EE 169 LECTURE 2

ASSIGNMENT

READ SECTION 2.0 - 2.3

SECTION 1.0 - 1.9 FOR INTEREST

2D SIGNALS

IMPULSES

BLADES

SAMPLING FUNCTIONS

2D RECTS AND SINCS

COMPLEX EXPONENTIALS AND SINUSOIDS

SPATIAL FREQUENCIES IN 2D

SEPERABLE SIGNALS

PERIODIC SIGNALS

2D EXTENSIONS OF FAMILIAR 1D SIGNALS

IMPULSE FUNCTION

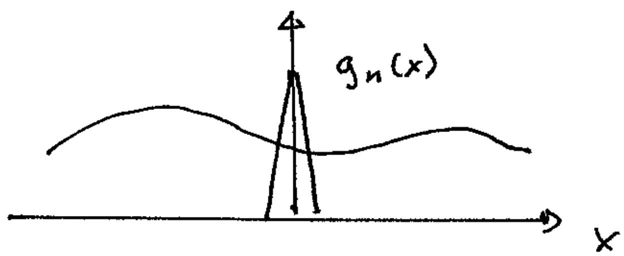
IN 1D, DEFINED BY

$$\int_{-\infty}^{\infty} f(x)\,\delta(x)\,dx = f(0)$$

$f(x)$ CONTINUOUS AT $x=0$

"SIFTING" PROPERTY

CONCEPTUALLY



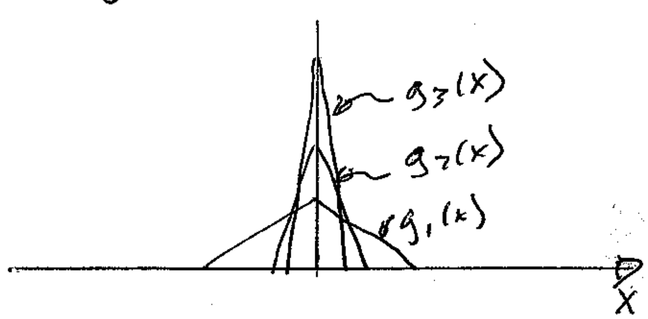
WHERE

$$\int_{-\infty}^{\infty} g_n(x) = 1$$

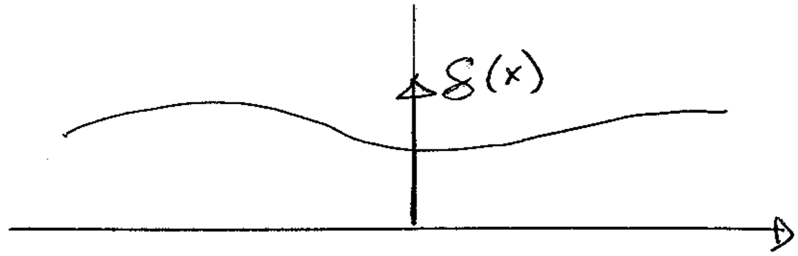
AND $g_n(x)$ GETS NARROWER AND TALLER AS $n \to \infty$

EXAMPLE

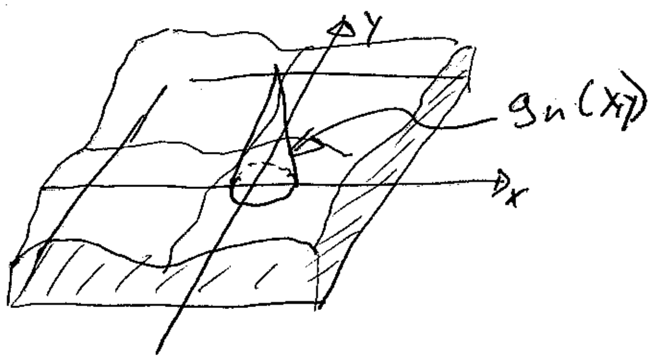
$$g_n(x) = n\,\Delta(nx)$$



DRAWN AS



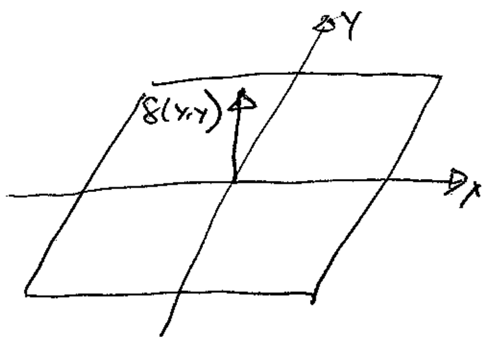
SAME IDEA IN 2D



$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_n(x,y)\,dx\,dy = 1$$

$$g_n(x,y) \to \delta(x,y)$$

$\delta(x,y)$ IS DRAWN AS



DEFINED AS

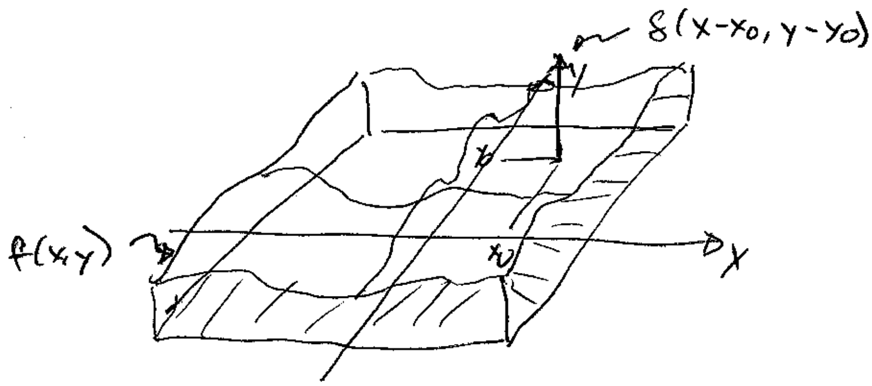
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x,y)\,dx\,dy = f(0,0)$$

FOR $f(x,y)$ CONTINUOUS AT $x=0$, $y=0$.

SHIFTING $\delta(x,y)$ TO (x_0, y_0) EXTRACTS VALUE OF $f(x,y)$ AT (x_0, y_0)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x-x_0, y-y_0)\,dx\,dy = f(x_0, y_0)$$

USEFUL FOR 2D SAMPLING



SCALING IMPULSES (WE AVOIDED THIS IN 102A!)

$$\delta(ax, by) = \frac{1}{|ab|}\,\delta(x,y)$$

IF WE MAKE δ "SMALLER", ITS VOLUME DECREASES PROPORTIONATELY. THINK OF $g_n(ax, ay)$ AS $n \to \infty$.

DOES

$$\delta(x,y) = \delta(x)\,\delta(y)\,?$$

TEST BY SUBSTITUTING INTO DEFINITION

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x)\delta(y)\,dx\,dy$$

$$= \int_{-\infty}^{\infty}\delta(y)\underbrace{\left[\int_{-\infty}^{\infty} f(x,y)\,\delta(x)\,dx\right]}_{f(0,y)}dy$$

$$= \int_{-\infty}^{\infty}\delta(y)\, f(0,y)\,dy$$

$$= f(0,0)$$

SO $\delta(x)\delta(y)$ DOES THE SAME THING AS $\delta(x,y)$

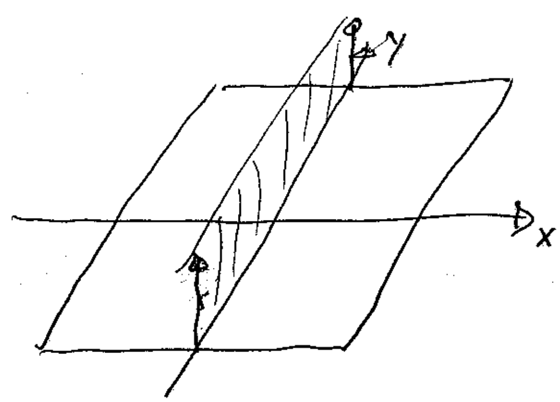
SO YES.

WHAT IS

$$f(x,y) = \delta(x)$$

AS A FUNCTION OF (x,y)? THINK OF THIS AS

$$f(x,y) = \delta(x)\,1(y)$$



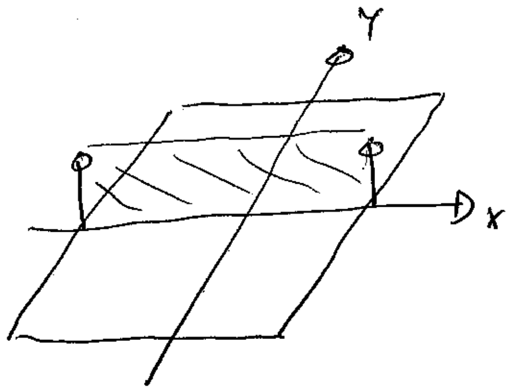
IMPULSE AT $x=0$
FOR ANY y

$\delta(\cdot)$ BLADE

SIMILARLY

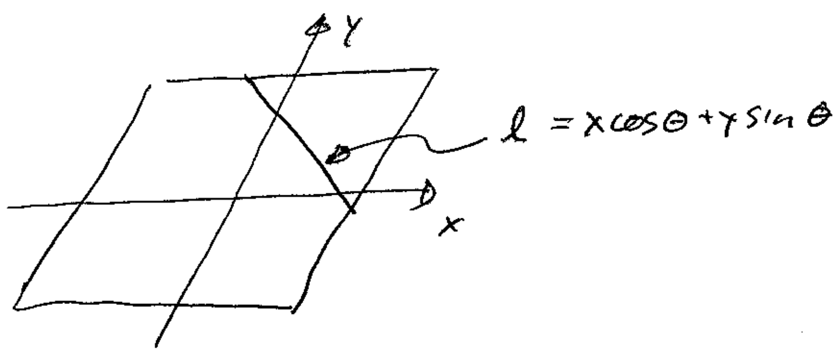
$$f(x,y) = \delta(y)$$

IS



HOW ABOUT OTHER ORIENTATIONS?

CONSIDER A LINE



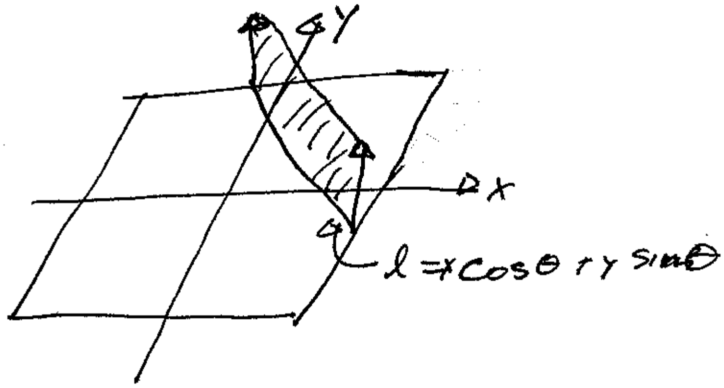
WHAT DOES

$$\delta(x\cos\theta + y\sin\theta - l)$$

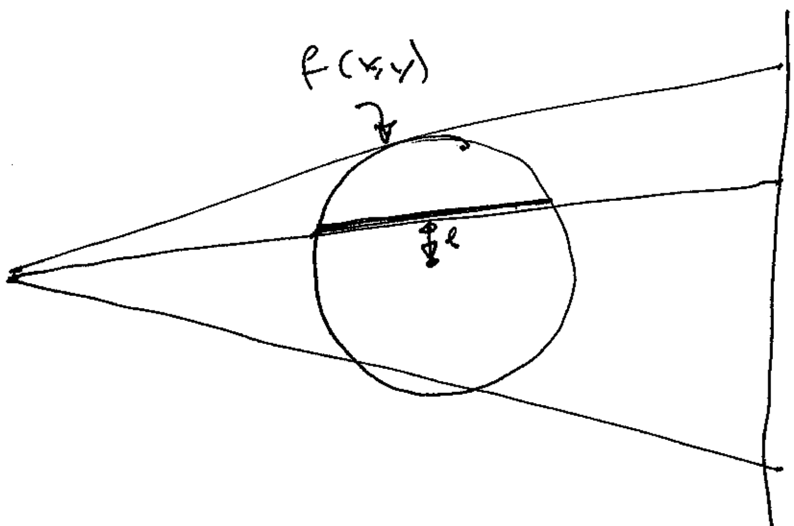
1D $\delta(\cdot)$, FUNCTION OF BOTH x, y

DO?

THIS IS AN IMPULSE WHERE ARGUMENT IS ZERO



USEFUL FOR MODELING PROJECTIONS, CT



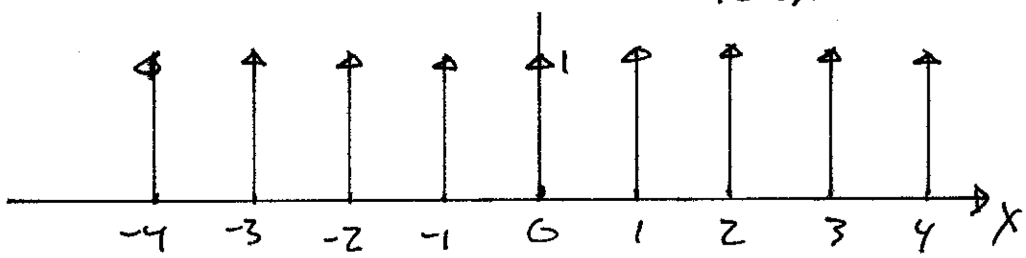
INTEGRAL ALONG RAY IS

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - l)\,dx\,dy$$

SAMPLING FUNCTIONS

ARRAYS OF $\delta(\cdot)$'s

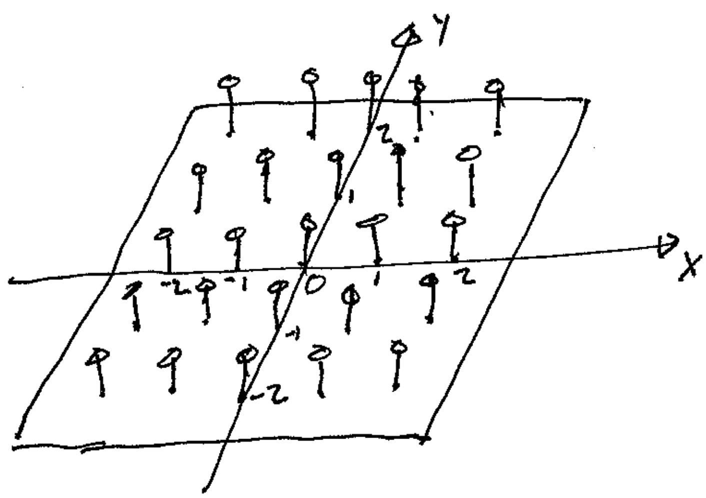
$$\mathrm{COMB}(x) = \sum_{n=-\infty}^{\infty} \delta(x-n) = \underbrace{\delta_1(x)}_{102A}$$



ALSO CALLED SHAH(x) OR $\text{Ш}(x)$

IN 2D

$$\mathrm{COMB}(x,y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \delta(x-n, y-m)$$



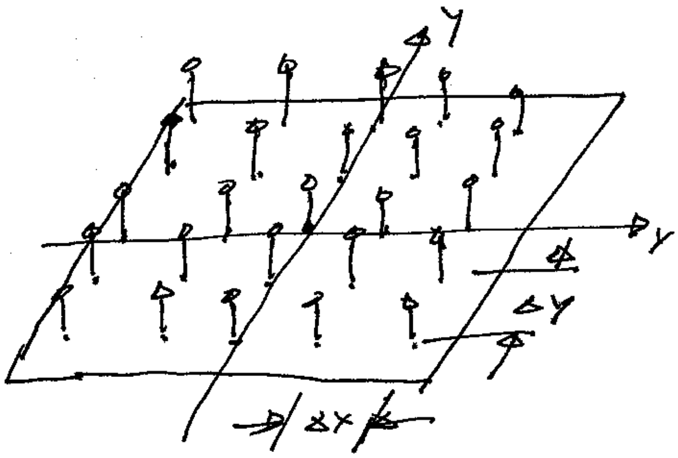
"BED OF NAILS"

OFTEN USED TO MODEL 2D SAMPLING

FOR SAMPLING WE WANT IMPULSE OF UNIT STRENGTH SPACED BY $\Delta x, \Delta y$.

DEFINE A SAMPLING FUNCTION AS

$$\delta_s(x,y;\Delta x,\Delta y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \delta(x - m\Delta x,\, y - n\Delta y)$$



WE CAN WRITE THIS AS

$$\delta_s(x,y;\Delta x,\Delta y) = \frac{1}{\Delta x \Delta y} \operatorname{comb}\left(\frac{x}{\Delta x}, \frac{y}{\Delta y}\right)$$

SINCE

$$\frac{1}{\Delta x \Delta y} \operatorname{comb}\left(\frac{x}{\Delta x}, \frac{y}{\Delta y}\right) = \frac{1}{\Delta x \Delta y} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \delta\left(\frac{x}{\Delta x} - n, \frac{y}{\Delta y} - m\right)$$

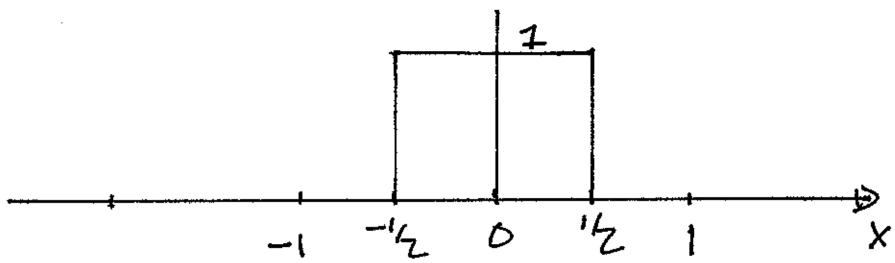
$$= \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \delta(x - n\Delta x,\, y - m\Delta y)$$

$$= \delta_s(x,y;\Delta x,\Delta y)$$

2D SINC AND RECT

IN 1D

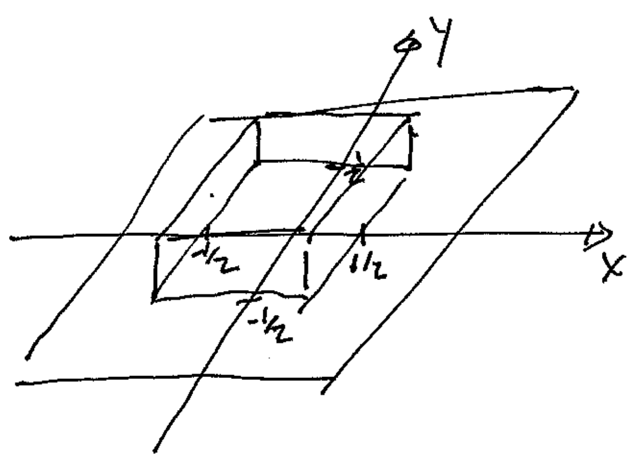
$$\mathrm{rect}(x) = \begin{cases} 1 & |x| < \frac{1}{2} \\ 0 & |x| > \frac{1}{2} \end{cases}$$



IN 2D

$$\mathrm{rect}(x,y) = \begin{cases} 1 & |x| < \frac{1}{2},\ |y| < \frac{1}{2} \\ 0 & \text{OTHERWISE} \end{cases}$$

$$= \mathrm{rect}(x)\,\mathrm{rect}(y)$$ SEPARABLE

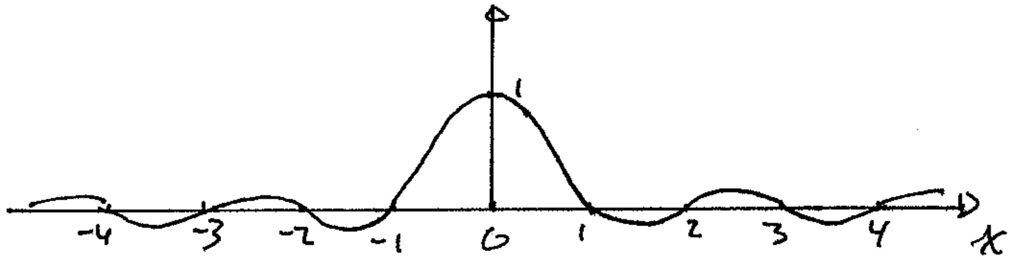


WHAT DOES THIS LOOK LIKE?

$\mathrm{rect}(r) \qquad r = \sqrt{x^2+y^2}$

SIMILARLY, IN 1D

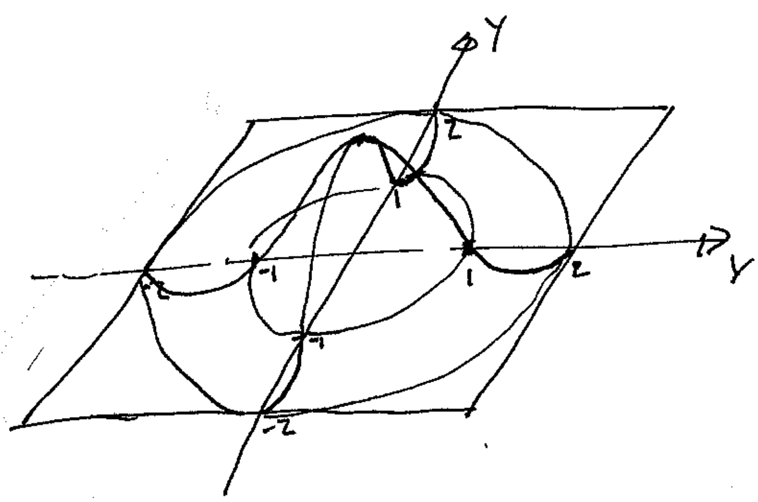
$$\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$$



IN 2D

$$\mathrm{sinc}(x,y) = \frac{\sin(\pi x)\,\sin(\pi y)}{\pi^2 x y}$$

$$= \mathrm{sinc}(x)\,\mathrm{sinc}(y)$$ SEPARABLE

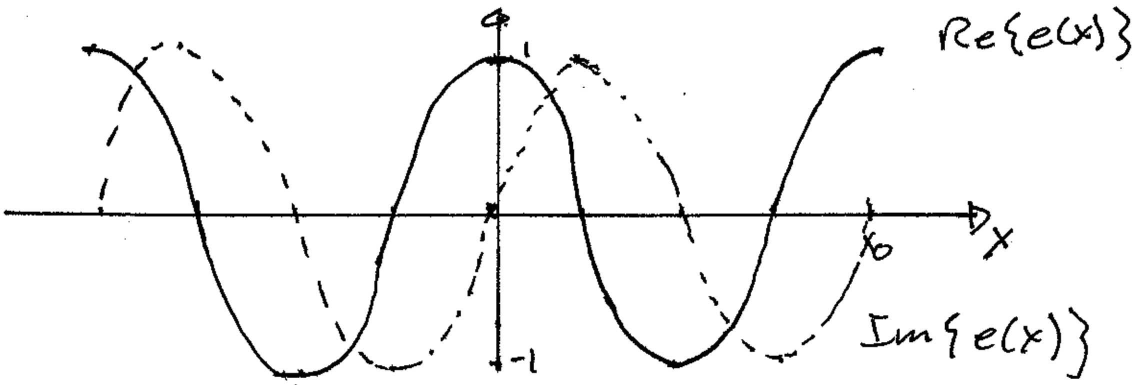


L2 :

COMPLEX EXPONENTIALS AND SINUSOIDS

IN 1D

$$e(x) = e^{j2\pi(u_0 x)}$$



x = SPACE (m)
u_0 = SPATIAL FREQUENCY (CYCLES/m)
x_0 = PERIOD, DISTANCE (m)

IN 2D

$$e(x,y) = e^{j2\pi(u_0 x + v_0 y)}$$

$$c(x,y) = \mathrm{Re}\{e(x,y)\}$$
$$= \cos(2\pi(u_0 x + v_0 y))$$

$$s(x,y) = \mathrm{Im}\{e(x,y)\}$$
$$= \sin(2\pi(u_0 x + v_0 y))$$

SPATIAL FREQUENCIES IN 2D

u_0, v_0 ARE SPATIAL FREQUENCIES

$$x_0 = \frac{1}{u_0}$$

$$y_0 = \frac{1}{v_0}$$

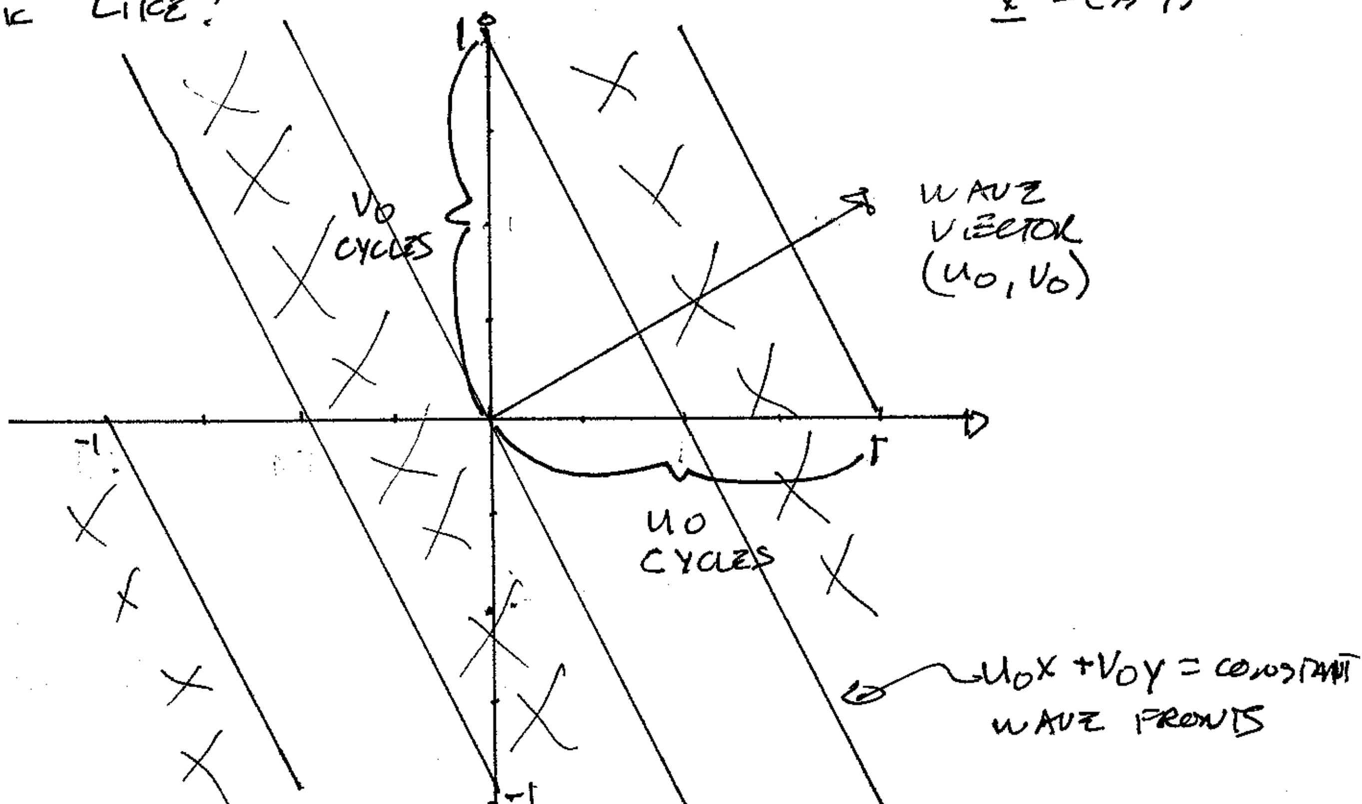
x_0, y_0 ARE SPATIAL PERIODS ALONG AXES x, y.

WHAT DOES

$$f(x,y) = \sin(2\pi(u_0 x + v_0 y)) = \sin(2\pi(\underline{u_0} \cdot \underline{x}))$$

$\underline{u_0} = (u_0, v_0)$
$\underline{x} = (x, y)$

LOOK LIKE?



WHAT SPATIAL FREQUENCY u_0, v_0 IS THIS?
WHAT IS x_0, y_0?

L7

SEPARABLE SIGNALS

$$f(x,y) = f_1(x)\,f_2(y)$$

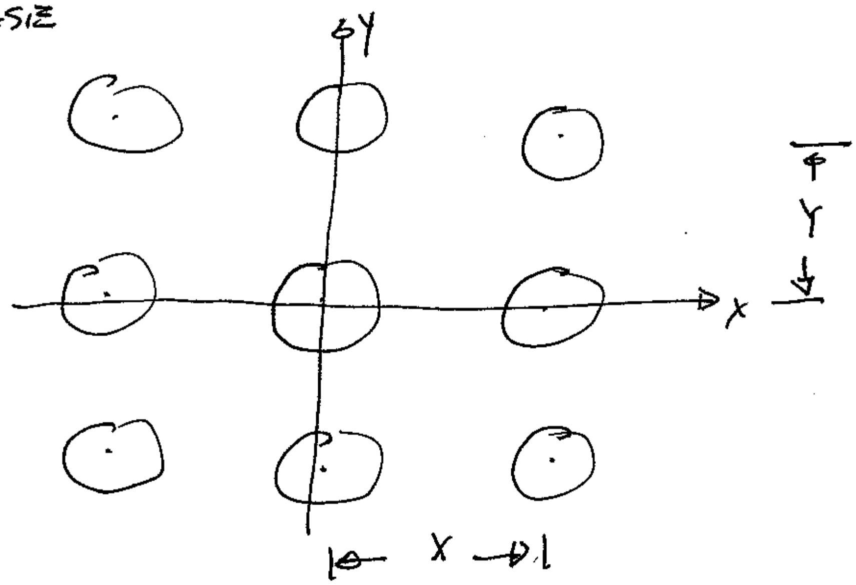
VERY USEFUL, EASY TO ANALYZE

PERIODIC SIGNALS

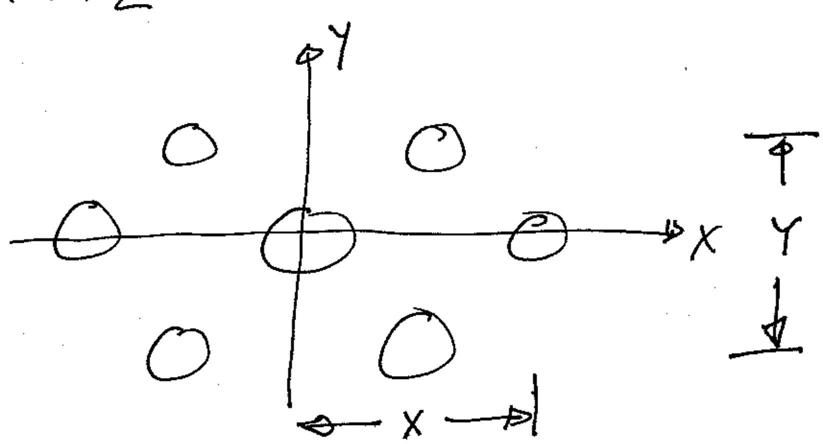
$$f(x,y) = f(x+X, y) = f(x, y+Y)$$

X, Y ARE PERIODS. EITHER OR BOTH MAY BE NON-ZERO

USUAL CASE



ANOTHER EXAMPLE



LZ