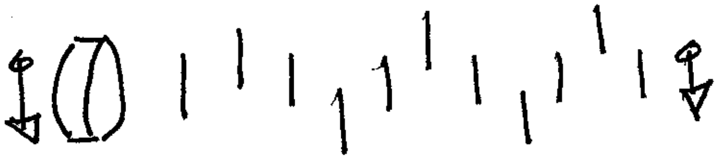
- NON-TOXIC, NO IONIZING RADIATION
- INEXPENSIVE
- PORTABLE
- MANY APPLICATIONS
  - CARDIAC
  - OB/GYN (FETAL)
  - ABDOMINAL
  - BREAST
- INTERESTING PHYSICS
  - TISSUE CHARACTERISTICS
  - CONTRAST AGENTS (BUBBLES)
  - HEATING / THERAPY
- INTERESTING EE PROBLEM
  - HIGH DATA RATES
  - COMPUTATIONALLY INTENSIVE SIGNAL PROCESSING
  - SMALL SIZE, LOW POWER

# ULTRASOUND WAVES

COMPRESSION WAVES (LONGITUDINAL)



SHEAR WAVES (TRANSVERSE)



IN TISSUE

COMPRESSION WAVES ARE FAST, USED IN IMAGING

SHEAR WAVES ARE SLOW, DECAY RAPIDLY USED IN TISSUE CHARACTERIZATION

WE'LL FOCUS ON COMPRESSION WAVES HERE

## SPEED OF SOUND IN TISSUE

$$c = \sqrt{\frac{1}{k\rho}}$$

$c$ = SPEED OF SOUND
$\rho$ = TISSUE DENSITY
$k$ = COMPRESSABILITY

AQUEOUS TISSUES HAVE SIMILAR SPEEDS (LIVER, BRAIN, KIDNEY)

BONE, LUNG, FAT HAVE DIFFERENT SPEEDS

## TYPICAL VALUES OF SPEED OF SOUND

| | |
|---|---|
| WATER | 1480 m/s |
| BLOOD | 1570 m/s |
| BRAIN | 1540 m/s |
| LIVER | 1570 m/s |
| MUSCLE | 1550 m/s |
| FAT | 1450 m/s |
| BONE | 4080 m/s |

## ULTRASOUND WAVES

ULTRASOUND WAVES ARE CHARACTERIZED BY

| | | |
|---|---|---|
| $u(x,y,z,t)$ | PARTICLE POSITION | (± 1nm) |
| $v(x,y,z,t)$ | PARTICLE VELOCITY | ( 1-10cm/sec)<br>NOT $c$ |
| $p(x,y,z,t)$ | PRESSURE | |

THESE ARE ALL VECTORS IN GENERAL, BUT SCALARS ARE ADEQUATE FOR COMPRESSION WAVES

PRESSURE IS

$$p = Zv$$

WHERE

$$Z = \rho c$$

IS THE CHARACTERISTIC IMPEDANCE

THINK:
$v$ ⇒ CURRENT
$p$ ⇒ VOLTAGE
$Z$ ⇒ IMPEDANCE

$\rho$ = DENSITY
$c$ = SPEED OF SOUND

ANY OF u, v, p SATISFY WAVE EQN

$$\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

LAPLACIAN OR "DEL SQUARED"

IN 1D

$$\frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

WHICH HAS A SOLUTION

$$p(z,t) = \underbrace{\phi_f(t - z/c)}_{\text{FORWARD PROPAGATING}} + \underbrace{\phi_b(t + z/c)}_{\text{BACKWARD PROPAGATING}}$$

EXAMPLE

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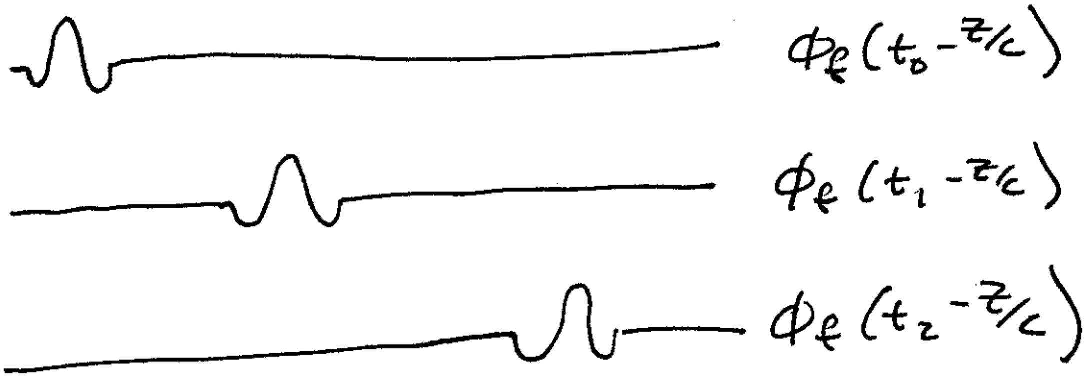


PULSE LAUNCHED BY TRANSDUCER PROPAGATES TO RIGHT

IMPORTANT SPECIAL CASE

$$p(z,t) = \phi_f(t - z/c) = \cos(k(z - ct))$$
$$= \cos(\underbrace{k}_{\substack{\text{SPATIAL}\\\text{FREQUENCY}}} z - \underbrace{kc}_{\substack{\text{TEMPORAL}\\\text{FREQUENCY}}} t)$$

## TEMPORAL FREQUENCY

## SPATIAL FREQUENCY

$$f = \frac{kc}{2\pi} \xRightarrow{\text{SOLVE FOR K}} k = \frac{2\pi f}{c}$$

## SPATIAL WAVELENGTH

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\frac{2\pi f}{c}} = \underline{\underline{\frac{c}{f}}}$$

## EXAMPLE

3.5 MHz ULTRASOUND IN LIVER (1570 m/s)

$$\lambda = \frac{1570 \text{ m/s}}{3.5 \times 10^{6} \text{ 1/s}} = 0.45 \times 10^{-3} \text{ m} = 0.45 \text{ mm}$$

THIS INDICATES THE TYPE OF RESOLUTION THAT MAY BE POSSIBLE.

## ACCOUSTIC INTENSITY

ACCOUSTIC INTENSITY IS

$$I = pv$$

THINK
$p$ = VOLTAGE
$v$ = CURRENT
$I$ = POWER

ACCOUSTIC ENERGY FLUX

## ULTRASOUND WAVE PROPAGATION

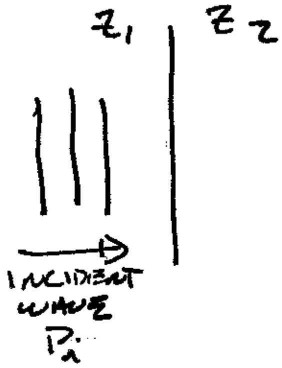
TWO MAJOR EFFECTS

- REFLECTION
- ATTENUATION
  - ABSORPTION
  - SCATTER

## REFLECTION

ULTRASOUND IS REFLECTED AT TISSUE BOUNDARIES WITH DIFFERENT ACCOUSTIC IMPEDANCES

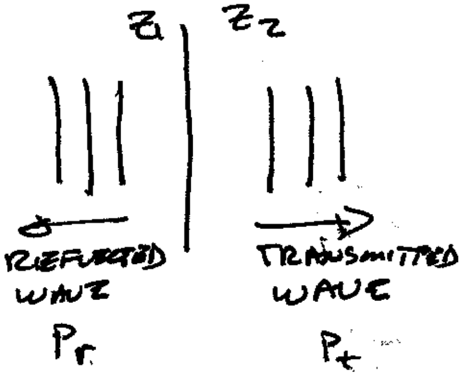
NORMAL INCIDENCE



REFLECTION COEFFICIENT

$$R_p = \frac{P_r}{P_1} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$R_I = R_p^2$$



TRANSMISSION COEFFICIENT

$$T_p = \frac{P_t}{P_i} = \frac{2Z_2}{Z_2 + Z_1}$$

$$T_I = T_p^2$$

## SPECIAL CASES

NO REFLECTION IF $Z_1 = Z_2$

LARGE REFLECTION IF $Z_1$ AND $Z_2$ DIFFER LARGELY

TISSUE / AIR INTERFACES

TISSUE / BONE INTERFACES

TYPICAL IMPEDANCES

| | | |
|---|---|---|
| KIDNEY | 1.62 | $(g/cm^2 s^{-1}) \times 10^5$ |
| LIVER | 1.65 | |
| BONE | 7.8 | |
| LUNG | 0.26 | |

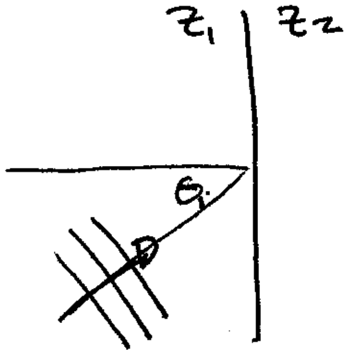
LIVER / LUNG

$$R_p = \frac{0.26 - 1.65}{0.26 + 1.65} = -0.73$$

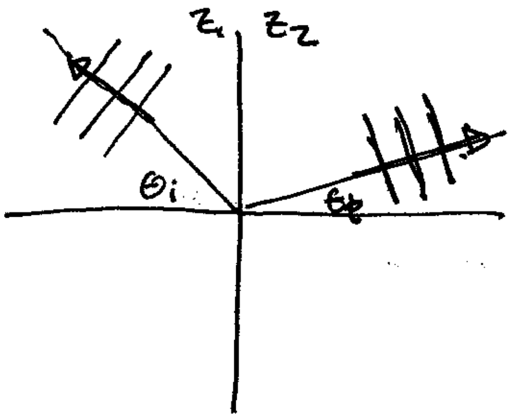
LIVER / KIDNEY

$$R_p = \frac{1.62 - 1.65}{1.62 + 1.65} = -0.01$$

## NON-NORMAL INCIDENCE



INCIDENT



REFLECTED TRANSMITTED

SNELL'S LAW

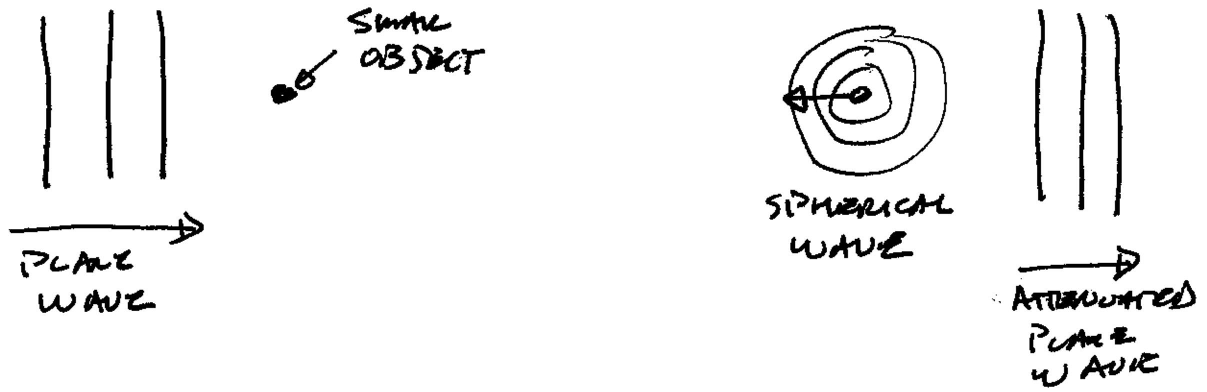
$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{c_1}{c_2}$$

<u>KEY POINT</u>: REFLECTION CAN DIRECT ENERGY <u>AWAY</u> FROM TRANSDUCER

# ATTENUATION

ABSORBTION – ENERGY DISSIPATED IN TISSUE AS ULTRASOUND WAVE PASSES

SCATTERING



IF OBJECTS ARE MUCH SMALLER THAN $\lambda$, THEY SCATTER ISOTROPICALLY.

EXAMPLE: BLOOD CELLS $\sim 10\,\mu m$

RESULT

- WAVEFRONT IS ATTENUATED
- ENERGY SCATTERED BACK TO SOURCE
  - TISSUE SIGNAL AND CONTRAST
  - SPECKLE, LOW SNR

# ATTENUATION OVERALL

GOOD MODEL

$$I(z) = I_0 e^{-\mu z}$$

$\mu$ INTENSITY ATTENUATION

$$p(z) = p_0 e^{-\alpha z}$$

$\alpha$ PRESSURE ATTENUATION

WHEN EXPRESSED IN dB/cm $\left(10 \log \frac{I(z)}{I_0} \text{ or } 20 \log \frac{p(z)}{p_0}\right)$

$$\alpha = \mu \qquad (\mathrm{dB/cm})$$

FOR MANY TISSUES, $a \sim f$

$$\alpha = a f$$

WHERE

| | a | |
|---|---|---|
| LIVER | 0.94 | dB/(cm·MHz) |
| KIDNEY | 1.0 | |
| BLOOD | 0.18 | |
| CARDIAC MUSCLE | 1.8 | |
| LUNG | 41.0 | |
| BONE | 20.0 | |

EXAMPLE

10 cm DEEP LESION
LIVER TISSUE
3 MHz ULTRASOUND

$$\alpha = \left(0.94 \frac{\mathrm{dB}}{\mathrm{cm \cdot MHz}}\right) \cdot (3\,\mathrm{MHz}) = 2.82\,\mathrm{dB/cm}$$

TOTAL ATTENUATION IS

$$(2.82\,\mathrm{dB/cm})(10\,\mathrm{cm})(2) = \underline{56.4\,\mathrm{dB}}$$