Online Appendix to
"Financing from Family and Friends"

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I. Additional proofs

Proof of Proposition 1. Writing out the utility functions in the constraints, the optimization problem reduces to

$$\begin{aligned} \underset{I_O, I_F, R_O, R_F}{\text{maximize}} \quad & qu(I_{ext} - I + R - R_O - R_F) + (1-q)u(I_{ext} - I) \\ s.t. \quad & R - B - R_O - R_F \geq 0 && (12) \\ & qR_F - I_F \geq 0 && (13) \\ & qR_O - I_O \geq 0 && (14) \\ & I_{ext} - I \geq 0 && (15) \end{aligned}$$

We omit the limited liability condition, as it is implied by the incentive compatibility constraint (12).

The objective function is strictly decreasing in both R_F and R_O, making it optimal to set R_F and R_O such that the participation constraints (13) and (14) are binding: $R_i = {}^{I_i}/_q$ for $i = F, O$. This reduces the problem to choosing I_F and I_O. Substituting $R_i = {}^{I_i}/_q$ into the objective function and the incentive compatibility constraint yields, respectively,

$$qu(I_{ext} - I + R - {}^{I_{ext}}/_q) + (1-q)u(I_{ext} - I)$$

and

$$R - B - {}^{I_{ext}}/_q \geq 0.$$

Thus, the choice between the two financing sources is irrelevant, as only I_{ext} matters. The latter expression for the incentive compatibility constraint is compatible with the funding constraint (15) if and only if $R - B - {}^{I}/_q \geq 0$, which yields the capital constraint (1).

To derive the risk constraint (2), note that the incentive compatibility constraint can equivalently be written as $R_E \geq B$. The amount of project risk transferred to investors is maximized for $R_E = R - {}^{I_{ext}}/_q$ as small as possible, i.e., $R_E = B$. Now distinguish two cases:

First, if $qR - I \geq \underline{x}$, an efficient risk allocation implies that E's lower bound on consumption is weakly higher than $\underline{x}$. For $R_E = B$, E's lowest consumption, i.e., her consumption if the project fails is $I_{ext} - I = q(R - B) - I$. This is weakly larger than $\underline{x}$ if and only if $B \leq R - {}^{\underline{x}}/_q - {}^{I}/_q$.

Second, if $qR - I < \underline{x}$, an efficient risk allocation implies that E's upper bound on consumption is weakly lower than $\underline{x}$. For $R_E = B$, E's highest consumption, i.e., her consumption if the project succeeds is $I_{ext} - I + R - B = qR - I + (1-q)B$. This is weakly smaller than $\underline{x}$ if and only if $B \leq \frac{\underline{x}-(qR-I)}{1-q}$. □

Proof of Lemma 3. We first show that family finance increases the range of B for which the project is feasible. Suppose the capital constraint (1) is violated, in which case $E(\overline{u}_E) = u(0) = 0$. To examine feasibility, suppose further that E uses the minimum amount of external financing: $I_O + I_F = I$. F's binding participation constraint is then

$$(1-\phi)\, E[u(W - I_F + \tilde{R}_F)] + \phi E[u(\tilde{R} - \tilde{R}_{ext})] = (1-\phi)u(W) + \phi u(0).$$

Note that F is never needy under any optimal contract (since he would not be needy even if he *donated* the entire amount I to E). Thus, $u'(x_F^s) = m$ for $s = h, l$. Using this and $R_O = {}^{I_O}\!/_{q}$, we can rearrange the above equality to

$$qR_F = I_F - \underbrace{\frac{\phi q u(R - I/q)}{(1-2\phi)\, m}}_{I_F^d}. \tag{16}$$

Notice that $R_F < 0$ for $I_F < I_F^d$. This means that F is willing to contribute up to I_F^d as a *donation*, which – strictly optimal for E to accept – defines a lower bound on "financing" from E.

Next consider E's incentive compatibility constraint, which for $I_O + I_F = I$ yields

$$(1-\phi)\, E[u(\tilde{R} - \tilde{R}_{ext})] + \phi E[u(W - I_F + \tilde{R}_F)] \geq (1-\phi)\, E[u(\tilde{B})] + \phi u(W - I_F)$$

Given our specification of project returns, payoffs are the same whether the project fails after working or shirking. Since this occurs with probability $1-q$ in either case, the payoffs conditional on "failure" conveniently cancel out of the incentive compatibility constraint. The constraint reduces to the requirement that, conditional on "success," working gives E a higher payoff than shirking does:

$$(1-\phi)\, u(R - R_{ext}) + \phi u(W - I_F + R_F) \geq (1-\phi)\, u(B) + \phi u(W - I_F).$$

For $I_F \leq I_F^d$, in which case $R_F = 0$, this becomes $(1-\phi)\, u(R - (I - I_F)/q) \geq (1-\phi)\, u(B)$, which is laxer for higher I_F. For $I_F > I_F^d$, note from (16) that $R_F = \frac{I_F - I_F^d}{q}$, so $\frac{\partial R_F}{\partial I_F} = \frac{1}{q}$. That is, the repayment on each marginal dollar above I_F^d is *not* priced at a discount. (Intuitively, a below-market rate loan from F can be seen as a donation of I_F^d combined with a loan of

$I_F - I_F^d$ priced at the market interest rate.) Thus, defining $\hat{I}_F = I_F - I_F^d$ and $\hat{R}_F = {}^{\hat{I}_F}/_q$, we can rewrite the incentive compatibility constraint for this case as

$$(1-\phi)\left[u(R - {}^{I_O}/_q - {}^{\hat{I}_F}/_q) - u(B)\right] \geq \phi\left[u(W - I_F^d - \hat{I}_F) - u(W - I_F^d - \hat{I}_F + {}^{\hat{I}_F}/_q)\right].$$

Consider the alternative financing $I'_F = I_F + \epsilon$ and $I'_O = I_O - \epsilon$ with $\epsilon > 0$. This leaves the left-hand side unaffected, but strictly decreases the right-hand side. That is, we can relax the constraint by substituting family finance for outside finance. It is most lax for $\hat{I}_F$ such that ${}^{\hat{I}_F}/_q = R$ (in which case $I_O = 0$). Still, even in this case, it need not be satisfied since the left-hand side strictly decreases in B. These observations imply:

1. There exists a unique I_F^{IC} such that the project is feasible if $I_F \geq I_F^{IC}$ and ${}^{I_F^{IC}}/_q \leq R$.

2. There exists a unique $B^* > \hat{B}$ such that the project is feasible for all $B \leq B^*$.

3. If the project can be financed, $I_F \geq \max\left\{I_F^d, I_F^{IC}\right\}$.

4. Thus, family finance can help overcome the capital constraint.

In the second part of the proof, we show that E strictly prefers to use family finance as long as the risk allocation is inefficient. We prove this by contradiction. Conjecture that the optimum involves an inefficient risk allocation with $I_O > 0$. Given the inefficiency, the incentive compatibility constraint must be binding:

$$(1-\phi)\, u(I_{ext} - I + R - R_{ext}) + \phi u(W - I_F + R_F) = (1-\phi)\, u(I_{ext} - I + B) + \phi u(W - I_F).$$

(We use again the fact that the failure outcomes cancel out.) Now consider the alternative financing $I'_F = I_F + \epsilon$ and $I'_O = I_O - \epsilon$ with $\epsilon > 0$ infinitesimal, and $R'_F = R_F + {}^{\epsilon}/_q$ and $R'_O = R_O - {}^{\epsilon}/_q$. By construction, I_{ext} and R_{ext} remain the same. But the left-hand side of the incentive compatibility constraint increases to

$$(1-\phi)\, u(I_{ext} - I + R - R_{ext}) + \phi u(W - I_F + R_F - \epsilon + {}^{\epsilon}/_q)$$

and the right-hand side decreases to $(1-\phi)\, u(I_{ext} - I + B) + \phi u(W - I_F - \epsilon)$. Thus, the incentive compatibility constraint becomes slack. Meanwhile, E's expected utility remains at $(1-\phi)\, E[u(I_{ext} - I + \tilde{R} - \tilde{R}_{ext})] + \phi E[u(W - I_F + \tilde{R}_F)]$, because both her own and F's expected consumption utilities are unaffected. (E's consumption remains the same in each state, whereas F's consumption remains the same only in expectation but F marginal consumption utility is the same in both states.) Therefore, contrary to the conjecture, the initial financing is not an optimum. Thus, if the project can be financed but an efficient risk allocation cannot be achieved, the entrepreneur uses only family finance.

We conclude this proof by showing that E is indifferent between all financing alternatives with $I_F \geq I_F^d$ that implement an efficient risk allocation. Consider a case where, at the optimum, the risk allocation is efficient, and hence the incentive compatibility constraint is slack (unless the set of optima is a singleton). Since the risk allocation is efficient, it must be that E's consumption is always above or always below $\underline{x}$, so that $u'(x_F^s)$ is constant across cash flow realizations. Her expected utility can therefore be written as

$$E(U_E|I_F, I_O) = (1-\phi)\, u(I_{ext} - I + R - qR_{ext}) + \phi u(W - I_F + qR_F).$$

Consider changes to the financing that leave the investors' participation constraints, which are binding at the optimum, unaffected. To account for changes in both directions, consider an initial financing with $I_F > I_F^d$ and $I_O > 0$, and infinitesimal changes $\epsilon_F, \epsilon_O \gtreqless 0$ such that $I_F' = I_F + \epsilon_F$ and $I_O' = I_O - \epsilon_O$ maintain efficiency of the risk allocation. Now consider the associated changes in prices. Clearly, $R_O' = I_O + {}^{\epsilon_O}/_{q}$. As regards the pricing by F, recall that $R_F = \frac{I_F - I_F^d}{q}$. Thus, $R_F' = \frac{I_F + \epsilon_F - I_F^d}{q}$ and $R_F' - R_F = {}^{\epsilon_F}/_{q}$. Thus, as a result of the change to financing, E's expected utility becomes

$$E(U_E|I_F', I_O') = (1-\phi)\, u(I_{ext} + \epsilon_F + \epsilon_O - I + R - q(R_{ext} + {}^{\epsilon_F}/_{q} + {}^{\epsilon_O}/_{q})) + \\ \phi u(W - I_F - \epsilon_F + q(R_F + {}^{\epsilon_F}/_{q})) = E(U_E|I_F, I_O).$$

Thus, she is indifferent to the change. If the project can be financed efficiently, the lower bound on family finance is therefore $I_F = \max\{I_F^d, I_F^r\}$ where I_F^r is the minimum amount of family finance needed to achieve an efficient risk allocation. $\square$

Proof of Lemma 6. In this proof, we abbreviate the value of social collateral by $k \equiv (a^+ - a^-)\frac{\lambda}{1+\lambda}(W - I)$ so that the gift exchange condition (11) can be written as

$$(1-a^+)\min\{R_F, \widetilde{R}\} + (c - a^+)\max\{R_F - \widetilde{R}, 0\} \leq k.$$

For a given R_F, there are three possible cases: (*i*) If (11) is violated for $\tilde{R} \geq R_F$, then it is also violated for all $\tilde{R} < R^F$ since $(1-a^+)\widetilde{R} + (c-a^+)(R_F - \widetilde{R}) > (1-a^+)R_F$ for $c \geq 1$. In this case, $R^c = \infty$ and (11) is violated for all $\tilde{R} \geq 0$. (*ii*) If (11) is satisfied for $\tilde{R} \geq R_F$, there may exist some $R^c \in (0, R_F)$ such that (11) is violated for $\tilde{R} < R^c$ since $(1-a^+)\widetilde{R} + (c-a^+)(R_F - \widetilde{R})$ is decreasing in $\tilde{R}$. (*iii*) Else, $R^c = 0$ and (11) is satisfied for all $\tilde{R} \geq 0$.

Now consider how R_F affects R^c: First, note that $\lim_{R_F \to 0}(1-a^+)\widetilde{R} + (c-a^+)(R_F - \widetilde{R}) = 0$ and $\lim_{R_F \to 0}(1-a^+)R_F = 0$. Thus, for low enough R_F, (11) is satisfied irrespective of $\tilde{R} \geq 0$. This is case (*iii*). $R^c = 0$ for all R_F for which this case applies. Second, there exist higher R_F

such that $(1-a^+)R_F \leq k$ but $(1-a^+)\widetilde{R} + (c-a^+)(R_F - \widetilde{R}) > k$ for some $\tilde{R}$. This is case (*ii*) where $0 < R^c < \infty$ and R^c can be backed out from the binding gift exchange condition $(1-a^+)R^c + (c-a^+)(R_F - R^c) = k$ as

$$R^c = \frac{(c-a^+)R_F - k}{c-1},$$

which is increasing in R_F for $c > 1$ since Assumption 2 implies $c > a^+$. Finally, for high enough R_F, $(1-a^+)R_F > 0$ also. This is case (*iii*). $R^c = \infty$ for all R_F for which this case applies. □

II. Equivalence of outside finance to family finance cum outside insurance

Suppose both the charitable insurance and constrained charity conditions hold so that pure family finance is strictly suboptimal. To establish the equivalence, we need to restrict attention to parameters under which (i) E never becomes (i.e., is fully "insured" against being) needy under the optimal financing contract ($qR - I \geq \underline{x}$) and (ii) F is wealthy enough to be able to fund the project and pay P to buy insurance for E against low consumption without becoming needy ($W - I - P \geq 0$). These conditions simply ensure that the comparison with insurance is sensible. Suppose F covers the entire investment outlay I and buys an insurance from O that, if the project fails, pays E the amount $2\underline{x}$. The associated insurance premium is $P = (1-q)2\underline{x}$. The family's expected utility under this set of contracts is

$$\begin{aligned} E[U(\tilde{\Pi}_{EF})] &= qU(W - I - P + R) + (1-q)U(W - I - P + 2\underline{x}) \\ &= U(q\,(W - I - P + R) + (1-q)(W - I - P + 2\underline{x})) \\ &= U(W + qR - I - P + (1-q)2\underline{x}) \\ &= U(W + qR - I) \end{aligned}$$

where the first step follows from $W - I - P \geq \underline{x}$, $R > qR - I \geq \underline{x}$, and U being linear for $\Pi_{EF} \geq 2\underline{x}$. That is, if F not only provides selfishly priced financing for the entire project but also charitably purchases insurance for E against "neediness," then the family is as well off as under pure outside finance. Going back to the main analysis in the text, this suggests that the family funds E preserves by using outside finance (instead of family finance) are best viewed as a source of "cheap" insurance. In fact, the above implies that any ex ante donation or "cheap" financing contract that does not also include *ex post state-contingent* transfers to E cannot replicate the optimum.

III. Shadow costs in a model with paternalistic altruism

The standard specification of altruism used in the paper creates an aversion to family finance only when the altruism is strong enough to induce charitable transfers, i.e., familial insurance. Here we explore an alternative specification of altruistic preferences that works through a different mechanism. The specification is

$$U_i = u_i(e_i x_i + (1 - e_i)x_j) \quad \text{for } i \neq j \tag{17}$$

where u_i is a concave function and $e_i \in (1/2, 1]$ again reflects the degree of egoism i exhibits vis-à-vis j. The difference to before is that, here, i internalizes j's consumption rather than j's utility. This gives rise to a form of disagreement that is absent under standard altruism: Let $x_i = \bar{x}_i$ and $x_j \in \{\tilde{x}_1, \tilde{x}_2\}$. Under standard altruism, i and j have the same preferences over the two lotteries $\tilde{x}_1$ and $\tilde{x}_2$ (abstracting from charitable transfers):

$$\begin{array}{rcl} e_i u_i(\bar{x}_i) + (1-e_i)E[u_j(\tilde{x}_1)] & \geq & e_i u_i(\bar{x}_i) + (1-e_i)E[u_j(\tilde{x}_2)] \\ & \Leftrightarrow & \\ (1-e_j)u_i(\bar{x}_i) + e_j E[u_j(\tilde{x}_1)] & \geq & (1-e_j)u_i(\bar{x}_i) + e_j E[u_j(\tilde{x}_2)] \\ & \Leftrightarrow & \\ E[u_j(\tilde{x}_1)] & \geq & E[u_j(\tilde{x}_2)] \end{array}$$

for all e_i, e_j, u_i, u_j, $\bar{\pi}_j$, and $\tilde{x}_j$. That is, a choice i makes on behalf of j is the same choice j would make for herself. By contrast, under preferences of the form (17),

$$\begin{array}{rcl} E\{u_i[e_i\bar{x}_i + (1-e_i)\tilde{x}_1]\} & \geq & E\{u_i[e_i\bar{x}_i + (1-e_i)\tilde{x}_2]\} \\ & \not\Leftrightarrow & \\ E\{u_j[(1-e_j)\bar{x}_i + e_j\tilde{x}_1]\} & \geq & E\{u_j[(1-e_j)\bar{x}_i + e_j\tilde{x}_2]\}. \end{array}$$

These preferences allow for "paternalism": i may make choices on behalf of j that j would not choose for herself. Similarly, i may grant j money only for specific purposes, whereas under non-paternalistic altruism, i would want j to spend the money on whatever j wants to use it for (so long as it is materially irrelevant to i).[30]

We assume that E and F exhibit symmetric paternalistic preferences vis-à-vis each other:

[30] Jacobsson et al. (2007) provide experimental evidence for paternalistic preferences. In their experiments, subjects can donate money or nicotine patches to a smoking diabetes patient. In one experiment, one group can donate only money and another group only patches. It turns out that average donations are 40% greater in the nicotine patches group. Moreover, when subjects can donate both nicotine patches and money, more than 90% of the donations are given in kind rather than cash. Under non-paternalistic preferences, subjects should prefer donating money.

$U_E(x_E, x_F) = u(ex_E + (1-e)x_F)$ and $U_F(x_F, x_E) = u(ex_F + (1-e)x_E)$. Note that $\partial U_i/\partial x_i \geq \partial U_i/\partial x_j$ for $e > 1/2$. Hence, ex post charitable transfers will not take place, which rules out familial insurance mechanisms. This formulation nevertheless creates a preference for outside finance through another mechanism. To show this most concisely, let us compare an unconstrained entrepreneur's expected utility under pure outside finance and pure family finance, assuming that both types of finance are equally priced $R_i = {}^{I_i}/_{q}$. If E sells the entire project cash flow to O, she receives $qR - I$ and her expected utility is

$$u(e(qR - I) + (1 - e)W). \tag{18}$$

If she sells the project to F instead, her expected utility is

$$qu(e(qR - I) + (1 - e)(W - qR + R)) + (1 - q)u(e(qR - I) + (1 - e)(W - qR)). \tag{19}$$

Let us refer to the argument inside E's social utility function (i.e., $ex_E + (1 - e)x_F$) as E's "social payoff." The expected value of E's social payoff is $e(qR - I) + (1 - e)W$, which equals E's social payoff in (18). Thus, if $u(.)$ is linear, (19) is equivalent to (18). However, if $u(.)$ is concave, it follows from Jensen's inequality that (19) is smaller than (18).

The intuition is that A experiences direct disutility from imposing risk on F, i.e., she "worries" about F's welfare. She experiences no such disutility from imposing risk on O, whom she is indifferent towards. Crucially, under paternalistic altruism, A's disutility from risking F's rather than O's wealth is not offset by her utility from repaying F rather than O – as it would be the case under non-paternalistic altruism.

Since the altruism is symmetric, F is also directly averse to any risk in E's consumption through her social utility function. Thus, under paternalistic altruism, family members are averse to risk in each other's consumption. We refer to this as *social risk aversion.* The benefit of outside finance in this setting is to transfer the risk out of the family circle. By contrast, family finance merely moves the risk around inside the family circle and hence represents an imperfect risk transfer.

For this reason, an unconstrained entrepreneur strictly prefers outside finance. However, when the entrepreneur is constrained, paternalistic altruism has the same advantages as standard altruism: it relaxes E's incentive compatibility constraint and lowers F's required return on a family loan. As a result, E may use some family finance, and family finance may be provided at a lower interest rate than outside finance. However, conditional on incentive compatibility, E still prefers outside finance because of social risk aversion.

IV. Financing and social debt when the gift exchange condition is violated

If the gift exchange condition (7) is satisfied, F will not receive any favors so that the relationship will suffer when the project fails. To emphasize this social "bankruptcy cost," we temporarily normalize E's consumption in the absence of the project to Z (rather than 0). After a project failure, the social utilities are then given by

$$\begin{aligned} U_E &= Z + f(a^+, a^-)(W - I_F) \\ U_F &= W - I_F + f(a^-, a^+)Z \end{aligned}$$

where $f(x, y) \equiv \frac{x+\lambda y}{1+\lambda}$. Note that $f(a^-, a^+) < f(a^+, a^-) < a^+$.

At the time of financing, F's participation constraint is therefore

$$q\left[W - I_F + R_F + a^+ (Z + R - R_O - R_F)\right] + (1 - q)\left[W - I_F + f(a^-, a^+)Z\right] \geq W + a^+ Z,$$

and with $R_O = I_O/q$, yields

$$R_F \geq \frac{I_F - a^+(qR - I_O) + (1 - q)\left[a^+ - f(a^-, a^+)\right] Z}{q\,(1 - a^+)}.$$

There are two notable differences to the case in which the gift exchange condition (7) holds. First, note that this expression would collapse to I_F/q for $a^+ = a^- = 0$, which reveals that the required repayment contains a default premium. Second, the expression $(1 - q)\left[a^+ - f(a^-, a^+)\right] Z > 0$ in the numerator is the expected value of the social "bankruptcy cost" borne by F in the case of default, which increases the required repayment. Last, notice the expression $-a^+(qR - I_O)$ in the numerator, which captures the effect that F is willing to lower the interest rate because she derives altruistic utility from helping E realize the project (which is also the case when (7) holds). Unlike in the case where (7) holds, this interest rate need not be lower than the one charged by an outside investor because not only is default a possibility but it comes with a social "bankruptcy cost" that outside finance is unburdened by.

Let us now turn to the incentive compatibility constraint. As before, we can restrict attention to the payoffs conditional on "success":

$$Z + R - R_O - R_F + a^+(W - I_F + R_F) \geq Z + B + f(a^+, a^-)(W - I_F).$$

With $R_O = I_O/q$ and $I = I_O + I_F$, this yields

$$R \geq B + \frac{I_O}{q} + \left(1 - a^+\right) R_F + \left[f(a^+, a^-) - a^+\right] (W - I + I_O).$$

Again, there are two notable differences to the case in which the gift exchange condition (7) holds. First, the expression $(1 - a^{+})\, R_F$ implies that a higher interest rate charged by F undermines (rather than improves) E's incentives. This is because, here, neither is liability unlimited nor does the social cost of a project failure increase in the amount owed to F (as it was with favors). Second, the expression $[f(a^{+}, a^{-}) - a^{+}]\,(W - I + I_O) < 0$, which relaxes the constraint, captures the positive incentive effect of the social "bankruptcy cost," i.e., that the threat of a damaged relationship reduces E's incentives to shirk.