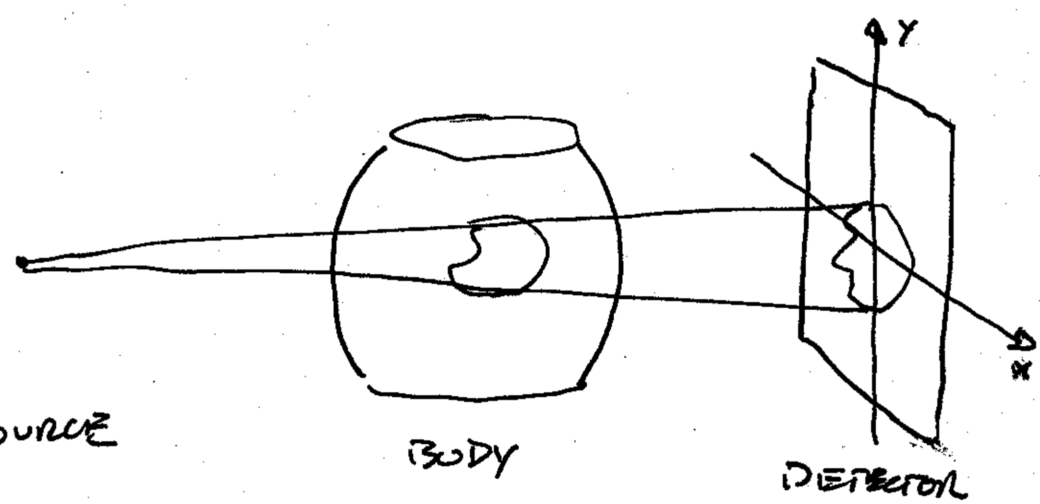
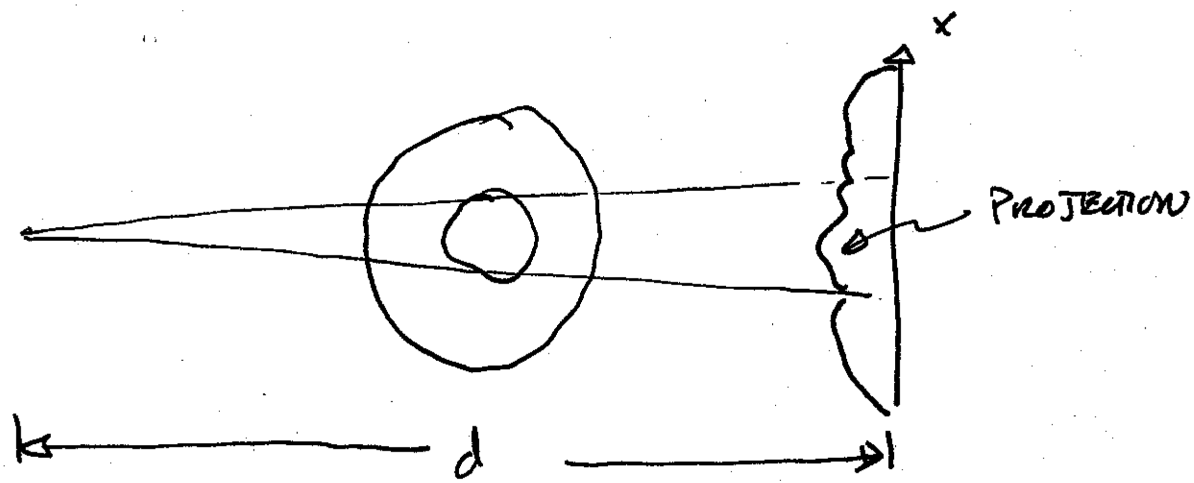
LECTURE 8

X-RAY COMPUTED TOMOGRAPHY

SO FAR WE HAVE BEEN LOOKING AT PROJECTION IMAGING



TOP VIEW

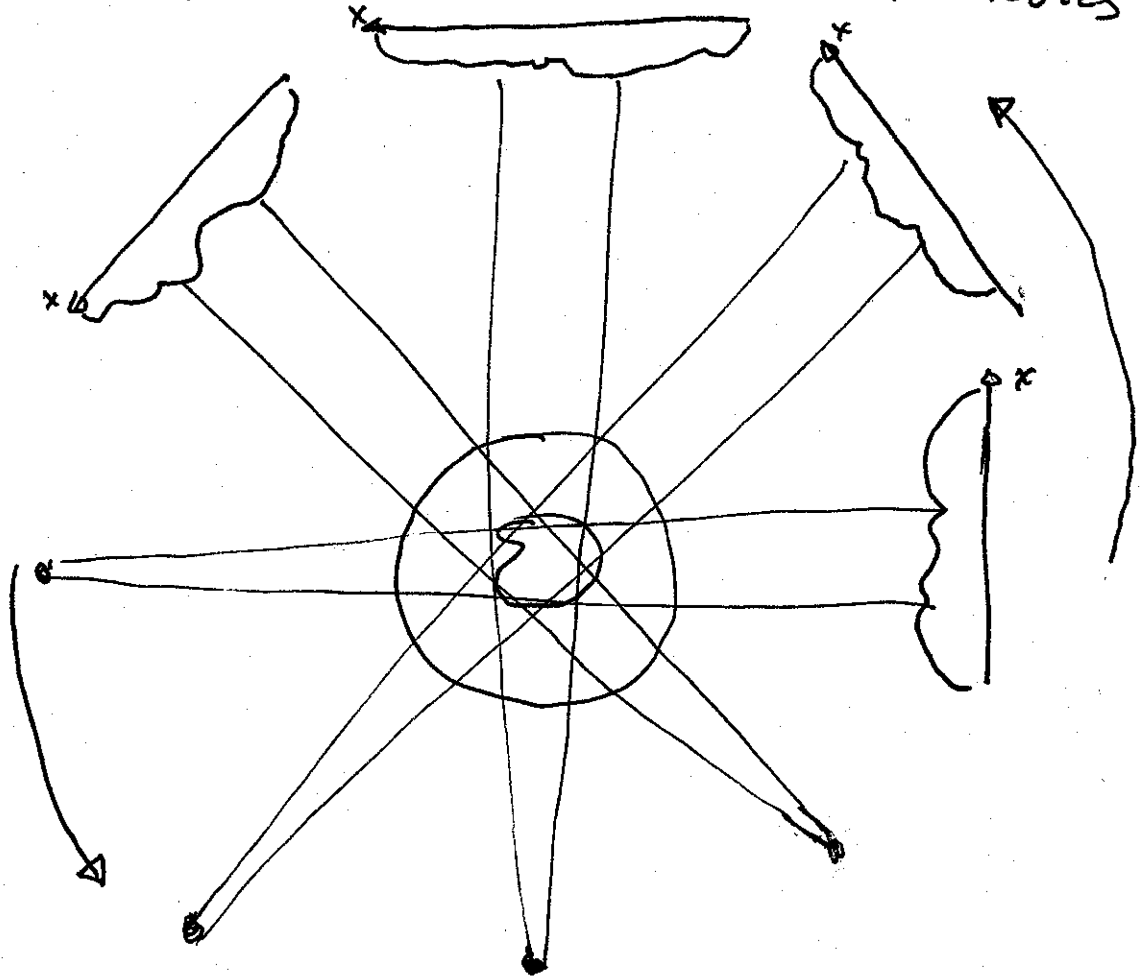


AT DETECTOR

$$I_d = \int_E \underbrace{S_o(E)}_{\text{SOURCE DENSITY}} \underbrace{E}_{\text{ENERGY}} \underbrace{e^{-\int_0^d \mu(s;E)\,ds}}_{\text{ATTENUATION}} dE$$

PATH s

IF WE HAD PROJECTIONS FROM MANY ANGLES



WE SHOULD BE ABLE TO FIGURE OUT WHAT THE OBJECT IS (WHAT $\mu(x,y)$ IS)

IN GENERAL, THIS IS A HARD PROBLEM.

SIMPLIFICATIONS

1) THE MEASUREMENT IS LINEAR IN $\mu(x,y)$

2) THE PROJECTION RAYS ARE PARALLEL

LINEAR MEASUREMENTS

INTENSITY AT DETECTOR, FROM ABOVE

$$I_d = \int_E S_0(E)\, E\, e^{-\int_0^d \mu(s;E)\,ds}\, dE$$

IF WE APPROXIMATE SOURCE BY ONE EFFECTIVE ENERGY $\bar{E}$

$$I_d = I_0\, e^{-\int_0^d \mu(s;\bar{E})\,ds}$$

THIS IS STILL AN EXPONENTIAL IN $\mu(s;\bar{E})$.

DIVIDE BY I_0, AND TAKE NEGATIVE LOG

$$\boxed{-\log\left(\frac{I_d}{I_0}\right) = \int_0^d \mu(s;\bar{E})\,ds}$$

MEASUREMENTS OF I_d HAVE BEEN CONVERTED INTO LINE INTEGRALS OF $\mu(s;\bar{E})$!

ASSUMPTIONS

ONE EFFECTIVE ENERGY

NOISE STATISTICS

I_d IS POISSON, APPROXIMATELY GAUSSIAN AT HIGH SNR

RECONSTRUCTION ASSUMES $-\log\left(\frac{I_d}{I_0}\right)$ IS GAUSSIAN, NOT TRUE

CT NUMBERS

μ IS THE QUANTITY WE WANT TO DISPLAY

USUALLY THIS IS SCALED TO <u>HOUNSFIELD</u> <u>UNITS</u>

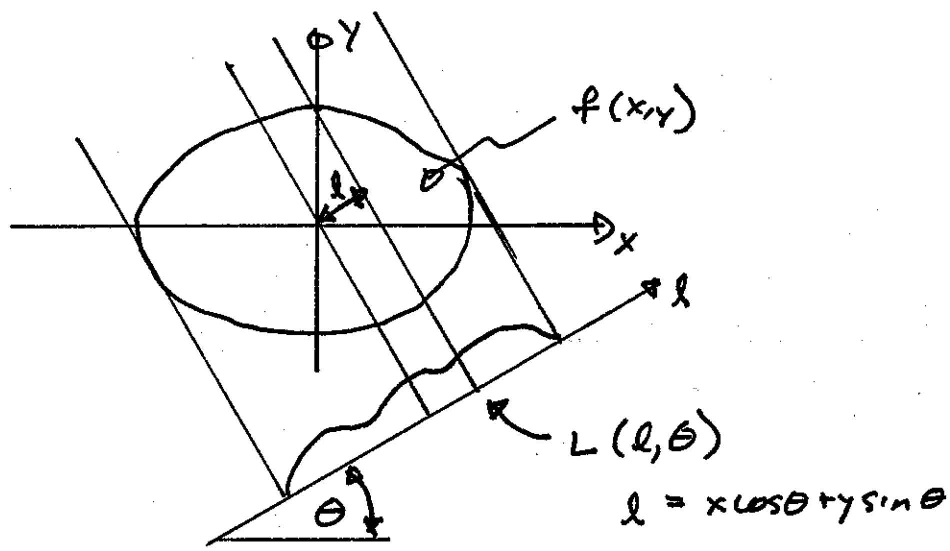
$$h = 1000 \times \left(\frac{\mu - \mu_{WATER}}{\mu_{WATER}} \right)$$

TYPICAL NUMBERS

AIR	-1000	(NO ATTENUATION)
SOFT TISSUE	-300 TO +400	
WATER	0	
BONE	1000	
METAL	3000	

PROJECTION DATA

FOR NOW, WE'LL ASSUME WE HAVE PARALLEL PROJECTION DATA



WE WANT TO DESCRIBE THE SAMPLE AT A DISTANCE l, AT A PROJECTION ANGLE θ.

THIS IS THE INTEGRAL ALONG $L(l,\theta)$. WE CAN DEFINE THE LINE AS

$$\delta(x\cos\theta + y\sin\theta - l)$$

THE PROJECTION IS THEN

$$g(l,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - l)\,dx\,dy$$

FOR CT

$f(x,y) = \mu(x,y;\bar{E})$ IMAGE OF ATTENUATION

$g(l,\theta) = -\log\left(\frac{I_d}{I_0}\right)$ MEASUREMENTS

THE FUNCTION $g(l,\theta)$ IS THE RADON TRANSFORM OF $f(x,y)$.

SOLVING FOR $f(x,y)$ FROM $g(l,\theta)$ IS AN INVERSE PROBLEM.

MANY DIFFERENT SOLUTIONS

WE WILL FOCUS ON TWO:

1) BACKPROJECTION
2) FOURIER INTERPOLATION

PROJECTION DATA IN FREQUENCY SPACE

WHAT DOES IMAGE DOMAIN PROJECTION CORRESPOND TO IN THE FREQUENCY DOMAIN?

CONSIDER THE 2D FOURIER TRANSFORM

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi(xu+yv)}\,dx\,dy$$

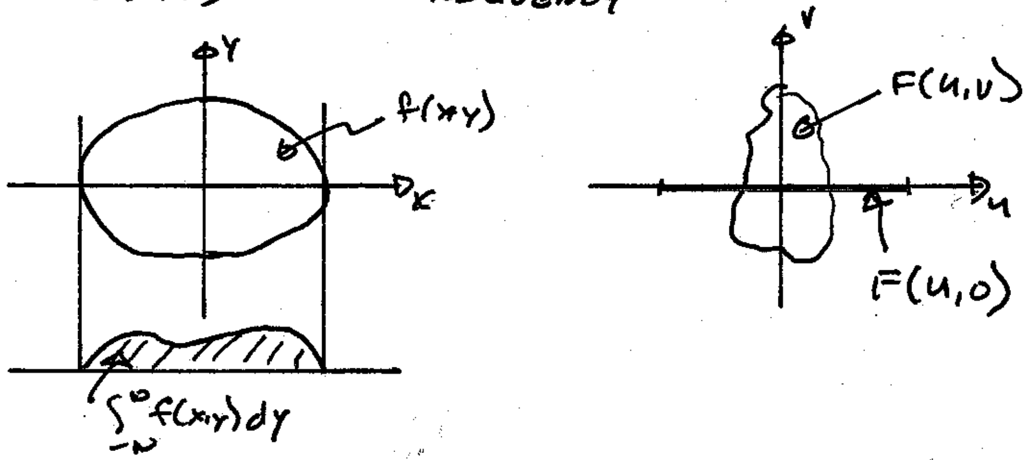
IF WE LET $v=0$

$$F(u,0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi(xu+0y)}\,dx\,dy$$

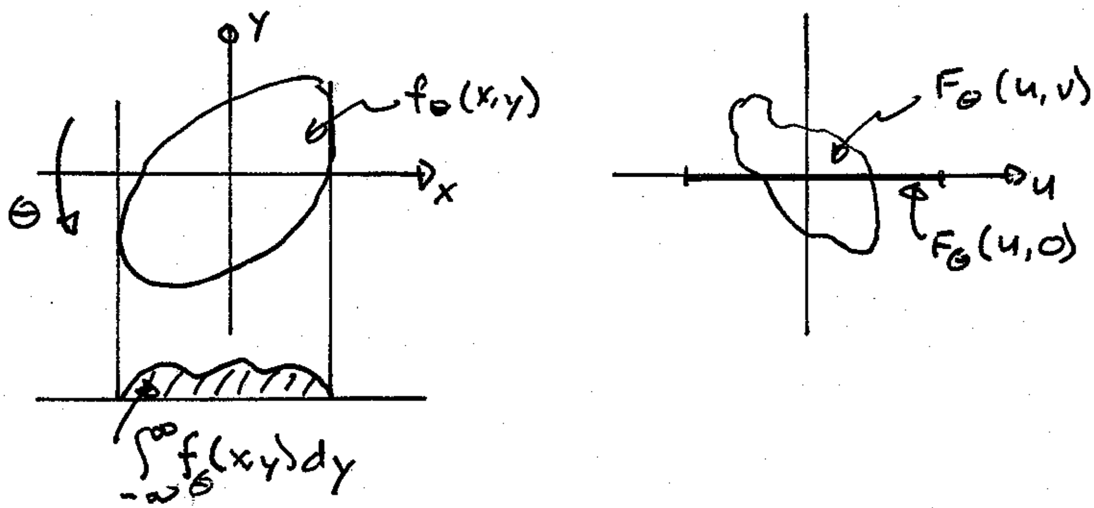
$$= \underbrace{\int_{-\infty}^{\infty} \underbrace{\left[\int_{-\infty}^{\infty} f(x,y)\,dy\right]}_{\text{PROJECTION ALONG } y} e^{-j2\pi xu}\,dx}_{\text{FOURIER TRANSFORM OF PROJECTION}}$$

L8

PROJECTING ALONG y CORRESPONDS TO SAMPLING ALONG u AXIS IN FREQUENCY

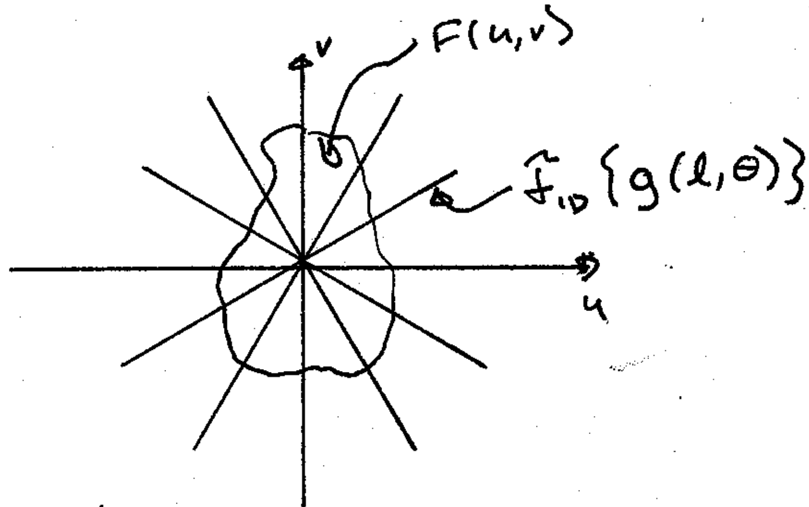


IF WE ROTATE THE OBJECT BY θ



THE TRANSFORM ALSO ROTATES, AND WE ACQUIRE ANOTHER SAMPLE OF A DIAMETER OF $F(u,v)$.

IF WE REPEAT FOR MANY Θ'S, WE GET



WE HAVE $F(u,v)$ ALONG A SET OF DIAMETERS IN SPATIAL FREQUENCY SPACE!

IF WE HAVE ENOUGH Θ'S, WE CAN INTERPOLATE TO RECONSTRUCT $F(u,v)$, INVERSE TRANSFORM TO FIND $f(x,y)$

ANOTHER PERSPECTIVE

TAKE THE 1D FOURIER TRANSFORM OF PROJECTION

$$\mathcal{F}_{1D}\{g(l,\theta)\} = \int_{-\infty}^{\infty} g(l,\theta)\, e^{-j2\pi q l}\, dl$$

$$= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - l)\,dx\,dy\right] e^{-j2\pi q l}\, dl$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) \underbrace{\left[\int_{-\infty}^{\infty} \delta(x\cos\theta + y\sin\theta - l)\, e^{-j2\pi q l}\, dl\right]}_{e^{-j2\pi q(x\cos\theta + y\sin\theta)}} dx\,dy$$

$$= \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi(x\overbrace{q\cos\theta}^{u} + y\overbrace{q\sin\theta}^{v})}\, dx\,dy}_{F(u,v)\big|_{\substack{u=q\cos\theta \\ v=q\sin\theta}}}$$

$$= F(q\cos\theta, q\sin\theta)$$

AGAIN, FOURIER TRANSFORM OF A PROJECTION IS SAMPLE ALONG DIAMETER IN SPATIAL FREQUENCY.

RECONSTRUCTION

FOURIER INTERPOLATION, INVERSE TRANSFORM
FILTERED BACKPROJECTION IN IMAGE DOMAIN
FAN BEAM DATA