

Lecture 16: Digital Carrier Modulation

John M Pauly

February 16, 2026

Digital Carrier Modulation

Binary modulation: one bit per symbol

- Amplitude shift keying (ASK)
- Frequency shift keying (FSK)
- Phase shift keying (PSK)

M-ary modulation: multiple bits per symbol

- Pulse amplitude modulation (PAM)
- Phase modulation: PSK, QPSK
- Frequency modulation: FSK, MFSK, GMFSK
- Many Frequencies: FDM, OFDM

Digital Carrier Modulation

In class so far we've just talked about baseband modulation of digital signals

- Ethernet
- Fiber
- USB

As you've seen in the labs, the same ideas work for sending bits over a carrier, such as RF

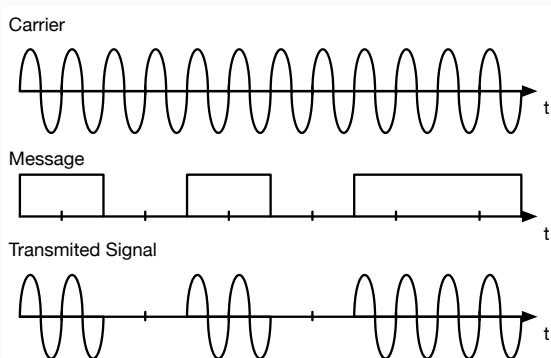
We'll look at several simple ways to encode bits on a carrier

- On-Off Keying (OOK)
- Phase-Shift Keying (PSK)
- Frequency Shift Keying (FSK)

These are designed to be simple to generate, and simple to decode.

On-Off Keying (OOK)

A simple version of ASK Modulated signal is $m(t) \cos 2\pi f_c t$.

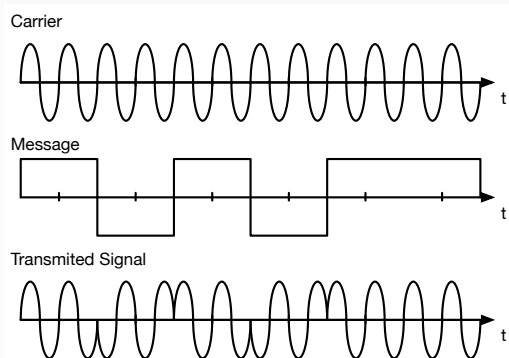


Easy to generate, gate an oscillator on and off

Easy to receive, a simple envelope detector suffices

Phase Shift Keying (PSK)

Binary PSK is the same as polar ASK. We'll see M-ary versions shortly



Usually an integer number of cycles of an offset frequency used

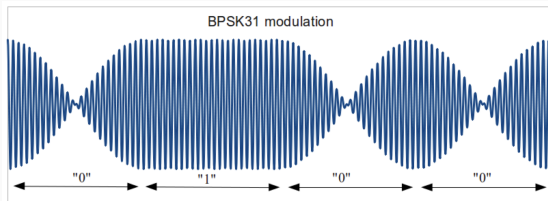
Phase changes at zero crossing to minimize bandwidth

Detection generally must be synchronous, since the envelope is constant

PSK Example: BPSK31

One PSK method is BPSK31, widely used in amateur radio.

A "1" is a constant phase interval, and a "0" is sent with a phase inversion



The shaped pulses minimize the bandwidth

After demodulating to baseband and lowpass filtering, the "1" interval will have an integrated value, and the "0" interval will integrate to zero.

Timing can be extracted from the zero crossings from the "0" symbols.

Figure from *Wikipedia*

Differential PSK (DPSK)

Just as in line coding, there are advantages to sending the *difference* between the transmitted bits.

Differential PSK encodes the bits as the phase difference between two PSK pulses.

An initial state (usually "0") is assumed at the beginning, then

- "1" is a change of phase,
- "0" is the same phase.

This doesn't need a synchronous receiver! The signal is its own reference.

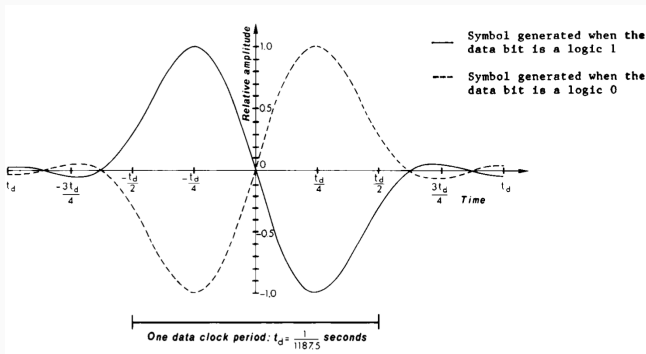
This is a significant benefit.

Still need to get the frequency right from the preamble or a tracking loop.

DPSK Example: Radio Broadcast Data System (RBDS)

FM radio service that provides song titles, music genre, station ID, etc

Manchester encoded, with raised cosine pulses

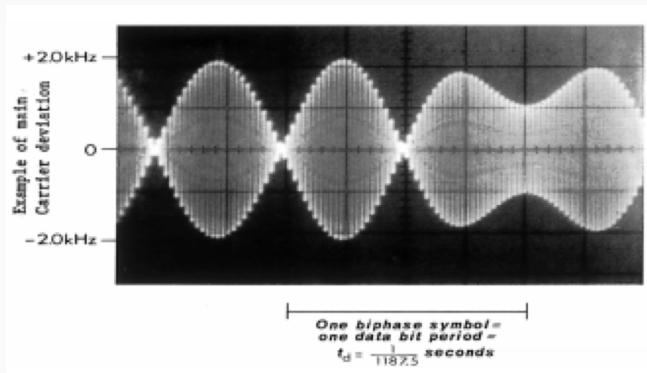


The shaped pulses minimize the bandwidth

Figure from *RBDS Standard Apr. 1998*

RBDS Waveform

The waveform looks like this

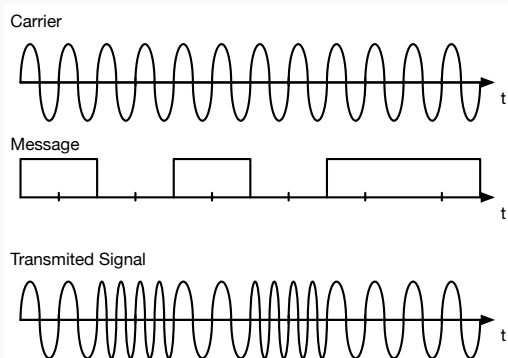


Data is extracted by comparing the phase of between Manchester pulses.

Timing is extracted from the zeros of the Manchester pulses.

Frequency Shift (FSK)

Binary FSK uses two frequencies for 1 and 0.

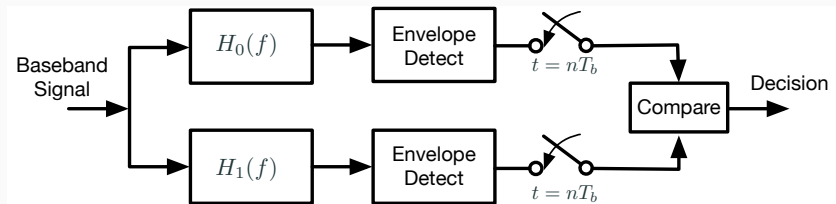


An integer number of cycles of each makes them orthogonal

Easy to receive, can be done with filters and an envelope detector (see this week's lab!). Does not need to be synchronous.

FSK Receivers

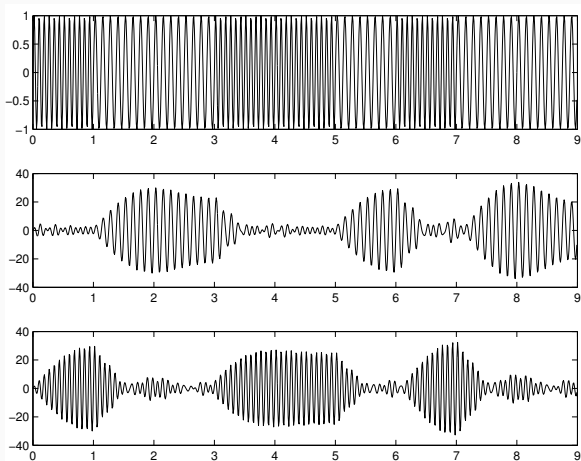
One way to receive FSK signals is to use narrowband filters for each tone



You can think of each channel as a matched filter.

Example: Bell 103 modem (V.21, 300 bps) uses 1270 Hz and 1070 Hz for originating station, only 3 or 4 cycles per bit

FSK Example: $f_0 = 8$, $f_1 = 12$



This would be followed by an envelope detector (rectifier and lowpass filter).

M-ary Digital Carrier Modulation

M-ary: many bits in a single symbol

Many encoding schemes use different features of the pulses

- Amplitude: PAM
- Phase: PSK, QPSK, OQPSK
- Frequency: FSK, MFSK, GMFSK

M-ary Signalling

Many ways to send multiple bits of information with each pulse, or symbol.

Options are different amplitudes of a common pulse, or sending different pulses

$$y(t) = \sum_k a_k p_k(t - kT_b)$$

where a_k is chosen from a set of more than two values (i.e., not just ± 1). Both a_k and $p_k(t)$ may be complex.

Some methods use a common pulse $p(t)$ with different amplitudes

- Amplitude shift keying (ASK) where a_k is real
- Quadrature amplitude modulation (QAM) where a_k is complex
- Phase shift keying (PSK) where a_k is complex, and unit magnitude

Often this pulse is a square root Nyquist pulse, covered last time.

M-ary Signalling (cont'd)

Other methods use different waveforms and amplitudes

- Frequency shift keying (FSK)
- Minimum shift keying (MSK and GMSK)
- Orthogonal frequency domain multiplexing (OFDM)

These are sets of waveforms that are usually chosen to be orthogonal to each other.

Amplitude Shift Keying (ASK)

One widely used encoding of two bits into four levels is

$$a_k = \begin{cases} -3 & \text{message bits 00} \\ -1 & \text{message bits 01} \\ +1 & \text{message bits 11} \\ +3 & \text{message bits 10} \end{cases}$$

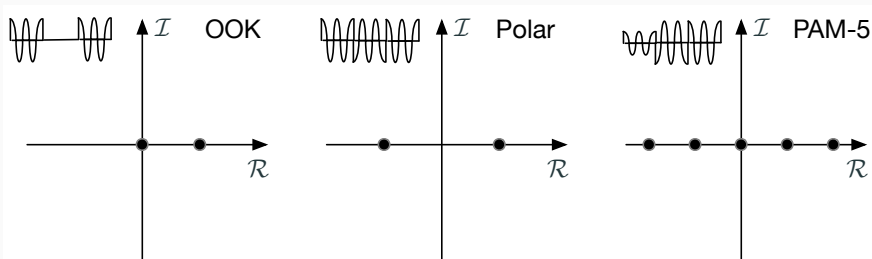
Note the order. Adjacent symbols differ by only one bit. This minimizes the impact of single bit errors. This is called a Grey code, which we will see more of soon.

ASK used in

- Ethernet 100Base-T (PAM-3), 1000Base-T (PAM-5), 10GBase-T (PAM-16)
- GDDR6X memory (used in the RTX 3090) with PAM-4.
- ATSC Digital TV, (PAM-8)

Constellation Plots

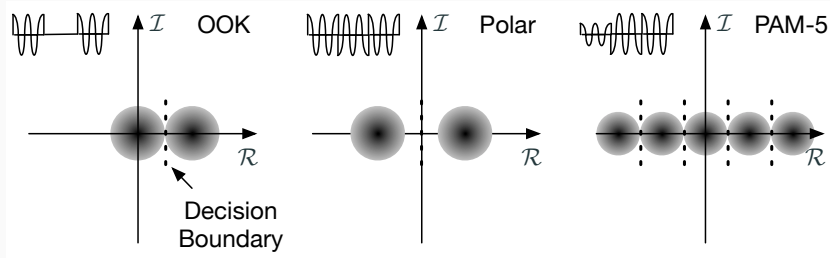
- For all of the methods that use a common pulse $p(t)$ we can characterize them by plotting the coefficients a_k
- In general a_k can be complex, so the plot is in the complex domain.
- Some examples so far:



- These are all on the real axis.

Constellation Plots

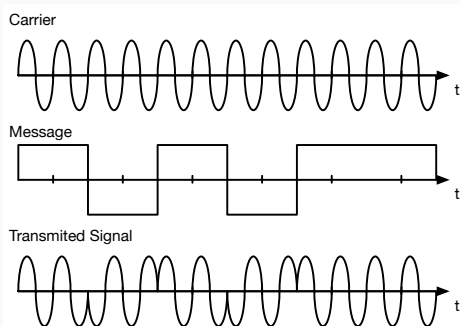
- Noise blurs the constellation causing decoding errors



- We want to pack more symbols in!
- Where should they go?

Binary Phase Shift Keying (PSK)

Recall that binary PSK is the same as polar ASK.



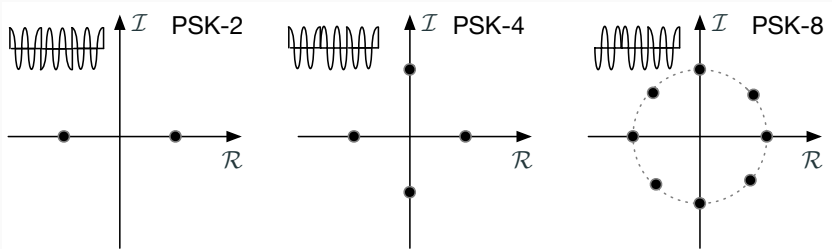
Here the phases are 0 and 180, which corresponds to amplitudes of ± 1 .

We can generalize this by using more phases.

This gets us off the real axis in the constellation plot.

M-ary Phase Shift Keying

We can add more symbols by using additional phase shifts



The waveforms for M symbols are

$$p_k(t) = \cos(2\pi f_c t / T_b + 2\pi k / M)$$

This is a coefficient $a_k = e^{j2\pi k / M}$ for the complex pulse $e^{j2\pi f_c t / T_b}$.

Generally need a synchronous receiver. Differential encoding (DPSK) makes this much easier.

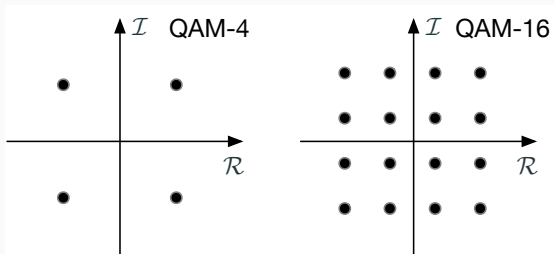
Quadrature AM (QAM)

Another way to fill in the space is to use both sines and cosines

$$p_k(t) = a_k \cos(2\pi f_c t/T_b) + b_k \sin(2\pi f_c t/T_b)$$

We plot the cosine term as real, and the sine term as imaginary

The constellations look like



Here the symbols are spaced rectilinearly and evenly, many other options

Quadrature AM (QAM)

If there are $N = 2^M$ coefficient values, each symbol can encode M bits

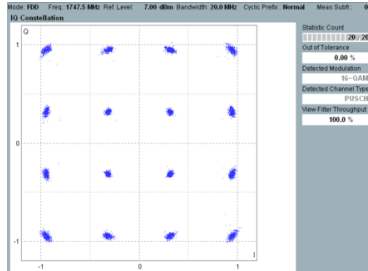
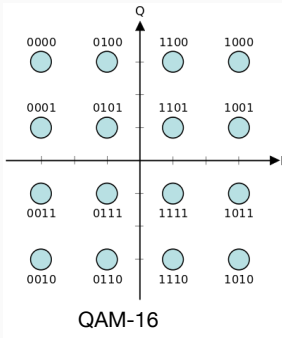
We can also consider each symbol to be a complex number!

- QAM-16 can be thought of as having a 2 bit real component and a 2 bit imaginary component.
- This will be useful for OFDM, which we'll cover later in the course.

QAM is Common for cable TV, where QAM-64 or QAM-256 are used.

If you put your modem into hardware configuration mode, it will tell you what it is using.

QAM Constellation Measurement



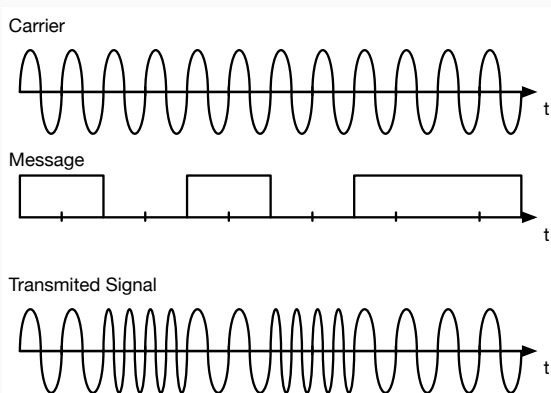
Experimental Measurement

Again, note the ordering. Adjacent symbols differ by only one bit. This is a 2D Gray code.

Errors due to noise will mostly be between adjacent symbols, which have less impact.

Binary Frequency Shift Keying (FSK)

Binary FSK uses two frequencies for 1 and zero.



We can generalize this to more bits by adding frequencies

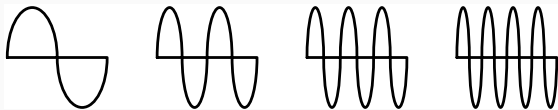
M-ary Frequency Shift Keying (MFSK)

For M-ary FSK we use several frequencies

$$p_k(t) = \cos(2\pi(f_c + k\Delta f)t)$$

where Δf is the spacing in frequency.

Decoding is easy if $(\Delta f)T_b = 1$, so that we have integer numbers of cycles of the cosine (or sine).



This is a modulation index $m = 1$, meaning the frequencies are multiples of one cycle. These are all orthogonal

The receiver can use an array of filters, one for each frequency, followed by envelope detectors, just as in the binary case.

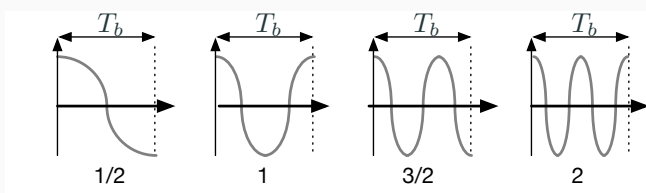
Digital Mobile Radio

- Digital Mobile Radio (DMR) uses 4-FSK
 - A very common digital “walkie talkie” system by Motorola
 - Popular with amateur radio
- C4FM
 - Variant of 4-FSK with continuous phase between symbols
 - P25 used by police, and many public works systems
 - Yaesu System Fusion for amateur radio
- Other digital handheld protocol is D-STAR, which is GMSK.

Minimum Shift Keying (MSK)

If we want to get the most bits/symbol across a channel for FSK, we'd like the frequencies to be as close together as possible

The minimum duration orthogonal signals are multiples of *half* a cycle



These are all orthogonal if we use only cosines (or only sines)

The modulation index is $m = 0.5$, and all the frequencies are multiples of that for one half cycle

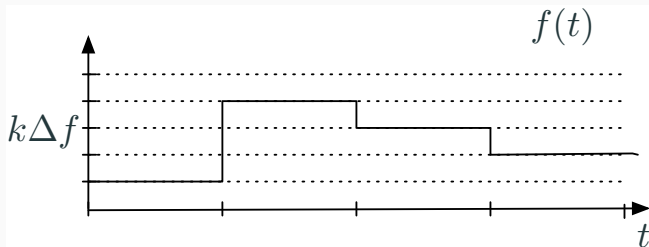
The receiver has to be more sophisticated, simple filters and envelope detectors won't suffice.

Gaussian Minimum Shift Keying (GMSK)

- FSK can be thought of as a time varying frequency waveform

$$y(t) = \cos(2\pi(f_c + f(t))t)$$

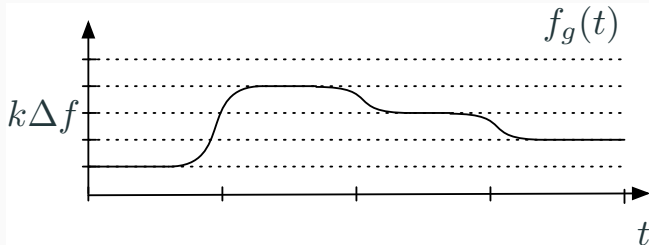
where $f(t)$ looks like this:



- The abrupt frequency changes cause sidelobes, and interfere with adjacent channels

Gaussian Minimum Shift Keying (GMSK)

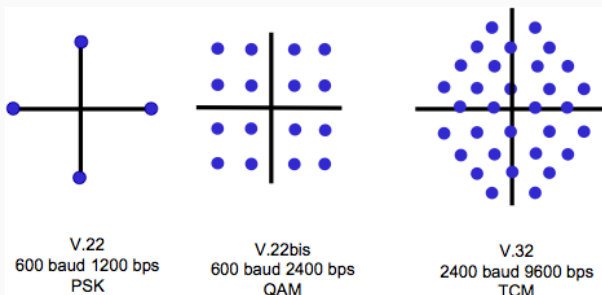
- To minimize this, the $f(t)$ waveform is first convolved with a Gaussian waveform to smooth it



- If this is used with Minimum Shift Keying, the results is Gaussian Minimum Shift Keying or GMSK
- This is used in GSM cell telephones, with a modulation index of $m = 0.3$, even less than MSK!
- The results is ISI, and mitigation strategies and error correction are required to make it work.

Phone Modems: PSK, FSK and QAM Examples

These were all used in telephone modem protocols



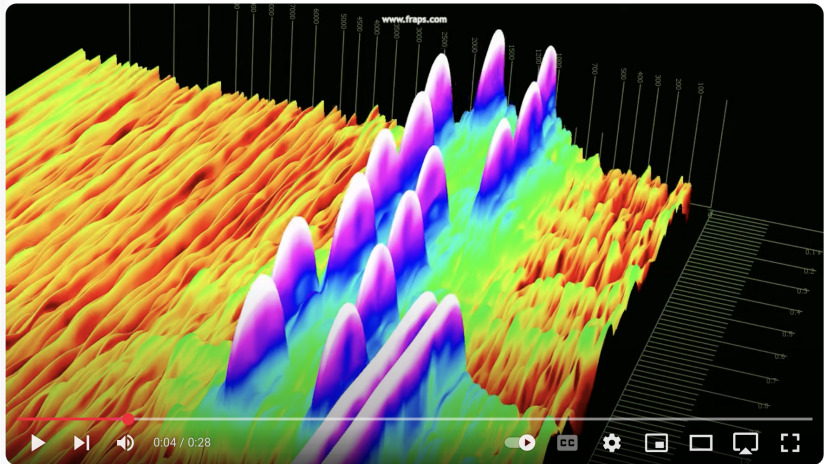
Recall that baud is symbols per second

V.32 has 32 values, or 5 bits/symbol. This is used as 4 bits plus an error correction bit

The symbol rate is 2400 baud (within the telephone frequency response)

The total bit rate is $2400 \times 4 = 9600$ bits/second

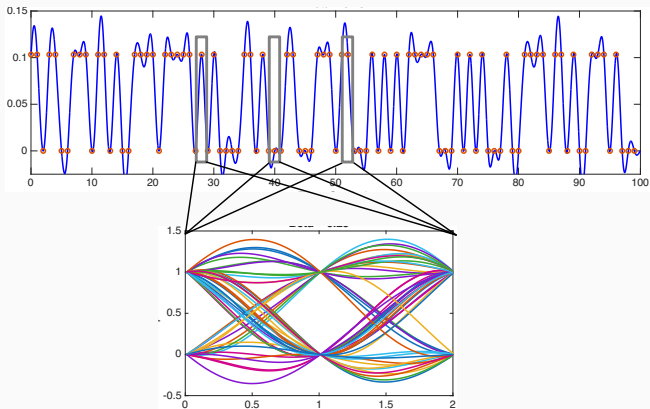
Phone Modem Sounds



https://youtu.be/vvr9AMWEU-c?si=b-01fCUH8zvmsob_

Decoding M-ary Signals

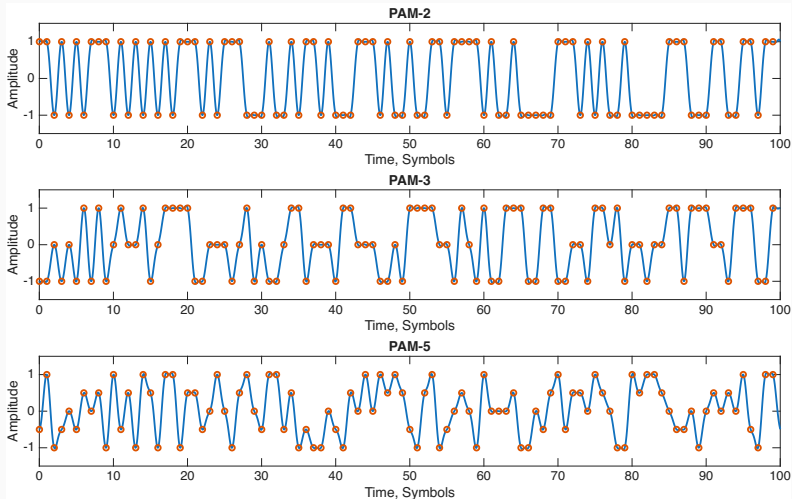
We looked at eye diagrams for binary encoding.



The particular Nyquist pulse we used changed how difficult the signal was to decode, both in noise and timing sensitivity.

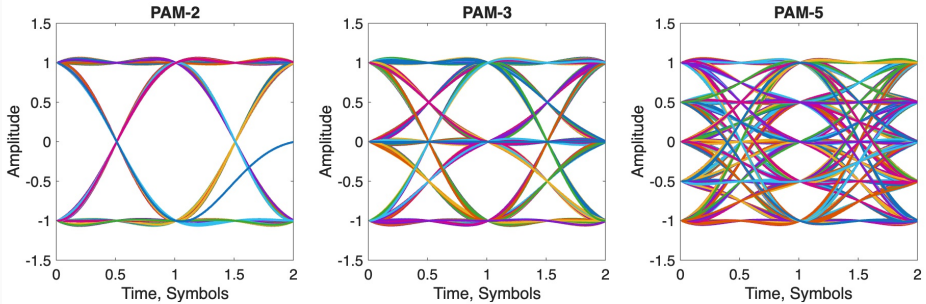
Decoding M-ary Signals (cont)

For M-ary signals things get more complicated. Here are PAM waveforms with an ideal Nyquist pulse ($\beta = 1.0$)



Decoding M-ary Signals (cont)

The eye diagrams for these look like this:

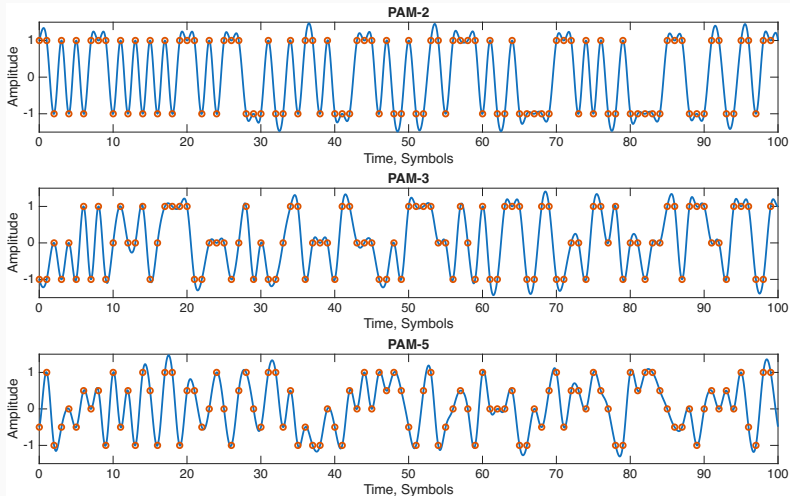


The vertical size of the eye decreases as expected. PAM-5 requires higher SNR than PAM-3 for the same performance

The timing sensitivity also gets much worse.

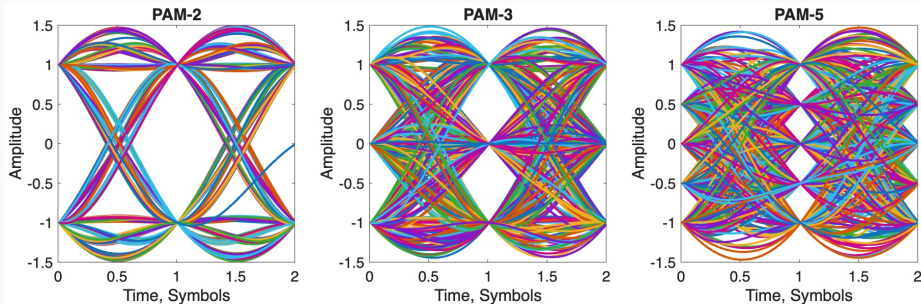
Decoding M-ary Signals (cont)

Here are PAM waveforms with a more reasonable Nyquist pulse ($\beta = 0.5$)



Decoding M-ary Signals (cont)

The eye diagrams for the Nyquist pulse with $\beta = 0.5$ look like this:



The vertical size of the eye is the same as when $\beta = 1$

The timing sensitivity also gets even worse. Accurate symbol timing will be critical for correct decoding.

So far we've looked at a number of digital modulation methods.

We can get more bits/s with

- Higher symbol rate (baud)
- More bits/symbol (M-ary encoding)

Fundamental limits are

- Channel bandwidth
- Noise, interference, and power
- Complexity and computation

Next time we will look at noise and power. What is the best we can do for accurately decoding symbols in noise?