

# Payments, Credit and Asset Prices\*

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## Abstract

This paper studies a modern monetary economy: trade in both goods and securities relies on money provided by intermediaries. While money is valued for its liquidity, its creation requires costly leverage. Inflation, security prices and the transmission of monetary policy then depend on the institutional details of the payment system. The price of a security is higher if it helps back inside money, and lower if more inside money is used to trade it. Inflation can be low in security market busts if bank portfolios suffer, but also in booms if trading absorbs more money. The government has multiple policy tools: in addition to the return on outside money, it affects the mix of securities used to back inside money.

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# 1 Introduction

In modern economies, transactions occur in two layers. In the *end user layer*, nonbanks – households, firms and institutional investors – pay for goods and securities with inside money, that is, payment instruments supplied by banks.<sup>1</sup> End users' payment instructions to banks in turn generate interbank transactions in the *bank layer*.<sup>2</sup> Interbank payments are often made with reserves – outside money – via central banks' real time gross settlement systems, but may also be handled through short-term credit including interbank netting arrangements.

Models of monetary policy typically abstract from these institutional features. While the New Keynesian approach minimizes the transactions role for money altogether, even models of money as a medium of exchange tend to focus on a single layer of transactions in which money is used to pay for goods. As a result, financial structure does not matter for securities prices, inflation, and the transmission of monetary policy. Moreover, policy is usually simple: it chooses either the supply of money or the interest-rate spread between money and other nominal assets. Real securities prices are typically determined as risk-adjusted present values, as in frictionless nonmonetary models.

This paper models the determination of securities prices and inflation in an economy with a layered payment system that supports trade in both goods and securities. In both the bank and end user layers, money is valued for its liquidity services, but its creation requires costly leverage. What happens in securities markets then matters for both the supply and the demand of inside money: securities are held by banks to back inside money, which is in turn used by other investors to pay for securities. As a result, securities prices, inflation, and policy transmission depend on the institutional details of the payment system.

In our model, the real value of a security is higher (and its rate of return lower) if it is held by banks or institutional investors who borrow from banks – in both cases the security is valued as collateral that backs inside money. At the same time, the real value of a security is lower (and its rate of return higher) if it is held by institutional investors who rely on inside money to trade it. Inflation is also subject to opposing money supply and demand effects: it falls if securities held by banks or their borrowers decrease in value, say due to an

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<sup>1</sup>Payment instruments include not only short-term demandable assets such as deposits and money-market fund shares, but also credit lines that can be drawn on demand such as credit cards. Credit lines also play a key role in payment for securities. For example, institutional investors have sweep arrangements with their custodian banks. Participants in the triparty repo market obtain intraday credit from clearing banks.

<sup>2</sup>Perhaps the most obvious example are direct payments out of bank deposit accounts by check or wire transfer: payments between customers of different banks generate interbank transfers of funds. In many securities markets, transactions are cleared by specialized financial market utilities such as clearinghouses that provide some netting of transactions. Institutional investors then settle netted positions with those utilities through payment instructions to their banks.

uncertainty shock, because a loss of collateral makes it more costly for banks to supply inside money. Inflation rises if securities that require money to trade fall in value, since lower money demand for securities trading effectively increases velocity in the goods market.

Our model summarizes the role of a layered payment system by two key aggregate bank balance sheet ratios. The *collateral ratio* equals risk-weighted assets divided by debt, or the inverse of leverage. It is chosen by banks to equate end users' liquidity benefit of extra inside money to banks' cost of issuing extra debt – a banking version of the tradeoff theory of capital structure. The *liquidity ratio* equals reserves divided by inside money, or the inverse of the money multiplier. It is chosen by banks to equate the liquidity benefit of extra reserves to the spread between the interest rate on short safe bonds and the reserve rate.

Our model distinguishes two regimes for liquidity management. Reserves are *scarce* if the liquidity ratio is small relative to the scale of liquidity shocks faced by the typical bank. There is then an active interbank market: in the event of large liquidity shocks, banks borrow overnight from other banks. Reserves are valued for their liquidity and the spread between the short rate and the reserve rate is positive. In contrast, reserves are *abundant* if the liquidity ratio is sufficiently high; the spread shrinks to zero and the interbank market shuts down.

The abundant reserves regime of our model thus describes spreads, bank balance sheets and interbank credit in the "liquidity trap" that many countries have entered in the wake of recent crises. A key difference to other liquidity trap models is that the cost of liquidity is zero only in the bank layer, where it is measured by the spread between short bonds and reserves. In the end user layer, the cost of liquidity is measured by the spread between deposits and assets directly held by households.<sup>3</sup> It is positive even in a liquidity trap since bank leverage remains costly.

Another key feature is that an economy can reach a liquidity trap not only because of expansionary monetary policy, but also because of a negative shock to the payoffs on securities that banks use to back inside money, for example claims on housing. Indeed, a shock that lowers expected payoffs or increases uncertainty about payoffs makes the production of inside money more costly and drives banks to rely relatively more on reserves in order to back inside money. A layered payment system can thus reach a liquidity trap without large inflation, even if there is no large injection of reserves and prices are flexible.

The determination of collateral and liquidity ratios in general equilibrium reflects two key principles. First, when banks have lower collateral ratios, they demand more reserves

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<sup>3</sup>In our model, households choose not to hold any short bonds: banks, who value bonds as collateral, bid up the bond price so the low rate of return makes bonds unattractive for households. Equilibrium thus features endogenous market segmentation: while bonds are priced by intermediaries, other assets such as bank equity are priced by households.

for precautionary reasons. The reason is that banks with lower collateral ratios face higher borrowing costs in the interbank market and therefore choose to hold more reserves as a buffer against large liquidity shocks. The equilibrium collateral and liquidity ratios must therefore lie on a *liquidity-management curve* that slopes down as long as reserves are scarce. Second, since reserves are also collateral, the balance sheet implies the ratios must always lie on an upward sloping *capital-structure curve*.

The two curves illustrate the two distinct policy tools available to the government if it wants to, say, tighten the stance of monetary policy. First, a higher *real return on reserves* – implemented either by paying more interest on reserves as the Fed has done in recent years or by targeting a lower inflation rate – only shifts the liquidity management curve up. If it is cheaper to back inside money with reserves, banks choose to increase both their liquidity ratio and their overall collateral ratio. Second, an open market sale of securities for reserves changes the *collateral mix* available to banks, which only shifts the capital structure curve up. With more securities available as collateral, banks increase their collateral ratio while reducing their liquidity ratio.

While both moves towards tighter policy increase the real short rate, lower bank leverage and are qualitatively deflationary, they differ in their effects on bank liquidity and inflation.<sup>4</sup> Indeed, while a higher real return on reserves induces banks to rely more on reserves to back inside money, an open market sale implies more reliance on bonds and hence a lower liquidity ratio. The mechanism by which tightening lowers inflation is therefore also different. With higher interest on reserves, inflation declines because banks shrink the money multiplier, the inverse of the liquidity ratio. In contrast, an open market sale leads to a higher money multiplier which partly offsets the deflationary impact of lower reserves.

The magnitude of the inflation response to either policy move depends further on details of the payment system. In particular, a higher reserve rate lowers inflation by less if banks hold more assets with nominally rigid payoffs, such as long-term debt that is not adjusted in response to short-term policy changes. Indeed, as banks increase their liquidity ratio and lower the money multiplier, the resulting deflationary pressure raises the value of nominally rigid collateral. As banks move along a steeper capital structure curve, the adjustment in the money multiplier is smaller and the overall inflation response is dampened.<sup>5</sup>

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<sup>4</sup>In contrast to many other models of monetary policy, the stance of policy in our model cannot be summarized by one interest rate (or the growth rate of reserves). Instead, it must be described by two variables that take into account both the role of the real return on reserves as well as the collateral mix. Within a particular model, there are tight relationships between prices and quantities so many pairs of variables “work”. For example, in an abundant reserves regime, we could use jointly the reserve rate and growth rate of nominal government liabilities, or the reserve rate and the interest rate on deposits. The key point is that the reserve rate alone is not sufficient to understand the transmission of policy.

<sup>5</sup>Our model also lends itself to the analysis of negative interest rates on reserves. Since there is no currency

Details of the payment system also matter for thinking about open market sales. Consider the concrete example of the Fed selling the massive portfolio of government bonds and other securities assembled through its quantitative easing programs. As more collateral sold by the Fed becomes available to banks and reserves are withdrawn, reserves should eventually become scarce, as they did when the Bank of Canada “unwound” its portfolio shortly after the financial crisis. Our model shows that the point at which this happens depends on the quality of other bank collateral as well as the netting arrangements banks set up to handle liquidity shocks. Knowing the threshold to scarcity is crucial for assessing the consequences for interest rates and inflation.

Our model assumes that markets are competitive and all prices are perfectly flexible. Banks and other financial firms maximize shareholder value and operate under constant returns to scale. Moreover, they do not face adjustment costs to equity. The effects we highlight thus do not follow from a scarcity of bank capital. We think of our model as one of large banks that provide payment services in a world where credit markets are highly securitized. This perspective also motivates our leading example for a shock: a change in uncertainty that moves asset premia.

Financial frictions are formally introduced as follows. First, nominal payment instruments and reserves relax liquidity constraints in the end user and bank layer, respectively. In this sense, those assets are more liquid than other assets. By assuming generalized cash-in-advance constraints for households and institutional investors, we abstract from effects of interest rates on the volume of transactions in units of goods and securities, respectively. While adding such effects is conceptually straightforward, say by assuming a utility function over goods and money with curvature in both arguments, our goal here is to provide a tractable setup that zeros in on novel effects for the demand and supply for inside money.

Second, banks and the government face an upward-sloping marginal cost of making commitments. For a bank, this “leverage cost” is smaller the larger and safer is its asset portfolio relative to its debt. It can have either an ex post or an ex ante interpretation. For example, if more levered banks are more likely to renege on certain promises, more labor may be required to ex post renegotiate those promises so that less labor is available for producing goods. Alternatively, more levered banks may have to exert more effort ex ante to produce costly signals of their credibility. For the government, we assume that leverage cost increases with the ratio of debt to consumption. It could be motivated by output losses from distortionary taxation.

An optimal payment system in our model minimizes the *total* cost of leverage – government

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– all payments must be made with inside money – negative interest on reserves works like a tax on banks. By the mechanism just described, a move to negative interest on reserves will do little to produce inflation if the banking system has a lot of nominally rigid assets.

plus banks – required to support the volume of transactions. Whether it is better to adopt a scarce or abundant reserves regime then depends on the relative leverage costs of banks versus the government. If the government can borrow more cheaply than banks, then it makes sense to move to abundant reserves, as several central banks have done recently. An extreme version would be narrow banking. In contrast, if government debt is costly, then it is beneficial to have banks rely more on collateral other than government debt or reserves. Since the optimal system depends on the quality of collateral, it may also make sense to switch between regimes over time in response to asset market events.

Our analysis below starts with a baseline model without aggregate uncertainty in which end users' demand for inside money and the supply of securities available as bank collateral do not respond to changes in the cost of liquidity. This model is already sufficient to show how a layered payment system changes the transmission of policy. We then add uncertainty about asset payoffs as well as two other additional type of institutional investors whose demand for loans or payment instruments is interest elastic.

We first consider *carry traders* who hold real assets and borrow against those assets using short-term credit supplied by banks. Carry traders have no demand for payment instruments, but supply collateral to banks in the form of short-term loans. An example are asset-management firms who finance securities holdings with repurchase agreements with banks (or other payment intermediaries like money market funds) via the triparty repo market. Carry traders make the supply of collateral respond to end users' cost of liquidity: if it becomes more costly to produce deposits, banks offer cheaper loans to carry traders to back inside money.

The new feature in an economy with carry traders is that the price level now depends on carry traders' demand for loans. For example, lower uncertainty increases the demand for loans and hence the quantity of collateral for banks, which creates an inflationary force by increasing the supply of payment instruments. An asset-price boom can thus be accompanied by inflation even if the supply of reserves as well as the amount of goods transacted remains constant and banks hold no uncertain securities themselves. Moreover, monetary policy that lowers the real short rate lowers carry traders' borrowing costs and boosts the aggregate market by allowing more leverage.

Second, we consider *active traders* who hold not only securities but also payment instruments, since they must occasionally rebalance their portfolio using cash payments. An example are asset management firms who sometimes want to exploit opportunities quickly before they can sell their current portfolio. Active traders' portfolio choices respond to the end user cost of liquidity – the deposit interest rate offered by banks or the fee for credit lines banks charge: if payment instruments are cheaper, active traders hold more of them, and the value

of their transactions is higher. The strength of their response depends importantly on how much netting takes place among active traders through intraday credit systems.

The new feature in an economy with active traders is that inflation now depends on active traders' demand for payment instruments. For example, lower uncertainty increases their demand for deposits and credit lines. As more of payment instruments provided by banks are used in asset market transactions, fewer instruments are used in goods market transactions, a deflationary force. During an asset price boom, we may thus see low inflation even if the supply of reserves increases. Moreover, monetary policy that lowers the real short term interest rate lowers active traders' trading costs and further boosts the aggregate market.

The broad questions we are interested in are the subject of a large literature. The main new features of our model are that *(i)* transactions occur in layers, with payment instruments (inside money) used exclusively in the end user layer and reserves (outside money) used exclusively in the bank layer, *(ii)* end users include institutional investors, and *(iii)* both banks and the government face leverage costs. Relative to earlier work, these properties change answers to policy questions as well as asset pricing results, as explained in more detail in Section 7.

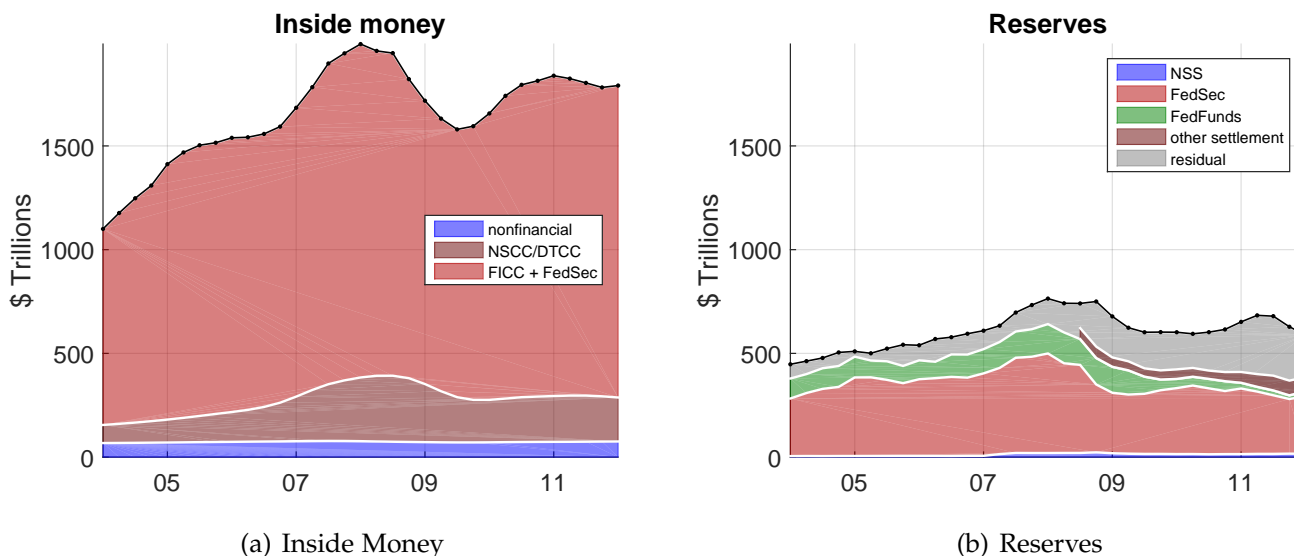
The paper is structured as follows. Section 2 presents a few facts about payments. Section 3 describes the model. Section 4 looks at the baseline model that features only households and banks. It shows how steady state equilibria can be studied graphically and considers different monetary policy tools. Section 6 introduces uncertainty and studies the link between the payment system and securities markets. It also extends the model to accommodate institutional investors as a second group of end users. Finally, Section 7 discusses the related literature.

## 2 Facts on payments

This section presents a number of facts that motivate our model. We combine data from the BIS Payments Statistics, the Payments Risk Committee sponsored by the Federal Reserve Bank of New York, the Federal Reserve Board's Flow of Funds Accounts and Call Reports, as well as publications of individual clearinghouse companies.

**Transactions in the end-user layer.** Figure 1 gives an impression of payments in the two layers in US dollars. The left-hand panel shows payments by bank customers with inside money, that is, payment instructions to various types of intermediaries. The blue area labeled "nonfinancial" adds up payments by cheque as well as various electronic means, notably Automated Clearinghouse (ACH) transfers as well as payments by credit card. While the area appears small in the figure, it does amount to several multiples of GDP. For example, in 2011, nonfinancial transactions were \$71 trillion whereas GDP was \$15 trillion. This is what one

Figure 1: Selected U.S. dollar transactions, quarterly at annual rates.



(a) Inside Money: “nonfinancial” = cheque and electronic payments reported by banks, “NSCC/DTCC” = securities transactions cleared by NSCC, “FICC+FedSec” = securities transactions cleared by FICC and Fedwire Securities Service. (b) Reserves: “NSS” = NSS settlement, “FedSec” = Fedwire Securities settlement, “FedFunds” = estimate of payments for Fed Funds borrowing by U.S. banks, “other settlement” = estimate of payments to financial market utilities.

would expect given that there are multiple stages of production and commerce before goods reach the consumer. Moreover, a share of trade in physical capital including real estate is also contained in this category.

Payment for assets in U.S. markets is organized by specialized financial market utilities who clear transactions and see them through to final settlement. A major player is the Depository Trust & Clearing Corporation (DTCC). One of its subsidiaries, the National Securities Clearing Corporation (NSCC) clears transactions on stock exchanges as well as over-the-counter trades in stocks, mutual fund shares and municipal and corporate bonds. NSCC cleared \$221 trillion worth of such trades in 2011. In the left hand panel of Figure 1, transactions cleared by NSCC are shaded in brown.

NSCC has a customer base (“membership”) of large financial institutions, in particular brokers and dealers. When a buyer and a seller member agree on a trade – either in an exchange or in an over-the-counter market – the trade is reported to NSCC which then inserts itself as a counterparty to both buyer and seller. In the short run, members thus effectively pay for assets with credit from NSCC. To alleviate counterparty risk, members post collateral that limits their position relative to NSCC. Over time, NSCC nets opposite trades by the same member. Peri-



odically, members settle net positions via payment instructions to members' bank which then make (receive) interbank payments to (from) DTCC. Netting implies that settlement payments amount to only a fraction of the dollar value of cleared transactions.

Another DTCC subsidiary, the Fixed Income Clearing Corporation (FICC) offers clearing for Treasury and agency securities. FICC payments are settled on the books of two "clearing banks", JP Morgan and Bank of New York Mellon. Interbank trades of Treasury and agency bonds can alternatively be made via the Fedwire Securities system offered by the Federal Reserve System to its member banks. The left hand panel of Figure 1 shows the sum of FICC and Fedwire Securities trades in red. This number is high partially because every repurchase agreement involves two separate security transactions (that is, the lender wires payment for a purchase to the borrower and the borrower wires payments back to the lender at maturity).

Figure 1 does not provide an exhaustive list of US dollar transactions. First, it leaves out financial market utilities handling derivatives and foreign exchange transactions. For example, the Continuous Linked Settlement (CLS) group is a clearinghouse for foreign exchange spot and swap transactions that handled trades worth \$1,440 trillion in 2011. Netting in these markets is very efficient so that CLS payments after netting were only \$3 trillion. Second, even for goods and assets covered, Figure 1 omits purchases made against credit from the seller that involves no payment instruction to a third party. This type of transaction includes trade credit arrangements. In asset markets, a share of bilateral repo trades between broker dealers and their clients is settled on the books of the broker dealers. Finally, the figure also leaves out transactions made with currency.

Even given these omissions, the message from the left panel of Figure is clear: transaction volume is large, and especially so in asset markets. The volume in asset markets also exhibits pronounced fluctuations in the recent boom-bust episode. We also emphasize that not all of these payment instructions are directly submitted to traditional banks. Financial market utilities that provide netting are also important. Moreover, customers of money market mutual funds may also pay by cheque or arrange ACH transfers. The payment instruction is then further relayed by the money market fund to its custodian bank.

**Transactions in the bank layer.** The right-hand panel of Figure 1 shows transactions over two settlement systems provided by the Federal Reserve Banks. The blue area represents interbank payments via the National Settlement Service, which allows for multilateral netting of payments by cheque and ACH. To a first approximation, one can think of it as the counterpart of the blue area in the left hand panel, that is, non-securities payments after netting. All other areas in the right panel represent interbank payments over Fedwire, the real time gross settlement system of the Federal Reserve. Fedwire is accessed by participating banks who send reserves to each other.

The coloring of areas is designed to indicate roughly how the interbank payments were generated. The red area represents payments for Treasury and agency securities over Fedwire Securities. Since there is no netting involved, large securities transfers correspond to large transfers of reserves. For the years after 2008, the brown area is an estimate of payments made over Fedwire to settle positions with financial market utilities. The estimate includes not only NSCC and FICC, but also CHIPS, a private large value transfer system used by about 50 large banks. CHIPS uses a netting algorithm to simplify payments among its member banks; in 2011, it handled \$440 trillion worth of transactions.

The green area in the figure represents payments for interbank credit in the Fed Funds market, also sent over Fedwire. As for repo transactions, a relatively small amount of outstanding overnight credit can generate a large number for annual Fedwire transfers. The transition from a regime of scarce reserves to one with abundant reserves after the financial crisis is apparent by the drop in Fed Funds transactions. The presence of government sponsored enterprises and Federal Home Loan banks implies that the Fed Funds market has not dried up completely.

The red and brown areas suggest that payment instructions generated by asset trading are responsible for a large share of interbank payments. This is true even though netting by financial market utilities reduced the cleared transactions from the left panel to much smaller numbers. At the same time, during times of scarce reserves, bank liquidity management via the Fed Funds market also generates a large chunk of payments. The figure also contains a gray area which we cannot assign to one of the payment types.

### 3 Model

Time is discrete, there is one good and there are no aggregate shocks. Output  $Y$  is constant. Figure 2 shows a schematic overview of the model. There are claims to future output that are "securitized", in the sense that they are tradable in securities markets. *Trees* promise a constant stream of goods  $x < Y$ . Nominal government debt takes the form of *reserves* or *short bonds* with one period maturity. Below we will also consider nominal private debt, which are trees that promise a constant nominal value  $X < PY$ . Households receive the rest of output which includes the share of output that is not securitized as well as dividends paid by trees that are off limits to banks.

Households invest in securities either directly or indirectly via banks. Banks are competitive, issue deposits as well as equity and maximize shareholder value. The only restrictions on investment are that households cannot directly hold reserves, and banks cannot hold bank equity or claims to the share of output that is not securitized. There are certain trees that banks are not allowed to buy and are thus held exclusively by households. To save on notation, we

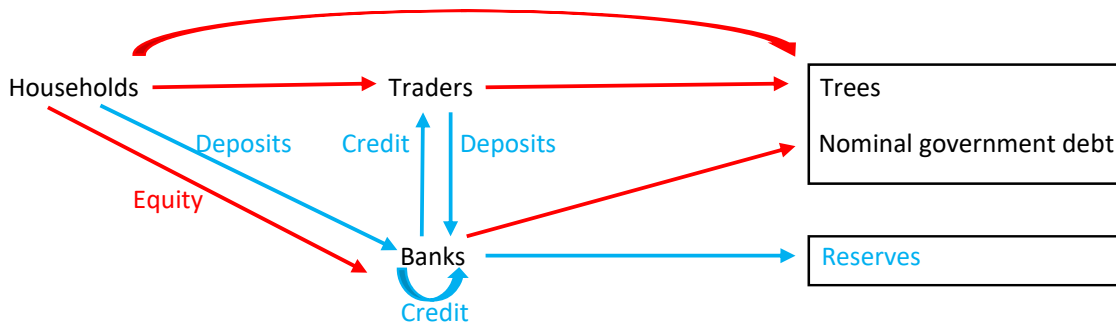


Figure 2: Schematic overview of the model with goods and assets trading

do not model these trees separately but simply include their dividends into the share of output that is not securitized. The share of securitized output plays a key role in our model, because it describes the amount of collateral that banks can potentially use to back inside money.

Tradeoffs in the model reflect two basic principles. First, some assets provide liquidity benefits. We capture a need for liquidity by cash-in-advance constraints in both layers of the model. In the end-user layer, households must pay for goods with deposits.<sup>6</sup> In the bank layer, banks face liquidity shocks because they execute payment instructions from households. As a result, they must make payments to each other with reserves that they hold or borrow from other banks in the interbank market. Investment indicated in blue in Figure 2 thus receives liquidity benefits.

The second principle is that it is costly for agents to commit to make future payments, and more so if they own fewer assets that can serve as collateral. Such “leverage costs” apply when banks issue deposits or when the government issue debt. They use up goods and hence lower consumption. The optimal asset structure and payment system therefore minimizes leverage costs. Moreover, banks’ receive collateral benefits on their investments.

The remainder of this section will analyze a version of the model in which payments are made for goods purchases. Section 6 will introduce another motive for payments: asset purchases. There, we will introduce the institutional traders illustrated in Figure 2, competitive firms held by households. These traders borrow from banks to finance their securities positions and use inside money to pay for their asset trades. Moreover, Section 6 will introduce uncertainty about future security payoffs. This extended version of the model determines how much inside money will be spent in goods markets versus asset markets, and thereby

<sup>6</sup>Section 3.6 introduces credit lines and shows that the model continues to work similarly. The key property of either payment instrument is that it provides liquidity to end users and requires costly commitment on the part of banks. Our model is about a modern economy where currency plays a negligible role in all (legal) transactions.

determine goods and asset price inflation.

### 3.1 Households

Households have linear utility with discount factor  $\beta$  and receive an exogenous amount  $\Omega_t$  of goods every period, which includes dividends of trees that cannot be bought by banks as well as a portion of output that is not securitized. Households enter period  $t$  with deposits  $D_{t-1}^h$  and buy consumption  $C_t$  at the nominal price  $P_t$  measured in units of reserves. Their liquidity constraint is

$$P_t C_t \leq D_{t-1}^h. \quad (1)$$

A cash-in-advance approach helps us zero in on the role of endogenous inside money. It is not difficult to extend the model so that the money demand by households is elastic, but it would make the new mechanisms in our model less transparent. Moreover, the key conclusions of our analysis extend to a model with curvature in the utility function.

In addition to deposits, households can invest in safe short bonds that earn an interest rate  $i_t$ . Households can also buy trees, which are infinitely lived assets that provide fruit  $x_t$  and trade at a nominal price  $Q_t$ . The household budget constraint is

$$\begin{aligned} P_t C_t = & P_t \Omega_t + D_{t-1}^h \left(1 + i_{t-1}^D\right) - D_t^h \\ & + (1 + i_{t-1}) B_{t-1}^h - B_t^h + (Q_t + P_t x_t) \theta_{t-1}^h - Q_t \theta_t^h \\ & + \text{dividends} + \text{government transfers}. \end{aligned} \quad (2)$$

Expenditure on goods must be financed through either (i) the sale of endowment, (ii) changes in household asset positions in deposits, short bonds or trees, or (iii) exogenous income from dividends, fees or government transfers, described in more detail below.<sup>7</sup> Households cannot borrow overnight or sell trees short, that is, we impose  $\theta^h, B^h, D^h \geq 0$ .

We denote households' marginal utility of wealth at date  $t$  by  $\omega_t$ , so the Lagrange multiplier on the budget constraint (2) is  $\omega_t/P_t$ . From the household first-order conditions derived in Appendix (A.1), the discount factor for the payoffs of real assets held by households is the marginal rate of substitution between wealth at date  $t$  and  $t+1$ , that is  $\hat{\beta}_t := \beta \omega_{t+1}/\omega_t$ . It is also convenient to define the associated nominal discount rate  $i_t^h := (1 + \pi_{t+1})/\hat{\beta}_t - 1$ , where  $\pi_{t+1}$  is the inflation rate between  $t$  and  $t+1$ .

With a binding cash-in-advance constraint, there is a wedge between the marginal utilities

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<sup>7</sup>We assume that an interest rate  $i^D$  is earned on deposits regardless of whether they are used for payment. This assumption helps simplify the algebra. More detailed modeling of the fee structure of deposit accounts is possibly interesting but not likely to be first order for the questions we address in this paper.

of wealth and consumption, that is,  $\omega_{t+1} < 1$ . The first-order conditions further imply

$$i_t^h - i_t^D = \frac{1}{\omega_{t+1}} - 1. \quad (3)$$

The cash-in-advance constraint thus binds as long as the *end-user cost of liquidity*, measured by the spread between households' nominal discount rate and the deposit rate, is positive. The spread  $i_t^h - i_t^D$  is the convenience yield of holding inside money for end users.

Equation (3) illustrates a key difference between our model and the baseline cash-in-advance model of Svensson (1985).<sup>8</sup> In Svensson, all money is currency that earns no interest. Moreover, there is no bank layer, so households hold short bonds directly. The cost of liquidity is then simply the interest rate on short bonds  $i_t$ . In contrast, the nominal discount rate  $i_t^h$  in our model may be higher than the short rate  $i_t$ , since households may choose not to hold short bonds in equilibrium. Moreover, households pay with inside money which earns the endogenous interest rate  $i^D$ .

## 3.2 Banks

Households own many competitive banks. The typical bank maximizes shareholder value

$$\sum_{t=1}^{\infty} \left( \prod_{\tau=0}^{t-1} \hat{\beta}_{\tau} \right) y_t^b, \quad (4)$$

which discounts (real) dividends  $y_t^b$  with households' marginal rate of substitution between wealth at date 0 and date  $t$ . Dividends are positive when banks distribute profits or negative when banks recapitalize. Table 1 illustrates a bank's balance sheet at the beginning of day  $t$ . The asset side consists of reserves, overnight lending, and trees. The liability side shows equity, overnight borrowing, and deposits.

Table 1: Bank balance sheet at beginning of day  $t$

Assets	Liabilities
Reserves $M_{t-1}$	Equity
Overnight lending $B_{t-1}$	Overnight borrowing $F_{t-1}$
Trees $Q_t \theta_{t-1}$	Deposits $D_{t-1}$

**Liquidity management.** The typical bank enters period  $t$  with deposits  $D_{t-1}$  and reserves

<sup>8</sup>The Svensson (1985) setup is the natural benchmark for us since we adopt the same timing: households must choose money one period in advance. In contrast, the Lucas (1980) model assumes that households can choose money within the period.

$M_{t-1}$ . We want to capture the fact that customer payment instructions may lead to payments between banks. For example, a payment made by debiting a deposit account may be credited to an account holder at a different bank. We thus assume that a bank receives an idiosyncratic withdrawal shock: an amount  $\tilde{\lambda}_t D_{t-1}$  must be sent to other banks, where  $\tilde{\lambda}$  is iid across banks with mean zero and cdf  $G$ . We assume that  $G$  is continuous and strictly increasing up to an upper bound  $\bar{\lambda} < 1$ .

In the cross section, some banks draw shocks  $\tilde{\lambda}_t > 0$  and must make payments, while other banks draw shocks  $\tilde{\lambda}_t < 0$  and thus receive payments. Since  $E[\tilde{\lambda}_t] = 0$ , any funds that leave one bank arrive at another bank; there is no aggregate flow into or out of the banking system. The distribution of  $\tilde{\lambda}_t$  depends on the structure of the banking system as well as the pattern of payment flows among customers.<sup>9</sup>

Banks that need to make a transfer  $\tilde{\lambda}_t D_{t-1} > 0$  can send reserves they have brought into the period, or they can borrow reserves from other banks. The bank liquidity constraint is

$$\tilde{\lambda}_t D_{t-1} \leq M_{t-1} + F_t, \quad (5)$$

where  $F_t \geq 0$  is new overnight borrowing of reserves from other banks. If the marginal cost of overnight borrowing is larger than other sources of funding available to the bank, it is optimal to borrow as little as necessary. Banks then choose a threshold rule: they do not borrow unless  $\tilde{\lambda}_t$  is so large that the withdrawal  $\tilde{\lambda}_t D_{t-1}$  exhausts their reserves.

For a bank that enters the period with reserves  $M_{t-1}$  and deposits  $D_{t-1}$ , the liquidity constraint (5) implies a threshold shock

$$\lambda_{t-1} := \frac{M_{t-1}}{D_{t-1}}. \quad (6)$$

We refer to  $\lambda_{t-1}$  as the *liquidity ratio* of a bank. It is the inverse of a money multiplier that relates the amount of inside money created to the quantity of reserves.

For a given liquidity ratio, the liquidity constraint (5) binds if the bank's liquidity shock is sufficiently large, that is,  $\tilde{\lambda}_t > \lambda_{t-1}$ . Reserves then provide a liquidity benefit, measured by the multiplier on the constraint. Moreover, the bank borrows reserves overnight

$$F_t = \tilde{\lambda}_t D_{t-1} - M_{t-1} = \left( \frac{\tilde{\lambda}_t}{\lambda_{t-1}} - 1 \right) M_{t-1} > 0. \quad (7)$$

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<sup>9</sup>The likelihood of payment shocks is the same across banks, regardless of bank size. Since the size distribution of banks is not determinate in equilibrium below, little is lost in thinking about equally sized banks. Alternatively, one may think about large banks consisting of small branches that cannot manage liquidity jointly but instead each must deal with their own shocks.

Since liquidity shocks are bounded above by  $\bar{\lambda}$ , banks can in principle choose a high enough liquidity ratio,  $\lambda_{t-1} > \bar{\lambda}$ , so that they never run out of reserves. Banks who do this have a zero multiplier on their liquidity constraint in all states next period and hence obtain no liquidity benefit from holding reserves.

**Portfolio and capital structure choice.** Banks adjust their portfolio and capital structure subject to leverage costs. They invest in reserves, overnight credit and trees while trading off returns, collateral values and liquidity benefits. They issue deposits and adjust equity capital, either through positive dividend payouts or negative recapitalizations  $y_t^b$ . Capital structure choices trade off returns, leverage costs, and liquidity costs.

The bank budget constraint, or cash flow statement, says that net payout to shareholders must be financed through changes in the bank's positions in reserves, deposits, overnight credit, or trees:

$$\begin{aligned}
P_t y_t^b = & M_{t-1} \left(1 + i_{t-1}^R\right) - M_t - D_{t-1} \left(1 + i_{t-1}^D\right) + D_t \\
& + (B_{t-1} - F_{t-1}) (1 + i_{t-1}) - (B_t - F_t) + ((Q_t + P_t x_t) \theta_{t-1} - Q_t \theta_t) \\
& - c(\kappa_t) (D_t + F_t).
\end{aligned} \tag{8}$$

In the second line,  $B \geq 0$  represents lending in overnight credit. The last line collects bank leverage costs and credit lines that banks use to pay those costs, both discussed in detail below.

The first and second lines in (8) collect payoffs from payment instruments and other assets, respectively. The bank receives interest  $i_{t-1}^R$  on reserves that are held overnight, regardless of whether those reserves were used to make payments. Similarly, the bank pays deposit interest  $i_{t-1}^D$  on deposits issued in the previous period, regardless of whether its customers used the deposits to make payments. Both conventions could be changed without changing the main points of the analysis, but at the cost of more cluttered notation.

**Leverage costs.** If the last line in the bank budget constraint (8) was omitted, the cost of debt would be independent of leverage. We assume instead that the commitment to make future payments is costly. It takes resources to convince overnight lenders that debt will be repaid, as well as to convince customers that the bank will indeed accept and execute payment instructions. Moreover, we assume that convincing lenders and customers is cheaper if the bank owns more assets to back the commitments, especially if those assets are safe.

The cost of commitment depends on the *collateral ratio*

$$\kappa_t := \frac{M_t + \rho Q_t \theta_t + B_t}{D_t + F_t}, \tag{9}$$

where  $\rho$  is a fixed parameter strictly between 0 and 1. The collateral ratio divides weighted assets by debt; its inverse is a measure of leverage. Banks have to purchase real resources  $c(\kappa_t)(D_t + F_t)/P_t$  in the goods market at date  $t$ . The cost function  $c$  is smooth, strictly decreasing and convex. We further assume below that it slopes down sufficiently fast so that banks choose  $\kappa_t > 1$ .

Bank assets in the numerator of (9) have a collateral value: the resources needed to convince customers about future commitments are smaller if the bank owns more assets. The weight  $\rho$  allows a distinction between safe assets (reserves and overnight lending) and trees, which we will later assume to be uncertain. The presence of a weight implies that leverage computed as the inverse of the collateral ratio does not generally correspond to accounting measures of leverage.<sup>10</sup>

**First order conditions.** A shareholder-value-maximizing firm compares the rates of return on all assets and liabilities to the rate of return on equity. At the optimal policy, all assets must earn rates of return that are smaller or equal to the rate of return on equity, with equality if the bank holds the asset. Otherwise the firm could issue more equity, buy the asset, and thereby add value. Similarly, all liabilities must earn rates of return that are large or equal to the return on equity, with equality if the bank indeed issues the liability. In our model, rates of return not only reflect pecuniary returns, but also the effect of leverage costs and the liquidity constraint.

The nominal rate of return on bank equity is  $i_t^h$ , because households must hold all bank equity in equilibrium. To illustrate the choice of assets, consider banks' first order conditions for short bonds and reserves, derived<sup>11</sup> in Appendix A.2:

$$\begin{aligned} i_t - c'(\kappa_t) &\leq i_t^h, \\ i_t^R - c'(\kappa_t) + E_t \tilde{\mu}_{t+1} &\leq i_t^h. \end{aligned} \tag{10}$$

The nominal bond return on the left-hand side of the first equation consists of a pecuniary return  $i_t$  plus a *collateral benefit*; since  $c$  is strictly decreasing and convex, the collateral benefit is positive and strictly decreasing in  $\kappa_t$ . Banks value bonds as collateral and hence require a lower pecuniary return to hold bonds, but less so if their collateral ratio is already higher.

The rate of return on reserves in the second equation includes the same collateral benefit, but also a *liquidity benefit* measured by the expected multiplier  $\tilde{\mu}_{t+1}$  on the future liquidity

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<sup>10</sup>Since leverage costs take up real resources, we need to address how banks pay for them. The details of this process are not essential and we choose an approach that simplifies formulas: we assume that banks do not face a cash-in-advance constraint for leverage costs.

<sup>11</sup>The FOCs in equations (10) approximate those in the appendix for a short period length, as shown in Proposition 3 below. To get from Appendix A.2 to equations (10), approximate  $(1+i)/(1+i^h)$  as  $1+i-i^h$ .



constraint (5). If banks hold both bonds and reserves (as will be true in equilibrium), the liquidity benefit can induce a spread  $i_t - i_t^R$  between the bond and reserve rate, which measures the convenience yield of holding reserves for banks. For a high enough liquidity ratio  $\lambda_t$ , however, the liquidity constraint never binds at date  $t + 1$ . The multiplier  $\tilde{\mu}_{t+1}$  is thus zero for sure, and there is no convenience yield,  $i_t = i_t^R$ .

The optimal choice of the collateral ratio follows from a bank-specific version of the tradeoff theory of capital structure. The banks' first order condition for deposits is

$$i_t^D + c(\kappa_t) - c'(\kappa_t)\kappa_t + E_t\tilde{\mu}_{t+1}\tilde{\lambda}_{t+1} \geq i_t^h. \quad (11)$$

At the optimal policy, the rate of return on deposits must be greater or equal to the rate of return on equity. It consists of the interest rate  $i_t^D$ , a marginal leverage cost that is decreasing in  $\kappa_t$  and a liquidity cost, captured by the multiplier on the future liquidity constraint  $\tilde{\mu}_{t+1}$ .

Since deposits provide liquidity benefits to households and therefore pay a lower rate than equity, the first dollar of deposits issued is always a "cheap" source of funding from the perspective of the bank. As the bank issues more deposits, however, its marginal leverage cost increases. Eventually the bank reaches an interior optimum for its leverage ratio. The difference to the standard tradeoff theory is that the conventional tax advantage of debt is replaced by the liquidity benefit of debt for end users. Moreover, issuing deposits may incur liquidity costs. Banks can mitigate those costs by holding more reserves; we return to the determination of the optimal liquidity ratio below.

### 3.3 Government

We treat the government as a single entity that comprises the central bank and the fiscal authority. The government issues reserves  $M_t$ , borrows  $B_t^g$  in the overnight market and chooses the reserve rate  $i_t^R$ . The government also makes lump-sum transfers to households so that its budget constraint is satisfied every period. Below we further consider particular policies that target endogenous variables such as the overnight interest rate. Such policies are still implemented using the basic tools  $M_t$ ,  $B_t^g$  and  $i_t^R$ . It is convenient to parametrize government policy by its two monetary policy tools  $M_t$  and  $i_t^R$  as well as the ratio of bonds to reserves  $b_t = B_t^g/M_t$ . The growth rate of reserves is  $g_t$ .

Just like financial firms, the government incurs a cost of issuing debt, above and beyond its pecuniary cost. The government differs from firms in that it has the power to tax and hence the (implicit) collateral that is available to it. We define the government's collateral ratio as  $\kappa_t^g = P_t C_t / M_t (1 + b_t)$  and denote the date  $t$  government leverage cost as  $c_g(\kappa_t^g) M_t (1 + b_t)$  where  $c_g$  is strictly decreasing and convex, as is the bank leverage cost function  $c$ . The more

real debt  $M_t(1 + b_t) / P_t$  the government issues relative to consumption, the more resources it must spend to convince lenders that it will repay.

### 3.4 Equilibrium

An equilibrium consists of interest rates  $i_t^h$ ,  $i_t$  and  $i_t^D$ , a tree price  $Q_t$ , a bank equity price  $q_t^b$ , a nominal price level as well as consumption, leverage cost, household portfolios and bank balance sheets such that markets clear at the optimal choices of banks and households, taking into account government policy. Tree market clearing requires that banks or households hold all trees. The overnight credit market clears if borrowing by banks plus government borrowing equals aggregate bank lending,  $B_t = B_t^S + F_t$ . Banks must hold all reserves.

The goods market clears if households consume the endowment and all fruit from trees, net of any resources spent by banks and the government as leverage costs. Since output is exogenous, only the use of goods for consumption or leverage cost is determined in equilibrium. For example, if banks and the government are more levered, then consumption must be lower.

We focus on equilibria in which inside money has a positive liquidity benefit. They arise if inside money is "scarce" because it is costly to produce. Scarcity of inside money has important implications for asset prices and portfolio allocations, summarized by

*Proposition 1 (Implications of scarce inside money). In any equilibrium with  $i_t^h - i_t^D > 0$ ,*

*(i) all cash-in-advance constraints bind and together imply the quantity equation*

$$P_t C_t = D_{t-1}, \quad (12)$$

*(ii) all bonds and trees are held by banks,*

*(iii) rates of return are ordered by*

$$i_t^R \leq i_t < \frac{Q_{t+1} + P_{t+1}x_{t+1}}{Q_t} - 1 < i_t^h,$$

*(iv) all banks choose the same collateral and liquidity ratios.*

*(v) There are two possible regimes for liquidity management: if reserves are abundant,  $\lambda_{t-1} \geq \bar{\lambda}$ , no bank borrows overnight; if reserves are scarce,  $\lambda_{t-1} < \bar{\lambda}$ , interbank credit is*

$$\frac{F_t}{D_{t-1}} = \int_{\lambda_{t-1}}^{\bar{\lambda}} (\bar{\lambda} - \lambda_{t-1}) dG(\tilde{\lambda}) := f(\lambda_{t-1}). \quad (13)$$

The spread  $i_t - i_t^R$  is positive if  $\lambda_{t-1} < \bar{\lambda}$  and zero otherwise.

(vi) Consumption is  $C_t = Y_t / (1 + \ell(\lambda_{t-1}, \lambda_t, \kappa_t, g_t, b_t))$ , where the leverage cost as a share of consumption is defined as

$$\begin{aligned} \ell(\lambda_{t-1}, \lambda_t, \kappa_t, g_t, b_t) := & c_g \left( \lambda_{t-1}^{-1} (1 + b_t)^{-1} (1 + g_t)^{-1} \right) \lambda_{t-1} (1 + b_t) (1 + g_t) \\ & + c(\kappa_t) \left( \frac{\lambda_{t-1}}{\lambda_t} (1 + g_t) + f(\lambda_{t-1}) \right). \end{aligned} \quad (14)$$

The proof is in Appendix A.3; we now provide some intuition for each property.

**Quantity theory.** The nominal price level is given by the quantity equation (12), as in a standard cash-in-advance model. The only difference is that the relevant quantity of money is the (endogenous) supply of inside money, and the relevant opportunity cost of money is not simply the nominal interest rate, but the convenience yield  $i_t^h - i_t^D$ . Nevertheless the argument for the convenience yield is standard: with a positive cost of liquidity, the condition (3) implies that households hold as few deposits as possible and the cash-in-advance constraint binds.

**Asset valuation** Property (ii) describes the valuation of securities held by banks. First, a positive convenience yield of inside money for end-users requires a nominal rate of return on reserves  $i_t^R$  below banks' nominal return on equity  $i_t^h$  (that is, their cost of capital). If instead the reserve rate were equal to  $i_t^h$ , then banks could achieve any collateral ratio by issuing equity and holding reserves. In such an economy, money would not be scarce and cash-in-advance constraints would not bind. We focus here on the empirically relevant case where the rate of return on equity exceeds the return on money. Since holding reserves is costly, banks hold a finite (real) quantity of reserves in equilibrium.

A key feature of our model is that asset valuation interacts with the payment system. This is a permanent effect on real asset prices: the collateral benefit of assets to banks in their first-order condition (10) also holds in steady state and introduces a permanent spread between the rate of return on assets held by banks and assets held directly by households. The magnitude of the collateral benefit depends on the health of the banking system measured by  $\kappa$ . It is thus reminiscent of existing models of "intermediary asset pricing". Next, we emphasize that it arises from an endogenous choice to use collateral to back inside money, not from exogenous restrictions on market participation.

**Market segmentation.** Collateral benefits give rise to endogenous market segmentation. As long as the quantity of collateral available to banks is finite, the collateral benefit on bonds in (10) is strictly positive. Since households do not enjoy collateral benefits, they hold bonds if and only if the pecuniary return  $i_t$  is exactly equal to the return on equity  $i_t^h$ . In equilibrium,

therefore, all bonds must be held by banks. Households view their rates of return as too low and cannot sell them short, hence they hold zero. Put differently, shareholders optimally decide to locate bonds (as well as trees that are also eligible as bank collateral) within banks.

**Aggregation.** Investment in competitive asset markets makes banks' technology linear. The individual bank's problem is thus homogenous of degree one in all of its balance sheet positions. In addition, the liquidity constraint (5) and leverage costs  $c$  only depend on balance sheet ratios. At the optimum, therefore, a bank's positions are indeterminate – only the ratios  $\kappa_t$  and  $\lambda_t$  are pinned down.

Moreover, there are no adjustment costs to any bank position; in particular, shareholders can costlessly recapitalize banks at any time. It follows that banks' choice of balance sheet ratios is purely forward looking, as illustrated by their first order conditions (10) and (11). Since all banks face the same prices, they also all choose the same ratios. This feature is convenient since it means that we do not have to keep track of a distribution of bank-level variables.

**Reserve regimes.** Since all banks choose the same liquidity ratio, we can define scarcity and abundance of reserves in terms of that single ratio. The expression for interbank credit (13) comes from aggregating (7) across banks. If reserves are abundant at date  $t$ ,  $\lambda_{t-1} \geq \bar{\lambda}$ , banks are in a liquidity trap: interbank credit is zero, and reserves carry no liquidity benefit. In this case, banks view reserves and bonds as perfect substitutes: they have the same collateral benefits and pay the same interest rate by (10). In contrast, households still view money as more liquid than any other asset.

**Leverage cost.** The total leverage cost incurred at date  $t$  adds up government and bank leverage costs. These costs depend on the current bank ratios, because the costs are for creating money to handle transactions at date  $t + 1$ . The more liquid the banking sector, the higher is the government leverage cost as a share of consumption. The better collateralized the banking sector, the lower is its leverage cost. When reserves are scarce, the real cost to borrow reserves overnight depends on reserves and deposits carried over from the previous period. This is why the leverage cost for banks also depends on  $\lambda_{t-1}$  and  $g_t$ .

### 3.5 Unanticipated shocks and comparison of steady states

To study the response of the economy to shocks, we use the aggregation result from Proposition 1 to characterize equilibrium as the solution to a system of nonlinear difference equations. It is convenient to eliminate the price level: the variables are the interest rates  $i_t^h$ ,  $i_t$  and  $i_t^D$ , the inflation rate  $\pi_t$ , the real price of trees  $q_t = Q_t/P_t$ , the marginal utility of wealth  $\omega_t$  and the bank ratios  $\kappa_t$  and  $\lambda_t$ . The bank liquidity ratio  $\lambda_t$  is the only endogenous state variable. The

exogenous state variables are output  $Y_t$  and reserves growth  $g_t$ . The derivation is contained in Appendix A.4.

In a steady state equilibrium, output, consumption and rates of return are constant. We restrict attention to policies that imply the same constant growth rate  $g$  for the nominal quantities  $M_t$  and  $B_t^s$  so the bond-reserve ratio  $b := B_t^s/M_t$  is constant as well. With fixed rates of return, the marginal rate of substitution of wealth across dates is given by  $\delta = 1/\beta - 1$  and inflation is  $\pi = g$ . The key bank ratios  $\lambda$  and  $\kappa$  are also constant but endogenously determined; they summarize the role of the payment system for asset prices and the price level.

Like the baseline Svensson model, our model has limited transition dynamics, described in *Proposition 2 (Simple dynamics)*. For any initial condition  $(\lambda_0, Y_0, g_0)$  sufficiently close to a steady state with  $(\lambda, Y, g)$ , there exists a one time government bond trade  $b_0$  such that the economy reaches the steady state after one period.

The price level jumps between dates 0 and 1 by the factor

$$1 + \pi_1 = (1 + g_0) \frac{\lambda_0 Y_0}{\lambda Y} \frac{1 + \ell(\lambda, \lambda, \kappa, g)}{1 + \ell(\lambda_0, \lambda, \kappa, g_0)}. \quad (15)$$

If reserves are abundant then  $b_0 = b$  and  $\ell(\lambda, \lambda, \kappa, g) = \ell(\lambda_0, \lambda, \kappa, g_0)$  for any  $g$  and  $g_0$ .

The proof is in Appendix A.5. In Svensson's model, an unanticipated shock only generates a one time jump in the price level from date 0 to date 1, after which the inflation rate continues at the new steady state rate. The reason the jump does not happen right away at date 0 is that the date 0 price level is predetermined by money holdings that agents bring into the period together with date 0 exogenous output.

In our model, there is more scope for dynamics when reserves are scarce, because real overnight credit depends on reserves and deposits carried over from the previous period. Since interbank credit is a relatively small share of bank assets, we view the potential transition dynamics as a technical feature of the model, rather than a substantively interesting one. We thus assume that an unanticipated shock is always accompanied by a government trade  $b_0$  that neutralizes the effect and allows immediate transition to the new steady state. The dynamics are then analogous to Svensson: there is a one time jump in the price level after which inflation reaches its steady state rate.

In what follows, we study an economy that is initially in steady state with  $(\lambda^*, Y^*, g^*)$  and experiences an unanticipated shock to the environment at date 0. The shock could be a change to future policy or asset payoffs, but it could also involve a one time injection of reserves  $g_0$ , say, or a temporary shock to current output  $Y_0$ . We know from Proposition 2 that much of

the response to the shock is described by comparative statics of steady states, from the initial steady state to the new steady state with  $(\lambda, Y, g)$  that is reached at date 1.

For inflation, it is interesting to further distinguish the short run and long run response to the shock. The long run response of inflation is the difference between  $g$  and  $g^*$ . In the short run, between dates 0 and 1, the price level jumps by the factor (15), for  $\lambda_0 = \lambda^*$ . It is important here that  $g_0$  can differ from both  $g$  and  $g^*$  if the shock involves a one time injection of reserves.

### 3.6 Credit lines as inside money

So far, our liquidity constraint requires households to pay with deposits. We now show that the basic logic of the model extends to another payment instrument that is important in practice, namely credit lines. The common denominator of both types of payment instruments is that banks make costly commitments to households that must be backed up by bank assets. In our analysis below, we thus apply the general term "inside money" to deposits and credit lines.

Suppose that households can pay either with deposits, or by drawing down a credit line  $L_{t-1}^h$ :

$$P_t C_t \leq D_{t-1}^h + L_{t-1}^h.$$

Credit lines must be arranged one period in advance with a bank, but require no investment – they represent intraday credit extended by banks on demand. In exchange for the commitment to accept payment instructions, banks charge a fee  $i_{t-1}^L L_{t-1}$  proportional to the credit amount.<sup>12</sup>

We consider equilibria such that the liquidity constraint binds and households are indifferent between alternative payment instruments. In any such equilibrium we have

$$i_{t-1}^h - i_{t-1}^D = i_{t-1}^L > 0. \tag{16}$$

Households who invest in deposits must provide funds a period in advance on which they receive the nominal rate  $i_{t-1}^D$ . Households who arrange a credit line can instead invest the funds in bank equity that yield  $i_{t-1}^h$ , but must then pay the fee  $i_{t-1}^L$ . The inequality says that inside money is costly and households' optimal choice is then to hold as little money as necessary.

On the bank side, we assume for simplicity that the same collateral is required to back a dollar of credit line and a dollar of deposits. Moreover, the distribution of liquidity shocks is the same per dollar of both payment instruments. Since the cost of liquidity (16) is also

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<sup>12</sup>We assume that an interest rate  $i^D$  is earned on deposits regardless of whether they are spent, and that the fee  $i^L$  is paid on credit lines regardless of whether they are drawn. These assumptions help simplify the algebra. More detailed modeling of the fee structure of different payment instruments is possibly interesting but not likely to be first order for the questions we address in this paper.

the same, banks are similarly indifferent between issuing the two instruments. In particular, both issuing deposits and issuing equity together with extending credit lines is cheaper than just issuing equity. Appendices A.1 and A.2 formally state problems with credit lines for households and banks, respectively, and works through the algebra.

## 4 Graphical analysis

To obtain simpler formulas for steady state values, and in line with our interpretation of the model period as short, we now let the period length shrink to zero. Appendix A.6 reformulates the model with a variable period length and characterizes the steady state in the limit. The result is summarized by

*Proposition 3. As the period length becomes short, steady state bank ratios must lie on a pair of curves in the  $(\lambda, \kappa)$ -plane, the liquidity management curve*

$$\delta - (i^R - \pi) = -c'(\kappa) + (1 - G(\lambda)) (c(\kappa) - c'(\kappa)(\kappa - 1)) \quad (17)$$

*and the capital structure curve*

$$\kappa = \frac{\lambda(1+b) + \rho \frac{1+\bar{\ell}(\lambda, \kappa)}{\delta + \rho c'(\kappa)} \frac{x}{Y} + f(\lambda)}{1 + f(\lambda)}, \quad (18)$$

where  $\bar{\ell}(\lambda, \kappa) = \ell(\lambda, \lambda, \kappa, 0)$  is the total steady state leverage cost as a share of consumption.

The liquidity management curve is downward sloping for  $\lambda < \bar{\lambda}$  and constant for  $\lambda \geq \bar{\lambda}$ . The capital structure curve is upward sloping if leverage costs are sufficiently small as a share of output. A unique steady state with  $\kappa > 1$  exists if holding reserves is sufficiently costly ( $i^R - \pi$  low) and bank collateral is sufficiently scarce ( $\rho x$  small).

The interest rate on bonds, the real price of trees and the interest rate on deposits are given by

$$i = \delta + \pi + c'(\kappa), \quad q = \frac{x}{\delta + \rho c'(\kappa)}, \quad (19)$$

$$i^D = \delta + \pi + c(\kappa) - c'(\kappa)\kappa + (c(\kappa) - c'(\kappa)(\kappa - 1)) \int_{\lambda}^{\bar{\lambda}} \tilde{\lambda} dG(\tilde{\lambda}). \quad (20)$$

The proof is in Appendix A.6. We now describe where the curves come from and then use them for graphical analysis. Figure 3 plots both curves. It divides the  $(\lambda, \kappa)$ -plane into two regions: in the white region with  $\lambda < \bar{\lambda}$ , reserves are scarce, banks' liquidity constraint sometimes bind and there is an active interbank credit market. In the yellow shaded region

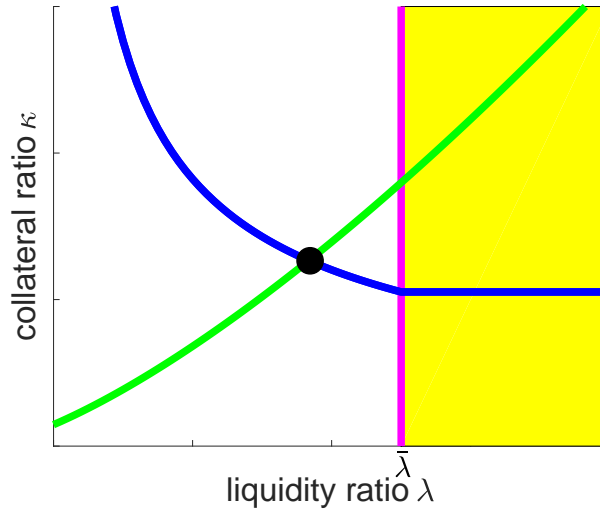


Figure 3: The blue line is the liquidity management curve. The green line is the capital structure curve. In the yellow shaded region, reserves are abundant,  $\lambda \geq \bar{\lambda}$ .

with  $\lambda \geq \bar{\lambda}$ , reserves are abundant, bank liquidity constraints never bind and no bank borrows overnight. Equilibrium could in principle be in either region.

The figure can also be used to track several other variables that are simple functions of  $\lambda$  and  $\kappa$ . From (19), the real interest rate on bonds is monotonically increasing in the collateral ratio, so it can be read along the vertical axis. Similarly, the tree price declines in  $\kappa$ . Moreover, the short run response of inflation in (15) depends importantly on the change in the money multiplier  $1/\lambda$  that can be inferred from movement along the horizontal axis.

The **liquidity management curve** is derived from banks' first order condition for reserves in (10). Banks' opportunity cost of holding reserves on the left-hand side is equal to the sum of the collateral benefit and the expected liquidity benefit of reserves on the right-hand side. A liquidity benefit accrues only if there is a chance that banks' liquidity constraint binds, that is, if  $\lambda < \bar{\lambda}$  and hence  $G(\lambda) < 1$ .

In general equilibrium, the liquidity benefit equals the leverage cost saved by not accessing the overnight credit market. The function  $c(\kappa) - c'(\kappa)(\kappa - 1)$  is decreasing in  $\kappa$ : if banks are better collateralized, the cost of overnight borrowing is lower. A bank that runs out of reserves must turn to the overnight market to make payments and incurs a marginal cost of leverage  $c(\kappa) - c'(\kappa)\kappa$ . At the same time, the borrowing bank's overnight credit serves as collateral for the lending bank. By (19), the collateral benefit of overnight credit is  $-c'(\kappa)$  and lowers the



equilibrium overnight rate. The cost of borrowing is the difference between the two terms.

Figure 3 displays the liquidity management curve as a dark blue line. Intuitively, the liquidity management answers the question: “how much liquidity  $\lambda$  should banks choose if their collateral ratio is  $\kappa$ ”? At a lower  $\kappa$ , borrowing overnight is more costly, so that banks choose to hold a higher liquidity ratio to lower the probability that they have to borrow. As a result, the curve slopes down. Once reserves are abundant, banks never borrow overnight and the optimal collateral ratio is a constant, determined by the opportunity cost of reserves.

**Banks’ demand for money.** The liquidity-management curve (17) can be viewed as a money demand function. From the bank Euler equation (10), the collateral ratio  $\kappa$  moves one-for-one with the overnight interest rate. Banks’ desired liquidity ratio  $\lambda$  is thus decreasing in the opportunity cost of holding reserves  $i - i^R$  as long as reserves are scarce. If leverage cost is a small share of output, the real quantity of deposits is effectively pinned down by output  $Y$ , and banks’ real demand for reserves  $\lambda C$  moves largely with the liquidity ratio: it slopes down when reserves are scarce and becomes perfectly elastic at  $i = i^R$  when reserves are abundant.

We emphasize that bank demand for outside money as described by the liquidity-management curve is quite different from the demand for inside money. In the current version of the model, the demand for inside money is inelastic at  $Y$ . As a result, the demand for outside money does not depend on the liquidity preference of households. Instead, it is shaped by banks’ technology for handling payments. For example, when bank’s money demand becomes flat depends on the maximal liquidity shock  $\bar{\lambda}$  banks can face, as opposed to say, the level of real balances at which households are satiated.

The **capital structure curve** exploits the fact that the ratios are connected via bank balance sheets: it says how large a liquidity ratio is needed to achieve a collateral ratio  $\kappa$  given the other collateral banks have access to. Formally, the numerator on the right-hand side sums up the value of bank collateral relative to deposits while the denominator does the same for debt. A simple special case is that of narrow bank that only holds reserves and government bonds: we then have  $\kappa = \lambda (1 + b)$ , a straight line.

Two more subtle effects further contribute to an upward slope. First, banks with higher liquidity ratios run out of reserves less often, which results in lower outstanding interbank credit  $f(\lambda)$  and hence a higher collateral ratio. Since every dollar of interbank credit is both an asset and a liability in the banking sector, a reduction in interbank credit also increases overall collateral. This effect makes the curve steeper but is active only when reserves are scarce.

The second effect is that the collateral ratio affects the market price of trees and hence the value of “real” collateral. Indeed, the second term in the numerator contains the ratio of the

value of trees  $q$  relative to consumption. If  $\kappa$  is higher, banks compete less for collateral: fruit are discounted at a higher rate and prices are lower. The result is again a steeper slope of the curve. The effect diminishes as  $\kappa$  rises since  $c$  is convex.

There is also a force that could in principle generate a downward slope. Since total steady state leverage cost is decreasing in both  $\kappa$  and  $\lambda$ , larger  $\kappa$  will generate lower consumption ( $\bar{\ell}(\lambda, \kappa)$  increases), and hence fewer transactions. As a result, more collateral is available relative to transactions, and if the effect is strong enough then less  $\lambda$  is needed to achieve the higher  $\kappa$ . However, this effect is small if leverage costs are a small share of output. Since we view the magnitude of leverage costs as similar to that of an interest rate spread, we are comfortable with assuming that they are sufficiently small.

We obtain a unique steady state if inside money is sufficiently scarce. We have already seen above that the return on reserves must be below  $i_t^h$  for an equilibrium in which insider money is scarce. If the difference is too small and other high quality real collateral is too readily available, then we may not have a steady state with that property. We thus focus on economies in which banks' access to real collateral is sufficiently limited.

## 4.1 Financial structure and the payment system

This section highlights three properties of a layered payment system as studied in this paper. First, the cost of liquidity differs across layers and policy only indirectly affects the cost of liquidity in the bank customer layer. Second, how many reserves it takes to achieve "abundance" depends on the organization of the banking system, in particular on interbank netting agreements. Finally, institutional details – in particular, netting and the denomination of collateral – matter for the cost of liquidity and the connection between asset prices and inflation.

**Cost of liquidity.** In many monetary models, the cost of liquidity to all agents is measured by the spread between the short interest rate and the interest rate on money. In a liquidity trap with  $i = i^R$ , bonds and money are then perfect substitutes for all agents. In a layered payment system, the spread  $i - i^R$  only represents the cost of liquidity of banks. When reserves are abundant, bonds and outside money are perfect substitutes *for banks*. However, the spread  $i - i^R$  does not matter directly to households, who do not hold reserves and do not participate in the overnight credit market.

The cost of liquidity for end users is given by the opportunity cost of deposits (20). In general, it depends on banks' liquidity and collateral ratios. It reflects the spread between bonds and reserves only to the extent banks pass on the cost of producing liquidity to their customers. For example, when reserves become abundant, the cost of liquidity falls. However, even in that case it remains positive: it is still true that liquidity relies on costly bank leverage,

so deposits and other assets are not perfect substitutes for households.

**Interbank netting and abundance vs scarcity of reserves.** How many reserves does it take to make reserves abundant? Relatedly, if a central bank wants to reduce its balance sheet to return to scarcity, how far does it have to go? Our model says that the answer depends crucially on how the banking system handles liquidity shocks. One concrete way in which banks influence the size of liquidity shocks is by participating in netting systems that economize on the use of reserves. We now illustrate the role of how the banking system is organized by introducing netting into the model.<sup>13</sup>

The liquidity constraint (5) assumes that all bank payments must be made with reserves. Suppose that banks can also use intraday credit  $I_t$  to pay, but that their intraday credit position is limited by a downpayment into the netting system that has to be made in reserves, either held overnight or borrowed from other banks. We thus replace the liquidity constraint by

$$\begin{aligned}\tilde{\lambda}_t D_{t-1} &= M_{t-1} + F_t + I_t, \\ I_t &\leq v (M_{t-1} + F_t)\end{aligned}$$

and subtract intraday credit  $I_t$  from the date  $t$  cash flow (8).

A bank that runs out of reserves will exhaust its intraday credit limit before tapping the overnight market, and will continue to borrow as little as possible. We can thus combine the two constraints into

$$\tilde{\lambda}_t D_t \leq (1 + v) (M_t + F_{t+1}). \quad (21)$$

The bank's problem is then to maximize shareholder value subject to (21). The appendix shows that the structure of the solution is the same as above, but with the new liquidity ratio (6) defined as  $\lambda_t := (1 + v) M_t / D_t$ . The price level is then determined as  $P_t = M_{t-1} (1 + v) / \lambda_{t-1} Y_t$ .

The new formula for  $\lambda_t$  links the amount of reserves required to achieve abundance (that is,  $\lambda_t \geq \bar{\lambda}$ ) to the extent of interbank netting: more efficient netting (higher  $v$ ) implies that fewer reserves are sufficient to deliver abundance. In general, the fewer interbank payments are required to handle a given volume of end-user transactions, the fewer reserves are needed a medium of exchange in the bank layer. At the same time, the extent of netting also affects the capital-structure curve, an effect discussed below.

**Nominal collateral, netting, interest rates and inflation.** In a layered payment system, bank balance sheets provide a link between asset prices – the value of collateral held by banks

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<sup>13</sup>More generally, the changes in the industrial organization of banking can have similar effects. For example, if banking becomes more concentrated through, say, a merger wave, we would also expect fewer interbank payments to handle the same flow of bank customers' transactions and hence a lower demand for reserves.

– and the nominal price level, which is determined by banks’ issuance of inside money. In our model, this link is represented by the slope of the capital-structure curve: it connects the collateral ratio  $\kappa$  – and hence, from the bank Euler equation and the valuation of trees (19), the rates of return on assets held by banks – to the liquidity ratio  $\lambda$  that drives the money multiplier and hence the price level. We now show how financial structure – in particular the denomination of collateral and the presence of netting – shapes this link.

To clarify the role of nominal collateral, we allow households to also issue bonds. This introduces a second type of bonds: it is issued by households and promises a payment stream of  $X_t = nM_t$  dollars. We think of this tree as, say, nominal long term mortgages; payments grow with the supply of reserves to ensure a constant share of nominal trees in steady state. Other than the denomination of payments, nominal trees work exactly like the “real trees” studied so far – both enter the collateral ratio with weight  $\rho$ . In equilibrium, nominal trees are thus held exclusively by banks, and their nominal value is  $nM_t / (\delta + \pi + \rho c'(\kappa))$ .

Allowing for netting and nominal collateral does not affect the liquidity-management curve: banks’ choice of ratios given prices does not change. However, the appendix derives the new capital-structure curve as

$$\kappa = \frac{\frac{\lambda}{1+v} \left( 1 + b + \rho \frac{n}{\delta+g+\rho c'(\kappa)} \right) + \rho \frac{1+\bar{c}(\lambda, \lambda, \kappa, g)}{\delta+\rho c'(\kappa)} \frac{x}{Y} + \frac{f(\lambda)}{1+v}}{1 + \frac{f(\lambda)}{1+v}}. \quad (22)$$

Comparing with our previous expression for the capital-structure curve (18), there are three differences: more efficient netting (higher  $\gamma$ ) makes the curve flatter, whereas more nominal collateral (higher  $n$ ) makes it steeper. Moreover, since nominal trees are long term, the growth rate of nominal liabilities now enters the equation as well.

Intuitively, financial structure determines the link between collateral and liquidity ratios because the price level feeds back to the real value of collateral on the balance sheet. Indeed, a lower money multiplier  $1/\lambda$  depresses the price level  $P_t = M_{t-1} (1 + \gamma) / Y_t \lambda_{t-1}$  and thereby increases the real value of all nominal assets. If there are nominal assets other than reserves, then this revaluation is stronger and it takes a smaller decline in  $1/\lambda$  to achieve any given increase in  $\kappa$ : the curve becomes steeper. In contrast, if there is more netting then a lower money multiplier  $1/\lambda$  depresses the price level by less, the revaluation is weaker and the curve is flatter.

Importantly, what matters for revaluation is only the denomination of collateral, not the identity of the borrower. The slope of the capital-structure curve increases both when there are more nominal trees, which we interpret as private liabilities denominated in dollars, and when there are more nominal government bonds (higher  $b$ ). The feature that the quantity of

nominal bonds matters for the price level connects our model to the fiscal theory of the price level. However, the role of government debt here is different: it is only one of the various assets that banks use to back inside money, and this is what gives it a special role.

## 5 Monetary/fiscal policy and interest on reserves

We now study the effect of government policy on equilibrium asset prices and inflation. The government has two distinct policy tools. First, it can set the real return on reserves  $r^R = i^R - \pi$ : it controls the nominal rate on reserves  $i^R$ , and it can change the inflation rate  $\pi$  via the growth rate of nominal government liabilities. Second, the government can alter the mix of collateral available to banks. For example, the central bank can perform open-market trades in government bonds, thus changing the bond-reserve ratio  $b$ . It could also purchase other assets such as trees.

A key feature of our model is that the two policy tools affect asset prices and inflation in different ways. A quick way to see this is that the real return on reserves  $r^R$  affects only the liquidity-management curve (17) but not the capital-structure curve (18), whereas the opposite is true for the bond-reserve ratio  $b$ . We now discuss the policy tools in turn. We pay particular attention to another key feature of the model: the magnitude of policy effects depends on financial structure.

### 5.1 The real return on reserves

Changes in the real return on reserves capture two policy shifts that may at first sight appear quite different. Before the financial crisis, most countries did not pay interest on reserves ( $i^R = 0$ ) and real returns on reserves differed only because of inflation: a higher growth rate of government liabilities increases the inflation rate and lowers the real return on reserves. More recently, many central banks operate in an environment of abundant reserves and actively change the nominal reserve rate  $i^R$  while committing to a stable path of inflation.

Our framework emphasizes the common denominator of these two policy changes. An expansionary policy lowers the real return on reserves and effectively taxes banks' liquidity more, which lowers banks' overall return on assets. Banks respond by adjusting their leverage and portfolio choice. To maintain a return  $\delta$  on equity, banks increase their leverage, or lower their collateral ratio  $\kappa$ . Mechanically, the liquidity-management curve shifts down, as in both panels of Figure 4, where the shifted curve is shown as a dashed line. Since banks value collateral less, they also lower their liquidity ratio – mechanically, we move down to a new equilibrium along the capital-structure curve. A lower real return on reserves thus reduces both ratios,  $\lambda$  and  $\kappa$ .

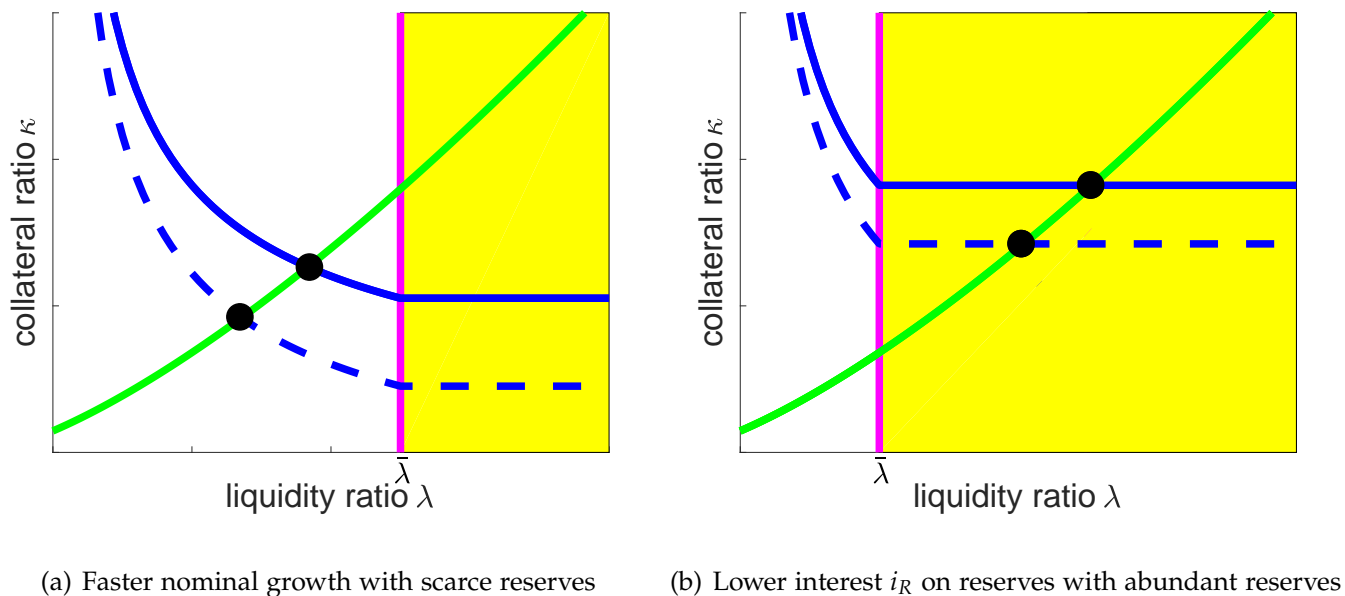
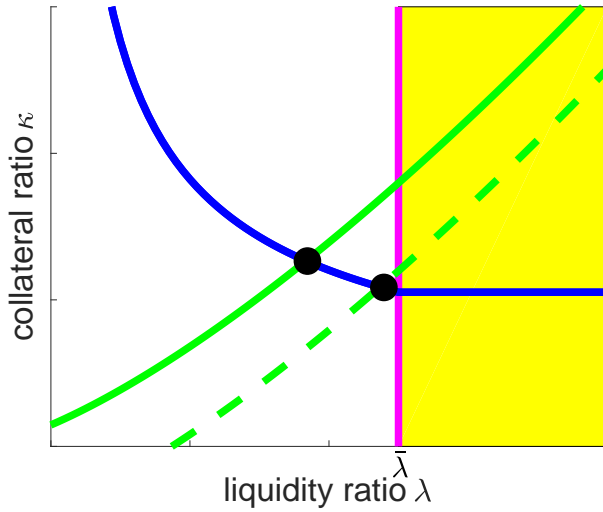


Figure 4: Expansionary monetary policy lowers the real reserve rate

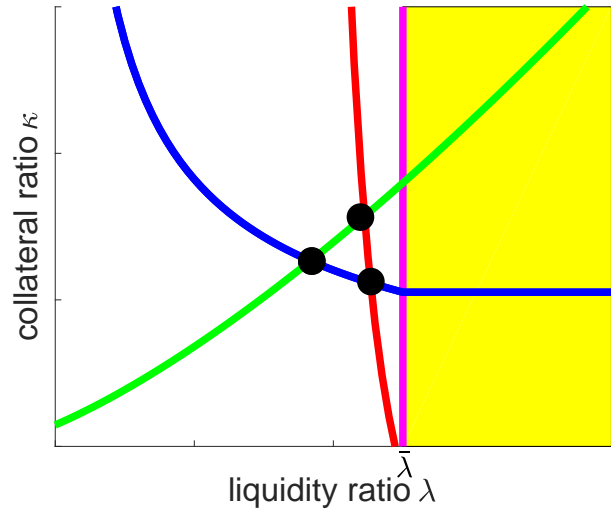
The magnitude of the impact of an expansionary monetary policy depends on the slope of the capital structure curve. When the curve is steep, the liquidity ratio  $\lambda$  drops by less and the money multiplier  $1/\lambda$  increases by less, leading to less inflation in the short run. The curve is steeper when banks hold more nominal collateral, or there is less efficient netting.

The policy also has a *permanent* effect on real asset prices – in particular the real overnight rate and deposit rate fall, and the price of trees increases. The reason is that a higher tax on bank assets makes deposit production more costly and banks pass the higher costs on to customers in the form of lower deposit rates, whether reserves are abundant or not. Moreover, the higher tax on bank assets reduces the collateral ratio, which makes collateral assets more valuable in the new steady state. This is in contrast to many models with sticky prices or segmented markets, where “liquidity effects” on the real interest rate are temporary phenomena. Permanent effects arise in our model because the opportunity costs of holding reserves change the way banks produce inside money, with effects on the cost of leverage and the value of collateral.

**Negative interest rates on reserves.** Our framework does not require that the nominal or real interest rate on reserves be positive. This is due to our technological assumption that banks rely on reserves for payment and always hold reserves in equilibrium. In particular, banks cannot handle payments or hold collateral by converting reserves into currency. We



(a) Open market operations with scarce reserves



(b) The red curve is equation (23)

Figure 5: More expansionary monetary policy and interest-rate policies

believe this is a sensible assumption as long as nominal rates on reserves are not too low. If the government chooses negative rates in Figure 4, it can in principle lower interest rates so as to move all the way into the scarce reserve region.

## 5.2 The collateral mix

The second policy tool of the government is changing the mix of collateral available to banks with open market trades. In a traditional open market purchase, the government buys back bonds from banks in exchange for reserves. In our framework, such a trade alters the balance sheet link between collateral and liquidity ratios: as bonds are no longer available to back inside money, it takes more reserves to achieve any given collateral ratio. Mechanically, the capital-structure curve shifts to the right.

The effect of traditional open market policy on bank portfolios depends crucially on whether reserves are scarce or abundant. With scarce reserves, banks with higher liquidity ratios have to borrow overnight less often and therefore choose lower collateral ratios as well. We thus move to a new equilibrium along the downward sloping portion of the liquidity-management curve, as in the left panel of Figure 5. In contrast, when reserves are abundant the collateral ratio remains fixed and banks absorb the additional reserves to increase the liquidity ratio  $\lambda$ .

More formally, consider a comparative static that increases reserves and offsets this change by an equal change in bonds: we move from an initial equilibrium with  $(B_0^g, M_0)$  to a new equilibrium with  $(\tilde{B}_0^g, \tilde{M}_0)$ , where  $\tilde{M}_0 - M_0 = B_0^g - \tilde{B}_0^g > 0$ . As before, reserves and bonds then grow at the same rate  $\pi$  throughout. For graphical analysis, this expansionary open-market operation can be summarized by a decrease in the bond-reserve ratio from  $b$  to  $\tilde{b}$  which shifts the capital-structure curve to the right by (18). There is no effect on the liquidity-management curve which does not depend on  $b$ .

If reserves are abundant in the initial equilibrium, then they must still be abundant in the new equilibrium. As a result, the policy has no effect on the collateral ratio and asset prices. With abundant reserves, both reserves and short bonds are equally valued by banks as collateral only; there is no liquidity benefit from reserves. A trade that leaves total government liabilities  $M_0 + B_0^g$  unchanged must therefore be irrelevant. This result is consistent with analysis in existing monetary models – here it is derived in a two layer system where the demand for outside money comes from banks. We emphasize that the irrelevance result does *not* hold for a sufficient large open market *sale*. This is because a large increase in  $b$  could move the new equilibrium into the scarce reserve region.

What about inflation? Any open market trade that keeps reserves abundant has no effect on the price level. This result follows from the capital structure curve (18) and the determination of the price level (15). By construction, the policy maintains the same nominal path of government liabilities, that is  $\tilde{M}_0(1 + \tilde{b}) = M_0(1 + b)$ . The initial growth rate of reserves is therefore  $1 + g_0 = (1 + b) / (1 + \tilde{b})$ . Moreover, banks prefer to maintain the same *real* value of government liabilities – they want to keep leverage constant. In the abundant reserve region, the liquidity-management curve is flat and  $f(\lambda) = 0$  so that  $\tilde{\lambda}(1 + \tilde{b}) = \lambda(1 + b)$ . From (15), the higher supply of outside money is therefore exactly offset by a drop in the money multiplier. Constant bank leverage further implies that asset prices do not move.

In contrast, when reserves are scarce as in Figure 5, then a purchase of bonds with reserves is inflationary in the short run and permanently lowers the real interest rate. The short run inflation response is subtle since the policy again both increases the quantity of outside money and lowers the money multiplier. However, with scarce reserves, banks with higher liquidity ratios choose to reduce the collateral ratio. Lower demand for collateral reduces the real value of government liabilities. The money multiplier thus falls by less than the growth rate of outside money and the price level increases in the short run.

Graphically, consider moving horizontally at the original  $\kappa$  to the new capital-structure curve. Such a move would increase the liquidity ratio to keep the real value of government liabilities  $\lambda(1 + b)$  constant, which is what happens with abundant reserves. In the scarce reserve region, however, the liquidity-management curve slopes down, so that the new equi-



librium liquidity ratio is lower. In particular, the percentage change in  $\lambda$  is now less than the change in outside money growth given by the policy.

### 5.3 Interest rate policies

Many central banks conduct monetary policy by following a nominal interest rate rule. In practice, the rule is typically implemented by open-market policy. For example, during the scarce reserves regime in place in the U.S. until 2008, the New York Fed's trading desk bought and sold bonds of various maturities in exchange for reserves in order to move the overnight interest rate (the Federal Funds rate) close to the Fed's target. More recently, as reserves have become abundant, the Fed Funds rate and the interest rate on reserves have been essentially the same, and the policy lever is the reserve rate. It is then tempting to simply transfer existing analysis of interest-rate rules to the abundant reserves environment even though the policy implementation is different.

In many monetary models, the details of how the central bank implements the interest-rate rule are irrelevant; the nominal interest rate alone summarizes the stance of monetary policy. In particular, many models use households' optimal choice between currency and short-term bonds to derive optimal real balances as a function of the nominal interest rates and consumption. At the same time, intertemporal asset pricing equations – and possibly price setting equations – imply a path for inflation. The path for the money supply can then be inferred ex post so as to generate the implied path for real balances, but is often omitted from the analysis altogether. In particular, it does not matter whether policy is implemented with open-market purchases or interest on reserves.

In our model, policy cannot be summarized by the nominal interest rate alone. As discussed above, policy matters in two ways. Policy can either change the real return on reserves or the collateral mix between reserves and government bonds, which matters as long as reserves are scarce. Both policy actions affect the overnight nominal interest rate. We now show that, with scarce reserves, the same nominal interest rate can be achieved with many combinations of interest on reserves and open-market purchases that have different implications for real interest rates and inflation. Moreover, we show that the reserve rate is not sufficient to characterize policy with abundant reserves.

**Scarce reserves.** We start from an initial equilibrium in the region with scarce reserves with some values for  $i^R$ ,  $\pi$  and  $b$  and an initial overnight rate. Holding fixed  $i^R$ , we now choose a new target overnight rate  $i$  that is below the initial overnight rate (but still above the reserve rate) and ask how  $\pi$  and  $b$  can change to implement it. To proceed graphically, we combine the first-order condition for overnight lending (10) and the liquidity-management curve (17)

to trace out all equilibrium pairs  $(\lambda, \kappa)$  that are consistent with the target spread  $i - i^R$ :

$$i - i^R = (1 - G(\lambda)) (c(\kappa) - c'(\kappa)(\kappa - 1)). \quad (23)$$

This spread is the opportunity cost of holding reserves rather than lending overnight. It must be equal to the liquidity benefit of reserves on the right-hand side since the collateral benefits of the two assets are the same.

The curve in  $(\lambda, \kappa)$  plane described by equation (23) is displayed as a red line in the right panel of Figure 5. It has four key properties. First, it is downward sloping, much like the liquidity-management curve with scarce reserves: at a given spread, banks with higher liquidity ratios choose lower collateral ratios since they run out of reserves less often. Second, the red curve never enters the abundant reserves region: as reserves become more abundant, collateral must fall to maintain a positive spread.

Third, the red curve lies above the liquidity-management curve at the initial equilibrium. This is because the new target overnight rate and hence the new target spread are below the initial overnight rate and spread, respectively. For the old target spread, the red line would pass through the initial equilibrium. To lower the spread, a given liquidity ratio  $\lambda$  must be associated with more collateral. Finally, we note that the red curve is independent of  $\pi$  and  $b$ , the two parameters describing policy with scarce reserves.

How can the government change policy to move to the new lower overnight rate, that is, to shift equilibrium onto the red line? The new equilibrium pair  $(\lambda, \kappa)$  must lie on the capital structure and liquidity-management curves. The policy analysis above thus suggests two simple options. First, the government could announce a lower growth rate of nominal liabilities  $\pi$ , and thereby shift the liquidity-management curve up until all three curves intersect. Second, the government could purchase bonds in the open market and thereby shift the capital-structure curve to the right until all three curves intersect.

Both policies produce the same change in the overnight nominal rate. At the same time, they have very different implications for the real interest rate and inflation. As discussed in the previous section, lower growth of government liabilities lowers inflation and increases the real interest rate. In contrast, open market purchases of bonds increase inflation and lower the real interest rate. With open market purchases, the inflation response is short term only, so that the new lower nominal interest target is achieved via a lower real rate. In contrast, with lower outside money growth the higher real rate is offset by lower inflation.

In addition to the two extreme policies just sketched, many other policies are also consistent with the new target nominal overnight rate. These policies combine open-market purchases with lower future growth of liabilities and thereby shift both curves at once, rather than one

at a time as for the extreme policies. The only requirement on the shifts is that the new equilibrium ends up on the curve described by equation (23). In particular, the same nominal interest rate is compatible with an entire range of bank liquidity ratios in equilibrium.

A key difference between our model and other models of scarce outside money is that the *only* medium of exchange for end users is inside money produced by banks. Outside money – here reserves – is only one input into the production of inside money. In particular banks also use government bonds as collateral to back inside money. As a result, the spread between the overnight rate and the reserve rate measures the scarcity of reserves for banks; it does not measure the scarcity of inside money in the economy as a whole. In particular, there is not a unique amount of reserves implied by a given volume of transactions and a spread.

**Abundant reserves.** Can the government describe policy with an interest rate rule for  $i = i^R$  when reserves are abundant? Equilibrium is described by (10) and (18), which determine  $\lambda$  and  $\kappa$  for a given real return on reserves. As a result, a nominal reserve rate alone cannot pin down bank liquidity and collateral. Similarly, a feedback rule that relates the rate on reserve to inflation, for example  $i = g(\pi)$ , does not uniquely determine  $\lambda$ ,  $\kappa$ , inflation and the real interest rate. This is true even if we directly select a rule for the real rate as a function of  $\pi$ , thus eliminating possible multiplicity coming from the shape of  $g$  that has been discussed in the literature.

Our model differs from other monetary models in what happens once outside money becomes abundant. Consider first models with bonds and currency only. At the zero lower bound in such models, bonds and outside money become perfect substitutes to bank customers, so the medium of exchange (currency) loses its liquidity benefit. In our model, end users hold neither bonds nor outside money – both are held only by banks. Equating  $i$  and  $i^R$  makes bonds and outside money perfect substitutes for banks, but does not remove the liquidity benefit of the medium of exchange, inside money produced by banks.

There are also models in which reserves, bonds and currency coexist. In such models,  $i = i^R > 0$  makes bonds and reserves perfect substitutes. At the same time, currency remains a scarce medium of exchange that is relevant for some transactions. The reserve rate represents the spread between reserves and the medium of exchange; it measures the scarcity of currency and relates the demand for real balances to real variables such as consumption. The tradeoff between currency and reserves is what enables those models to work with interest rate rules in the usual way even when reserves are abundant.

## 5.4 Optimal policy

The optimal payment system minimizes the loss of resources due to leverage. Our technological assumptions say that a given volume of transactions  $T$  requires a fixed amount of deposits supplied by banks. The question for society is how those deposits should be backed. The steady state consumption loss is summarized by the leverage cost as a share of consumption  $\bar{\ell}(\lambda, \kappa)$  defined in Proposition 3. Provided that the leverage cost of the government slopes up fast enough, the indifference curves of this loss function are upward sloping and convex, as shown in Figure 6.

We consider the best steady state equilibrium that the government can select by choice of its two policy instruments, the collateral mix represented by  $b$  and the real return on reserves  $i_R - \pi$ , which determines the opportunity costs of holding reserves. The optimal policy problem is to choose those instruments together with  $\lambda$  and  $\kappa$  to minimize losses  $\bar{\ell}(\lambda, \kappa)$  subject to the capital-structure curve (18) and the liquidity-management curve (17).

If the government can freely choose the ratio of bonds to reserves, it is optimal to set  $b = 0$ . Indeed, while bonds and reserves provide the same collateral services, reserves also provide liquidity services, which lowers the need for interbank borrowing and hence costly bank leverage. Since the model focuses on the provision of payment services, there is no benefit of government bonds per se, nothing is lost by just issuing reserves. More generally, the way fiscal policy is conducted independently of monetary policy may imply that there is a constraint on  $b$ . We can then view the welfare costs as a function in  $\lambda$  and  $\kappa$  with  $b$  a fixed parameter.

The return on reserves directly affects neither the welfare cost nor the capital-structure curve. We can therefore find the optimal solution in two steps. We first find a point  $(\lambda, \kappa)$  on the capital-structure curve (for given  $b$ ) that minimizes  $\bar{\ell}(\lambda, \kappa)$ . The optimal real return on reserves is then whatever return shifts the liquidity-management curve so that the equilibrium occurs precisely at that optimal point. If the indifference curves are convex and the capital-structure curve is curved less – which is a reasonable assumption if the effects of interbank credit are relatively small – then we obtain an interior solution as shown in Figure 6.

**Should reserves be abundant?** The figure suggests that this is not necessarily the case. If the government leverage cost curve slopes upward very steeply, then it may be optimal to run a system with scarce reserves, in which real government leverage is much lower than the amount of debt needed to run the payment system. In this case, it is better to have banks rely on other collateral in order to back inside money. However, if the government can borrow cheaply at will, so that its leverage cost is close to zero, then it makes sense to move towards narrow banking where reserves make up the lion's share of bank portfolios.

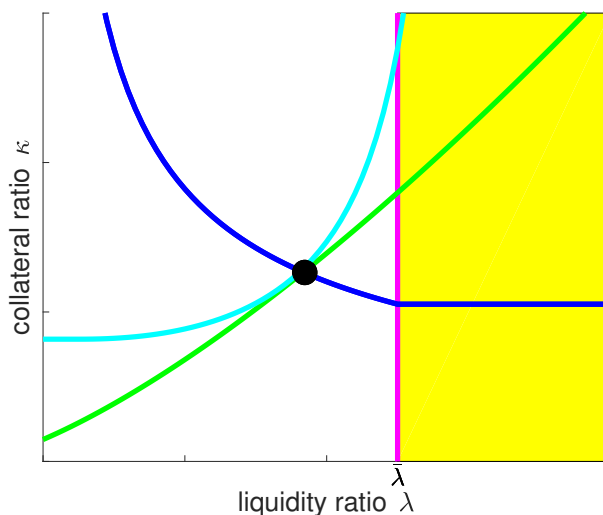


Figure 6: The cyan line is an indifference curve of the loss function  $\bar{\ell}(\lambda, \kappa)$  from Proposition 3.

## 6 Securities markets and the payment system

This section introduces uncertainty premia, the key source of fluctuations in asset prices, and institutional investors. We then study the effect of uncertainty on the supply of inside money as well as unconventional monetary policy – the government buys trees that carry uncertainty premia. Moreover, we study a version of the model with two types of asset management firms: *carry traders* buy trees on margin, whereas *active traders* face liquidity constraints for some asset purchases. Both types of firms otherwise work like banks: they are competitive firms owned by households that have access to a subset of trees and maximize shareholder value.

### 6.1 Introducing uncertainty and collateral quality

We capture a change in uncertainty as a change in beliefs about asset payoffs: we assume that households behave *as if* tree dividends  $x$  are permanently lower by  $s$  percent from the next date on. Actual tree payoffs remain constant throughout. One way to think about these beliefs is that households are simply pessimistic. Our preferred interpretation is ambiguity aversion: households contemplate a range of models for payoffs, and evaluate consumption plans using the worst case model. In either case, the key effect of pessimistic valuation is to generate premia on assets: an observer (such as an econometrician measuring the equity premium) will observe low prices relative to payoffs and hence high average returns.

To define equilibrium, we must take a stand not only on beliefs about exogenous variables, but also about endogenous variables such as the nominal price level and asset prices. We follow Ilut and Schneider (2015) who also capture the presence of uncertainty with low

subjective mean beliefs about exogenous variables: beliefs about endogenous variables follow from agents' knowledge of the structure of the economy. In particular, agents know the policy rule of the government and that banks maximize shareholder value given the households' discount factor. Households' worst case beliefs thus also affect bank decisions; shareholder value (4) is replaced by its worst case expectation.

Appendix A.7 characterizes steady state equilibrium with uncertainty. Mechanically, agents live in a steady state with all variables constant, yet act *as if* the economy is on a transition path to a worst case steady state with lower tree payoffs in the future. In general, dynamics are characterized by first finding the transition path to derive the law of motion for endogenous variables, and then combining that law of motion with the true dynamics of the exogenous variables. We show that the transition path converges to the worst case steady state after one period, and that the bank ratios in the actual and worst case steady state coincide. These properties allow simple graphical analysis, as in the case without uncertainty.

**Uncertainty premia on assets.** Asset prices reflect uncertainty in two ways. First, agents act *as if* payoffs will be lower and hence value trees less. At the same time, however, they discount future payoffs at a lower rate. This is because they understand the quantity equation, know that the government is committed to a certain growth rate  $g$  of nominal liabilities, and therefore fear that lower overall output generates higher inflation. Real returns on nominal assets thus reflect *worst case* inflation  $\tilde{\pi}$ , which is higher than actual inflation. In particular, since the nominal rate on reserves is fixed, the cost of liquidity next period is perceived to be high.

To see the effect of both forces on asset prices, consider the value of a tree held by households. A drop in tree payoff of  $s$  percent implies that agents expect output to be permanently lower by  $\tau s$  percent, where  $\tau = x / (\Omega + x)$  is the share of tree payoffs in output. If a tree was held by households, its steady state price would be

$$\frac{Q}{P} = u(s) \frac{x}{\delta}; \quad u(s) = \frac{1-s}{1-\tau s}. \quad (24)$$

The factor  $u$  reflects compensation for uncertainty. If  $s = 0$ , then  $u(s) = 1$ , and the price is the present value  $x/\delta$ . The same result obtains if  $\tau = 1$ : if all output comes from trees, then the cash flow and discount rate effects on asset prices exactly offset. In the interesting case where tree payoffs represent some intermediate share of output,  $u$  is strictly between zero and one, so uncertainty lowers prices.

To see how uncertainty generates premia on assets, consider an econometrician who ob-

serves tree prices as well as payoffs. The return on trees measured in steady state is

$$\frac{Q/P + x}{Q/P} = 1 + \delta/u(s).$$

As payoff uncertainty  $s$  increases,  $u$  declines and the return on the tree increases to compensate investors. If there was also a second “safe” tree held by households that earns exactly the discount rate, the econometrician would measure an equity premium on the uncertain tree. In terms of comparative statics, an increase in uncertainty captured by an increase in  $s$  leads to higher premia and lower prices.

**Uncertainty and collateral quality.** It is natural to assume that uncertain trees are also worse collateral. To capture this effect, we make the weight that trees receive in the aggregation of collateral explicitly a decreasing function  $\rho(s)$  of payoff uncertainty  $s$ . A change in uncertainty thus has two effects on banks’ tree portfolios. There is a direct effect on prices that lowers the total value of collateral available to banks. In addition, uncertainty makes trees worse collateral per dollar of funds invested in them.

Since uncertain trees still provide some collateral benefits, banks continue to hold all trees in an equilibrium with uncertainty. From the first order condition for trees – derived in the appendix – the value of a tree held by banks is

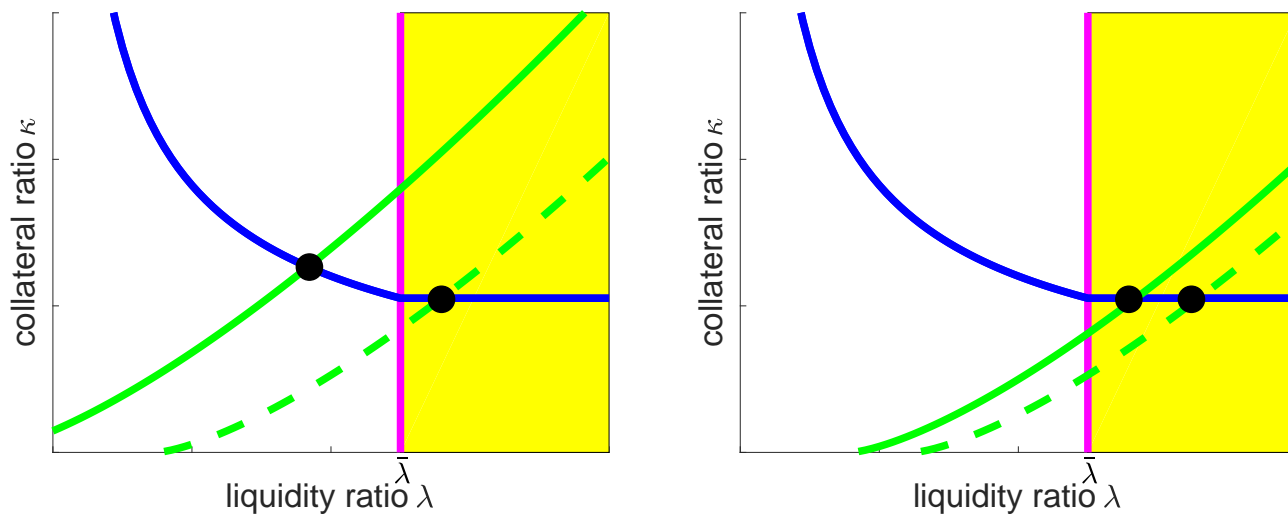
$$q = \frac{u(s)x}{\delta + \rho(s)c'(\kappa)}. \quad (25)$$

Comparing to the frictionless price in (24), an increase in uncertainty lowers the price more since it must compensate banks not only for a lower expected payoff, but also for lower collateral quality.

**Characterizing equilibrium with uncertainty** We can study equilibria with uncertainty using the same graphical analysis as in the previous section. The appendix shows that we only need to replace the value of trees  $q$  in the capital-structure curve (18) by the new valuation formula (25). The liquidity-management curve and the determination of the price level are not affected by changes in  $s$ . The richer model allows us to discuss the effects of an increase in uncertainty on asset prices and inflation; this is taken up in the next section.

## 6.2 An increase in uncertainty and policy responses

How does the payment system respond to an increase in uncertainty? An increase in  $s$  lowers the value of uncertain trees (25) and hence the amount of collateral banks can use to back deposits. Since less other collateral is available, it takes a larger liquidity ratio to achieve any



(a) Higher uncertainty shifts the capital structure curve.

(b) Quantitative easing: central bank buys trees.

Figure 7: Higher uncertainty and unconventional monetary policy.

given collateral ratio  $\kappa$ , and the capital structure moves to the right as in Figure 7. As long as reserves are scarce, banks hold more liquidity and thus choose lower collateral ratios.

The spillover from asset markets to the payment system via bank balance sheets thus leads to deflation as the money multiplier declines. At the same time, the scarcity of collateral pushes the real interest rate down. If the uncertainty shock is sufficiently strong, the economy can move all the way into the abundant reserve region where the overnight interbank market shuts down. We can thus arrive at abundance of reserves even if policy (described the standard central bank tools  $i_R - \pi$  and  $b$ ) does not change.

An increase in uncertainty is an attractive candidate for a shock that could have occurred at the beginning of the recent financial crisis. It is consistent with an increase in asset premia, a drop in uncertain asset prices, a decline in the overnight interest rate all the way to the reserve rate as well as a decline in bank collateral and an effective shutdown of interbank Federal Funds lending. However, we did not see a large deflation – after an initial small drop in late 2008 the price level remained quite stable over time.

**One-time expansion of government liabilities.** A candidate for the absence of strong deflation during the financial crisis is a one-time increase in nominal government liabilities. Suppose that the Treasury issues a lot of new debt, some of which is then purchased by the central bank so as to keep the ratio  $b$  of debt held by the public to reserves constant. Suppose



further that this is perceived as a one time change, with a stable path of nominal liabilities thereafter.

Bank portfolios do not react to a one-time increase in government liabilities; our two curves do not shift. An increase in government liabilities is neutral: the only response is that the price level increases proportionately to maintain the same real value of government liabilities. The real interest rate and leverage do not change, and the economy remains in the abundant reserves regime. A joint increase in uncertainty and government debt can thus move the economy into a period of abundant reserves with low asset prices, collateral and real rates, but stable inflation.

**Unconventional monetary policy.** An alternative response by central banks to a decline in asset values has been to purchase low quality collateral, such as risky mortgage backed securities. We start from an initial equilibrium with abundant reserves  $M_0$ , price level  $P_0$  and collateral ratio  $\kappa$ . Suppose the government injects reserves to purchase all risky trees from banks' balance sheet, that is, new reserves are chosen such that, at the new equilibrium with reserves  $\tilde{M}_0$ , price level  $\tilde{P}_0$  and collateral ratio  $\tilde{\kappa}$ , we have

$$\tilde{M}_0 - M_0 = \frac{\tilde{P}_0 u(s) x}{\delta + \rho(s) c'(\tilde{\kappa})}.$$

The effect of the purchase is displayed in the right panel of Figure 7. As trees are removed from bank balance sheets, the capital-structure curve moves to the right.

After the additional injection, reserves continue to be abundant, so the policy has no effect on the collateral ratio. However, the policy does increase the price level and thus counteracts the deflationary effect of higher uncertainty. In the new equilibrium, collateral is  $\tilde{M}_0/\tilde{P}_0$  which equals the real value of old reserves  $M_0/\tilde{P}_0$  plus the full real value of trees. In contrast, collateral in the initial equilibrium was given by the real value of reserves  $M_0/P_0$  as well as the value of trees multiplied by the collateral quality weight  $\rho(s) < 1$ . Since the collateral ratio is the same in the two equilibria, it must be that  $\tilde{P}_0 > P_0$ .

Unconventional policy thus works by replacing low quality real collateral on bank-balance sheets with high quality nominal collateral. Since reserves continue to be abundant and the real return on reserves stays the same, this does not actually lead to an increase in real collateral. However, backed by the new reserves, banks provide more inside money, which is inflationary in the short run. Compared with an outright increase in reserves, the inflationary effect of tree purchases is smaller as trees are removed from the collateral pool.

Tree purchases by the central bank have two additional, more subtle, effects. First, it makes the capital-structure curve steeper, which reduces the impact of any changes in the reserve

rate on short-run inflation. Second, removing trees from bank balance sheets reduces banks' exposure to further shocks to asset quality. For example, suppose that once all trees have been bought by the government, the uncertainty shock is reversed and asset prices recover. The payment system would not benefit from this recovery as banks no longer have any trees on their balance sheet. In this case, the economy will remain in an abundant reserves environment after a financial crisis.

### 6.3 Carry traders

So far, the effect of asset values on bank balance sheets is direct: it requires bank investment in trees. In this section, we introduce institutional investors who borrow short-term from banks in order to invest in trees. We call these investors "carry traders" – they do not actively trade trees but roll over their debt. This creates an additional link between asset markets and banks that operates even if banks only engage in short-term lending. Monetary policy can affect carry traders' funding cost.

Carry traders are competitive firms that issue equity and borrow overnight. They invest in a special set of trees that banks do not have access to. We denote the fruit from carry trade trees by  $x^*$ . Like banks, carry traders face leverage costs, captured by a decreasing convex function  $c^*$  that could be different from the cost function  $c_b$  assumed for banks.<sup>14</sup> The collateral ratio of a carry trader is defined as the market value of his tree holdings divided by overnight credit  $F_t^*$ :

$$\kappa_t^* = \frac{Q_t^* \theta_t^*}{F_t^*}.$$

We assume further that carry traders are more optimistic about the payoff of trees than households: they perceive uncertainty  $s^* < s$  whereas households perceive  $s$ . The idea here is that the firm employs specialized employees whom households trust to make asset-management decisions. As a result, the spread relevant for investment in carry trader trees — indirectly through investment by carry traders — carries the lower uncertainty premium  $s^*$ .

**Optimal investment and borrowing.** Carry traders' first-order condition for overnight borrowing resembles that of banks (equation (A.8) in the appendix), except that it does not provide liquidity benefits: the return on equity must be smaller than the real overnight rate plus the marginal cost of leverage. We focus on steady states only and drop time subscripts. Since we already know that the real rate is lower than  $\delta$  in equilibrium, it is always optimal

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<sup>14</sup>We do not consider welfare effects of leverage for carry traders – instead we focus on the positive implications of margin trading. We thus assume for simplicity that leverage costs of carry traders are paid lump sum to households so that they have no impact on welfare.

for carry traders to borrow and we directly write the condition as an equality:

$$\delta = i - \pi + c^* (\kappa) - c^{*'}(\kappa^*)\kappa^*. \quad (26)$$

Carry traders' collateral ratio is higher in equilibrium when interest rates are high.

Like banks, carry traders hold all trees accessible to them. This is due not only to the collateral benefit conveyed by trees, but also to carry traders' relative optimism. From carry traders' first order condition, the value of trees held by carry traders

$$q^* = \frac{u^* x^*}{\delta + c^{*'}(\kappa^*)}; \quad u^* = \frac{1 - s^*}{1 - s_y}$$

where  $s_y$  is the worst case loss of output; it depends on the weighted average of losses on the different trees in the economy. When interest rates or uncertainty is low, carry traders apply a lower effective discount rate to trees, which results in higher tree prices.

The amount of carry trader borrowing in steady state equilibrium is

$$F^* = q^* / \kappa^*. \quad (27)$$

Lower interest rates increase both leverage and the value of collateral, and therefore increase borrowing. Moreover, an increase in uncertainty (that is, an increase in  $s^*$ ) lowers collateral values and borrowing.

**Equilibrium with carry traders.** Our graphical analysis of equilibrium remains qualitatively similar when carry traders are added to the model. The only change is that carry trader borrowing now enters on the asset side of the banking sector. We can thus add in the numerator of the collateral ratio (18) a term  $B^* (\kappa)$  that expresses carry trader borrowing as a function of bank collateral. We obtain the function  $B^*$  by substituting for  $\kappa^*$  in (27) from (26) and then substituting for the interest rate from the bank first-order condition from (10). The function  $B^*$  is decreasing: if banks have more collateral, the interest is higher and carry traders borrow less. The leverage cost incurred by carry traders also lowers equilibrium consumption – as with the level of bank leverage costs above, we treat this effect as negligible for the shape of the capital structure curve.

We can now revisit the effect of changes in beliefs and monetary policy in an economy with carry traders. Suppose first that, starting from an equilibrium with scarce reserves, there is an increase in uncertainty. The new effect is that, as carry traders value trees less, they demand fewer loans from banks. This lowers bank collateral and shifts the capital-structure curve to the right. The liquidity-management curve does not change. In the new equilibrium, bank

collateral is even lower and the interest rate is lower, as is the price level. The deflationary effect therefore amplifies the increase in uncertainty about bank trees considered earlier. The additional prediction is that we should see a decline in funding of institutional investors with short-term credit from payment intermediaries, such as a decline in repo extended by money-market mutual funds to broker-dealers.

It is also interesting to reconsider the effect of monetary policy. Suppose policy engineers a change in the mix of bank assets or their value that lowers the real overnight interest rate. Carry traders borrow more and bid up the prices of the trees they invest in. As one segment of the tree market thus increases in value, the aggregate value of trees also rises: there is a tree market boom. Importantly, this is not a real interest rate effect: the discount rate of households, which is used to value trees held by households, is unchanged. The effect comes solely from the effect of monetary policy on the overnight rate and hence on carry traders' funding costs.

## 6.4 Active traders

Carry traders provide collateral to banks and thus affect the supply of inside money. We now introduce another group of institutional investors with a demand for inside money. "Active traders" periodically rebalance their portfolios as their view of tree payoffs changes, and they require cash to pay for new trees. The money demand of active traders counteracts the supply side effects due to changes in the collateral values or the borrowing by carry traders. At the same time, monetary policy also affects their funding costs.

Active traders are competitive firms that issue equity and hold inside money as well as a special set of trees denoted  $\hat{\Theta}$ . There are many active traders and each is optimistic about one particular "favorite" tree: the trader perceives uncertainty  $\hat{s} < s$  about this tree. In contrast, households and other active traders perceive uncertainty  $s$ , but they trust that when they invest through active traders, they are exposed to less uncertainty,  $\hat{s} < s$ . Active traders also perceive uncertainty  $s$  about all other trees. Every period, the identity of the favorite tree within the subset  $\hat{\Theta}$  changes to some other tree in the subset in an iid fashion.

To generate a need for inside money, we assume that active traders must pay for new tree purchases with prearranged payment instruments or intraday credit. Active trader  $i$  faces the liquidity constraint

$$\int_{\hat{\Theta}} Q_t^j \hat{\theta}_{i,j,t} dj = I_{i,t} + \hat{D}_{i,t-1} + \hat{L}_{i,t-1},$$

where  $I_{i,t}$  is the intraday credit position,  $\hat{D}_{i,t-1}$  are deposits that the fund keeps at its bank together with credit lines  $\hat{L}_{i,t-1}$ .

Like a bank, active trader  $i$  faces a limit on intraday credit

$$I_{i,t} \leq \hat{\nu}(\hat{D}_{i,t-1} + \hat{L}_{i,t-1}),$$

where  $\hat{\nu}$  is a parameter that governs netting in tree transactions. It is generally different from the parameter  $\nu$  that was introduced to describe netting among banks in (21), since it captures netting by a clearing and settlement system for the securities that active traders invest in.

Active traders choose inside money, trees and their shareholder payout. We focus on equilibria in which every active trader always holds only his favorite tree; we can assume that the perceived uncertainty on other trees is high enough. Since money is costly – the deposit rate is below the discount rate – active traders hold as little money as necessary in order to purchase the entire outstanding amount of their new favorite tree in case the identity of their favorite tree changes. It follows that the intraday credit limit binds in equilibrium, a form of “cash-in-the-market pricing”.

**Optimal investment and deposits.** Much like households, active traders equate their marginal liquidity benefit to the marginal cost of money holdings, given by the opportunity cost of deposits  $i_t^h - i_t^D$  or equivalently the interest rate on credit lines  $i_{t-1}^L$ . The liquidity benefit in turn is due to traders’ ability to invest in their favorite tree, which carries a return that compensates them for cost of liquidity. As in the previous section, tree prices also reflect uncertainty about output and hence inflation. It is again convenient to denote the worst loss of output by  $s_y$  which depends on the weighted average of losses on the different trees in the economy.

The steady state price of trees held by active traders can then be written as

$$\hat{q} = \hat{u} \frac{\hat{x}}{\delta + \frac{i^L}{1+\hat{\nu}}}; \quad \hat{u} := \frac{1 - \hat{s}}{1 - s_y}.$$

Here the factor  $\hat{u}$  is again compensation for uncertainty, which is less than one if the expected payoff loss on active trader trees is larger than the expected output drop. The second term in the denominator shows that prices reflect traders’ need for inside money: prices are higher when their cost of liquidity  $i^L$  is lower and netting is more efficient (higher  $\hat{\nu}$ ).

Equilibrium money holdings by active traders are proportional to the market value of active traders’ favorite trees:

$$\hat{D} + \hat{L} = \frac{\hat{q}}{1 + \hat{\gamma}}.$$

The money demand by active traders is interest elastic, in contrast to the inelastic demand from households. This is a stark way to capture the idea that financial institutions responds

more strongly to changes in liquidity costs.

Since the household and active trader sector differ in their money demand, the share of inside money used in the goods versus the asset market changes over time. We define active traders' money share as

$$\hat{\alpha} = \frac{\hat{D} + \hat{L}}{\hat{D} + \hat{L}} = \frac{\hat{q} / (1 + \hat{v})}{C + \hat{q} / (1 + \hat{v})}.$$

If bank customers' liquidity becomes cheaper, the value of active traders' trees increases and their share of the total supply of inside money goes up. In equilibrium, the share is an increasing function  $\hat{\alpha}(\lambda, \kappa)$  of the two bank ratios  $\lambda$  and  $\kappa$ , assuming as before that the impact of  $\lambda$  and  $\kappa$  on equilibrium consumption is small. The key force here is that higher  $\kappa$  and  $\lambda$  lower the cost of end-user liquidity and hence boost the value of active traders' trees.

**Equilibrium with active traders.** We focus on local changes to equilibria with abundant reserves. Our graphical analysis of equilibrium bank ratios remains qualitatively similar when active traders are added to the model. The key difference is the effect of active traders on the nominal price level. In steady state, the price level grows at the rate  $g$  according to

$$P_t = \frac{M_t}{\lambda C} (1 - \hat{\alpha}(\lambda, \kappa)).$$

A higher active trader share thus works like lower velocity. If the cost of liquidity is lower, then active traders absorb more inside money. As less money is used in the goods market, the price level declines.

We can now revisit the effect of changes in beliefs and monetary policy in an economy with active traders. Suppose first that there is an increase in uncertainty. As active traders value trees less, they demand less money. This increases the collateral ratio of banks and shifts the capital-structure curve to the left. The liquidity-management curve does not change. In the new equilibrium, bank collateral and the interest rate are higher, as is the price level. In other words, active traders are a force that generates the opposite response to a change in uncertainty from banks and carry traders. Since in the typical economy all traders are present to some extent, we can conclude that their relative strength is important. An additional prediction is that we should see a decline in money – either deposits or credit lines – provided to institutional investors.

We can also reconsider the effect of monetary policy. Suppose once more that policy lowers the real overnight interest rate. The opportunity cost of holding deposits falls and active traders demand more money. At the same time, they bid up the prices of the trees they invest in. Again a segment of the tree market increases in value and the aggregate value of trees also rises: there is a tree market boom. Again the effect is not due a change in the discount rate,

but instead a change in the funding cost: here it affects active traders' strategy which requires money in order to wait for trading opportunities.

## 7 Related literature

In this section we discuss how our results relate to existing work in monetary economics.

### *Balance sheet effects and government liabilities*

In our model, welfare costs derive from "balance sheet effects" and policy matters by changing the asset mix in the economy. This theme is familiar from other work on unconventional monetary policy. Several papers study setups where banks are important to channel funds to certain borrowers. By purchasing the bonds of these borrowers, policy can effectively substitute public credit when weak balance sheets constrain private credit (e.g. (Cúrdia and Woodford, 2010), (Christiano and Ikeda, 2011), (Gertler and Karadi, 2011), (Gertler, Kiyotaki and Queralto, 2012).<sup>15</sup> Our model differs from this literature in how banks add value – their special ability is not lending, but the handling of payment instructions.

Since the price level depends on the supply of payment instruments, shocks to bank assets have deflationary effects in our model. If all payment instruments are taken to be deposits, we obtain a collapse of the money multiplier along the lines of Friedman and Schwartz (1963). Brunnermeier and Sannikov (2016) also consider the link between asset values and the supply of inside money by banks. In their model, banks' special ability is to build diversified portfolios and deposits are a perfect substitute to outside money as a store of value. In contrast, in our model inside money is a medium of exchange for bank customers, and outside money works like an intermediate good for producing inside money, rather than a substitute.

### *Asset pricing and money*

In our model, collateral benefits generate market segmentation. We thus arrive endogenously at "intermediary asset pricing" equations that are reminiscent of those in He and Krishnamurthy (2013) or Bocola (2016). Unlike our banks, banks in those models are investors with limited net worth who are assumed to hold some assets because they have special investment abilities.

The interaction of liquidity and collateral benefits in our model also generates permanent liquidity effects. In contrast, the literature on monetary policy with partially segmented asset markets (for example, Lucas 1990, Alvarez, Atkeson and Kehoe 2002) obtains temporary liquidity effects; collateral benefits play no role there.

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<sup>15</sup>Buera and Nicolini (2014) also consider the effect on monetary policy on balance sheets in a model of entrepreneurs who face collateral and cash-in-advance constraints.

Asset values in our model depend on the cost of inside money to institutional investors. A permanent effect of monetary policy on asset values also obtains in Lagos and Zhang (2014) where the inflation tax discourages trade between heterogeneous investors; this alters which investor prices assets in equilibrium. The effect we derive is different because the cost of liquidity to bank customers is not captured by the inflation tax; instead the cost of inside money depends on banks' cost of leverage. In particular, our mechanism is also operative when reserves are abundant.

### *Bank liquidity management and monetary policy*

With scarce reserves, bank liquidity management matters for asset valuation, policy impact and welfare in our model. The liquidity management problem arises because banks cannot perfectly insure against liquidity shocks due to customer payment instructions, as in Bhattacharya and Gale (1987).<sup>16</sup> Recent work has discussed the interaction of monetary policy and liquidity management with scarce versus abundant reserves (for example, Whitesell 2006, Keister, Martin and McAndrews 2008)<sup>17</sup>. While these papers consider more detail that is useful to understand the cross section of banks, our stylized model tries to capture the main tradeoff and its interaction with other features of the economy.

Several papers have incorporated bank liquidity management into DSGE models. Cúrdia and Woodford (2011) study optimal monetary policy in a New Keynesian model.<sup>18</sup> In their setup, reserve policy can be stated in terms of a rule for the overnight interest rate and the reserves rate; there is no need to formulate policy in terms of the quantity of reserves. Our setup is different because of market segmentation: the nominal interest rate is not directly connected to a household marginal rate of substitution, but rather to bank leverage. Rules for interest rates are then not enough to characterize the behavior of inflation – the supply of nominal government liabilities is also relevant.

Bianchi and Bigio (2014) study a quantitative model in which banks have a special ability to lend and face a perfectly elastic demand for debt as well as idiosyncratic liquidity shocks. Monetary policy changes the tradeoff between reserves and interbank credit and hence the

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<sup>16</sup>A related literature asks whether government supplied liquidity is useful when firms cannot perfectly insure shocks to investment opportunities (for example, (Woodford, 1990), (Holmström and Tirole, 1998)). An alternative approach to bank liquidity, following Diamond and Dybvig (1983), considers optimal contracts offered by banks to end users. This approach typically abstracts from interbank transactions; the focus is instead on optimal dependence of contracts on end user liquidity needs, given information problems as well as the scope for multiple equilibria that include bank runs.

<sup>17</sup>Similar tradeoffs have been developed in the literature on the dynamics of the Federal Funds market (e.g. Ho and Saunders 1985, Hamilton 1996, Afonso and Lagos 2015.)

<sup>18</sup>Other notions of bank liquidity have also been explored in DSGE settings. Gertler and Kiyotaki (2011) consider a model in which bank borrowing not only depends on bank net worth but also is fragile and subject to runs. Del Negro, Eggertsson, Ferrero and Kiyotaki (2017) study a model in which assets become illiquid in the sense of being harder to sell, as in Kiyotaki and Moore (2008).



willingness of banks to make loans. In contrast, the demand for inside money in our model comes from its role as a medium of exchange for goods and securities; monetary policy affects the cost of money holdings to bank customers, not only to banks.

In Drechsler, Savov and Schnabl (2018), banks are investors with relatively low risk aversion who issue debt subject to aggregate liquidity shocks. Monetary policy changes the cost of self-insurance via reserves and thereby affects banks' willingness to take leveraged positions in risky assets as well as the risk premium on those assets. In our model, monetary policy affects not only the funding cost of banks, but also that of banks' institutional clients; the two channels have opposite effects on uncertainty premia.

The role of interest on reserves as a policy tool has recently received renewed attention. A number of papers ask when the price level remains determinate ((Sargent and Wallace, 1985), (Hornstein, 2010), (Ennis, 2018)). Woodford (2012) and Ireland (2014) consider macroeconomic effects of interest on reserves in a New Keynesian framework. Kashyap and Stein (2012) consider a model with a financial sector; they emphasize the presence of quantity and price tools for macroeconomic and financial stability, respectively.

#### *Multiple media of exchange and liquidity premia*

While our model allows for both deposits and credit lines in the bank-customer layer, we assume that those instruments are perfect substitutes. An interesting related literature asks which instruments are used in which transactions. In particular, Telyukova and Wright (2008) consider a model in which both credit and money are used and explain apparently puzzling cost differences with convenience yields. Lucas and Nicolini (2015) distinguish currency and interest bearing accounts and show that a model that makes this distinction can better explain the relationship between interest rates and payment instruments. Nosal and Rocheteau (2011) survey models of payment systems.

In our model, asset values reflect collateral benefits to banks and hence indirectly benefits to the payments system. Moreover, government policy can matter by changing the scarcity of collateral that effectively backs inside money. Similar themes appear in "new monetarist" models with multiple media of exchange; these are covered in a recent survey by Lagos, Rocheteau and Wright (2017). In models based on Lagos and Wright (2005), assets that are useful in decentralized exchange earn lower returns. Several papers have recently studied collateralized IOUs as media of exchange, following Kiyotaki and Moore (2005).

For example, in Williamson (2012), Williamson (2014) some payments are made with claims on bank portfolios that contain money, government bonds or private assets; banks moreover provide insurance to individuals against liquidity shocks. Rocheteau, Wright and Xiao (2018) consider payment via money or government bonds (or, equivalently in their setup, IOUs se-

cured by bonds). These models give rise to regimes of scarcity or abundance for each medium of exchange. The real effects of scarcity can be different for, say, bonds and money because money is used to purchase a different set of goods.

While we also study how policy affects the scarcity of different assets like bonds and reserves, the mechanisms we emphasize as well as our welfare conclusions differ in important ways.<sup>19</sup> Indeed, in our model only one medium of exchange helps in bank-customer transactions: inside money supplied by banks. Since any bank commitment is costly, inside money is never abundant and collateral to back it is always scarce – only the degree of scarcity changes and affects welfare. In contrast, reserves can be scarce or abundant depending on their role in bank liquidity management.

In our two layer setup, whether scarcity affects asset prices or welfare thus depends crucially on features of the banking system. For example, scarcity of bonds has different effects from scarcity of reserves because reserves change banks' liquidity management problem and the leverage costs of tapping the overnight market. In addition, the price level in our model is related to the supply of inside money by banks and hence the nominal collateral that banks hold. For example, the quantity of nominal collateral available to banks shapes the price level response to policy.

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<sup>19</sup>In addition, the new monetarist literature considers explicit models of decentralized exchange, rather than reduced form liquidity constraints as we do. At the same time, Andolfatto and Williamson (2015) consider a cash-in-advance model with bonds and money and show that several key effects from the more complex papers can be seen already there.

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# A Appendix

In this appendix we provide all the equations to accompany the analysis in the text. Sections A.1 and A.2 derive household and bank first order conditions, respectively. Section A.3 proves Proposition 1. Section A.4 derives a system of equations characterizing equilibrium. Sections A.5 and A.6 proves Propositions 2 and 3, respectively. Section A.7 introduces uncertainty and Section A.8 adds active traders.

## A.1 Household optimization

Households can invest in deposits, bonds, trees and bank equity. In addition, they can take out credit lines. We define  $Y_t^b$  as the aggregate real dividends of the banking sector, and write the price of a claim to  $Y_t^b$  as  $Q_t^b$ . Households then choose  $(C_t, D_t^h, L_t^h, B_t^h, \theta_t^h, \theta_t^{h,b})$  to maximize utility

$$\sum_{t=0}^{\infty} \beta^t C_t$$

subject to the budget constraint

$$\begin{aligned} P_t C_t + B_t^h + D_t^h + Q_t \theta_t^h + Q_t^b \theta_t^{h,b} + i_{t-1}^L L_{t-1}^h \\ \leq P_t \Omega_t + B_{t-1}^h (1 + i_{t-1}) + D_{t-1}^h (1 + i_{t-1}^D) + (Q_t + P_t x_t) \theta_{t-1}^h + (Q_t^b + P_t Y_t^b) \theta_{t-1}^{h,b}. \end{aligned}$$

Households further have to respect the cash-in-advance constraint

$$P_t C_t \leq D_{t-1}^h + L_{t-1}^h.$$

At date  $t$ , purchases of consumption goods are constrained by deposits and credit lines arranged the period before. The fee  $i_{t-1}^L$  for a credit line that allows for consumption at date  $t$  enters the date  $t$  budget constraint.

We write the multiplier on the budget constraint as  $\omega_t/P_t$  where  $\omega_t$  is the marginal utility of wealth. Similarly, we write the multiplier on the cash-in-advance constraint as  $\mu_t/P_t$ . It is also convenient to define real bank equity and tree prices  $q_t^b = Q_t^b/P_t$  and  $q_t = Q_t/P_t$ . The household first order conditions for consumption, bonds, deposits, credit lines, bank equity

and trees are then

$$\begin{aligned}
1 &= \omega_t + \mu_t, \\
\omega_t &\geq \beta \omega_{t+1} \frac{1 + i_t}{1 + \pi_{t+1}}, \\
\omega_t &\geq \beta \left( \omega_{t+1} \frac{1 + i_t^D}{1 + \pi_{t+1}} + \frac{\mu_{t+1}}{1 + \pi_{t+1}} \right), \\
\omega_{t+1} i_t^L &\geq \mu_{t+1}, \\
q_t^b \omega_t &\geq \beta \omega_{t+1} (q_{t+1}^b + Y_{t+1}^b), \\
q_t \omega_t &\geq \beta \omega_{t+1} (q_{t+1} + x_{t+1}).
\end{aligned}$$

The FOCs for assets are inequalities since no asset can be sold short. The FOC for an asset is satisfied with equality if the household indeed holds the asset. Similarly, the inequality for credit lines is satisfied with equality if households actually arrange any credit lines.

As usual, the pricing kernel for assets with real payoffs is given by the marginal rate of substitution for wealth at date  $t$  versus  $t + 1$ . We define

$$\hat{\beta}_t := \beta \frac{\omega_{t+1}}{\omega_t} \tag{A.1}$$

as the date  $t$  price of a hypothetical asset that pays off one unit of consumption goods at date  $t$ . Because of the cash-in-advance constraint, the marginal utilities of wealth and consumption need not coincide in general. In steady state, however, we always have  $\hat{\beta}_t = \beta$ .

Deposits and credit lines are perfect substitutes as payment instruments. As a result, households are indifferent as long as they imply the same liquidity cost – otherwise they pick only the cheaper instrument. Indifference obtains if

$$\frac{1 + \pi_{t+1}}{\hat{\beta}_t} - (1 + i_t^D) = i_t^L. \tag{A.2}$$

The liquidity cost from deposits on the left hand side reflects the spread between a nominal asset that does not provide liquidity (and hence has real return  $1/\hat{\beta}_t$ ) and deposits. The liquidity cost from credit lines on the right hand side reflects the cost of arranging a credit line.

The cash-in-advance constraint binds as long as the cost of liquidity is positive. Below we will focus on the case where the liquidity prices  $i^D$  and  $i^L$  always satisfy (A.2). We denote the

nominal rate on an asset held by households as

$$i_t^h = \frac{1 + \pi_{t+1}}{\hat{\beta}_t} - 1, \quad (\text{A.3})$$

so that the household cost of liquidity is given by the spread  $i_t^h - i_t^D$ .

The first order conditions further imply that the date  $t$  cost of liquidity reflects the difference between the marginal utilities of wealth and consumption at date  $t + 1$ :

$$i_t^h - i_t^D = \frac{1}{\omega_{t+1}} - 1. \quad (\text{A.4})$$

If liquidity is more expensive then it is more difficult to transform wealth into consumption and the gap between these marginal utilities widens.

## A.2 Bank optimization

We now consider the bank optimization problem. We allow for credit lines and interbank netting. To account for these features, we rewrite the bank liquidity constraint and the definition of the collateral ratio as

$$\begin{aligned} \tilde{\lambda}_t(D_{t-1} + L_{t-1}) &\leq (1 + \nu)(M_{t-1} + F_t), \\ \kappa_t &= \frac{M_t + \rho Q_t \theta_t + B_t}{D_t + L_t + F_t}. \end{aligned} \quad (\text{A.5})$$

Here  $\nu \geq 0$  is a parameter that captures the efficiency of netting arrangements among banks.

Banks maximize (4) subject to (A.5) and the budget constraint

$$\begin{aligned} P_t y_t^b &= i_{t-1}^L L_{t-1} + M_{t-1} (1 + i_{t-1}^R) - M_t - D_{t-1} (1 + i_{t-1}^D) + D_t \\ &\quad + (B_{t-1} - F_{t-1}) (1 + i_{t-1}) - (B_t - F_t) + ((Q_t + P_t x_t) \theta_{t-1} - Q_t \theta_t) \\ &\quad - c(\kappa_t) (D_t + L_t + F_t). \end{aligned} \quad (\text{A.6})$$

**First order conditions.** We write the multiplier on the banks' liquidity constraint (the first inequality in A.5) as  $\tilde{\mu}_t/P_t$ . We also define the price of trees in units of consumption goods as  $q_t = Q_t/P_t$ . On the asset side, banks' first order conditions for reserves, trees and bonds



chosen at date  $t$  are

$$\begin{aligned}
\hat{\beta}_t \frac{1 + i_t^R}{1 + \pi_{t+1}} - c'(\kappa_t) + (1 + \nu) \hat{\beta}_t \frac{E_t \tilde{\mu}_{t+1}}{1 + \pi_{t+1}} &\leq 1, \\
\hat{\beta}_t \frac{q_{t+1} + x_{t+1}}{q_t} - \rho c'(\kappa_t) &\leq 1, \\
\hat{\beta}_t \frac{1 + i_t}{1 + \pi_{t+1}} - c'(\kappa_t) &\leq 1.
\end{aligned} \tag{A.7}$$

Since banks cannot sell assets short, the FOCs are inequalities. The FOC for an asset holds with equality if the bank actually participates in the market for that asset.

On the liability side, the FOCs for deposits and overnight credit are

$$\begin{aligned}
\hat{\beta}_t \frac{1 + i_t^D}{1 + \pi_{t+1}} + c(\kappa_t) - c'(\kappa_t) \kappa_t + \hat{\beta}_t \frac{E_t \tilde{\mu}_{t+1} \tilde{\lambda}_{t+1}}{1 + \pi_{t+1}} &\geq 1, \\
\hat{\beta}_t \frac{1 + i_t}{1 + \pi_{t+1}} + c(\kappa_t) - c'(\kappa_t) \kappa_t - \tilde{\mu}_t (1 + \nu) &\geq 1.
\end{aligned} \tag{A.8}$$

Again, the FOCs are inequalities that are satisfied with equality if the bank actually issues the liability. Comparing the last equations in (A.7) and (A.8), we have that in the range where  $\kappa_t > 1$ , banks borrow overnight only if the liquidity constraint binds.

Finally the first order condition for credit lines is

$$\hat{\beta}_t \frac{-i_t^L}{1 + \pi_{t+1}} + c(\kappa_t) - c'(\kappa_t) \kappa_t + \hat{\beta}_t \frac{E_t \tilde{\mu}_{t+1} \tilde{\lambda}_{t+1}}{1 + \pi_{t+1}} \geq 0.$$

As long as (A.2) holds, banks are indifferent between offering credit lines and deposits as payment instruments.

### A.3 Proof of Proposition 1

We prove Proposition 1 for the case without credit lines. As discussed in the previous section, (A.2) implies that both households and banks are indifferent between credit lines and deposits. For any equilibrium without credit lines, we can therefore find a suitable fee  $i_t^L$  such that there is an equilibrium for the economy with credit lines.

An equilibrium consists of consumption, prices, household portfolio and bank balance sheet positions such that households and banks optimize and markets clear.

Property (i) follows from the household FOCs in Appendix A.1. In particular, equation (A.4) says that  $i^h > i_t^D$  implies  $\omega_{t+1} < 1$  which means that the multiplier  $\mu_{t+1}$  on the cash-in-

advance constraint must be positive.

To show property (ii), we first note that some banks must be active in equilibrium. Indeed, banks are the only issuers of deposits which are necessary for consumption. Moreover, banks are the only agents who can invest in reserves. It follows that the first FOCs in both (A.7) and (A.8) must hold with equality for some banks.

It follows that  $i_t^R < i_t^h$ . Indeed, if this is not true, then the bank FOC for reserves in (A.7) cannot hold for any finite collateral ratio.

There cannot be an equilibrium in which households hold trees or bonds. Indeed, suppose households were to hold, say, bonds at date  $t$ . From the household FOCs, the real interest rate on bonds would have to be  $1/\hat{\beta}_t - 1$ . Since active banks have finite collateral ratios  $\kappa_t$  and the leverage cost function is strictly increasing, the FOC for bonds is then violated for any active bank. An analogous argument works for trees.

Given that banks hold reserves, bonds and trees, the FOCs for these assets must hold with equality. Property (iii) then follows directly from these equalities.

Consider property (iv). In any equilibrium with  $i_t^R \leq i_t$ , all banks choose the same collateral ratios. Indeed, suppose there are two banks such that bank 1 chooses a higher collateral ratio than bank 2. Since  $c$  is strictly convex,  $-c'(\kappa)$  is strictly decreasing. The FOCs for bonds and trees thus imply that bank 1 cannot hold either of these assets. Bank 1 must therefore hold only reserves. However, if its entire asset portfolio consists of reserves, then its expected multiplier  $\tilde{\mu}_{t+1}$  is zero. But since  $i_t^R \leq i_t$ , bank 1 should not hold any reserves, a contradiction.

In any equilibrium with  $i_t^R \leq i_t$ , all banks also choose the same liquidity ratios  $\lambda_t$ . Indeed, since banks choose the same collateral ratios, (A.8) together with the first-order condition for bonds (A.7) with equality implies that the Lagrange multiplier on the liquidity constraint is the same for all constrained banks and equal to

$$\tilde{\mu}_t^c := \frac{1}{1+\nu} (c(\kappa_t) - c'(\kappa_t)(\kappa_t - 1)). \quad (\text{A.9})$$

Since moreover the distribution of liquidity shocks is iid across banks, the conditional distribution of  $\tilde{\mu}$  one period ahead is also the same: the multiplier is zero with probability  $G(\lambda)$  and equal to  $\tilde{\mu}_t^c$  otherwise. Since banks hold reserves, the first FOC in (A.7) holds with equality and implies equal  $\lambda$ s across banks.

Property (v) follows from the first order condition, as explained in the text.

Property (vi) starts from the market clearing condition which equates output to consump-

tion and leverage costs

$$Y_t = C_t + c(\kappa_t) (D_t + F_t) / P_t + c_g(\kappa_t^s) (M_t + B_t^s) / P_t$$

With the definitions for the liquidity ratio  $\lambda_t = M_t / D_t$  and the growth rate of reserves  $g_t = M_t / M_{t-1} - 1$ , we can write the ratio of real bank debt to consumption as

$$\frac{(D_t + F_t) / P_t}{C_t} = \frac{D_t + F_t}{D_{t-1}} = \frac{D_t}{M_t} \frac{(1 + g_t) M_{t-1}}{D_{t-1}} + f(\lambda_{t-1}) = \frac{\lambda_{t-1}}{\lambda_t} (1 + g_t) + f(\lambda_{t-1})$$

Similarly, we can write the ratio of real government debt to consumption as

$$\frac{(M_t + B_t^s) / P_t}{C_t} = \frac{M_t (1 + b_t)}{D_{t-1}} = \lambda_{t-1} (1 + g_t) (1 + b_t)$$

The inverse of the ratio of real government debt to consumption is the government's collateral ratio  $\kappa_t^s$ . Inserting these expressions back into market clearing, and solving for consumption, we get Property (vi). ■

#### A.4 Equilibrium as solution to a difference equation

An equilibrium is described by sequences for consumption  $C_t$ , the interest rates  $i_t^h$ ,  $i_t$  and  $i_t^D$ , the inflation rate  $\pi_t$ , the tree price  $q_t$ , the price of bank equity  $q_t^b$ , the household marginal utility of wealth  $\omega_t$ , liquidity and collateral ratios  $\lambda_t$  and  $\kappa_t$  as well as the multiplier for constrained banks  $\mu_t^c$ . We fix exogenous paths of output  $Y_t$  as well as the growth rate of the reserves and bonds  $g_t$ .

The difference equation consists of three blocks. The bank block consists of the banks' first order conditions for assets and liabilities (A.7)-(A.8) that hold with equality:

$$\frac{1 + i_t^R}{1 + i_t^h} - c'(\kappa_t) + (1 + \nu) \frac{(1 - G(\lambda_t)) \tilde{\mu}_{t+1}^c}{1 + i_t^h} = 1, \quad (\text{A.10})$$

$$\frac{1 + \pi_{t+1} q_{t+1} + x_{t+1}}{1 + i_t^h} - \rho c'(\kappa_t) = 1, \quad (\text{A.11})$$

$$\frac{1 + i_t}{1 + i_t^h} - c'(\kappa_t) = 1, \quad (\text{A.12})$$

$$\frac{1 + i_t^D}{1 + i_t^h} + c(\kappa_t) - c'(\kappa_t) \kappa_t + \tilde{\mu}_{t+1}^c \frac{\int_{\lambda_t}^{\tilde{\lambda}} \tilde{\lambda} dG(\tilde{\lambda})}{1 + i_t^h} = 1, \quad (\text{A.13})$$

$$\frac{1 + i_t}{1 + i_t^h} + c(\kappa_t) - c'(\kappa_t) \kappa_t - \tilde{\mu}_t^c (1 + \nu) = 1. \quad (\text{A.14})$$

The household block consists of equilibrium consumption with the function  $\ell$  defined in Proposition 1 as well as the household first order condition for deposits, the definition of  $i_t^h$  as households' nominal discount rate, and the price of bank equity:

$$C_t = Y_t / (1 + \ell(\lambda_{t-1}, \lambda_t, \kappa_t, g_t, b_t)), \quad (\text{A.15})$$

$$i_t^h - i_t^D = \frac{1}{\omega_{t+1}} - 1, \quad (\text{A.16})$$

$$\frac{1 + \pi_{t+1}}{1 + i_t^h} = \beta \frac{\omega_{t+1}}{\omega_t}, \quad (\text{A.17})$$

$$q_t^b = \beta \frac{\omega_{t+1}}{\omega_t} (q_{t+1}^b + Y_{t+1}^b). \quad (\text{A.18})$$

The third block of equations uses market clearing to relate the collateral ratio, liquidity ratio and output. To derive it, consider first market clearing in the overnight credit market. From the proof of property (ii) above, banks borrow overnight only if the liquidity constraint binds. The aggregate volume of overnight bank borrowing relative to deposits is therefore

$$\frac{F_t}{D_{t-1}} = \frac{1}{1 + v} \int_{\lambda_{t-1}}^{\bar{\lambda}} (\bar{\lambda} - \lambda_{t-1}) dG(\bar{\lambda}) =: \frac{1}{1 + v} f(\lambda_{t-1}). \quad (\text{A.19})$$

The function  $f$  is decreasing in  $\lambda_{t-1}$ : if interest rates are such that banks hold a lot of reserves, then  $\lambda_{t-1}$  is high and banks rarely run out of reserves, so outstanding interbank credit is low. In this sense, reserves and overnight borrowing are substitutes in liquidity management.

The third block of equations is

$$\kappa_t = \frac{\lambda_{t-1} (1 + b_t) \frac{1+g_t}{1+v} + \rho q_t / C_t + f(\lambda_{t-1}) \frac{1}{1+v}}{(1 + g_t) \frac{\lambda_{t-1}}{\lambda_t} + f(\lambda_{t-1}) \frac{1}{1+v}}, \quad (\text{A.20})$$

$$(1 + \pi_{t+1}) \frac{C_{t+1}}{C_t} = (1 + g_t) \frac{\lambda_{t-1}}{\lambda_t}. \quad (\text{A.21})$$

The first equation represents the aggregate relationship between  $\kappa$  and  $\lambda$  from bank balance sheets. To derive it, we start from the definition of the collateral ratio and substitute aggregate overnight credit (A.19) as well as the market value of trees, and then express both the numerator and denominator in terms of ratios. The second equation describes the evolution of inflation with a binding household cash-in-advance constraint.

The system is comprised of the ten equations (A.10)-(A.14), (A.15)-(A.18) and (A.20)-(A.21). It has only one endogenous state variable  $\lambda_{t-1}$ .

A steady state requires a constant interest rate on reserves, a constant growth rate of re-

serves and bonds  $g$  as well as constant output  $Y$  and fruit from trees  $x$ . Endogenous variables are then pinned down by  $g = \pi$ ,  $1 + i^h = (1 + \pi) / \beta$ ,  $i^h - i^D = 1/\omega - 1$ ,  $q^b = \beta (q^b + Y^b)$ , as well as the bank first order conditions

$$\begin{aligned} \frac{1 + i^R}{1 + i^h} - c'(\kappa) + (1 + \nu) \frac{(1 - G(\lambda)) \tilde{\mu}^c}{1 + i^h} &= 1, \\ \beta \frac{q + x}{q} - \rho c'(\kappa) &= 1, \\ \frac{1 + i}{1 + i^h} - c'(\kappa) &= 1, \\ \frac{1 + i^D}{1 + i^h} + c(\kappa) - c'(\kappa) \kappa + \tilde{\mu}^c \frac{\int_{\lambda}^{\bar{\lambda}} \tilde{\lambda} dG(\tilde{\lambda})}{1 + i^h} &= 1, \\ \frac{1 + i}{1 + i^h} + c(\kappa) - c'(\kappa) \kappa - \tilde{\mu}^c (1 + \nu) &= 1, \end{aligned}$$

and the balance sheet condition

$$\kappa = \frac{\lambda (1 + b) \frac{1+g}{1+\nu} + \rho q / C + f(\lambda) \frac{1}{1+\nu}}{1 + g + f(\lambda) \frac{1}{1+\nu}}. \quad (\text{A.22})$$

## A.5 Proof of Proposition 2

We fix an initial condition  $\lambda_{-1}$  close to the steady state and look for a solution to the difference equation such that all variables except the inflation rate  $\pi_1$  are at steady state from  $t = 1$  on. We know from (A.20) that the state variable  $\lambda_0$  must already be at its steady state value. We also have from (A.16) that  $i_0^h - i_0^D$  already achieves its steady state value.

We now work through the equations for date 0 to find the remaining date 0 endogenous variables together with the policy intervention  $b_0$ . We first note that (A.10), (A.12) and (A.13) are jointly satisfied at the steady state values  $i_0^h = i^h$ ,  $i_0 = i$  and  $\kappa_0 = \kappa$ . From (A.15) and (A.21), inflation  $\pi_1$  is given by

$$1 + \pi_1 = (1 + g_0) \frac{\lambda_{-1} Y_0}{\lambda Y} \frac{1 + \ell(\lambda, \lambda, \kappa, g)}{1 + \ell(\lambda_{-1}, \lambda, \kappa, g_0)}. \quad (\text{A.23})$$

Substituting into (A.11) and using the fact that  $q_1$  and  $x_1$  achieve their steady state values, we obtain

$$q_0 = \frac{1 + \pi_1}{1 + i^h} \frac{(q_1 + x)}{1 + \rho c'(\kappa)} = q \frac{1 + \pi_1}{1 + g}. \quad (\text{A.24})$$

The date 0 price  $q_0$  thus differs from the steady state price  $q_1$  only if the short run inflation

rate  $\pi_1$  differs from the long run inflation rate  $g$ .

For small enough deviations from steady state, we can find multipliers  $\tilde{\mu}_0^c \geq 0$  and  $\omega_0 < 1$  to satisfy (A.14) and (A.18), respectively. It thus remains to check whether (A.20) holds. Using the above expressions for  $q_0$  and  $\pi_1$ , we obtain

$$q_0/C_0 = q \frac{1 + \pi_1}{1 + g} \frac{1 + \ell(\lambda_{-1}, \lambda, \kappa, g_0)}{Y_0} = \frac{q}{C} \frac{1 + g_0}{1 + g} \frac{\lambda_{-1}}{\lambda}$$

and substituting into (A.20) we have

$$\kappa = \frac{\lambda_{-1} (1 + b_0) \frac{1+g_0}{1+v} + \rho \frac{1+g_0}{1+g} \frac{\lambda_{-1}}{\lambda} \frac{q}{C} + f(\lambda_{-1}) \frac{1}{1+v}}{(1 + g_0) \frac{\lambda_{-1}}{\lambda} + f(\lambda_{-1}) \frac{1}{1+v}}. \quad (\text{A.25})$$

For an initial condition close enough to steady state, we can solve for an initial government asset mix  $b_0 \geq 0$  to make the equation hold.

If reserves are abundant at the initial condition  $\lambda_{-1}$  then  $b_0 = 0$  for any  $g_0$  and  $Y_0$  since the  $\lambda_{-1}$  and  $g_0$  cancel so the equation reduces to (A.22). With abundant reserves, no government trade at date 0 is needed. We further have  $\ell(\lambda_{-1}, \lambda, \kappa, g_0) = \ell(\lambda, \lambda, \kappa, g)$ . ■

We note a second useful special case of the proposition: a one time transitory shock to output. If  $g_0 = g$  and  $\lambda_{-1} = \lambda$ , then (A.25) implies  $b_0 = b$  for any  $Y_0$ . In other words, an unanticipated transitory shock to output similarly does not require a government trade – the bank ratios remain in steady state, but there is a one time jump in the price level. The date 1 inflation rate in this case is proportional to the output change: from (15), we have  $1 + \pi_1 = (1 + g) Y_0 / Y$ .

## A.6 Proof of Proposition 3

For period length  $\Delta$ , we consider the household problem

$$\sum \prod \beta^{\Delta t} U(C_t) \Delta$$

subject to the budget constraint and cash-in-advance constraint

$$\begin{aligned} P_t C_t \Delta + B_t + D_t + Q_t \theta_t + Q_t^b \theta_t^b + i_t^L L_{t-\Delta} \\ \leq P_t \Omega_t + B_{t-\Delta} (1 + i_{t-\Delta} \Delta) + D_{t-\Delta} (1 + i_{t-\Delta}^D) + (Q_t + P_t x \Delta) \theta_{t-\Delta} + (Q_t^b + P_t Y_t^b \Delta) \theta_{t-\Delta}^b \\ P_t C_t \Delta \leq (D_{t-\Delta} + L_{t-\Delta}) \Delta \end{aligned}$$

The banks problem is to maximize shareholder value

$$\sum \prod \beta^{\Delta t} y_t^b \Delta$$

where cash flow is

$$\begin{aligned} P_t y_t^b \Delta &= i_{t-1}^L \Delta L_{t-\Delta} + M_{t-1} \left( 1 + i_{t-\Delta}^R \Delta \right) - M_t - D_{t-\Delta} \left( 1 + i_{t-\Delta}^D \right) + D_t \\ &+ (B_{t-\Delta} - F_{t-\Delta}) (1 + i_{t-\Delta} \Delta) - (B_t - F_t) + ((Q_t + P_t x_t \Delta) \theta_{t-\Delta} - Q_t \theta_t) \\ &- c(\kappa_t) \Delta (D_t + L_t) + F_t. \end{aligned}$$

and the constraints are

$$\begin{aligned} \tilde{\lambda}_t (D_{t-\Delta} + L_{t-\Delta}) &\leq (1 + \nu) (M_{t-\Delta} + F_t) \\ \kappa_t &= \frac{M_t + \rho Q_t \theta_t + B_t}{D_t + L_t + F_t} \end{aligned}$$

For given  $\Delta$ , the difference equation characterizing equilibrium can be derived in the same way as in the proof of Proposition 1. We are interested only in the equations determining steady state. We know that the steady state discount factor is  $1 / (1 + \delta \Delta)$ .

For the first block – derived from bank first order conditions – we start from the analogue to (A.10)-(A.14) for general  $\Delta$ , set all variables to steady state and multiply through by  $(1 + \delta \Delta)$  and  $(1 + \pi \Delta)$  whenever these variables appear in a denominator:

$$\begin{aligned} 1 + i^R \Delta - c'(\kappa) \Delta + (1 + \nu) (1 - G(\lambda)) \tilde{\mu}^c \Delta &= (1 + \delta \Delta) (1 + \pi \Delta) \\ \frac{q + x \Delta}{q} - \rho c'(\kappa) \Delta &= 1 + \delta \Delta \\ 1 + i \Delta - c'(\kappa) \Delta &= (1 + \delta \Delta) (1 + \pi \Delta) \\ (i^h - i^D) \Delta + (c(\kappa) - c'(\kappa) \kappa) \Delta + \tilde{\mu}^c \Delta \int_{\lambda}^{\tilde{\lambda}} \tilde{\lambda} dG(\tilde{\lambda}) &= 0 \\ 1 + i \Delta + c(\kappa) - c'(\kappa) \kappa - \tilde{\mu}^c \Delta (1 + \nu) &= (1 + \delta \Delta) (1 + \pi \Delta) \end{aligned}$$

From the second block, we have

$$\begin{aligned} C \Delta &= Y \Delta \left( c_g \left( \lambda^{-1} (1 + b)^{-1} \right) \lambda (1 + b) + c(\kappa) \left( 1 + f(\lambda) \frac{\lambda}{\lambda (1 + g \Delta)} \right) \right) \\ (i^h - i^D) \Delta &= \left( \frac{1}{\omega} - 1 \right) \Delta \end{aligned}$$

The third block becomes

$$\kappa = \frac{\lambda(1+b) \frac{1+g\Delta}{1+v} + \rho \frac{q}{\Omega+x} + f(\lambda) \frac{1}{1+v}}{1+g\Delta + f(\lambda) \frac{1}{1+v}}$$

$$1+g = 1+\pi$$

We now take the limit as  $\Delta \rightarrow 0$ . The bank block simplifies to

$$\begin{aligned} i^R - c'(\kappa) + (1+\nu)(1-G(\lambda))\tilde{\mu}^c &= \delta + \pi \\ x - \rho c'(\kappa)q &= q\delta\Delta \\ i - c'(\kappa) &= \delta + \pi \\ (i^h - i^D) + (c(\kappa) - c'(\kappa)\kappa) + \tilde{\mu}^c \int_{\lambda}^{\bar{\lambda}} \tilde{\lambda} dG(\tilde{\lambda}) &= 0 \\ i + (c(\kappa) - c'(\kappa)\kappa) - \tilde{\mu}^c(1+\nu) &= \delta + \pi. \end{aligned}$$

Substituting the fifth and third equation into the first, we obtain the liquidity management curve

$$\delta = i^R - \pi - c'(\kappa) + (1-G(\lambda))(c(\kappa) - c'(\kappa)(\kappa-1)). \quad (\text{A.26})$$

From the second equation in the bank block, the tree price is

$$q = \frac{x}{\delta - \rho c'(\kappa)}$$

From the second block, we have  $C = Y / (1 + \bar{\ell}(\lambda, \kappa))$ , where

$$\bar{\ell}(\lambda, \kappa) := c_g \left( \lambda^{-1} (1+b)^{-1} \right) \lambda(1+b) + c(\kappa)(1+f(\lambda)) = \ell(\lambda, \lambda, \kappa, 0)$$

Substituting into the third block, we obtain the capital structure curve

$$\kappa = \frac{\lambda(1+b) \frac{1}{1+v} + \rho \frac{1+\bar{\ell}(\lambda, \kappa)}{\delta - \rho c'(\kappa)} \frac{x}{Y} + f(\lambda) \frac{1}{1+v}}{1+f(\lambda) \frac{1}{1+v}}$$

The liquidity management curve is flat in  $(\kappa, \lambda)$ -plane if  $\lambda \geq \bar{\lambda}$  since the  $G(\lambda) = 0$ . For  $\lambda < \bar{\lambda}$ , it is decreasing since  $G$  is increasing,  $c''(\kappa) > 0$  and

$$\frac{d}{d\kappa} (c(\kappa) - c'(\kappa)(\kappa-1)) = c'(\kappa) - c'(\kappa) - c''(\kappa)(\kappa-1) < 0$$



for  $\kappa > 1$ . Ignoring the small effect from leverage cost  $\bar{\ell}(\lambda, \kappa)$ ,  $f'(\lambda) \leq 0$  and  $c''(\kappa) > 0$  imply that the capital structure curve is monotonically increasing in  $(\kappa, \lambda)$  plane.

If  $\delta - (i^R - \pi) < -c'(1)$ , all points on the liquidity management curve satisfy  $\kappa > 1$ . If

$$1 > \frac{\rho \frac{1+\bar{\ell}(0,1)}{\delta-\rho c'(1)} \frac{x}{Y} + f(0) \frac{1}{1+v}}{1 + f(0) \frac{1}{1+v}},$$

the capital structure is below one at  $\lambda = 0$ . This is true in particular if  $\rho$  or  $x/Y$  become small. ■

## A.7 Steady state with uncertainty

We find a steady state with uncertainty in three steps, following Ilut and Schneider (2015). We first determine the “worst case” steady state, that is, the steady state to which the economy would converge were the worst case scenario contemplated by agents to actually occur. Here the worst case scenario is that the payoff of the trees is multiplied by  $1 - s$ .

The second step characterizes agents’ *perceptions* of the equilibrium dynamics. Since agents see the actual tree payoffs  $x$ , they behave *as if* they are on a transition path from a “better” initial condition to the worst case steady state. The third step then characterizes the actual dynamics by combining the law of motion for the endogenous variables implied by the second step with the actual dynamics of the exogenous variables.

For the first step, worst case steady state prices and bank ratios follow from the equations derived in Proposition 3, with  $x$  replaced by  $x(1 - s)$ . We indicate worst case steady state values by stars. In particular, the worst case steady state ratios  $\lambda^*$  and  $\kappa^*$  are determined by the intersection of the liquidity management curve (A.26) and the capital-structure (A.22), with  $x$  replaced by  $x(1 - s)$ . Since output and consumption are constant in the worst case steady state, inflation is given by the growth rate of nominal liabilities  $g$ , as in the steady state without uncertainty.

For the second step, consider agents’ *perceptions* of the equilibrium dynamics. Agents observe every period the true tree payoff  $x$  which is larger than the worst case steady state payoff. As a result, agents always act *as if* they are on a transition path away from an initial transitory shock  $x$  that is higher than the (perceived) steady state value  $x(1 - s)$ . Moreover, the actual steady state value of the endogenous state variable  $\lambda$  must be such that agents’ behavior leaves  $\lambda$  constant over time *given their worst case beliefs*.

Formally, we are looking for a solution to the system of difference equations derived in Appendix A.4 together with an initial condition for the endogenous state variable  $\lambda$  such that

(i) the solution converges to the worst case steady state, and (ii) the endogenous state variable  $\lambda$  is constant in the first period of the transition path. We now use Proposition 2 to show that there is a solution with initial condition  $\lambda = \lambda^*$ , that is, the worst case and actual steady state value of the liquidity ratio are identical.

Consider the special case of Proposition 2 with the steady state values given by the worst case steady state values  $(\lambda^*, Y^*, g)$ , and initial conditions  $\lambda_0 = \lambda^*$ ,  $Y_0 = \Omega + x$ , and  $g_0 = g$ . As noted below the proof of Proposition 2 in Appendix A.5, for this case the government bond-reserve ratio at date 0 is  $b_0 = b^*$ , the steady state ratio. Moreover, the inflation rate is given by  $(1 + g) Y_0 / Y^*$  at date 1 and settles at  $g$  thereafter.

We have thus found a transition path with the required properties. Starting from the worst case steady state  $\lambda = \lambda^*$ , a transitory shock that increases all tree payoffs by a factor of  $1 / (1 - s)$  thus induces banks to maintain the liquidity ratio constant at  $\lambda^*$ .<sup>20</sup> From the proof of Proposition 2, we also know that the collateral ratio  $\kappa$  is also unchanged in response to the shock. We can therefore conclude that the actual steady state ratios  $(\lambda, \kappa)$  are identical to the worst case steady state ratios  $(\lambda^*, \kappa^*)$ .

Since the transition path captures agents' perceptions, we can define the steady state *perceived inflation* rate as.

$$1 + \tilde{\pi} = (1 + g) \frac{Y_0}{Y^*} = (1 + g) \frac{\Omega + x}{\Omega + x(1 - s)} \quad (\text{A.27})$$

at date 1 and settles at  $g$  thereafter.

The third step is simplified by the fact that the actual evolution of output – the ambiguous exogenous variable – enters only one equation, namely (A.21). The last equation relates inflation to output growth. It implies that the *actual* steady state inflation rate equal to  $g$  and therefore lower than agents' perceived inflation  $\tilde{\pi}$ . This is because agents fear a drop in output that will translate into higher inflation for given growth of nominal liabilities.

#### *Graphical analysis and steady state asset prices*

To derive the steady state price of trees, we use (A.24) from the proof of Proposition 2 together with the definition  $\tau = x / (\Omega + x)$  and make explicit the dependence of the weight  $\rho$  on the spread  $s$ :

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<sup>20</sup>Intuitively, since output is expected to fall agents worry that inflation is temporarily higher than the growth rate of nominal government liabilities and hence expect a higher cost of liquidity. As a result, future tree payoffs are discounted at a lower rate and the ratio of tree value to output is the same as in the worst case steady state.

$$\begin{aligned}
q &= \frac{1 + \tilde{\pi}}{1 + g} q^* = \frac{\Omega + x}{\Omega + x(1 - s)} \frac{x(1 - s)}{\delta + \rho(s) c'(\kappa)} \\
&= \frac{x}{\delta + \rho(s) c'(\kappa)} \frac{1 - s}{1 - \tau s} = u(s) \frac{x}{\delta + \rho(s) c'(\kappa)}
\end{aligned} \tag{A.28}$$

We also note that while the realized real interest rate is  $i - g = \delta + \text{mb}(\kappa)$  as before, the ex ante real interest rate  $i - \pi_1 = \hat{\delta} - c'(\kappa)$  reflects the lower discount rate induced by output and hence inflation uncertainty.

With this formula in hand, the graphical analysis of optimal ratios works the same way as in the absence of uncertainty. As the period length becomes short, the CS curve is described by (18) with the new price formula (A.28). The liquidity-management curve is described by (A.26), as in the case without uncertainty.

## A.8 Equilibrium with active traders

Active traders maximize shareholder value, with nominal dividends given by

$$(\hat{Q}_t + \hat{x}_t)\theta_{t-1} - \hat{Q}_t\theta_t - i_{t-1}^L \hat{L}_{t-1} - \hat{D}_t + \hat{D}_{t-1} (1 + i_{t-1}^D)$$

where  $\theta_t$  is the number of trees bought at date  $t$  and  $\hat{L}_t$  is the credit line arranged at  $t - 1$  in order to pay for trees at  $t$  and  $\hat{D}_t$  are deposits chosen at  $t$ .

The first order conditions for active traders can be written as

$$\begin{aligned}
i_t^L &= i_t^h - i_t^D = \hat{\mu}_{t+1} (1 + \hat{\nu}) \\
\hat{q}_t (1 + \hat{\mu}_t) &= \frac{1 + \pi_{t+1}}{1 + i_t^h} (\hat{q}_{t+1} + \hat{x}_{t+1} (1 - \hat{s}))
\end{aligned} \tag{A.29}$$

where  $\hat{\mu}_t$  is the multiplier on active traders' liquidity constraint multiplied by the price level. The interest rate on credit lines – bank customers' cost of liquidity – is equated to active traders' liquidity benefit. The expected payoffs of active trader trees in the second equation incorporate the worst case payoff on trees.

To capture the quantity of deposits absorbed by active traders, we define their share  $\alpha_t = (\hat{D}_t + \hat{L}_t) / (D_t + L_t)$ . The equilibrium share satisfies

$$\alpha_{t-1} = \frac{\hat{q}_t}{\hat{q}_t + (1 + \hat{\nu}) C_t} \tag{A.30}$$

Active traders absorb more inside money if the netting system is less efficient (low  $\hat{\nu}$ ).

Since money held by active traders does not directly affect the price level, the leverage and inflation equations contain only the share of inside money that circulates in the goods market:

$$\begin{aligned} \kappa_t &= \frac{\lambda_{t-1} (1 + b_t) \frac{1+g_t}{1+\hat{\nu}} + \rho \frac{q_t(1-\alpha_{t-1})}{C_t} + f(\lambda_{t-1}) \frac{1}{1+\hat{\nu}}}{(1 + g_t) \frac{\lambda_{t-1}}{\lambda_t} + f(\lambda_{t-1}) \frac{1}{1+\hat{\nu}}} \\ (1 + \pi_{t+1}) \frac{C_{t+1}}{C_t} &= (1 + g_t) \frac{\lambda_{t-1}}{\lambda_t} \frac{1 - \alpha_{t-1}}{1 - \alpha_t} \end{aligned} \quad (\text{A.31})$$

The system of difference equations with active traders is thus given by (A.10)-(A.14), (A.15)-(A.18), (A.29), (A.30) and (A.31). Endogenous state variables are now both the banks' liquidity ratio  $\lambda$  and the active trader money share  $\alpha$ .

#### *Worst case steady state*

To characterize steady state equilibrium, the first step is again to derive the worst case steady state, that is, the steady state if  $\hat{x}_t = x^* = \hat{x}(1 - \hat{s})$  and  $Y_t = Y^*$ , where the worst case output reflects low payoffs on all trees including bank trees and active trader trees. The bank equations (A.10)-(A.14) are unchanged, so the bank ratios  $(\lambda, \kappa)$  still lie on the liquidity-management curve described by (A.26). The steady state liquidity benefit, and money share as well as the price of trees held by active traders satisfy

$$\begin{aligned} i^{L*} &= \hat{\mu}^* (1 + \hat{\gamma}), \\ \hat{q}^* &= \frac{\hat{x}(1 - \hat{s})}{\delta + \hat{\mu}^*}, \\ \alpha^* &= \frac{\hat{q}^*}{\hat{q}^* + (1 + \hat{\nu}) C^*}. \end{aligned}$$

To describe the capital-structure curve, we first write the cost of bank customers' liquidity and the active trader money share as functions of the bank ratios  $\lambda$  and  $\kappa$ :

$$\begin{aligned} \tilde{i}^L(\lambda^*, \kappa^*) &:= -c'(\kappa^*) + (1 + \nu) (c(\kappa^*) - c'(\kappa^*) (\kappa^* - 1)) \int_{\lambda^*}^{\bar{\lambda}} \tilde{\lambda} dG(\tilde{\lambda}), \\ \tilde{\alpha}(\lambda^*, \kappa^*) &:= \frac{\hat{x}(1 - s)}{\hat{x}(1 - s) + C^* (1 + \hat{\nu}) (\delta + \tilde{i}_L(\lambda^*, \kappa^*))}. \end{aligned}$$

The function  $\tilde{i}_L$  is decreasing in both arguments: higher liquidity or collateral ratios lower bank customers' cost of liquidity. As a result, the active trader money share is increasing in both  $\lambda^*$  and  $\kappa^*$ . In the region of  $(\lambda, \kappa)$ -plane where reserves are abundant both functions

depend on  $\lambda$  only via a negligible effect on  $C^*$ .

We find the worst case bank ratios from the intersection of the liquidity-management curve and the new capital-structure curve described by

$$\kappa^* = \frac{e^g \lambda (1 + B^g/M) \frac{\bar{v}}{1+\gamma} + \rho \bar{v} \frac{x^b(1-s)}{\delta-c'(\kappa)} \frac{1}{C^*} (1 - \tilde{\alpha}(\lambda^*, \kappa^*)) + \frac{\bar{v}}{1+\gamma} f(\lambda^*)}{1 + g + \frac{\bar{v}}{1+\gamma} f(\lambda^*)}. \quad (\text{A.32})$$

The difference to (18) is the presence of the factor  $1 - \alpha$  in the numerator. In the region where reserves are abundant,  $\tilde{i}^L$  is independent of  $\lambda$  and the curve is upward sloping in the  $(\lambda, \kappa)$ -plane without further assumptions. With scarce reserves, the presence of active traders reduces the slope of the curve. To guarantee an upward slope, we need the share of nominal assets sufficiently large or the payoff of active traders small enough relative to output.

#### *Actual steady state*

The second step in the characterization of equilibrium is to find the transition path from the actual to the worst case steady state. We conjecture again that this transition takes only one period, so that  $\lambda = \lambda^*$  and  $\alpha = \alpha^*$ . The bank block of the difference equations has not changed and delivers actual steady state  $i$ ,  $\kappa$  and  $i^L$ . To derive the value of bank trees, we apply the same argument as in Section A.7 but use the new equation for inflation – the second equation in (A.31) – to obtain

$$q = q^* \frac{C}{C^*} \frac{\lambda}{\lambda^*} \frac{1 - \alpha^*}{1 - \alpha}.$$

Substituting into the first equation in (A.31), the actual steady state ratios  $\lambda$  and  $\alpha$  satisfy

$$\kappa = \frac{e^g \lambda (1 + B^g/M) + \rho \bar{v} \frac{x^b(1-s)}{\delta-c'(\kappa)} \frac{1-\alpha^*}{C^*} \frac{\lambda}{\lambda^*} + \frac{\bar{v}}{1+\gamma} f(\lambda)}{1 + g \frac{\lambda}{\lambda^*} + \frac{\bar{v}}{1+\gamma} f(\lambda)}.$$

The equation is satisfied if  $\lambda = \lambda^*$  since we have  $\alpha^* = \tilde{\alpha}(\lambda^*, \kappa^*)$  and the equation is otherwise identical to (A.32).

It remains to check the second equation in (A.29) as well as (A.30). The actual steady state value of active trader trees follows from (A.30) as

$$\hat{q} = \frac{\alpha^*}{1 - \alpha^*} (1 + \hat{v}) C.$$

Given this value, we can find the actual steady state multiplier  $\hat{\mu}$  to satisfy (A.29). We have shown that if the initial conditions are given by the worst case steady value  $\alpha^*$  and  $\lambda^*$  and there is a transitory shock that increases tree payoffs, agents respond by choosing again worst

case steady state  $\alpha$  and  $\lambda$ .

We summarize the actual steady state for the economy with active traders as follows. Actual inflation is given by the growth rate of nominal government liabilities  $\pi = g$ . Since  $\kappa = \kappa^*$  and  $\lambda = \lambda^*$ , the actual steady state bank ratios are determined from the intersection of the liquidity management and capital-structure curves (A.26) and (A.32). The value of active trader trees and the evolution of the nominal price level are given by

$$\hat{q} = \hat{q}^* \frac{C}{C^*} = (1 - \hat{s}) \frac{Y}{Y^*} \frac{\hat{x}}{\delta + \hat{\mu}^*},$$

$$P_t = \frac{1 - \tilde{\alpha}(\lambda, \kappa)}{\lambda} \frac{1 + v}{1 + b_t} \frac{M_t + B_t^g}{Y}.$$