LECTURE 9
X-RAY CT RECONSTRUCTION

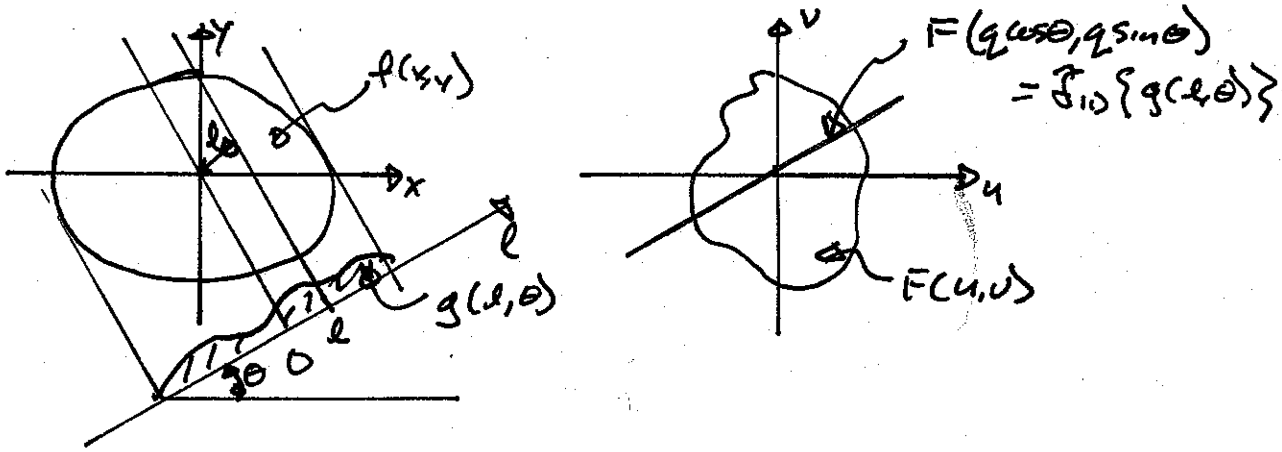
LAST TIME:

1) X-RAY MEASUREMENTS CAN BE CONVERTED INTO LINEAR MEASUREMENTS OF THE ATTENUATION

$$-\log\left(\frac{I_d}{I_0}\right) = \int_0^d \mu(s;\bar{E})\,ds$$

THIS IS THE LINE INTEGRAL OF $\mu(s;\bar{E})$ ALONG RAY s.

2) PROJECTION AT AN ANGLE θ CORRESPONDS TO A DIAMETER IN SPATIAL FREQUENCY



IF WE COLLECT MANY PROJECTIONS AT DIFFERENT θ's, WE SHOULD BE ABLE TO RECOVER $F(u,v)$ AND $f(x,y)$.

RECONSTRUCTION

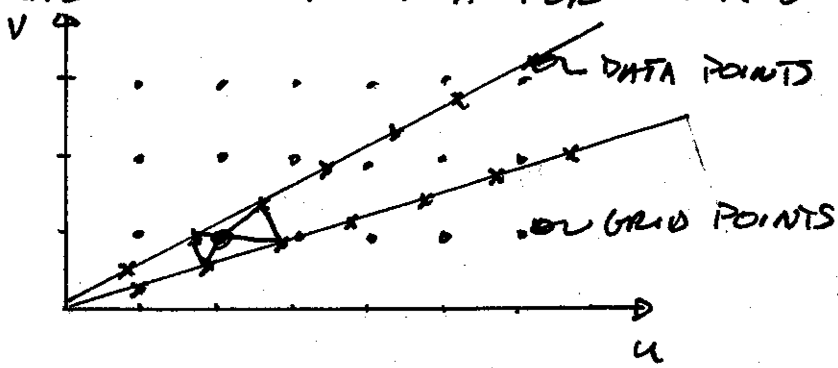
SIMPLE APPROACH: FREQUENCY DOMAIN INTERPOLATION

1) TRANSFORM PROJECTIONS
2) INTERPOLATE ONTO 2D SPATIAL FREQUENCY GRID
3) INVERSE 2D FFT

HOW DO YOU DO THE INTERPOLATION?

VERSION 1

INTERPOLATE NEAREST DATA FOR EACH GRID POINT



WORKS WELL.

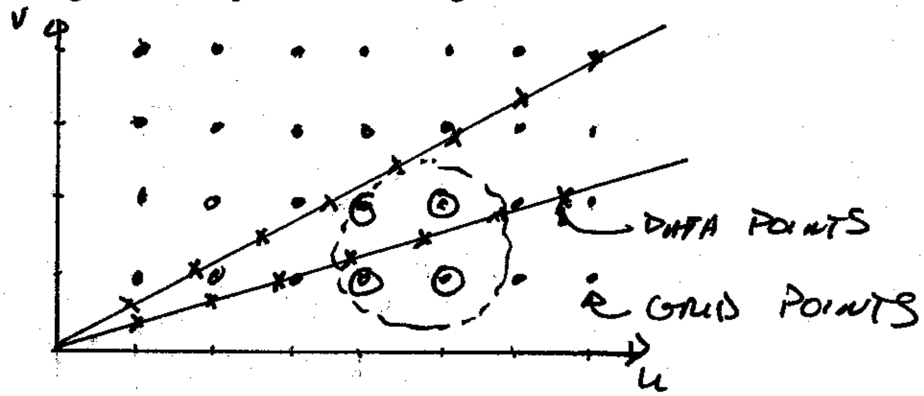
NOT ALL DATA POINTS ARE USED

AT ORIGIN THERE ARE MANY MORE DATA POINTS THAN GRID POINTS

IDEALLY, WE WANT TO USE ALL OF THE MEASUREMENTS.

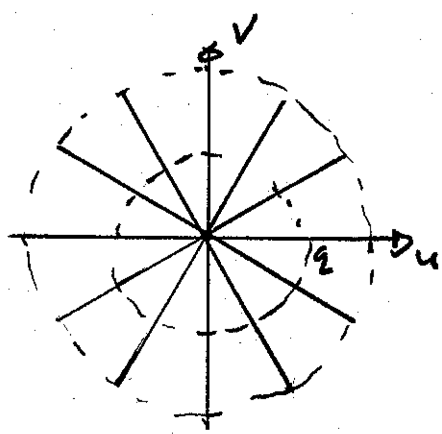
VERSION 2

ADD CONTRIBUTION OF EACH DATA POINT TO ALL NEIGHBORING GRID POINTS



ALL DATA CONTRIBUTES.

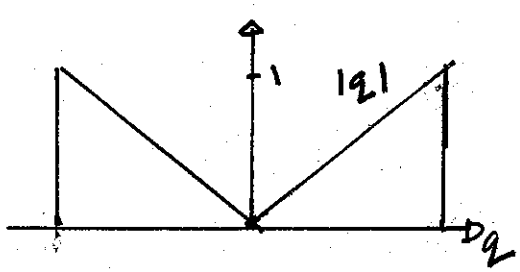
TOO MUCH DATA AT ORIGIN



AT RADIUS q THERE ARE $2N$ SAMPLES, AND A SAMPLE DENSITY OF

$$\frac{2N}{2\pi q}$$

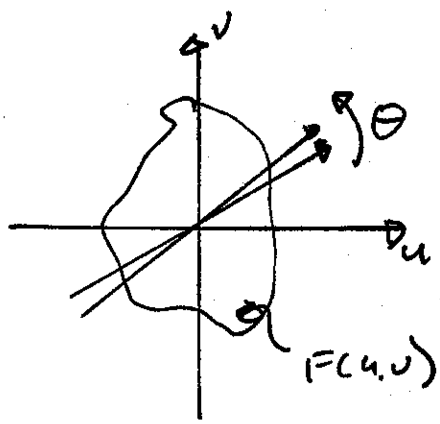
AS q GETS SMALL, DENSITY GETS LARGE



WE CORRECT FOR THIS BY MULTIPLYING BY $|q|$.

"RHO" FILTER

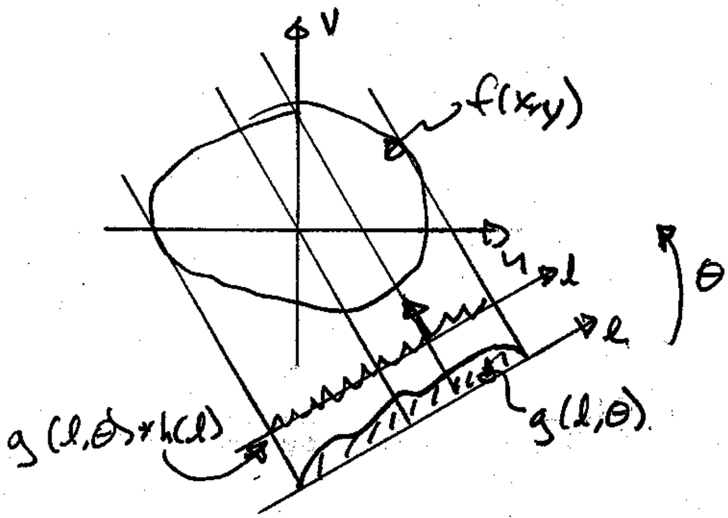
SUMMARY OF FOURIER DOMAIN INTERPOLATION



1) TRANSFORM PROJECTIONS
2) MULTIPLY BY $|q|$
3) ADD TO 2D GRID
4) INVERSE 2D FFT

ALTERNATIVE APPROACH

WHAT DO THESE STEPS CORRESPOND TO IN THE IMAGE DOMAIN?



1) NOTHING
2) CONVOLVE WITH $\mathcal{F}_{1D}^{-1}\{|q|\}$
3) SPREAD FILTERED PROJECTION ACROSS IMAGE
   BACKPROJECTION
4) NOTHING

THIS IS CALLED

FILTERED BACKPROJECTION
CONVOLUTION BACKPROJECTION

WIDELY USED.

ISSUES

FOURIER DOMAIN INTERPOLATION

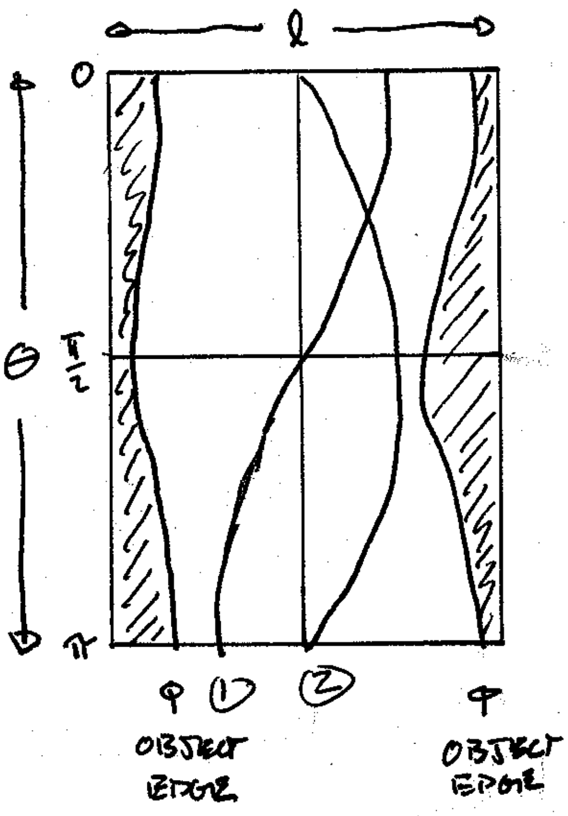
- ACCURACY OF RHO FILTER
- ACCURACY OF INTERPOLATION

FILTERED BACKPROJECTION

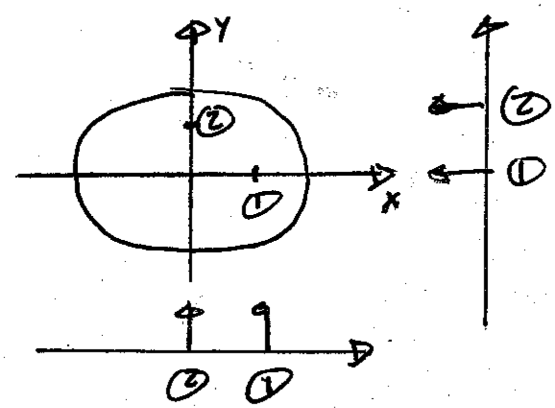
- ACCURACY OF CONVOLUTION FILTER
- INTERPOLATION OF RAYS ONTO 2D GRID

FILTERED BACKPROJECTION IMPLEMENTATION

INITIAL DATA IS IN (ℓ, θ) SPACE: RADON SPACE



EACH POINT IN OBJECT TRACES OUT A SINUSOID IN RADON TRANSFORM



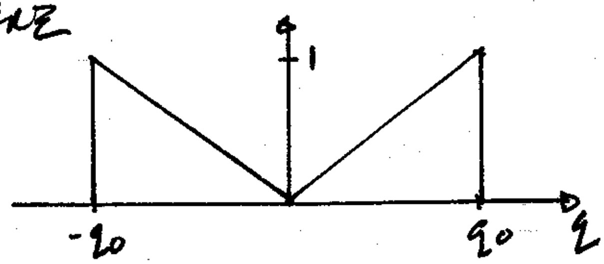
THIS IS CALLED A SINOGRAM

RHO FILTER

CONVOLVE EACH PROJECTION (ROW OF SINOGRAM) WITH

$$h(l) = \mathcal{F}_{1D}^{-1}\{H(q)\}$$

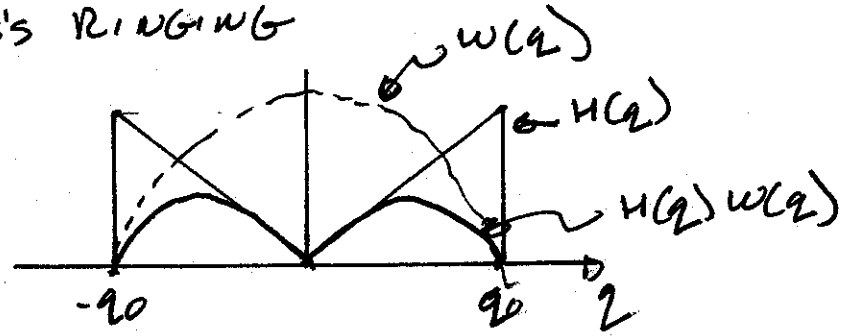
WHERE



$$H(q) = \frac{|q|}{q_0}\,\mathrm{rect}\left(\frac{q}{2q_0}\right)$$

ON THE MIDTERM YOU SOLVED FOR THE DISCRETE VERSION OF $h(l)$, WHICH HAS A SIMPLE FORM.

OFTEN A WINDOW FUNCTION IS INCLUDED TO REDUCE GIBB'S RINGING

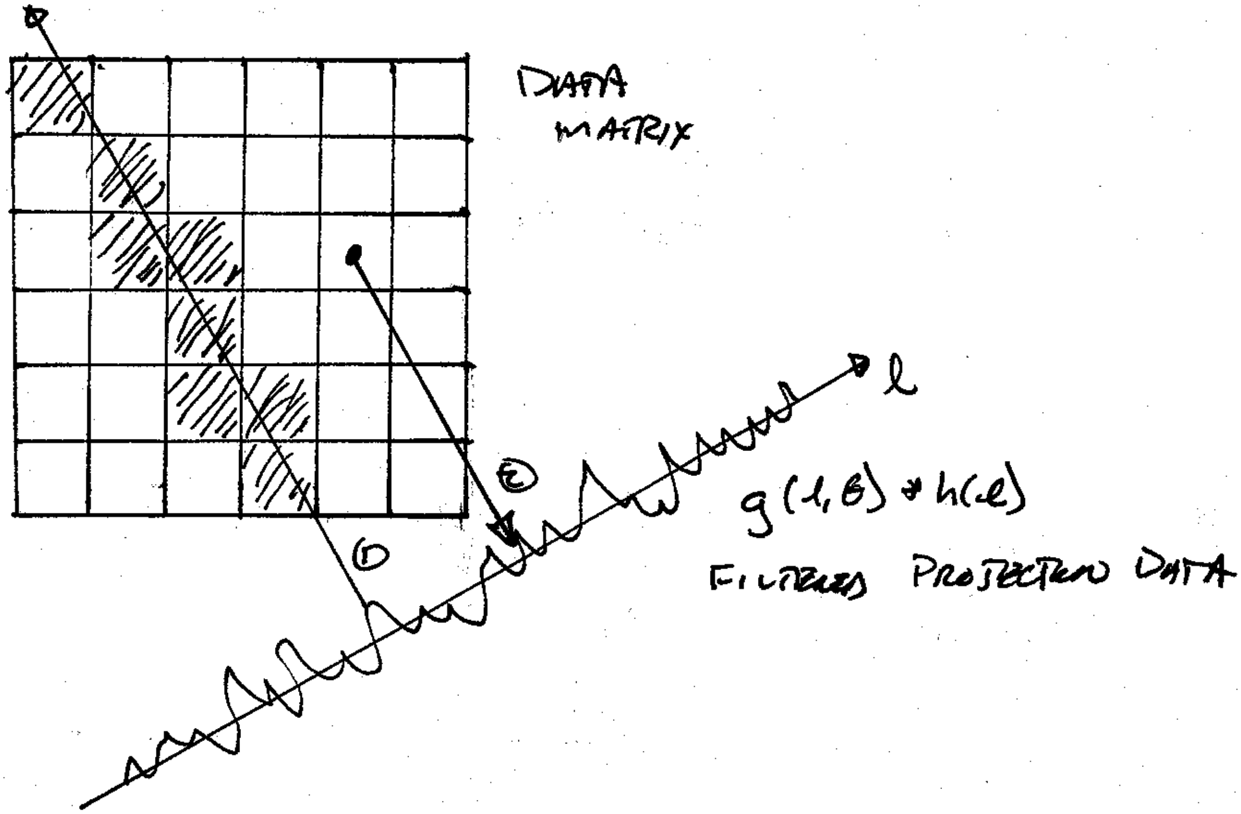


REDUCES RESOLUTION.

RHO FILTER CAN BE IMPLEMENTED A CONVOLUTION IN PROJECTION DOMAIN OR MULTIPLICATION IN FREQUENCY DOMAIN (HW).

RESULT IS A FILTERED SINOGRAM

BACKPROJECTION



TWO APPROACHES

1) PUSH PROJECTION THROUGH MATRIX, ADDING TO THE PIXELS IT CROSSES, WEIGHTED BY HOW CLOSE IT PASSES

2) TAKE THE LOCATION OF EACH PIXEL, AND INTERPOLATE INTO THE FILTERED PROJECTION DATA (HW)

SAME RESULT EITHER WAY.

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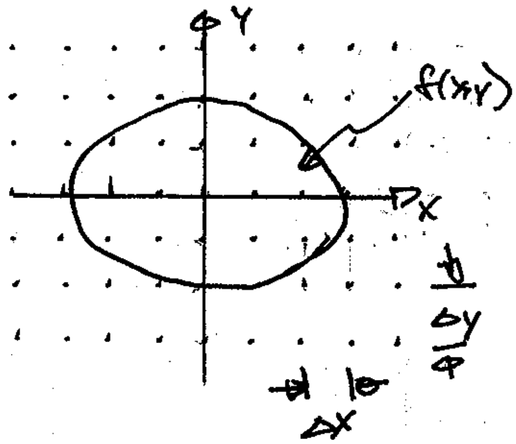
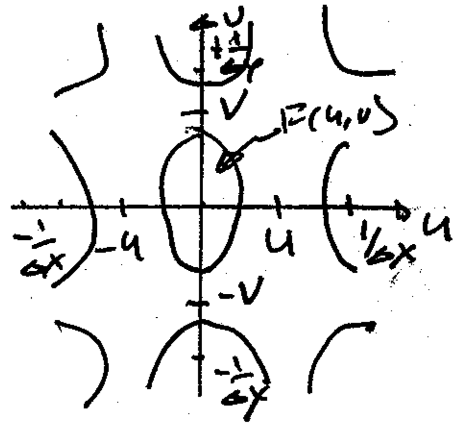
SAMPLING REQUIREMENTS

SAMPLING IN SPACE

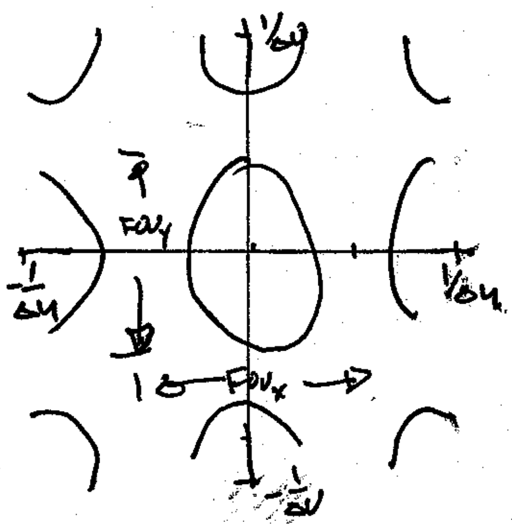


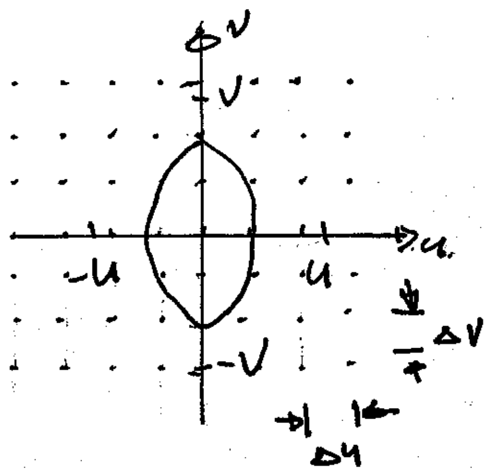


$$\frac{1}{\Delta x} > 2u$$

$$\Delta x < \frac{1}{2u}$$

SAMPLING IN SPATIAL FREQUENCY



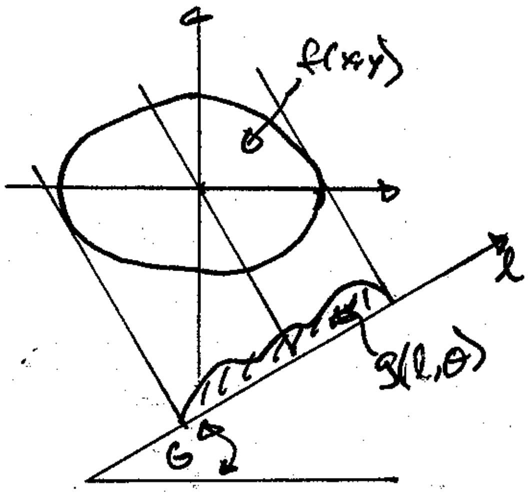


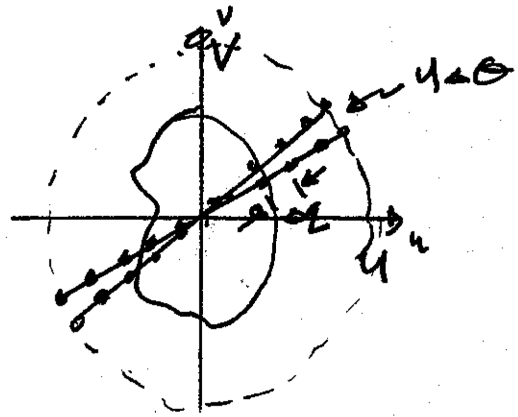
$$\frac{1}{\Delta u} > FOV_x$$

$$\Delta u < \frac{1}{FOV_x}$$

L9

FOR PROJECTION DATA





IF $FOV_x = FOV_y = FOV$, THE $U = V$

FOR A GIVEN FOV, WE NEED

$$\Delta q \leq \frac{1}{FOV}$$

THIS REQUIRES

$$N_q = \frac{2U}{\Delta q}$$

SAMPLES. AT THE EDGE OF SPATIAL FREQUENCY WE NEED

$$U \Delta\theta \leq \frac{1}{FOV}$$

WHERE

$$\Delta\theta = \frac{\pi}{N_\theta}$$

THEN

$$\Delta q = u \, \Delta \theta$$

$$\frac{2\cancel{u}}{N_q} = \cancel{u} \frac{\pi}{N_\theta}$$

$$\boxed{N_\theta = \frac{\pi}{2} N_q}$$

WE NEED $\frac{\pi}{2} \simeq 1.57$ TIMES MORE PROJECTIONS THAN SAMPLES ALONG PROJECTION.

<u>EXAMPLE</u>

WE ACQUIRE 256 SAMPLE PROJECTIONS OVER FOV.

WE THEN NEED

$$N_\theta = \frac{\pi}{2} 256 = 402$$

PROJECTIONS TO MEET NYQUIST REQUIREMENTS.

L9