Lecture 3: AM Modulation

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Channels, Power Spectrum, and AM Modulation

Today:

Amplitude Modulation

Quadrature Receivers

Modulators

Commercial AM

Single Sideband Modulation (SSB)

Covers *Madhow* sections 3.1–3.2

Baseband Communication

The *baseband* is the frequency band of the original signal.

- Telephones: 300–3700 Hz
- High-fidelity audio: 0–20 KHz
- Television (NTSC) video: 0–4.3 MHz
- Ethernet (10 Mbs): 0–20 MHz

Baseband communication usually requires wire (single, twisted pair, coax).

Multiple baseband signals cannot share a channel without time division multiplexing (TDM).

Carrier Communication

Carrier communication uses modulation to shift spectrum of signal.

- Wireless communication requires frequencies higher than baseband
- Multiple signals can be sent at same time using different frequencies: frequency division multiplexing (FDM)

In carrier communication, the signal modulates a sinusoidal carrier.
The signal modifies the amplitude, frequency, or phase of carrier.

$$s(t) = A(t)\cos\big(2\pi f_c(t)t + \phi(t)\big)$$

- amplitude modulation: $A(t)$ is proportional to $m(t)$
- frequency modulation: $f_c(t)$ is proportional to $m(t)$
- phase modulation: $\phi(t)$ is proportional to $m(t)$

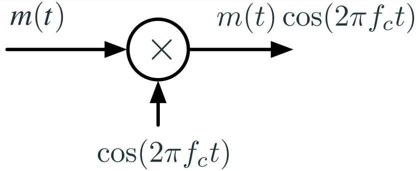
Frequency and phase modulation are called *angle modulation*.

Double-Sideband Suppressed Carrier (DSB-SC) Modulation

- This is a complicated way of saying "multiply by a cosine."
- We have a message (baseband) signal, and cosine $\cos(2\pi f_c t)$ at a frequency f_c.
- The modulated message signal and its Fourier transform are

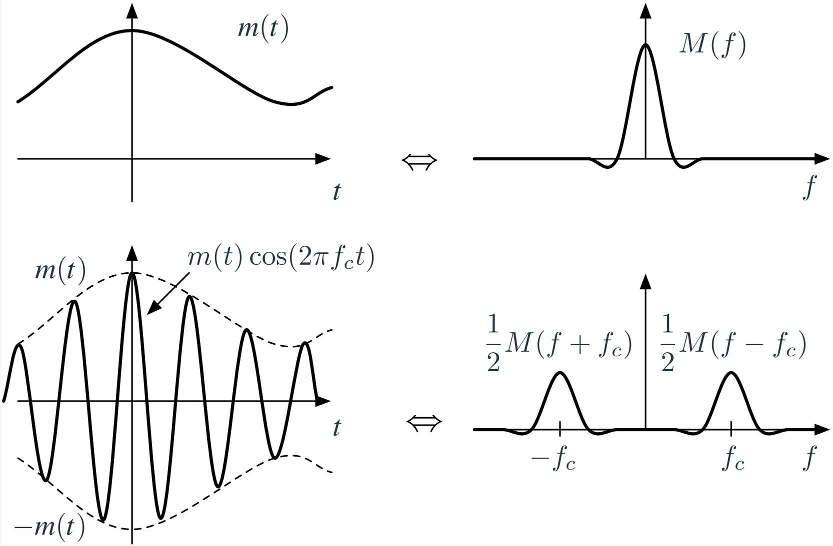
$$m(t)\cos(2\pi f_c t) \Leftrightarrow \frac{1}{2}\left[M(f+f_c)+M(f-f_c)\right]$$

where $m(t) \Leftrightarrow M(f)$. The block diagram is



- This signal and its spectrum are illustrated on the next page:

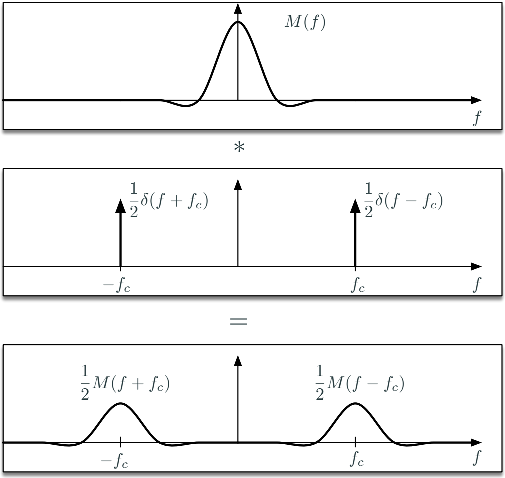
DSB-SC



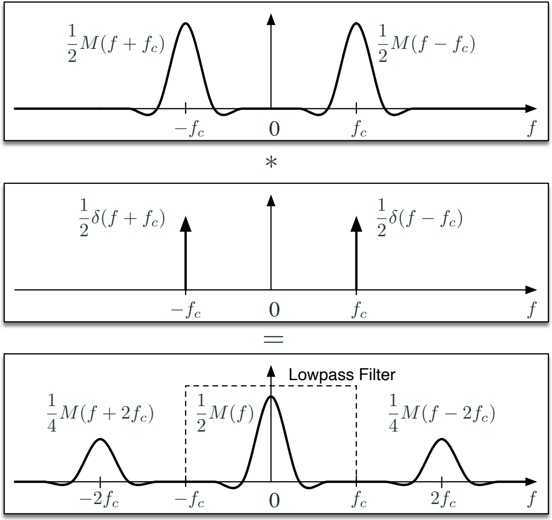
$m(t)$
$M(f)$
t
f
$\Leftrightarrow$
$m(t)\cos(2\pi f_c t)$
$m(t)$
$-m(t)$
$\frac{1}{2}M(f+f_c)$
$\frac{1}{2}M(f-f_c)$
$-f_c$
f_c

We can think of modulation as frequency domain convolution.

$$\mathcal{F}\left[m(t)\cos(2\pi f_c t)\right] = M(f) * \left(\frac{1}{2}\delta(f+f_c) + \frac{1}{2}\delta(f-f_c)\right)$$



To demodulate this signal, consider what happens if we multiply again by $\cos(2\pi f_c t)$. Again, we can think of this as a convolution in the frequency domain:



After the convolution there is a replica of the spectrum centered at $f = 0$, which we can extract with a lowpass filter.

The modulated signal spectrum is

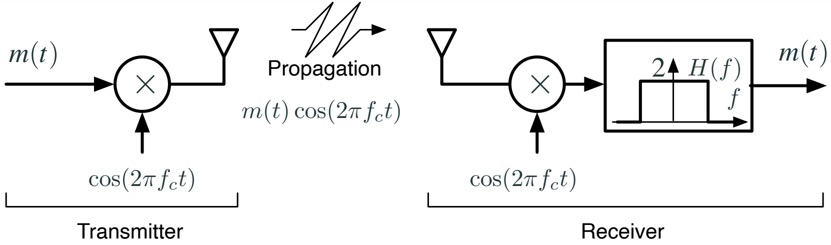
$$\mathcal{F}\left[m(t)\cos(2\pi f_c t)\right] = \frac{1}{2}M(f + f_c) + \frac{1}{2}M(f - f_c)$$

Multiplying this by $\cos(2\pi f_c t)$ corresponds to convolving in frequency,

$$\begin{aligned}
\mathcal{F}\left[m(t)\cos^2(2\pi f_c t)\right] &= \left[\frac{1}{2}M(f + f_c)) + \frac{1}{2}M(f - f_c)\right] \\
&\quad * \left[\frac{1}{2}\delta(f + f_c) + \frac{1}{2}\delta(f - f_c)\right] \\
&= \frac{1}{4}\left[M(f + f_c) + M(f - f_c))\right] \\
&\quad * \left[\delta(f + f_c) + \delta(f - f_c)\right] \\
&= \frac{1}{4}M(f + 2f_c) + \frac{1}{2}M(f) + \frac{1}{4}M(f - 2f_c)
\end{aligned}$$

Lowpass filtering extracts the $M(f$ term, recovering the original message.

The block diagram of the entire system is now:



- This is a *synchronous* receiver, meaning the transmitter and receiver must be in phase. Synchronising the receiver requires a more complex system.
- In practice the propagation delay is unknown and time varying, and the transmitter and receiver phase can drift with respect to each other.

DSB-SC Synchronization and Quadrature Receivers

Often we won't know the phase of the signal we are receiving, and this can cause problems with a synchronous receiver.

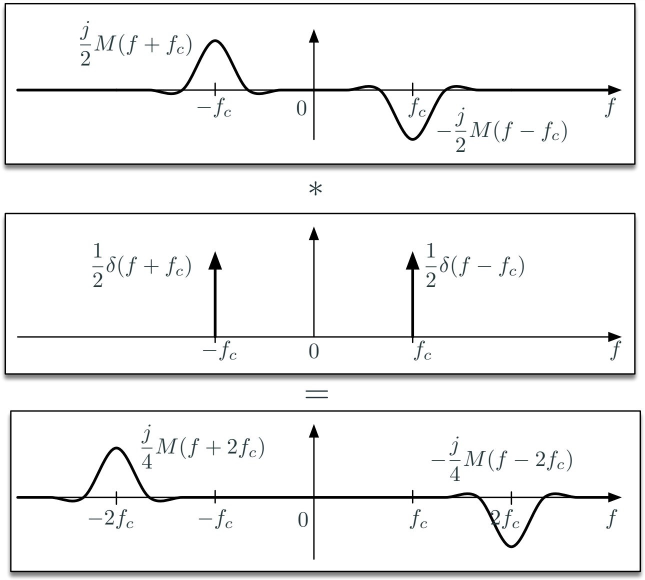
Consider the case where the transmitter has a phase of $-\pi/2$, so that the modulated signal is

$$m(t)\cos(2\pi f_c t - \pi/2) = m(t)\sin(2\pi f_c t).$$

The spectrum of the transmitted signal is now

$$\mathcal{F}\left[m(t)\sin(2\pi f_c t)\right] = \frac{j}{2}M(f+f_c) - \frac{j}{2}M(f-f_c)).$$

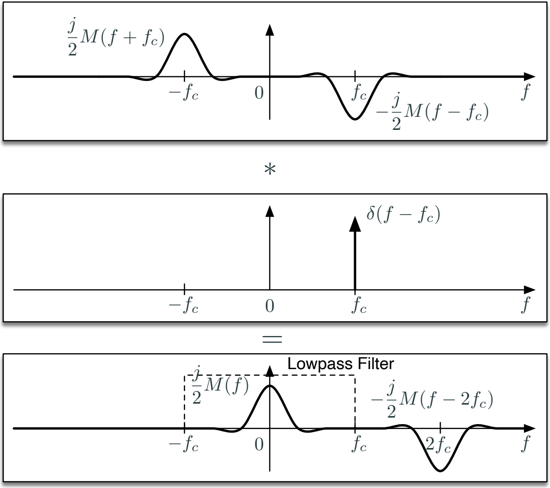
If we demodulate with a cosine, the result is shown in the next plot:



The baseband signal we want cancels!

One solution is to demodulate with a complex exponential

$$e^{j2\pi f_c t} \Leftrightarrow \delta(f - f_c)$$



This also works for an arbitrary phase shift.

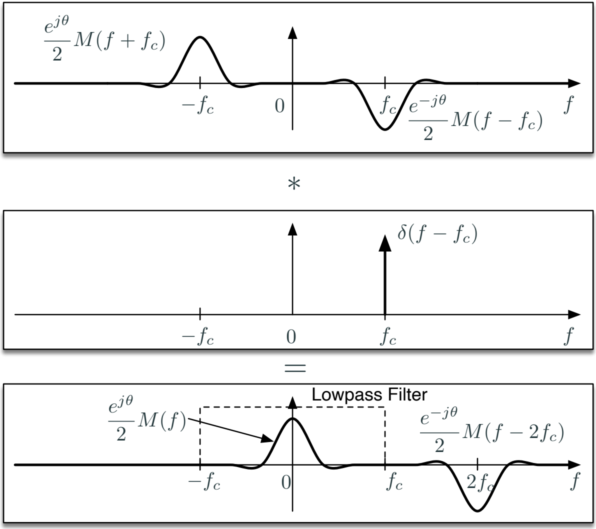
If the input has a phase shift of $-\theta$, the spectrum of the carrier is

$$\begin{aligned}\mathcal{F}\left[\cos(2\pi f_c t - \theta)\right] &= \mathcal{F}\left[\cos(2\pi f_c t)\cos\theta + \sin(2\pi f_c t)\sin\theta\right] \\ &= \frac{1}{2}\cos\theta\left[\delta(f+f_c) + \delta(f-f_c)\right] \\ &\quad\; + \frac{1}{2}\sin\theta\left[j\delta(f+f_c) - j\delta(f-f_c)\right] \\ &= \frac{1}{2}\left[e^{j\theta}\delta(f+f_c) + e^{-j\theta}\delta(f-f_c)\right]\end{aligned}$$

and the spectrum of the modulated signal is

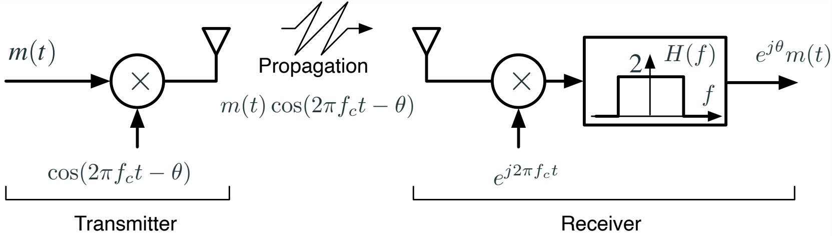
$$\mathcal{F}\left[m(t)\cos(2\pi f_c t - \theta)\right] = \frac{e^{j\theta}}{2}M(f+f_c) + \frac{e^{-j\theta}}{2}M(f-f_c)$$

Demodulating with a complex exponential can be plotted as:



The lowpass filter extracts $\frac{e^{j\theta}}{2}M(j\omega)$ corresponding to the complex signal $\frac{e^{j\theta}}{2}m(t)$.

The system now looks like



- This is a *quadrature receiver*, common in radar, sonar, ultrasound, and MRI systems. Often $m(t)$ is a simple pulse, and the interesting information is in θ, such as doppler shift for weather radar.
- This also common in communications systems. You can buy a digital chip that implements a quadrature receiver for your cell phone.
- The cost is that the receiver has to be implemented for complex signals. This is done the way you do it, by keeping track of two real signals, the real part and the imaginary part (the I and Q channels).

Modulators

- Modulators are the key element in transmitters and receivers
- There are lots of ways to make modulators
- Often the problem is how not to make a modulator, say when you are designing an amplifier.
- We will look at some very common types
  - Just about any non-linearity
  - Multipliers such as choppers

Modulators Using Nonlinearities

Suppose we have the non-linear input-output characteristic:

$$y(t) = ax(t) + bx^2(t)$$

Let

$$\begin{aligned} x_1(t) &= \cos(2\pi f_c t) + m(t) \\ x_2(t) &= \cos(2\pi f_c t) - m(t) \end{aligned}$$

Then, it we apply $x_1(t)$ and $x_2(t)$ to the non-linear modulator, and look at the difference

$$\begin{aligned} y_1(t) - y_2(t) = a\,(\cos(2\pi f_c t) + m(t)) + b\,(\cos(2\pi f_c t) + m(t))^2 \\ - a\,(\cos(2\pi f_c t) - m(t)) - b\,(\cos(2\pi f_c t) - m(t))^2 \\ = 2a\,m(t) + 4b\,m(t)\cos(2\pi f_c t) \end{aligned}$$

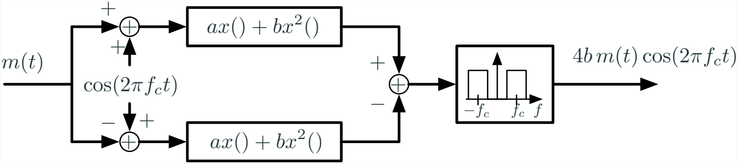
Convince yourself this is true!

From the previous page

$$y_1(t) - y_2(t) = 2a\,m(t) + 4b\,m(t)\cos(2\pi f_c t)$$

This has the term we want at $\omega_c = 2\pi f_c$, plus another copy of the message at baseband.

The unwanted baseband component is blocked by a bandpass filter. This could be the antenna or the amplifier.



Or we can just forget about the baseband signal, it won't propagate!

Switching Modulators

Multiply message by a simple periodic function.

Suppose $w(t)$ is periodic with a fundament frequency f_c:

$$w(t) = \sum_{n=-\infty}^{\infty} D_n e^{j2\pi f_c n t}$$

Each term is an impulse in frequency at multiple of f_s. Then

$$m(t)w(t) = m(t) \sum_{n=-\infty}^{\infty} D_n e^{j2\pi f_c n t}$$

By the convolution theorem, the spectrum of $m(t)w(t)$ consists of $M(f)$ shifted to $\pm f_c, \pm 2f_c, \pm 3f_c, \ldots$

For example if $w(t)$ is a 50% duty cycle square wave centered at $t = 0$,

$$w(t) = \frac{1}{2} + \frac{1}{\pi} \sum_{n=-\infty}^{\infty} \frac{1}{n} e^{j2\pi f_c n t};\ n \text{ odd}$$

In general, the spectrum of $m(t)w(t)$ is then

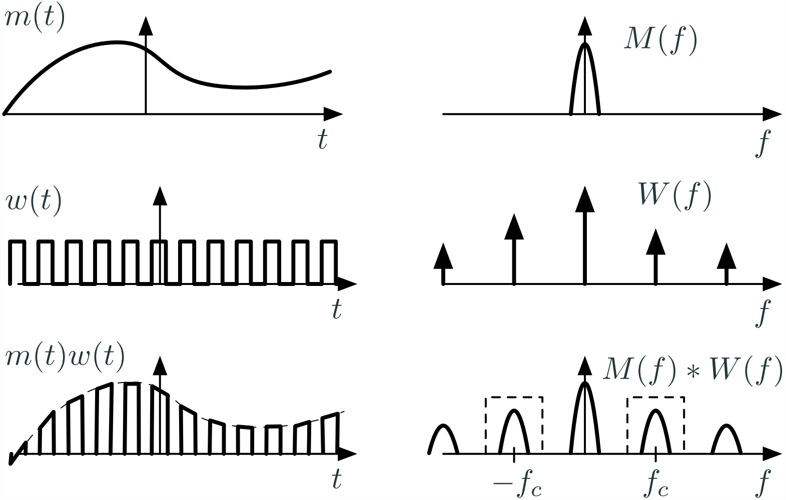
$$\begin{aligned} \mathcal{F}\{m(t)w(t)\} &= M(f) * W(f) \\ &= \sum_{n=-\infty}^{\infty} D_n M(f - nf_c) \end{aligned}$$

There are replicas at multiples of f_c.

I can choose any of these provided D_n doesn't happen to be zero.

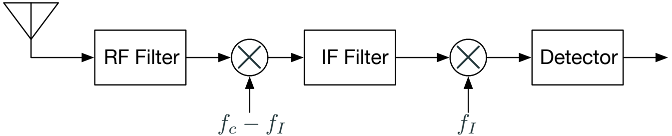
The next page illustrates this modulation method

Switching Modulator



Typical Radio Receiver

Assume we want to listen to a radio signal at f_c. This is a typical receiver.

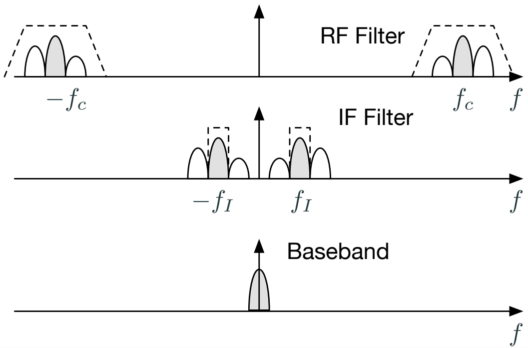


The input RF at f_c is mixed down to a fixed intermediate frequency f_I

- RF filter is not very selective
- First modulation frequency is adjustable
- The IF filter is selective
- Everything from the IF filter onward doesn't change with tuning

Typical Radio Receiver Spectrum

The spectrum of the signals in the receiver look like this:

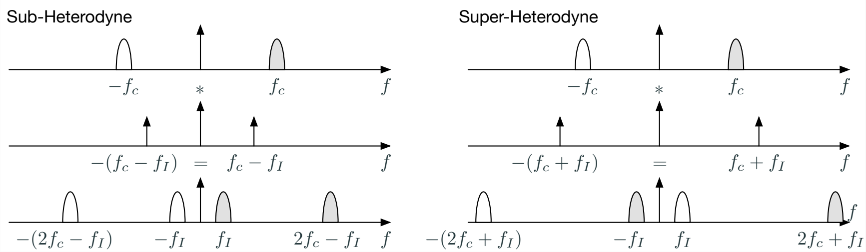


The RF filter selects some part of the band of interest, while the IF filter selects the signal you are interested in.

Your SDR samples the IF directly, and do the rest in software.

Frequency Translation

The key to this receiver is being able to translate signals in frequency.



- To help keep track of what is happening, one of the bands has been shaded gray. In fact, both are the same.
- Both produce the same IF signals.

AM Modulation

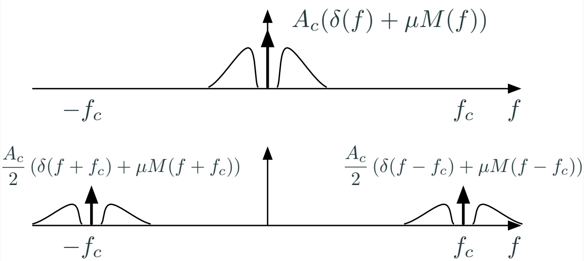
AM modulation is a form of amplitude modulation. For $\mu > 0$,

$$s(t) = (A_c + m(t))\cos(2\pi f_c t) = A_c(1 + \mu m(t))\cos(2\pi f_c t)$$

We need bandwidth of $m(t) \ll f_c$ and modulation index $\mu < 1$.

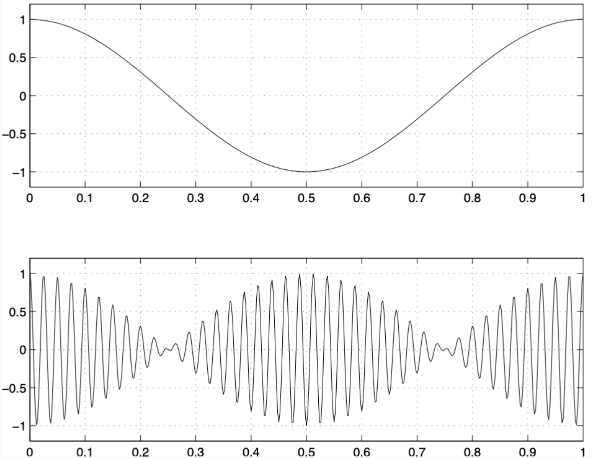
Spectrum of modulated signal:

$$S(f) = \frac{A_c}{2}\left(\delta(f + f_c) + \delta(f - f_c)\right) + \frac{A_c\mu}{2}\left(M(f + f_c) + M(f - f_c)\right)$$



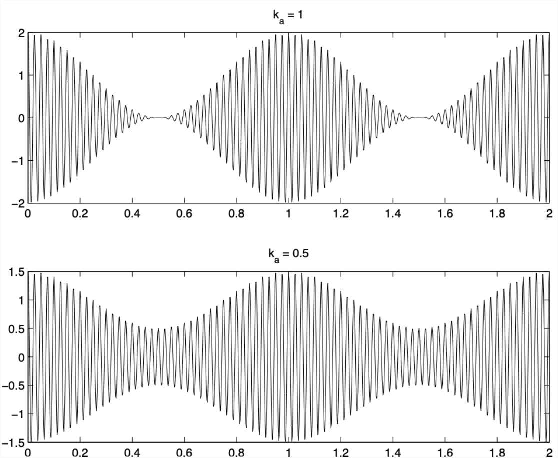
DSB-SC vs. AM

DSB-SC modulated signals undergo phase reversal when $m(t)$ changes sign. It is difficult to extract carrier from received signal.



DSB-SC vs. AM (cont.)

In AM, the carrier signal is modulated by $A_c + m(t) = A_c(1 + \mu m(t))$.
Examples: $\mu = 1$ and $\mu = 0.5$.



Envelope Detection of AM Signals

The term *detection* means extracting signal from received data. In some cases it means demodulation.

Suppose that a signal $x(t)$ can be written as

$$x(t) = E(t)\cos(2\pi f_c t)$$

where $E(t)$ varies slowly compared to the carrier $\cos(2\pi f_c t)$.

Then $|E(t)|$ is called the envelope of $x(t)$.

For envelope detection to work, we need

- $f_c \gg$ bandwidth of $m(t)$

  Otherwise positive and negative spectral components overlap.

- $A + m(t) \geq 0$

  Otherwise phase reversals occur when $A + m(t) < 0$.

Modulation index

The maximum deviation of $m(t)$ from zero is

$$m_p = \max(|m(t)|)$$

The *modulation index* of the modulated signal is defined by

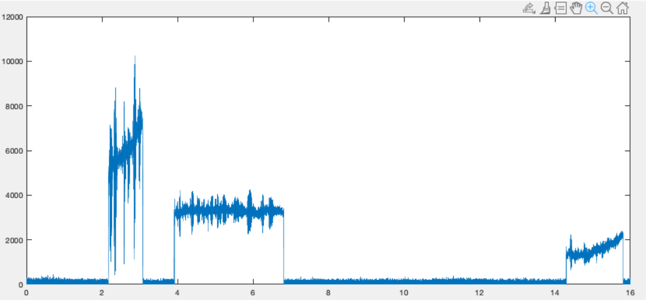
$$\mu = \frac{m_p}{A}$$

Larger modulation index reduces power but makes demodulation harder.

Broadcast AM stations use modulation index close to 1. Input signals are controlled using automatic gain control (AGC).

Modulation Index Example

This is an captured airband signal

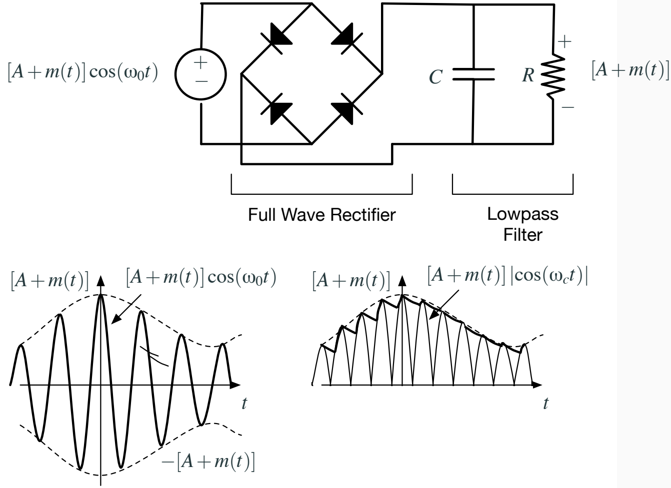


You can clearly see the carrier, and when it is keyed on and off.

The first transmission has a much higher modulation index than the second two. Why might this be?

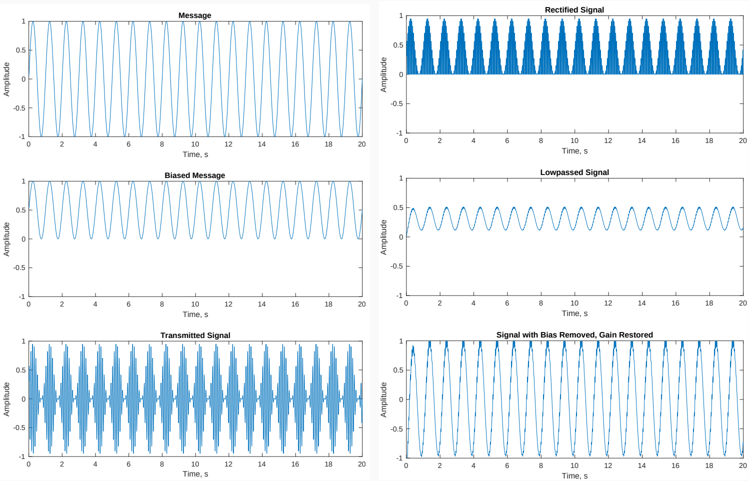
Envelope Detector for AM

Rectify the RF signal, then lowpass filter:



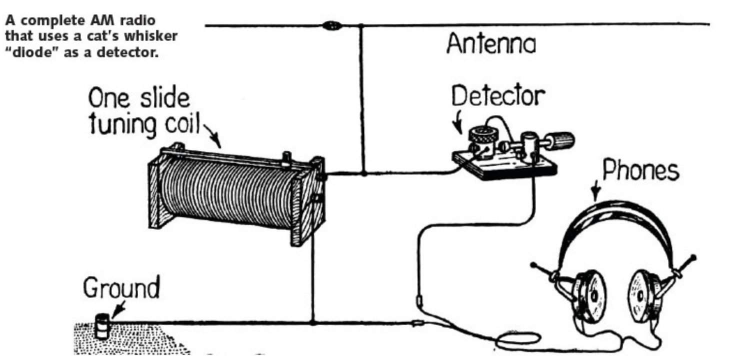
AM Demodulation Experiment

$$m(t) = \cos 2\pi t\,,\ f_c = 10\,,\ h(t) = \frac{1}{RC} e^{-t/RC} u(t)$$



Cat's Whiskers (Crystal) Radio

This radio was powered only by received radio energy.



Wikipedia: Crystal radio wiring pictorial based on Figure 33 in Gernsback's 1922 book *Radio For All* (copyright expired) with "Aerial" changed to Antenna by J.A. Davidson.

The point-contact semiconductor detector was subsequently resurrected around World War II because of the military requirement for microwave radar detectors.

Power of AM Signals

The power of an AM signal is the sum of the power of two components.

$$\phi_{AM}(t) = (A + m(t))\cos(2\pi f_c t) = \underbrace{A\cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{m(t)\cos(2\pi f_c t)}_{\text{sidebands}}$$

The carrier and sideband signals are orthogonal, so powers add. Carrier power is

$$P_c = A^2 \int_0^T \cos^2(2\pi f_c t)\; dt = \frac{1}{2}A^2$$

Signal power after modulation is 1/2 the original message power

$$P_s = \tfrac{1}{2} P_m\,,$$

where message power is average power as T gets large,

$$P_m = \overline{m^2(t)} = \frac{1}{T}\int_{t_0}^{t_0+T} m^2(t)\; dt$$

E.g., the power of a tone $\cos(2\pi f_m t)$ is $\frac{1}{2}$.

Power of AM Signals (cont.)

The carrier tone simplifies demodulation but carries no information.

The power efficiency is defined by

$$\eta = \frac{\text{useful power}}{\text{total power}} = \frac{P_s}{P_c + P_s}$$

where

$$P_c = A^2/2$$

and

$$P_s = \frac{1}{2}P_m = \frac{1}{2}\overline{m^2(t)}.$$

Then

$$\eta = \frac{\frac{1}{2}\overline{m^2(t)}}{\frac{A^2}{2} + \frac{1}{2}\overline{m^2(t)}} = \frac{\overline{m^2(t)}}{A^2 + \overline{m^2(t)}}$$

Power of AM Signals (cont.)

Examples: Tone modulation

$$m(t) = \mu A \cos(2\pi f_m t)$$

where $0 < \mu \leq 1$ and f_m is the audio tone frequency. Then

$$\overline{m^2(t)} = \frac{1}{2}(\mu A)^2$$

and

$$\eta = \frac{\frac{1}{2}(\mu A)^2}{A^2 + \frac{1}{2}(\mu A)^2} = \frac{\mu^2}{2 + \mu^2}$$

Since μ must be less than one, the maximum value is $1/3$.

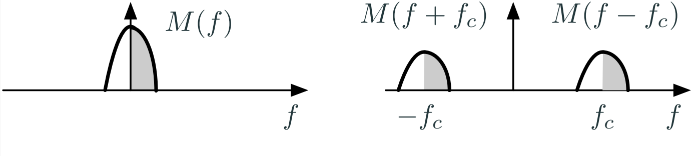
Efficiency falls off rapidly as μ decreases. For $\mu = 0.5$,

$$\eta = \frac{(0.5)^2}{2 + (0.5)^2} = \frac{1}{9}$$

AM is inefficient in both power and bandwidth.

Single Sideband (SSB)

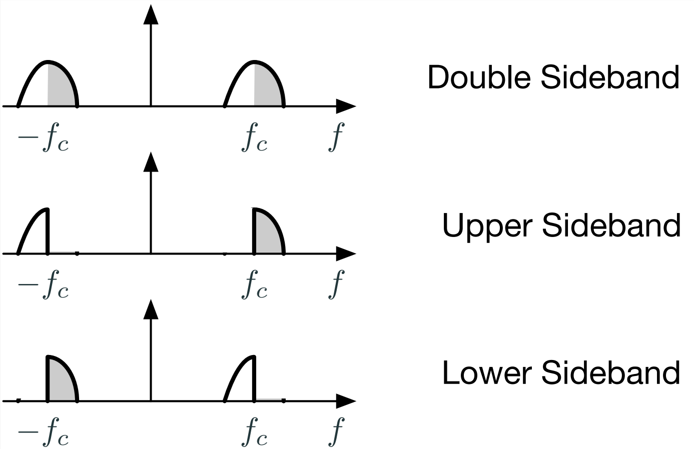
DSB-SC is spectrally inefficient. It uses twice the bandwidth of the message.



The signal can be reconstructed from either the upper sideband (USB) or lower sideband.

SSB transmits a bandpass filtered version of the modulated signal.

Single Sideband (cont.)



Single Sideband Modulation and Demodulation

- SSB can be transmitted using a DSB-SC modulator with a narrower bandpass filter. For USB, center frequency is

  $$\tilde{f}_c = f_c + \tfrac{1}{2}B$$

  and cutoff frequency is $B/2$.

  The single sideband filter must roll off quickly to eliminate unwanted contributions from the other sideband.

  Message frequencies near 0 will be affected by the non-ideal filter.

- SSB demodulation can use a DSB-SC demodulator with no change.

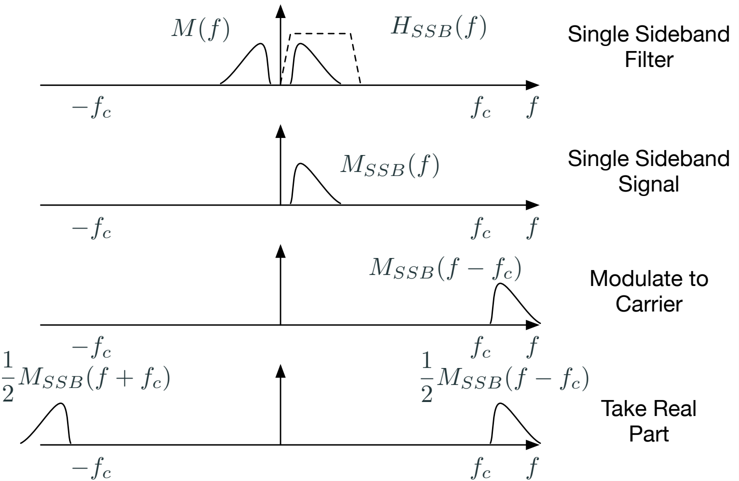
  The input to the lowpass filter is different from that of DSB-SC.

Which SSB Sideband?

- Transmitter and receiver must agree on use of LSB vs. USB.
- SSB is common for amateur radio
  - Below 10 MHz : LSB
  - Above 10 MHz : USB
  - Exception for 5 MHz :USB
  - Exception for digital modes : USB
- SSB also common for shortwave
  - 120m (2300-2495 kHz): LSB
  - 90m (3200-3400 kHz): LSB
  - 75m (3900-4000 kHz): USB
  - 60m (4750-5060 kHz): LSB
  - 49m (5900-6200 kHz): USB
  - 41m (7200-7450 kHz): USB

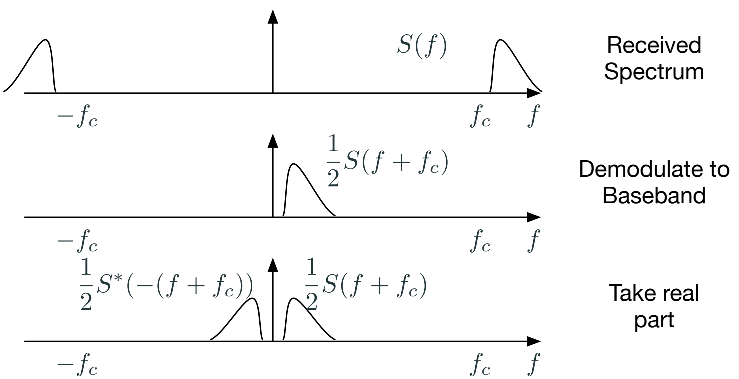
  Your radio knows!

SSB Modulation



SSB Demodulation

To decode the SSB signal, we just reverse the operations



- Ideally we want a synchronous demodulator
- In practice, f_c is estimated by the sound of the signal
- An error of 50 Hz is quite noticeable

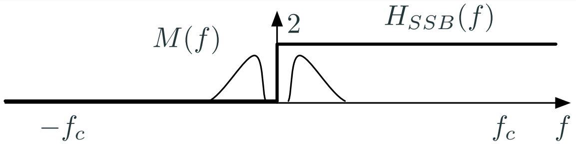
SSB in Time Domain

The upper sideband is the output of filtering a modulated signal $m(t)\cos\omega_c t$ with an ideal bandpass filter:

$$H_{\text{SSB}}(f) = \begin{cases} 2 & f > 0 \\ 0 & f < 0 \end{cases}$$

This is

$$H_{\text{SSB}}(f) = 2u(f)$$



The impulse response of this filter is

$$h_{SSB}(t) = \mathcal{F}^{-1}\left\{2u(f)\right\}$$

Hilbert Transform

We know

$$u(t) \Leftrightarrow \frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$$

so by duality, and multiplying by 2

$$\delta(t) + \frac{j}{\pi t} \Leftrightarrow 2u(f)$$

The impulse response of the filter is

$$h_{SSB}(t) = \delta(t) + \frac{j}{\pi t}$$

If $m(t)$ is the input signal, the single sideband signal is

$$m(t) * h_{SSB}(t) = m(t) * \left(\delta(t) + \frac{j}{\pi t}\right) = m(t) + j\left(m(t) * \frac{1}{\pi t}\right)$$

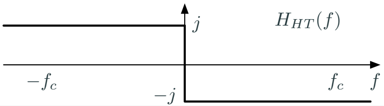
The last term is the Hilbert transform of $m(t)$

$$m_h(t) = m(t) * \frac{1}{\pi t}$$

The transfer function of the Hilbert transform is

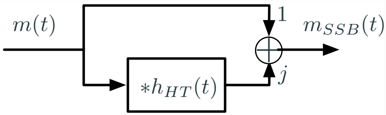
$$H_{HT}(f) = -j\text{sgn}(f) = \begin{cases} -j = & f > 0 \\ j = & f < 0 \end{cases}$$

which looks like



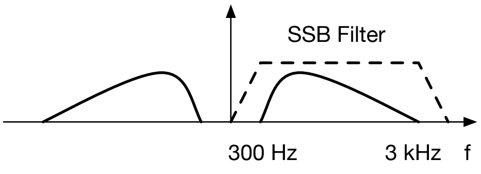
What happens if $m(t) = \cos(2\pi ft)$, or $m(t) = \sin(2\pi ft)$?

The block diagram is



Vestigial Sideband Modulation (VSB)

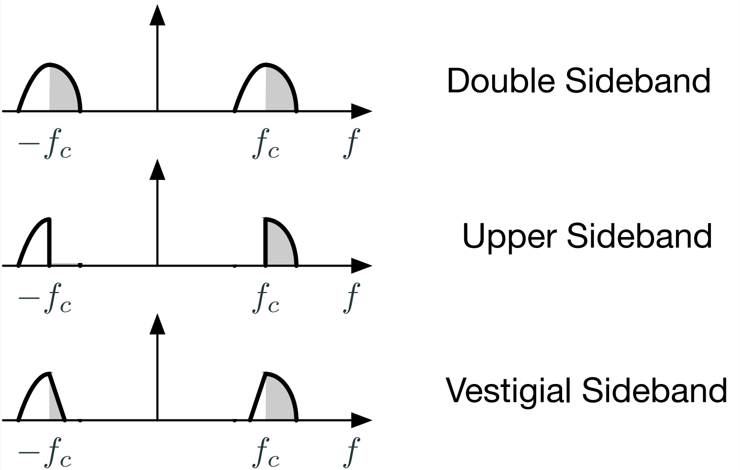
- SSB relies on being able to filter out one sideband. For audio this is possible because the voice spectrum drops off below 300 Hz, allowing space for a transition band



- This is not possible for other signals, like video, that have strong components at low frequencies.

VSB Idea

The solution is *Vestigial Sideband Modulation, VSB* where a small portion (a vestige) of the unneeded sideband. This reduces DC distortion.



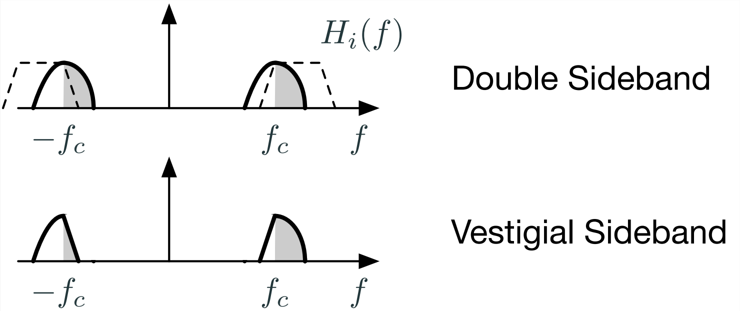
- VSB signals are generated using standard AM or DSB-SC modulation, then passing modulated signal through a sideband shaping filter.
- The signal can be designed so that demodulation uses either standard AM or DSB-SC demodulation, depending on whether a carrier tone is transmitted.
- VSB modulation with envelope detection are used to modulate image in analog TV signals. (The audio signal is modulated using FM.)

VSB Modulator

The transmitted signal has spectrum

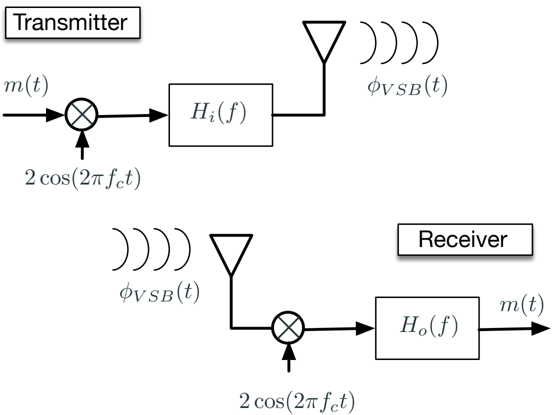
$$\Phi_{VSB}(f) = \big(M(f+f_c) + M(f-f_c)\big)H_i(f)$$

where $H_i(f)$ is the *shaping filter* for the VSB modulator.



VSB System

We transmit the VSB signal $\phi_{VSB}(t)$,



How do we choose the receiver filter $H_o(f)$ so that we get the original message back?

VSB Receiver

- The intermediate signal after the demodulator is

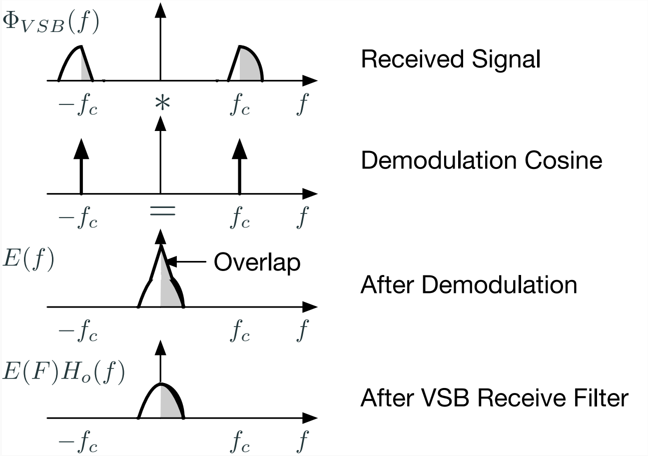
$$e(t) = \phi_{VSB}(t) \cdot 2\cos\omega_c t$$

has spectrum

$$\Phi_{VSB}(f + f_c) + \Phi_{VSB}(f - f_c)$$

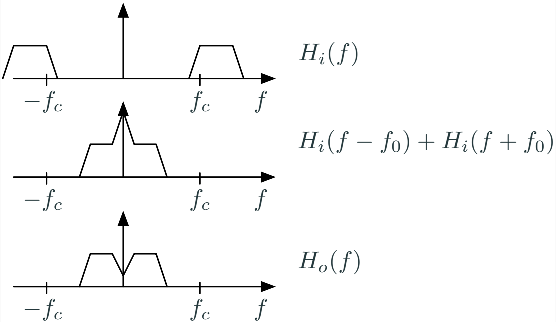
- This has two copies of the signal that are shifted to baseband, but unfortunately they overlap!
- This is then filtered by $H_o(f)$.

This looks like:



The filter $H_o(f)$ needs to compensate for the fact that the two sidebands overlap when demodulated to baseband.

VSB Receive Filter



We can recover $m(t)$ by using a filter $H_o(f)$ defined by

$$H_o(f) = \frac{1}{H_i(f+f_c)+H_i(f-f_c)}, \quad |f| \le B$$

Note that the division is only done over the signal bandwidth!

How could we design $H_i(f)$ to make our lives easier?

VSB Encoding and Decoding

- There are lots of other ways to encode and decode VSB, especially if we are using SDR's.
- You'll see one in the next homework that uses complex modulation, a different filter, and a neat Fourier transform symmetry trick.
- VSB signals turn up in many different places

- The analog TV system NTSC used VSB to save bandwidth
- VSB is widely used in Magnetic Resonance Imaging (MRI) to reduce the amount of data you need to collect

Quadrature Amplitude Modulation (QAM)

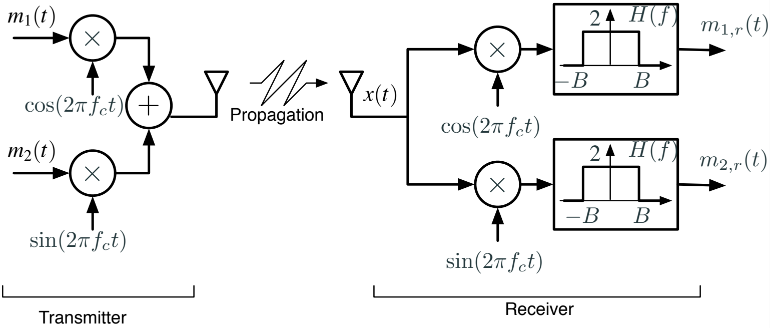
- DSB-SC modulates a real signal with bandwidth $2B$ to a transmitted signal with bandwidth $2B$, but half the spectrum is redundant
- SSB reduces the transmitted bandwidth to B, but
  - requires more complex modulator
  - reduces SNR (for a fixed carrier amplitude)
- Quadrature amplitude modulation uses the $2B$ transmitter bandwidth to send two independent (real) signals:

$$m_{QAM,c}(t) = m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)$$

- QAM has the same *spectral efficiency* as SSB but does not need sharp band-pass filters
- QAM is used in almost all digital communication methods, including telephone modems, cable TV, satellite TV

QAM Modulator and Demodulator

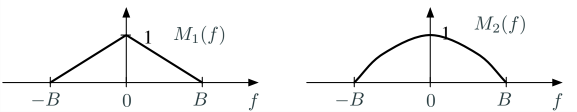
Two real messages, $m_1(t)$ and $m_2(t)$. m_1 is modulated on a cosine, and $m_2(t)$ is modulated on a sine.



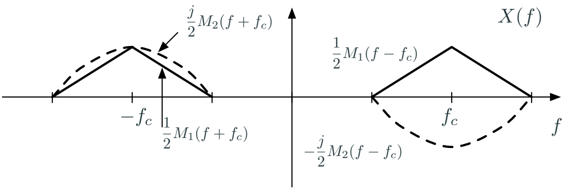
Note that we need a synchronous receiver, or the two channels will interfere.

What happens with a $90°$ phase shift?

If the input spectra look like



Then the transmitted spectrum looks like



Then demodulating with a cosine will give me $M_1(t)$ at baseband, and demodulating with a sine will give me $M_2(t)$.

QAM

- One way to think about this system is that we send $m_1(t)$ on the real, or in phase channel (modulate and demodulate with $\cos(2\pi f_c t)$)
- The second message $m_2(t)$ is sent on the imaginary, or quadrature channel (modulate and demodulate with $\sin(2\pi f_c t)$)
- There are generalizations that use many phases and amplitudes to send lots of digital bits at once.
- This is widely used for cable TV, such as QAM-64. We'll see this later in the course.

AM Modulation

- Many different ways to encode information as amplitude
  - AM
  - DSB-SC AM
  - SSB
  - VSB
  - QAM
- Common issues
  - Synchronization
  - Bandwidth
- Next: Encoding information in frequency