Protection for Sale with Unemployment

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Abstract

To match the observed patterns of protection over business cycle, I add unemployment into two models of trade policy: ‘Protection for Sale’ (1994) and ‘Trade Wars and Trade Talks’ (1995) by Grossman and Helpman. The source of unemployment is financial frictions: as in Aghion et. al. (2009) firms with profits less than a random threshold are liquidated. With small shocks protection goes up to save employment and efficient firms. With large shocks protection can go down, that is consistent with hump-shape pattern in the literature on declining industries. I also provide a simplified numerical example and ideas how to estimate and calibrate the model.

1 Introduction

During crises, most governments turn to protectionism. This pattern was visible during the Great Depression (Eichengreen, 2009) and, to the lesser extent, during the global financial crisis of 2007-2010 (Evenett, 2009, 2010, 2011).

Many papers on trade policy are based on lobbying framework suggested by Bernheim and Whinston (1986) and first used in a model of trade policy in a paper ‘Protection for Sale’ (Grossman, Helpman, 1994, GH94). Main motives of policymakers to provide protection to domestic industries are well explained by GH94 model, and the model is consistent with data.

My model is based on GH94, and all I do is I add unemployment. Economic part of the model is a general equilibrium in a small open economy with many consumers and producers. I concentrate on financial nature of business cycle\(^1\), and follow Aghion et. al. (2009) in assuming a presence ‘liquidity shocks’: a firm survives only if it is able to raise enough cash to pay the shock. The shock may be interpreted as (1) higher liquidity constraints that make firms to finance working capital by cash flow, (2) more expensive loan service or higher requirements for collateral, or (3) problems with suppliers and customers. If a firm doesn’t survive, it goes bankrupt, liquidates and dismisses all its workers. During recession many firms face high shocks, and on the macro-level we observe higher unemployment and lower production.

Trade policy is determined in a truthful equilibrium of a lobbying game in which some industries (organized in ‘lobbies’) offer policymaker contributions which depend on trade policy. Each lobby maximizes its

\(^1\)Costinot (2008) studies trade policy in search & match model a la Pissarides (2000), and finds that optimal trade barriers are higher in industries with more labor market frictions and so with higher unemployment. This result helps explain one more incentive to protectionism over business cycle.
own welfare net of contributions. Policymaker maximizes a weighted sum of social welfare and contributions from all lobbies. Financial shocks add uncertainty to GH94 model. Policy is determined before liquidity shocks are realized, and all the agents maximize their ex ante payoffs, taking into account distribution of shocks, that is common knowledge.

The model allows to distinguish different motives for protection. The GH94 equilibrium condition represents two motives: to increase profits of firms by raising trade barriers and to drop prices for the benefit of consumers by lowering the barriers.

Optimal policy for a small open economy is free trade, and it is always chosen if there is no lobbying. Unemployment changes all of this. Even without lobbying protection is positive to save firms, to save employment, and to help industries less affected by crisis create more jobs. Note that there is no harm of such a policy for the rest of the world because the economy is small: so we are in a rare case when the first best policy is not free trade. With lobbying protection goes up for organized industries but down for not organized ones. Also, the original political economy reason for protection is now weaker (demand for protection may go down because the gains are uncertain).

The model is also consistent with the idea that optimal policy allows the recession to throw away inefficient firms: more protection is provided to those firms whose shocks impose lower costs on the society.

The paper is related to literature on declining industries protection, which studies trade policy response to fall in world price in a particular industry. One puzzle in this literature is that models explaining higher protection of declining industries are not consistent with observed hump-shaped pattern of protection, according to which most protection is given to "average" declining industries\(^2\). My model demonstrates hump-shaped protection, so unemployment added to a standard GH94 framework resolves the puzzle.

In one extension I relax small open economy assumption to study trade wars and trade talks. The lobbying setup is the same inside each country, and trade policy is determined in equilibrium of a game between two policymakers. The game is either non-cooperative (‘Trade Wars’) or cooperative (‘Trade Talks’). The results are exactly the same as before.

The rest of the paper is organized as follows. First, I recall a basic GH94 setup, add unemployment and compare equilibrium with and without unemployment. Then I add the difference between efficient and inefficient firms. In an extension I consider trade wars and trade talks. Finally, I discuss possible ways to take the model to data.

2 The Model

2.1 Consumption

The setup follows GH94. Consider a small open economy populated by a unit continuum of individuals with identical preferences represented by a quasi-linear utility function

\[ u = x_0 + \sum_{i=1}^{n} u_i(x_i) \]

where good \( x_0 \) is a numeraire good, and goods \( x_i, i = 1, 2, ..., n \) are sold at domestic prices \( p_i \). All functions \( u_i(\cdot) \) are increasing, differentiable, and strictly concave.

In this setup the demand for good \( i \) depends on its price \( p_i \) only and is given by the function \( d_i(p_i) \), that is simply the inverse of the marginal utility \( u_i'(\cdot) \). The demand for the numeraire good is then \( x_0 = E - \sum_{i=1}^{n} p_i d_i(p_i) \), where \( E \) is the income of the individual. The consumer’s surplus is therefore

\[ S(p) = \sum_{i=1}^{n} u_i(d_i(p_i)) - \sum_{i=1}^{n} p_i d_i(p_i) \]

where \( p = (p_1, p_2, ..., p_n) \), and the indirect utility is \( V(p, E) = E + s(p) \).

### 2.2 Production

The economy produces \( n \) tradable goods from capital and labor with standard neoclassical production functions \( F_i(K_i, L_i) \) with constant returns to scale, and a numeraire non-tradable good 0 is produced from labor only with constant returns to scale and input-output ratio equal to 1. Labor market is competitive, and labor supply is large enough for the numeraire good to be always produced, so that the wage is equal to 1 in all industries. Capital supply is fixed and capital is specific to each industry, it may represent any specific factors needed in production. The economy is open and small, so all firms take prices as given. Denote \( \pi_i(p_i) \) the aggregate reward to the specific factor in industry \( i \), i.e. its total revenue \( p_j y_j(p_j) \) minus labor costs \( 1 \times L_i(p_j) \) Then the supply in this industry can be written as \( y_i(p_i) = \pi'_i(p_i) \).

### 2.3 Trade policy

Let world price of a good \( i \) equal \( p_i^* \). Then the domestic price is the world price plus tariff

\[ p_i = p_i^* + t_i \]

Domestic prices higher than world prices mean that there are import tariffs for that goods, domestic prices below world prices correspond to export subsidies. For simplicity I normalize units of all goods so that world price for unit of any good equals 1. Then \( p_i = 1 + t_i \).

The net revenue from all taxes and subsidies is redistributed uniformly among population. As population size is normalized to 1, the total and per capita transfer are equal to this net revenue

\[ R(p) = \sum_{i=1}^{n} (p_i - 1)(d_i(p_i) - y_i(p_i)) \]

where \( p_i - 1 \) is tariff rate and \( d_i(p_i) - y_i(p_i) = m_i(p_i) \) is import of good \( i \).
2.4 Welfare

An individual has three sources of income: wage, income from capital, and government transfers.

Capital ownership in each industry $i$ is concentrated in hands of some share $\alpha_i \in [0, 1]$ of the population (and distributed uniformly among people in this group). Then $\alpha_i = 0$ means the industry is extremely concentrated, and $\alpha_i = 1$ means ownership is spread uniformly among the whole population. Each individual may own capital in not more than one industry.

Then the total welfare of capital owners in industry $i$ is

$$W_i(p) = l_i + \pi_i(p_i) + \alpha_i(R(p) + S(p))$$  \hspace{1cm} (2.1)

The social welfare is\(^3\)

$$W(p) = 1 + \sum_{i=1}^{n} \pi_i(p_i) + R(p) + S(p)$$  \hspace{1cm} (2.2)

2.5 Unemployment

To have unemployment we need two things: negative shocks and frictions to keep the wage from falling down to clear the labor market.

In modelling shocks I use approach, a bit similar to Aghion, Hemous, and Kharroubi (2009). To keep labor market from clearing I assume sticky wages in the short run and restrict labor mobility: labor is still mobile among nonnumeraire industries, but the mobility is limited when it comes to numeraire sector. For now I assume that the unemployed cannot get a job in numeraire sector in the short run, but this assumption can be relaxed by assuming some moderate costs of getting this job (in terms of time or money).

Following AHK(2009), I assume that a recession imposes negative liquidity shocks on firms. If a firm is able to raise enough money to pay the shock, it survives. Otherwise, the firm can’t continue working, liquidates, and dismisses all its employees. The nature of the shock may be: problems with financing working capital, higher expenses on loan service (due to fall in value of the collateral, rise in interest rate for loans with floating rate, or for other reasons), liquidation of a supplier or default in payment from a customer. I assume liquidity constraints to be so strict that the only source of financing the shock is current profits\(^4\).

The size of the shock $c$ may be different among firms, let it be a random variable distributed with cdf $F(c)$, with corresponding pdf $f(c)$ and mean $\bar{c}$.

A firm in industry $i$ survives if and only if its profits $\pi_i(p_i)$ is sufficient to pay the shock: $\pi_i(p_i) > c_i$. Then the ex ante survival probability is $q_i(p_i) = F(\pi_i(p_i))$. First assume that the costs are inevitable (i.e. even if the firm liquidates, it pays $c$) or repayable (i.e. the firm gets $c$ back in short time after it has paid, so $c$

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\(^3\)Note that the social welfare is not necessarily the sum of capital owners’ welfare, because there might be people who own no capital, so we can’t simply state that $W(p) = \sum_{i=1}^{n} W_i(p)$

\(^4\)Another way is to allow firm to get a loan. If the size of the loan is proportional to profits, say it is $\nu \pi$, then the results will be qualitatively the same, and quantitatively the only difference would be that the effect of the shock will be "divided" by $(\nu + 1)$.
is not sunk costs but liquidity requirement a firm should satisfy to survive the hard times). This assumption is discussed and relaxed in the next chapters.

Note that now \( q_i(p_i) \) is simultaneously the probability of three events: a firm \( i \) employs its optimal number of workers \( L_i(p_i) \), it makes production \( y_i(p_i) \), and it obtains profits \( \pi_i(p_i) \). Therefore, an average person in our economy is employed with probability equal to the expected employment rate \(^5\)

\[
\overline{q}(p) = \sum_{j=1}^{n} q_j(p_j) L_j(p_j) \tag{2.3}
\]

The expected output of industry \( i \) is

\[
Ey_i(p_i) = q_i(p_i) y_i(p_i) \tag{2.4}
\]

The expected profits are

\[
E\pi_i(p_i) = q_i(p_i) \pi_i(p_i) - \bar{c} \tag{2.5}
\]

The consumption of all non-numeraire goods depends on their prices only and will change only to the extent to which prices will change. So, the expected import is

\[
\overline{m}_i(p_i) = d_i(p_i) - q_i(p_i) y_i(p_i) \tag{2.6}
\]

and the (expected) transfers are

\[
\overline{R}(p) = \sum_{i=1}^{n} (p_i - 1)(d_i(p_i) - q_i(p_i) y_i(p_i)) \tag{2.7}
\]

The (expected) welfare of capital owners in industry \( i \) is now

\[
\overline{W}_i(p) = \alpha_i \overline{q}(p) + q_i(p_i) \pi_i(p_i) - \bar{c} + \alpha_i(\overline{R}(p) + S(p)) \tag{2.8}
\]

The (expected) social welfare is

\[
\overline{W}(p) = \overline{q}(p) + \sum_{i=1}^{n} q_i(p_i) \pi_i(p_i) - n\bar{c} + \overline{R}(p) + S(p) \tag{2.9}
\]

2.6 Political choice and contributions

Trade policy is determined by a politician. In some exogenously given set \( L \) of industries capital owners are organized in lobbies, and these lobbies can offer contributions to the politician. The size of contributions depends on policy, that can be equivalently determined by tariffs or by domestic prices.

The politician maximizes a weighted sum of contributions and social welfare:

\[
G = \sum_{i \in L} C_i(p) + a\overline{W}(p) \tag{2.10}
\]

\(^5\)It also easy to consider a case in which all capital owners work in their specific industry, so that their chances of being employed are \( q_i(p_i) \) rather than \( \overline{q}(p) \).
where $C_i(p)$ is the contribution offered by lobby in industry $i$ for a price vector $p$ (if industry is not organized, $C_i(p) = 0$ for all $p$), $a$ is the weight on social welfare, that is a measure of accountability of the politician.

The timing in GH94 model is

1. Lobbies offer contributions
2. Politician chooses policy
3. Contributions are paid, production and consumption are made, taxes and transfers are paid, and all the agents get their payoffs.

The equilibrium concept is SPNE.

As it is shown in Grossman, Helpman (1994), the equilibrium in such a game is characterized by locally truthful contribution schedules, i.e. $\nabla C_i(p^o) = \nabla W_i(p^o)$ for all organized industries (where $p^o$ is the vector of domestic prices in the equilibrium), and the equilibrium tariff $t^o = p^o - 1$ is given by the first order condition in maximizing 2.10: $\sum_{i=1}^{n} \nabla C_i(p^o) + a \nabla W(p^o) = 0$, or

$$\sum_{i=1}^{n} \nabla W_i(p^o) + a \nabla W(p^o) = 0$$

The timing in model with unemployment differs from the above in one way: liquidity shocks are realized right after contributions are paid and before production:

1. Lobbies offer contributions
2. Politician chooses policy
3. Contributions are paid
4. Liquidity shocks are realized
5. Firms whose profits would be insufficient to pay the shocks, go bankrupt and liquidate dismissing all their workers; firms who survive pay shocks and make production; consumption is made, taxes and transfers are paid, and all the agents get their payoffs.

So, policy decisions are made and contributions are paid before liquidity shocks are realized. We use Perfect Bayesian Nash equilibrium concept here.

The first order condition for the optimal tariff is similar to 2.11:

$$\sum_{i \in L} \nabla \tilde{W}_i(p^o) + a \nabla \tilde{W}(p^o) = 0$$

(2.12)

The differences will be seen when we calculate all the gradients in 2.11 and 2.12, and get the explicit formulas for equilibrium tariffs.

### 2.7 Equilibrium trade policy: the basic model

It follows from 2.1 that

$$\frac{\partial W_i(p)}{\partial p_j} = (\delta_{ij} - \alpha_i)y_j(p_j) + \alpha_i(p_j - 1)m'_j(p_j)$$

(2.13)
where \( \delta_{ij} \) equals 1 for \( i = j \) and 0 otherwise. The sum for all organized industries is

\[
\sum_{i \in L} \frac{\partial W_i(p)}{\partial p_j} = (I_j - \alpha_L) y_j(p_j) + \alpha_L (p_j - 1)m_j'(p_j)
\]  

(2.14)

where \( I_j = \sum_{i \in Lobby} \delta_{ij} \) equals 1 if industry \( j \) is organized and 0 otherwise, \( \alpha_L = \sum_{i \in L} \alpha_i \) is the share of population, organized in lobbies.

From 2.2 we get

\[
\frac{\partial W(p)}{\partial p_j} = (p_j - 1)m_j'(p_j)
\]  

(2.15)

Substituting this into ?? gives the final expression for the equilibrium tariff in GH94 model.

\[
t^*_j = \frac{I_j - \alpha_L}{\alpha + \alpha_L}\left(\frac{y_j}{y_j - m_j'}\right)
\]  

(2.16)

The intuition behind 2.16 is the following. The organized industries (i.e. industries with \( I_j = 1 \)) pay for protection and get positive import tariffs/export subsidies. Not organized industries suffer from import subsidies/export tariffs since all the organized groups lobby for lower prices of all consumption goods that they don’t produce. The higher is the accountability of the politician (\( a \)), the less is the impact of lobbying, and so the closer is the equilibrium tariff to the social optimum, i.e. free trade. The more populous is the lobby (higher \( \alpha_L \)), the more it suffers from the deadweight loss of protection and so the less willing it is to pay for protection. Conversely, the more concentrated the lobby is (lower \( \alpha_L \)), the less it’s concerned about deadweight losses, and the more it bids for protection. The higher is the size of the industry \( y_j \), the more contributions are paid, giving more incentives for the politician to provide protection. Finally, the slope of import demand in the denominator reminds that 2.16 is just a modified Ramsey rule.

One problem with this setup is that although not all industries are well organized in practice, we observe positive protection for nearly all industries, including clearly not organized ones. The model predicts negative protection for these industries. Other problem is, contrary to the model predictions, we observe sizeable positive protection of some industries even in democratic countries where government officials are highly accountable to the public. Our extension of the model addresses these issues and demonstrates how the observed patterns of protection may exist in the equilibrium.

### 2.8 Equilibrium trade policy with unemployment

Now follow the same steps for the model with unemployment.

First, substituting 2.3 and 2.7 in 2.8 and differentiating the result we obtain

\[
\frac{\partial \bar{W}_i(p)}{\partial p_j} = \delta_{ij} - \alpha_i q_j(p_j) y_j(p_j) + \alpha_i (p_j - 1)m_j'(p_j) + \alpha_i q_j'(p_j) L_j(p_j) + \alpha_i q_j(p_j) L_j'(p_j) + \delta_{ij} q_j'(p_j) \pi_j(p_j)
\]

(2.17)

where \( q_j(p_j) = F_c(\pi_j(p_j)) \) measures the survival chances of the industry \( j \) that depend on its profits, and through it - on the price \( p_j \) that is affected by protection, \( q_j'(p_j) = f_c(\pi_j(p_j)) y_j(p_j) \) is the effect of a
small increase in protection of industry \( j \) on its survival chances, \( q_j'(p_j)L_j(p_j) = \frac{\partial \pi_j(p)}{\partial p_j} \) is the effect of a small increase in protection of industry \( j \) on the employment rate in the whole economy due to higher survival chances of the industry.

The term \( q_j(p_j)L_j'(p_j) \) measures an increase in employment in the whole economy because of higher labor demand in the industry. This term is relevant as long as higher labor demand helps decrease unemployment, i.e. as long as if \( q_i(p_i) < 1 \) in some industry (not necessarily in industry \( j \)), and it would disappear in case of full employment.

Summing up 2.17 for all organised industries we get

\[
\sum_{i \in L} \frac{\partial \bar{W}_i(p)}{\partial p_j} = (I_j - \alpha_L)q_j(p_j)y_j(p_j) + \alpha_L(p_j - 1)m_j'(p_j) + \alpha_Lq_j'(p_j)L_j(p_j) + \alpha_Lq_j(p_j)L_j'(p_j) + L_jq_j'(p_j)\pi_j(p_j)
\]

(2.18)

Substitution of 2.7 into 2.9 gives

\[
\frac{\partial \bar{W}(p)}{\partial p_j} = (p_j - 1)m_j'(p_j) + q_j'(p_j)L_j(p_j) + q_j(p_j)L_j'(p_j) + q_j'(p_j)\pi_j(p_j)
\]

(2.19)

Plug 2.18 and 2.19 into the equilibrium condition 2.12 to obtain the main result:

\[
\bar{t}_j^e = I_j - \alpha_L \left( \frac{q_jy_j + q_j'\pi_j}{m_j'} \right) + q_j' \frac{L_j + \pi_j}{-m_j'} + q_j \frac{L_j'}{-m_j'}
\]

(2.20)

There are two main differences between 2.16 and 2.20.

First, consider the term \( q_jy_j + q_j'\pi_j \). The equilibrium is characterised by locally truthful contribution schedules, i.e. the lobbies are willing to pay exactly additional income they receive. In GH94 it’s simply \( \pi_j'(p_j) = y_j(p_j) \). In our model, lobbies pay for an increase in expected profits, \( q_j\pi_j \), and this increase is exactly \( (q_j(p_j)\pi_j(p_j))' = q_jy_j + q_j'\pi_j \). The intuition behind this sum is that lobbies are willing to pay not only for an increase in profits \( \pi_j \) but also for higher chances to survive. The higher is the profits (higher \( \pi_j \)), or the more efficient is help (higher \( q_j' \)), the more they will pay for raising their chances. But the lower are their chances (lower \( q_j \) the less they will pay for an increase in profits (\( \pi_j' = y_j \)), i.e. the basic political motive for protection (higher profits) becomes weaker.

Second, the optimal tariff even in absence of lobbying is positive and equals \( q_j \frac{L_j + \pi_j}{-m_j} + q_j \frac{L_j'}{-m_j} > 0 \). The sum \( (L_j + \pi_j) \) is the total income generated by industry \( j \): labor income plus profits (wage equals 1). If an industry does not survive the crisis, this income is lost. If the government can help an industry to survive \( (q_j' > 0) \), it should provide protection. The more jobs are created in the industry (higher \( L_j \)), the more profitable it is (higher \( \pi_j \)), the higher protection should be provided (not infinite protection because the costs are DWL). Additionally, note that organized groups lobby only for profits, not for employment, that is why we don’t have \( L \) in the ‘political’ part of the formula6.

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6This is the result of an assumption that job distribution does not depend on distribution of capital ownership, i.e. when a person chooses a job he/she treats equally all industries, no matter whether he/she owns specific capital in some particular industry. If people tend to work in industries where they own capital, they would take this into account when lobbying for policy. In the extreme case when all capital owners work in their industries, the term \( L_j \) appears in the numerator of the first term: \( q_jy_j + q_j'\pi_j + L_j \).
Now consider the impact of recession on protection.

At normal times $q_j(p_j)$ is close to 1 and $q_j'(p_j)$ is near 0. In recession the risk of bankruptcy $q_j(p_j)$ goes up, and help $q_j'(p_j)$ starts to matter more. So, first, lobbies pay less for an increase in profits ($q_j y_j$ falls) but they are ready to pay for their resque ($q_j'(p_j)$ goes up). Second, the whole society demands protection ($q_j (L_j + \pi_j)$ goes up). In general, the change in protection is uncertain, because there are forces that increase and decrease it. But it seems reasonable that motives to save business ($q_j' \pi_j$), incomes and jobs ($q_j' (L_j + \pi_j)$) are very likely to overweigh the fall in willingness to pay for higher profits ($q_j y_j$).

The last term in 2.20, $q_j \frac{L_j}{\pi_j}$, is relevant only if unemployment is positive, i.e. $q_i(p_i) < 1$ for some industry $i$. Interestingly, there is a discontinuity here: even a minor risk in the economy shifts trade barriers up by some positive number, and for a particular industry $j$ this shift depends on its ability to create jobs. Surprisingly, it depends negatively on the extent of crisis in this particular industry. The reason is, this additional protection is provided not to help a particular industry survive, but rather to create more jobs, and for this purpose the strongest industries are the best.

2.9 When the response to recession is stronger: with or without lobbying?

The comparison is determined by the first parts of expressions 2.16 and 2.20: the impact of recession is higher higher with lobbying if and only if

$$q_j y_j + q_j' \pi_j > y_j$$

or, taking into account that $q_j(p_j) = F(\pi_j(p_j))$, $q_j'(p_j) = y_j(p_j)f(\pi_j)$ since $\pi_j'(p_j) = y_j(p_j)$, this condition is equivalent to

$$h(\pi_j)\pi_j > 1$$

where $h(\pi_j) = \frac{f(\pi_j)}{1-F(\pi_j)}$ is the hazard rate of crisis failures in the industry. Hazard rate here measures the effectiveness of protection as an instrument to help firms survive. Intuitively, if the shock hits firms on the edge of bankruptsy (high $h(\pi_j)$) and if stakes are sufficiently high (high $\pi_j$), then firms are willing to pay much money for a small increase in protection.

2.10 Hump-shaped protection

Protection is hump-shaped when (1) DWL from protection is close to zero for small protection and grows much when protection is high (quadratic DWL satisfies this), (2) $q_j = y_j(p_j)f(\pi_j)$ is hump-shaped if $f(\pi_j)$ is hump shaped enough (which is likely to be true for lognormal or Frechet distribution).

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7A possible analogy with health or labor economics may be the following. $1 - F(\pi_j)$ is the share of firms which will not survive, and $f(\pi_j)$ measures how many of them will suffer shock close to their profits ($c = \pi_j$) and so may die just on the edge or be saved at low cost. Hazard rate $h(\pi_j) = \frac{f(\pi_j)}{1-F(\pi_j)}$ tells which share of firms under risk can be resqued cheaply.

8I don’t have formal proof of hump-shaped protection yet, but simulations show this pattern is likely to be true.
Does the policy allow recessions to sweep away inefficient firms?

In previous chapter we considered a case when total costs of a recession for the economy are exogenous. In our setup it means that costs $c$ are either inevitable (firm pays them even if it goes bankrupt) or repayable (firm will get $c$ back after the crisis\(^9\)). In such a setup bankruptcy of any firm is clearly undesirable for the economy, the crisis has disadvantages only, it creates only losses of employment and income, and the policy should be aimed at reducing the scope of crisis (though taking into account DWL of protection). But there is a point of view according to which a recession may have also good consequences: it sweeps away inefficient firms, or makes firms to close inefficient branches or product lines, that increase average productivity of the economy.

To model this I assume that when a firm liquidates it does not have to pay the costs $c_i$. So, if for some firm $i$ costs of are higher than profits $c_i > \pi_i(p_i)$, then bankruptcy is efficient for that firm because it saves money for its owner. Higher protection here may cause additional losses because the firms who survive pay their shocks.

However, efficient protection is not zero even in this setup, because not all bankruptcies that take place are efficient for the society. The condition $c_i > \pi_i$ is not enough to guarantee that a bankruptcy is efficient for the society, because it does not take into account losses for workers dismissed from the firm. A bankruptcy is efficient for the society only if $c_i > \pi_i + L_i$. So, liquidation of firms with $\pi_i < c_i < \pi_i + L_i$ is efficient for their owners but imposes negative externality on the society.

Therefore there is still a space for protection - to save employment, but it is quite limited because protection has addidional deadweight losses - higher costs.

Formally, to account for efficient bankruptcy we should find the expected costs of crisis as a conditional expectation $\mathbb{E}[c|c > \pi_i(p_i)] = 0 \times \text{prob}[c > \pi_i(p_i)] + E[c|c < \pi_i(p_i)] \times \text{prob}[c < \pi_i(p_i)] = \int_0^{\pi_i(p_i)} cf(c) dc$

The (expected) welfare of capital owners in industry $i$ now becomes

$$\bar{W}_i(p) = \alpha_iq_i(p_i) + q_i(p_i)\pi_i(p_i) - \pi_i(p_i) + \alpha_i(\bar{R}(p) + S(p))$$

The (expected) social welfare is

$$\bar{W}(p) = \bar{q}(p) + \sum_{i=1}^{n} q_i(p_i)\pi_i(p_i) - \sum_{i=1}^{n} \pi_i(p_i) + \bar{R}(p) + S(p)$$  \hspace{1cm} (3.1)$$

The derivatives are

$$\sum_{i \in I} \frac{\partial \bar{W}_i(p)}{\partial p_j} = (I_j - \alpha_L)q_j(p_j)y_j(p_j) + \alpha_L(p_j - 1)m_j'(p_j) + \alpha_Lq_j'(p_j)L_j(p_j) + \alpha_Lq_j(p_j)L_j'(p_j) + I_jq_j'(p_j)p_j + I_jq_j'(p_j)\pi_j(p_j)$$  \hspace{1cm} (3.2)$$

\(^9\)i.e. $c$ is not cost, but liquidity necessary to survive the hard times. For example, it is cash to finance working capital, which can be easily borrowed in good times, but not when loans are less available.
\[
\frac{\partial \tilde{W}(p)}{\partial p} = (p_j - 1)m_j'(p_j) + q_j'(p_j)L_j(p_j) + q_j(p_j)L_j'(p_j) + q_j(p_j)\pi_j(p_j) - c_j'(\pi_j(p_j)) \tag{3.3}
\]

The equilibrium tariff is

\[
\tilde{t}_j = \frac{I_j - \alpha_L}{a + \alpha_L} \left( \frac{q_j y_j + q_j' \pi_j - c_j'}{-m_j'} \right) + \frac{q_j'L_j + q_j L'_j - c_j'}{-m_j'} \tag{3.4}
\]

It follows from 3.4 that equilibrium trade policy reflects the ideas discussed above: both efficient tariff \( \frac{q_j'L_j + q_j' \pi_j - c_j'}{-m_j'} \) and the bias due to lobbying \( \frac{I_j - \alpha_L}{a + \alpha_L} \left( \frac{q_j y_j + q_j' \pi_j - c_j'}{-m_j'} \right) \) are lower to the extent to which protection increases the average costs of crisis.

Now calculate \( c_j'('\pi_j(p_j)') \). Surprisingly, \( q_j' \pi_j \) and \( c_j' \) cancel each other. Indeed, \( c_j'('\pi_j(p_j)') = \left( \frac{\pi_i(p_i)}{\int_0^f c(f)df} \right)' = \pi_i(p_i)f(\pi_i(p_i)) \) and \( q_j' \pi_i = f(\pi_i(p_i))\pi_i(p_i) \), because \( q_i(\pi_i(p_i)) = F(\pi_i(p_i)) \). The intuition is: additional costs are the costs of the firms at the margin of bankruptcy: they got the shock close to their profits \( \pi_j \), and the measure of firms rescued by the policy is \( q_j' \), then we just multiply those numbers to calculate the total increase in the social costs.

So, the final formula is simpler than before

\[
\tilde{t}_j = \frac{I_j - \alpha_L}{a + \alpha_L} \left( \frac{q_j y_j + q_j' \pi_j - c_j'}{-m_j'} \right) + \frac{q_j'L_j + q_j L'_j - c_j'}{-m_j'} \tag{3.5}
\]

Now assume that a share \( \beta \) of costs \( c \) is returned to firms just after the shock. Recall, the sources of the costs may be very different, and some of them are likely to be not additional sunk costs, but only requirements for cash to have in the pocket. E.g. if a crisis makes firms to finance working capital from their own cash flow (not to borrow), then additional costs in the end are much less than the whole working capital required. Equivalently, we may assume that all costs return after some time, and \( \beta \) shows how they are discounted (the longer is the time, and the higher is discount rate, the lower is \( \beta \)).

In this case, the expression for tariff is

\[
\tilde{t}_j = \frac{I_j - \alpha_L}{a + \alpha_L} \left( \frac{q_j y_j + \beta q_j' \pi_j}{-m_j'} \right) + \frac{q_j'L_j + q_j L'_j + \beta q_j' \pi_j}{-m_j'} \tag{3.6}
\]

Therefore, the motive to save business is taken into account only to the extent to which the business is efficient, i.e. to which the costs are not sunk. If an industry can return larger share of the costs in shorter time, it will get more support. Without lobbying it’s true because such a policy is efficient in the sense that rescuing only efficient firms maximizes social welfare. With lobbying it’s also true because inefficient firms get less from protection and so are less willing to pay for protection.

### 3.1 Numerical example
Here I provide an oversimplified example to emphasize the main intuition of the above.

A firm generates added value $R = 100$ and pays wage $W = 60$. So the profits are $\pi = 40$. The social surplus from this firm is equal to $SW = W + \pi = R = 100$.

The firm can be hit by a shock $C \geq 0$ and survives if and only if $C \leq 40$.

In case 1 the shock is $C_1 = 10$, so the firm survives, and the social welfare is $SW = 100 - 10 = 90$.

In case 2 the shock is $C_2 = 50$, the firm is liquidated: it does not produce and does not pay $C_2$, social welfare is $SW = 0$.

In case 3 the shock is $C_2 = 150$, the firm is liquidated again and the social welfare is $SW = 0$.

Assume there is no lobbying so that policymaker maximizes social welfare. The policymaker observes $C$ and provides support to the firm: gives it a subsidy $S$ that is financed by a non-distortary lump-sum tax. What is the optimal support in each of the cases above?

In case 1 the support is not needed: the firm survives anyway, and any subsidy just redistributes money to the firm without any change in social welfare.

In case 2 the support $S \geq 10$ would rescue the firm and the social welfare would be $SW = 100 - 50 = 50 > 0$, so the protection helps improve social welfare in this case.

In case 3 the support $S \geq 110$ would not rescue the firm but the social welfare would be $SW = 100 - 150 = -50 < 0$, so the protection makes the society worse off.

The optimal support as a function of the shock is $S(C) = \begin{cases} 
0, & C \leq 40 \\
C - 40, & 40 < C \leq 100 \\
0, & C > 100 
\end{cases}$

This is an example of how we can get a hump-shaped protection in the most simple way. Also, the only motive for protection is to save jobs because the only problem is that the firm cannot use the wage to pay the shock.
To make the same modification as in the chapter about inefficient firms, assume that a fraction $\beta = 0.2$ of the shock does not represent social loss (it is a pure redistribution to, e.g. banks that charge higher interest, or the firm will get money back in future).

Then it’s optimal to provide protection up to the point when $(1 - \beta)C = R$, i.e. $C = \frac{R}{1 - \beta} = 125$. Now there is an additional motive to save the firm itself, not only employment.

When $\beta$ goes up to 1, the maximum support approaches infinity. This does not happen in our model at least because the say to give support is not distortionary. The pattern of protection could become smoother because in our model the decision about it is made before the shock is realized. With lobbying, the policymaker overestimates benefits to the firm so that he could potentially provide higher protection than without lobbying.

4 Trade wars and trade talks with unemployment

Here I make a similar modification of a 2-country model of Grossman, Helpman (1995) and show how interaction between trade partners changes during crises. First, consider a model of Trade Wars, in which countries choose tariffs non-cooperatively and simultaneously, maximizing the expected value function like 2.10.

The equilibrium tariff rates in the basic model (GH95) are

$$
\tau^*_j - 1 = \frac{I_j}{a + \alpha_L} \left( \frac{y_j}{-m_j^j \rho_j} \right) + \frac{1}{e_j^*} \tag{4.1}
$$

$$
\tau^{*a}_j - 1 = \frac{I^*_j - \alpha^*_L}{a^* + \alpha^*_L} \left( \frac{y^*_j}{-m^*_j \rho_j} \right) + \frac{1}{e_j} \tag{4.2}
$$

where * denotes the Foreign country in all equations, $\rho_j$ is the equilibrium world price of good $j$, tariffs are proportional: domestic price in home country is $\tau^*_j \rho_j$, and in foreign country is $\tau^{*a}_j \rho_j$.

The only difference between these tariffs with tariffs in small open economies is the inverse foreign export elasticities on the right. These elasticities express the result that a country with monopsonic market power should impose positive tariffs in order to balance DWL from tariff and gains from lower world price. The optimal tariff is then the inverse of the foreign export supply elasticity. So, here the tariff is the result of both political motives and terms-of-trade motives.

The corresponding equations in model with unemployment, which is GH95 modified in the same way as GH94 (for simplicity I consider the first case, i.e. without efficient/inefficient firms) are

$$
\tau^*_j - 1 = \frac{I_j - \alpha_L}{a + \alpha_L} \left( \frac{q_j \pi_j + q_j y_j}{-m_j^j \rho_j} \right) + \frac{1}{e_j} \tag{4.3}
$$

$$
\tau^{*a}_j - 1 = \frac{I^*_j - \alpha^*_L}{a^* + \alpha^*_L} \left( \frac{q^*_j \pi^*_j + q^*_j y^*_j}{-m^*_j \rho_j} \right) + \frac{1}{e_j} \tag{4.4}
$$
The difference with the basic model is the same as before. Political support part is changed to allow for less motives to pay for higher profits \((q'_j \pi_j)\) and additional motives to pay for rescue from the bankruptcy threat \((q_j y_j)\). The last term \(q'_j L_j + q'_j \pi_j + q_j L'_j\) is the same as before (taking into account new notation) and captures motives of a benevolent government to protect employment and incomes.

Another way to study a multi-country interactions is to consider *Trade Talks* - a cooperative game of governments.

The formula to the equilibrium difference in tariffs in the basic GH95 model is

\[
\tau^*_j - \tau^*_o = \frac{I_j - \alpha_L}{a + \alpha_L} \left( \frac{y_j}{-m'_j \rho_j} \right) - \frac{I^*_j - \alpha^*_L}{a^* + \alpha^*_L} \left( \frac{y'_j}{-m'_j \rho_j} \right) \tag{4.5}
\]

Export elasticities disappeared: the terms-of-trade were present only because players didn’t take into account DWL they imposed on trade partners.

The corresponding expression in our model is

\[
\tau^*_o - \tau^*_o = \frac{I_j - \alpha_L}{a + \alpha_L} \left( \frac{q'_j \pi_j + q_j y_j}{-m'_j \rho_j} \right) + \frac{q'_j L_j + q'_j \pi_j + q_j L'_j}{-m'_j \rho_j} - \frac{I^*_j - \alpha^*_L}{a^* + \alpha^*_L} \left( \frac{q'_j ^* \pi_j^* + q'_j ^* y_j^*}{-m'_j \rho_j} \right) - \frac{q'_j ^* L_j^* + q'_j ^* \pi_j^* + q_j ^* L'_j}{-m'_j \rho_j} \tag{4.6}
\]

\[
\tau^*_o - \tau^*_o = \frac{I_j - \alpha_L}{a + \alpha_L} \left( \frac{q'_j \pi_j + q_j y_j}{-m'_j \rho_j} \right) \tag{4.7}
\]

The results are the same as before.

### 5 Estimation ideas

The most straightforward approach to estimation is to follow Goldberg and Maggi (1999).

They estimate the formula for tariff from GH94:

\[
t^*_j = \frac{I_j - \alpha_L}{a + \alpha_L} \left( \frac{y_j}{-m'_j} \right)
\]

that can be equivalently written as

\[
\frac{t^*_j}{1 + t^*_j} e_j = \frac{I_j - \alpha_L}{a + \alpha_L} z_j = \frac{1}{a + \alpha_L} I_j z_j + \frac{-\alpha_L}{a + \alpha_L} z_j
\]

where \(e_j\) is import elasticity with respect to price, \(z_j = \frac{y_j}{m'_j}\) is import penetration. They estimate the equation

\[
\frac{t^*_j}{1 + t^*_j} e_j = \delta I_j z_j + \gamma z_j + \epsilon_j
\]

where \(\epsilon_j\) is an error. Import elasticities are moved to the left hand side because they are estimated with large error, and this helps to avoid bias from observation errors in independent variables.
The problems with this approach are standard. Import penetration is endogenous: even in this simple model it is a function of the tariff. Import elasticities and the dummy for 'whether an industry is organized or not' can also be endogenous. For instance, if we calculate the dummy as a function of observed contributions (as it is done in Goldberg and Maggi 1999), it will be determined simultaneously with the tariff rate.

Keeping in mind these problems, we proceed to the model with unemployment. The corresponding equation for the tariff rate

\[
t^o_j = \frac{I_j - \alpha L_j}{a + \alpha L_j} \left( q_j^o y_j + \beta q_j^o \pi_j \right) + \frac{q_j^L L_j + q_j^L' L_j' + \beta q_j^o \pi_j}{-m_j^o}
\]

written in the fascion of Goldberg and Maggi (1999) is

\[
\frac{t^o_j}{1 + t^o_j} e_j = \frac{I_j - \alpha L_j}{a + \alpha L_j} \left( q_j^o y_j + \beta q_j^o \pi_j \right) + \frac{q_j^L L_j + q_j^L' L_j' + \beta q_j^o \pi_j}{m_j^o}
\]

5.1 Approach 1

Assume for a moment that we can estimate the function \(q_j(\cdot)\) and \(L(\cdot)\) separately so that \(q_j\) and \(q_j'\) as well as \(L_j\) are known when we are estimating this regression. Then we express it in the following form

\[
\frac{t^o_j}{1 + t^o_j} e_j - \frac{(q_j L_j)^o}{m_j} = \frac{1}{a + \alpha L_j} I_j q_j y_j + \frac{-\alpha L_j}{a + \alpha L_j} q_j^o y_j + \frac{\beta}{a + \alpha L_j} q_j^o \pi_j + \frac{-\beta \alpha L_j q_j^o \pi_j}{a + \alpha L_j m_j}
\]

and then estimate a regression

\[
\frac{t^o_j}{1 + t^o_j} e_j - \frac{(q_j L_j)^o}{m_j} = \delta I_j q_j y_j + \gamma q_j^o y_j + \eta I_j q_j^o \pi_j + \frac{\delta \alpha L_j q_j^o \pi_j}{m_j} + \frac{\delta \beta \alpha L_j q_j^o \pi_j}{m_j} + \epsilon_j
\]

with the restriction \(\delta/\gamma = \eta/\theta\)

5.2 Approach 2

Now what if we cannot estimate \(q_j(\cdot)\) up front? Then we could try to estimate it simultaneously with all the rest. Recall that \(q_j(p) = F(\pi_j(p))\), \(q_j'(p) = F'(\pi_j(p))y(p)\), where \(F\) is the cdf of the shock. Set a functional form for \(F\) to be \(F = F(x, \psi)\), where \(\psi\) is a vector of unknown parameters. Then we could estimate a nonlinear equation

\[
\frac{t^o_j}{1 + t^o_j} e_j = \frac{1}{a + \alpha L_j} I_j F(\pi_j, \psi) y_j + \frac{-\alpha L_j}{a + \alpha L_j} F(\pi_j, \psi) y_j + \frac{\beta}{a + \alpha L_j} I_j F'(\pi_j, \psi) \pi_j + \frac{-\beta \alpha L_j F'(\pi_j, \psi) \pi_j}{m_j}
\]

\[+ \frac{\beta \alpha L_j}{a + \alpha L_j} F'(\pi_j, \psi) \pi_j + \frac{\beta \alpha L_j}{a + \alpha L_j} F'(\pi_j, \psi) L_j + \frac{\beta \alpha L_j}{a + \alpha L_j} F'(\pi_j, \psi) L_j' + \epsilon_j
\]

\(^{10}\)Estimation of survival brobability is discussed below. Estimation of the labor demand is not but it should also be possible
To have exact identification we could set the dimension of $\psi$ to be 3, e.g. consider $F$ to be a cdf of Frechet distribution (its support is positive real numbers and it is skewed to the right that is common for financial variables).

5.3 Approach 3

We could also not estimate $L(\cdot)$ up front but do everything at once. We observe $L_j$, so the only problem is we don’t know $L_j' = \frac{\partial L_j}{\partial p_j}$. Suppose the production function in industry $j$ is $A_j L_j^{\alpha}$. So, elasticities wrt labor are the same across industries but productivity parameters $A_j$ are heterogenous. Then the labor demand would be $L_j = \left( \frac{m_j A_j}{w} \right)^{\frac{1}{1+\alpha}}$, where the wage is assumed to be equal to 1 throughout the whole paper and employment is equal to labor demand. Then $L_j' = \frac{\partial L_j}{\partial p_j} = \frac{L_j}{p_j} = \frac{L_j}{1+\alpha}$. Plugging this into the above equation and solving for $t_j^o$ gives

$$t_j^o = \frac{1}{a+\alpha L} I_j F(\pi_j, \psi) z_j + \frac{-\alpha L}{a+\alpha L} F(\pi_j, \psi) z_j + \frac{\beta}{a+\alpha L} I_j F'(\pi_j, \psi) \pi_j + \frac{\beta L_j}{a+\alpha L} F'(\pi_j, \psi) \pi_j + \frac{F'(\pi_j, \psi) L_j}{m_j} + L_j$$

that is a highly nonlinear equation. Therefore it may be more reasonable to assume some reduced form linear demand for labor $L_j(p_j) = d + bp_j$, and estimate its derivative $b$, i.e. assuming $L_j'$ is constant that is the same for all industries. Note that we are going to plug only the derivative $b$ into the main equation in place of $L_j'$ (we plug the observed $L_j$ not $a + bp_j$). Then we would have

$$t_j^o + \frac{1}{1 + t_j^o} e_j = \frac{1}{a+\alpha L} I_j F(\pi_j, \psi) z_j + \frac{-\alpha L}{a+\alpha L} F(\pi_j, \psi) z_j + \frac{\beta}{a+\alpha L} I_j F'(\pi_j, \psi) \pi_j + \frac{\beta L_j}{a+\alpha L} F'(\pi_j, \psi) \pi_j + \frac{F'(\pi_j, \psi) L_j}{m_j} + F'(\pi_j, \psi) b_i + \frac{F'(\pi_j, \psi) L_j}{m_j}$$

and need to parameterize $F$ with 2-dimensional $\psi$. For instance, we can take $F$ to be cdf of lognormal distribution, that also proved to work well for financial data.

5.4 Estimating survival probability

We can refer to vast literature on predicting bankruptcy to estimate survival probability $q_j$. Classic references are Altman (1968), Altman (1977), Ohlson (1980), and Zmijewski (1984).

In our model $q_j (p_j)$ is exactly the predicted survival probability that could be a noisy and biased estimate but is the estimate used by the agents making decisions. So the idea behind this estimation is to take the approach that the most well-known and widely used in industry and by policymakers. This way we will get $q_j$ as a function of the observables used in the corresponding models. We can then evaluate it to get a number for each industry and plug it into our regression equation. The models are for a particular companies, so we could use data from Compustat and/or Bloomberg to estimate it for many companies and then take average or median for each industry.
The next step is to estimate \( q_j = \frac{\partial q_j}{\partial p_j} \). One approach is just to evaluate \( q_j \) for several periods of time and then regress it onto \( p_j \) or our measure of tariff \( t_j = p_j - 1 \). Another approach is to recall that \( q_j^* = F_j^*(\pi_j(p_j)y(p_j) = F_j^*y_j \) and estimate \( F_j^* \). This may be easier because the models used to predict bankruptcy frequently use profits as one of the observables (so we can take the derivative exactly) or otherwise we can regress \( q_j \) onto \( \pi_j \) on which we have better data than on \( p_j \) or on a proxy for \( t_j \).

We should also account for the fact that bankruptcy does not always mean liquidation of the firm (for now I don’t know how to address this).

5.5 Gravity model with unemployment

A very different, more reduced form approach is to estimate an adjusted gravity model of trade with unemployment.

Gravity model with additional variables is used widely for the purposes of very different research (Frankel, Romer 1999, Rose 2004, Head, Mayer, Ries 2010, Yu 2010).

The basic model specification is

\[
\log(\text{Im}_{odt}) = \beta_0 + \beta_1 \log(\text{GDP}_{ot}) + \beta_2 \log(\text{GDP}_{dt}) - \beta_3 \log(\text{dist}_{od}) + X_{odt}' \gamma + \varepsilon_{odt}, \tag{5.1}
\]

where

- \( \text{Im}_{odt} \) is the value of import from country \( o \) (Origin) to country \( d \) (Destination) in the year \( t \)
- \( \text{GDP}_{ot} \) and \( \text{GDP}_{dt} \) are respectively, nominal gross domestic product of Origin and Destination in the year \( t \)
- \( \text{dist}_{od} \) is the distance between countries
- \( X_{odt}' \) is the set of controls including dummies on the common border, the common language, regional trade agreements and colonial history, log gdp per capita in Origin and Destination.

I include 2 new variables and estimate the following model

\[
\log(\text{Im}_{odt}) = \beta_0 + \beta_1 \log(\text{GDP}_{ot}) + \beta_2 \log(\text{GDP}_{dt}) - \beta_3 \log(\text{dist}_{od}) + \beta_{uo} u_{ot} + \beta_{ud} u_{dt} + X_{odt}' \gamma + \varepsilon_{odt} \tag{5.2}
\]

where \( u_{ot} \) and \( u_{dt} \) are the rates of unemployment in the origin and destination countries, respectively.

The expected signs are:

\[
\begin{align*}
\beta_{uo} &> 0 \\
\beta_{ud} &< 0
\end{align*} \tag{5.3}
\]

The negative coefficient on the unemployment in the destination country comes directly from higher protection of domestic industries from competition with import. The positive coefficient on the unemployment in the origin country comes from the support given to exporting industries to help them hire the unemployed people. The second coefficient will of course capture also lower wages (positive bias: export goes up because
costs go down) and various problems in the origin that caused recession, e.g. financial frictions (negative bias: export firms go bankrupt and export falls down).

The data could be taken from UN comtrade, World Bank World Development Indicators, and a publicly available dataset of, e.g. Rose 2004 (dummies for common borders, common language, trade agreements). To update the data on trade agreements we could use WTO website.