Trend and Cycle in Bond Premia*

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Abstract

Common statistical measures of bond risk premia are volatile and countercyclical. This paper uses survey data on interest rate forecasts to construct subjective bond risk premia. Subjective premia are less volatile and not very cyclical; instead they are high only around the early 1980s. The reason for the discrepancy is that survey forecasts of interest rates are made as if both the level and the slope of the yield curve are more persistent than under common statistical models.

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1 Introduction

Many studies have documented that excess returns on long term bonds are predictable.[1] Indeed, postwar interest rate data reveal two striking patterns. On the one hand, high excess returns on long term bonds are often preceded by a high spread between the long and short Treasury interest rates (a steep ”slope” of the yield curve). On the other hand, higher excess returns are often preceded by a higher overall level of interest rates. An investor who understood these two patterns would have predicted high excess returns on long bonds in times of steep slope (for example, right after recessions) and in times of high level (for example, in the early 1980s). Over the postwar period, the investor could have made a fortune from a “carry trade” in long bonds (borrowing short term and investing in long bonds in times of steep slope or high level).

Why did investors not exploit these predictability patterns and thus make them disappear? There are two candidate reasons. The first is beliefs: investors' historical predictions of excess returns may have been different from predictions found in today's statistical analysis. Investors may not have recognized the same patterns that we see today with the benefit of hindsight, at least not to the same extent. The other candidate reason is risk assessment. Even if investors' historical predictions of excess returns were sometimes high, they may chosen not to trade on their predictions, because they also perceived the carry trade to be more risky, or because they had become more risk averse.

Most quantitative asset pricing studies focus exclusively on risk assessment as the reason for predictability. They assume that investors' historical predictions were identical to (in-sample) predictions derived today from statistical models. By construction, this approach rules out beliefs as a reason for predictability. Instead, a lot of research effort has been directed at documenting and explaining changes in risk assessment (e.g., Campbell and Cochrane 1995, Bansal and Yaron 2004, Wachter 2006, Gabaix 2012.)

The goal of this paper is to use survey data to study the relative importance of beliefs and risk assessment for bond return predictability. We use survey data on both interest rate and inflation forecasts to document historical predictions of excess returns and to compare them to predictions from statistical models. There are two main results. First, historical predicted excess returns vary less than statistical predictions (by roughly one half). Second, historical predicted excess returns move less with the business cycle, but have a larger low frequency component that is correlated with inflation.

We establish these facts by (i) direct comparison of survey forecasts and fitted values from standard predictability regressions and (ii) estimating a time series model that jointly describes yields and survey forecasts. The advantage of using a time series model is that we can compress information contained in yields of many maturities as well as forecasts of those yields for many different forecast horizons. The basic principle that emerges is that survey forecasts of interest rates are made *as if* both the level and the slope of the yield

[1] As a concrete example, consider the excess return on a 10 year Treasury bond held over one year. It is defined as the return earned by holding the bond over the course of a year minus the one year interest rate available at the beginning of the year.

curve were more persistent than they appear with hindsight. This discrepancy is what makes survey-based expected excess returns less volatile and less cyclical.

The findings of the paper imply that both candidate reasons for predictability patterns are important and that structural models should account for both. Indeed, a consumption based asset pricing model says that excess returns R^e_{t+1} should satisfy $E^*_t\left[R^e_{t+1}M_{t+1}\right]=0$, where M_{t+1} is marginal utility and where the expectation is taken *under the subjective belief of the investor.* The expected excess return measured today with hindsight by an econometrician can thus be decomposed into

$$E_t\left[R^e_{t+1}\right]=\underbrace{E_t\left[R^e_{t+1}\right]-E^*_t\left[R^e_{t+1}\right]}_{\text{difference in predictions}}\underbrace{-\operatorname{cov}^*_t\left(M_{t+1},R^e_{t+1}\right)/E^*_tM_{t+1}}_{\text{risk premium}}. \tag{1}$$

The standard approach to explaining predictability patterns (that is, changes in $E_tR^e_{t+1}$) assumes that the first component is zero and that changes in risk assessment move around risk premia. Our findings call for a theory that can jointly account for both components.

We build on a small literature which has shown that measuring beliefs directly from surveys can help understand asset pricing puzzles. Froot (1989) argued that evidence against the expectations hypothesis of the term structure might be due to the failure of the (auxiliary) rational expectations assumption imposed in the tests rather than to failures of the expectations hypothesis itself. He used the Goldsmith-Nagan survey to measure interest rate forecasts and found that the failure of the expectations hypothesis for long bonds can be attributed to expectational errors. The findings from our model confirm Froot's results while including the BlueChip data set that allows for a longer sample as well as more forecast horizons and maturities. Moreover, our estimation jointly uses all data and recovers and characterizes the kernel M.[2] Several authors have explored the role of expectational errors in foreign exchange markets. Frankel and Froot (1987) show that much of the forward discount can be attributed to expectational errors. Gourinchas and Tornell (2004) use survey data to show that deviations from rational expectations can rationalize the forward premium and delayed overshooting puzzles. Bacchetta, Mertens, and van Wincoop (2009) study expectational errors across a large number of asset markets.

Our paper is also related to recent work on bond pricing with learning. A number of papers have estimated the distribution of yields and macro variables using learning schemes. A typical finding is that deviations from the expectation hypothesis can be smaller if expectations of future interest rates are computed under a learning scheme. Fuhrer (1996) uses a regime-switching approach. Cogley (2005) estimates a VAR with drifting coefficients. Kozicki and Tinsley (2001) and DeWachter and Lyrio (2006) add stochastic trends to a VAR. Laubach, Tetlow, and Williams (2007) consider adaptive learning in an affine term struc-

[2] Kim and Orphanides (2007) estimate a term structure model using data on both interest rates and interest rate forecasts. They show that incorporating survey forecasts into the estimation sharpens the estimates of risk premia in small samples. In our language, they obtain more precise estimates of "statistical premia"; they are not interested in the properties of subjective risk premia. Chernov and Mueller (2012) adopt a similar approach for inflation forecasting. Wright (2011) uses survey forecasts on three-month interest rates to document the presence of inflation risk premia in the United States and other countries.

ture setup. They find that the learning model delivers better out-of-sample forecasts than models with fixed coefficients. In Piazzesi and Schneider (2006), adaptive learning about consumption and inflation helps understand yield volatility.

The rest of the paper is structured as follows. Section 2 documents properties of survey forecasts. Subsection 2.2.1 takes a first look at the raw survey data for selected maturities and forecasting horizons. Subsection 2.3 compresses the information from the surveys using a time series model.

2 Stylized Facts

We consider "zero coupon" bonds that pay off only once at some specified maturity date. Let $P_t^{(n)}$ denote the date t price of a zero coupon bond with maturity n that is, the bond pays off one dollar n periods from date t. The yield to maturity on this bond is defined as $i_t^{(n)} = -\log P_t^{(n)}/n$. It represents the per period interest rate earned from holding the bond to maturity if gains are continuously compounded.

The log excess return from holding the n period bond from date t to date $t+1$ is denoted $rx_{t,t+1}^{(n)}$. It is defined as the log capital gain on the n-period bond less the one period interest rate:

$$rx_{t,t+1}^{(n)} = \log P_{t+1}^{(n-1)} - \log P_t^{(n)} - i_t^{(1)}.$$

Excess returns over longer holding periods are defined analogously. We use lower case letters for logarithms, e.g. $p_t^{(n)} = \log P_t^{(n)}$ and so on. The log excess return on an n-period bond held from t to $t+h$ is thus

$$rx_{t,t+h}^{(n)} = p_{t+h}^{(n-h)} - p_t^{(n)} - hi_t^{(h)}, \tag{2}$$

where $i_t^{(h)}$ is the date t yield to maturity on an h-period bond. A high excess return on a long bond is earned if the capital gain exceeds the short term interest rate.

Excess returns and forward rates

Excess returns on long bonds can also be written in terms of forward rates. A forward contract fixes a price (or, equivalently, an interest rate) for purchase or sale of a specified bond at some specified date in the future. Let $F_t^{(n-h,h)}$ denote the price fixed at date t for an $n-h$ period bond to be purchased at date $t+h$. In a frictionless market, locking in the purchase of an $n-h$ period bond for date $t+h$ is the same as borrowing $P_t^{(n)}$ dollars at the h-period rate at date t and using the borrowed money to purchase one n-period bond. In both cases, one ends up paying some amount of money at date $t+h$ in exchange for an $n-h$ period bond. The absence of arbitrage thus requires that the payments are the same under the two strategies, that is, the forward price to be paid at date $t+h$ is the same as the repayment on the h-period loan at date $t+h$:

$$F_t^{(n-h,h)} = P_t^{(n)} \exp\left(hi_t^{(h)}\right). \tag{3}$$

Let $f_t^{(n-h,h)} = -\log\, F_t^{(n-h,h)} / \, (n-h)$ denote the yield to maturity on the forward contract, or the "forward rate". Condition (3) implies that the forward rate can be expressed as a linear function of the n- and h-period interest rates

$$\begin{aligned} f_t^{(n-h,h)} &= \frac{n}{n-h} i_t^{(n)} - \frac{h}{n-h} i_t^{(h)} \\ &= i_t^{(h)} + \frac{n}{n-h} \left(i_t^{(n)} - i_t^{(h)} \right). \end{aligned}$$

For a large maturity n of the long bond and a small maturity h of the forward contract, the forward rate is essentially equal to the long bond rate. In contrast, if n is large but h is close to n, the forward rate behaves like a scaled version of the spread between the two rates.

It now follows that the excess return (2) can be rewritten as the (scaled) difference between the forward and the future spot rate:

$$rx_{t,t+h}^{(n)} = (n-h) \, (f_t^{(n-h,h)} - i_{t+h}^{(n-h)})$$

Excess returns are thus high if forward rates are higher than the subsequently realized spot rates.

Statistical and historical expected excess returns

This paper is about conditional expectations of excess returns. In particular, we want to distinguish between expected excess returns implied by a statistical model and historical predictions of excess returns. We write $E_t rx_{t,t+h}^{(n)}$ to denote the date t prediction from a statistical model, for example, a regression of log excess returns on variables known at date t. This is the object that the literature has been interested in: the stylized fact "predictability of excess returns on long bonds" means that $E_t rx_{t,t+h}^{(n)}$ moves around over time. Of course, the behavior of the conditional expectation depends on the particular statistical model used to compute it, and we will discuss several alternative models. In the literature, predictability has been established for a wide range of models.

We reserve the notation $E_t^* rx_{t,t+h}^{(n)}$ for historical predictions, or "true" investor expectations, of excess returns made at date t. To relate the two types of predictions, we decompose the expected excess return measured by an econometrician as

$$\begin{aligned} \underbrace{E_t \left[rx_{t,t+h}^{(n)} \right]} &= E_t^* \left[rx_{t,t+h}^{(n)} \right] + E_t \left[rx_{t,t+h}^{(n)} \right] - E_t^* \left[rx_{t,t+h}^{(n)} \right] \qquad (4) \\ &= E_t^* \left[rx_{t,t+h}^{(n)} \right] + E_t \left[p_{t+h}^{(n-h)} \right] - E_t^* \left[p_{t+h}^{(n-h)} \right] \\ &= \underbrace{E_t^* \left[rx_{t,t+h}^{(n)} \right]} + (n-h) \underbrace{\left(E_t^* \left[i_{t+h}^{(n-h)} \right] - E_t \left[i_{t+h}^{(n-h)} \right] \right)} \end{aligned}$$

statistical premium = subjective premium + subj. − stat. interest-rate expectation

Here the second line reflects the fact that the only uncertain part in the excess return is the future bond price, and the third line follows from the definition of the yield to maturity. The decomposition shows that statistical expected excess returns can move around either because actual historical expected excess returns move around, or because statistical interest rate forecasts differ from historical interest rate forecasts.

In the next two subsections, we measure the two terms in the decomposition (4) with data on actual interest rates as well as survey forecasts. In subsection 2.2.1, we take the simplest possible approach. We run regressions of excess returns on a set of date t interest rates to construct measures of $E_t rx_{t,t+h}^{(n)}$ and $E_t i_{t+h}^{(n-h)}$, and we use the median survey forecast of $i_{t+h}^{(n-h)}$ as a measure of $E_t^* i_{t+h}^{(n-h)}$. This approach delivers a decomposition for a given bond maturity and a given forecast horizon, and thus provides a first look at the data. This approach is limited by the nature of survey forecast data, which provide long samples for only a few bond maturities and forecast horizons. Moreover, it does not simultaneously process the information contained in the surveys for the many maturities and horizons that are available.

To compress all the available information from surveys, subsection 2.3 estimates a statistical model that describes the joint dynamics of actual interest rates and survey forecasts. The model is a standard linear state space system – both interest rates and survey forecasts are represented as linear functions of a small number of factors. We show that such a system does a decent job in describing the joint dynamics of interest rates and forecasts. We then proceed to use the system to show decompositions (4) for bonds and holding periods for which raw survey data are not available.

2.1 A first look at the data

We measure expectations of interest rates with survey data from two sources. Both sources conduct comparable surveys that ask approximately 40 financial market professionals for their interest-rate expectations at the end of each quarter and record the median survey response. Our first source are the Goldsmith-Nagan surveys that were started in mid-1969 and continued until the end of 1986. These surveys ask participants about their one-quarter ahead and two-quarter ahead expectations of various interest rates, including the 3-month Treasury bill, the 12-month Treasury bill rate, and a longer maturity Treasury bond rate. Our second source are Bluechip Financial Forecasts, a survey that was started in 1983 and continues until today. This survey asks participants for a wider range of expectation horizons (from one to six quarters ahead) and about a larger set of interest rates. The most recent surveys always include 3-month, 6-month and 1-year Treasury bills, and the 2-year, 5-year, 10-year and 30-year Treasury bonds.

To measure interest-rate expectations from a statistical model, we estimate unrestricted VAR dynamics for a vector of interest rates with quarterly data over the sample 1952:2-2012:4 and compute their implied forecasts. Later, in section 2.3, we will impose more structure on the VAR by assuming the absence of arbitrage and using a lower number of variables in the VAR, and thereby check the robustness of the empirical findings we document here. The vector of interest rates Y includes the 1-year, 2-year, 3-year, 4-year, 5-year, 10-year and 20-year zero-coupon yields. We use data on nominal zero-coupon bond yields with longer maturities from McCulloch and Kwon (1993). The sample for these data is 1952:2 - 1990:4.

We augment these data with the new Gurkaynak, Sack, and Wright (2006) data starting in 1991:1. We compute the forecasts by running OLS directly on the system $Y_{t+h} = \mu + \phi Y_t + \varepsilon_{t+h}$, so that we can compute the h-horizon forecast simply as $\mu + \phi Y_t$.

We can evaluate equation (4) based on our survey measures of subjective interest-rate expectations $E_t^* i_{t+h}^{(n-h)}$ and the VAR measures of expectations $E_t i_{t+h}^{(n-h)}$ for different maturities n and different horizons h. Figure 1 plots the left-hand side of equation (4), expected excess returns under VAR beliefs as a black line, and the second term on the right-hand side of the equation, the difference between subjective and VAR interest-rate expectations, as a gray line. For the short post-1983 sample for which we have Bluechip data, we have data for many maturities n and many forecasting horizons h. The lower two panels of Figure 1 use maturities $n = 3$ years and 11 years and a horizon of $h = 1$ year, so that we deal with expectations of the $n - h = 2$ year and 10 year interest rate. These combinations of n and h are in the Bluechip survey, and the VAR includes these two maturities as well so that the computation of expectations is easy.

For the long post-1970 sample, we need to combine data from the Goldsmith-Nagan and Bluechip surveys. The upper left panel shows the $n = 1.5$ year bond and $h = 6$ month holding period from the estimated VAR (which includes the $n - h = 1$ year yield.) This works because both surveys include the $n - h = 1$ year interest rate and a $h =$ 6-month horizon. The VAR delivers a 6-month ahead expectation of the 1-year interest rate. For long bonds, we do not have consistent survey data over this long sample. To get a rough idea of long-rate expectations during the Great Inflation, we take the Goldsmith-Nagan data on expected mortgage-rate changes and the Bluechip data on expected 30-year Treasury-yield over the next $h = 2$ quarters and add them to the current 20-year zero-coupon yield. The VAR produces a $h = 2$ quarter ahead forecast of the 20-year yield.

Figure 1 also shows NBER recessions as shaded areas. The plots indicate that expected excess returns computed from the VAR and the difference between survey and VAR interest-rate expectations have common business-cycle movements. The patterns appear more clearly in the lower panels which use longer (1 year) horizons. This is not surprising in light of the existing predictability literature which documents that expected excess returns on bonds and other assets are countercyclical when we look at longer holding periods, such as one year (e.g., Cochrane and Piazzesi 2005.) In particular, expected excess returns are high right after recession troughs. The lower panels show indeed high values for both series around and after the 1991, 2001 and 2009 recessions. The series are also high in 1984 and 1996, which are years of slower growth (as indicated, for example, by employment numbers) although they were not classified as recessions.

For shorter holding periods, the patterns are also there in the data but they are much weaker. However, the upper panels show additional recessions where similar patterns appear. For example, the two series in both panels are high in the 1970, 1974, 1980 and 1982 recessions or shortly afterwards. (As we can see in the upper panels, expected excess returns for short holding periods are large when annualized. Of course, the risk involved in these investment strategies is high, and so they are not necessarily attractive.)

The above findings depend on a particular choice of a statistical model, or VAR. We can

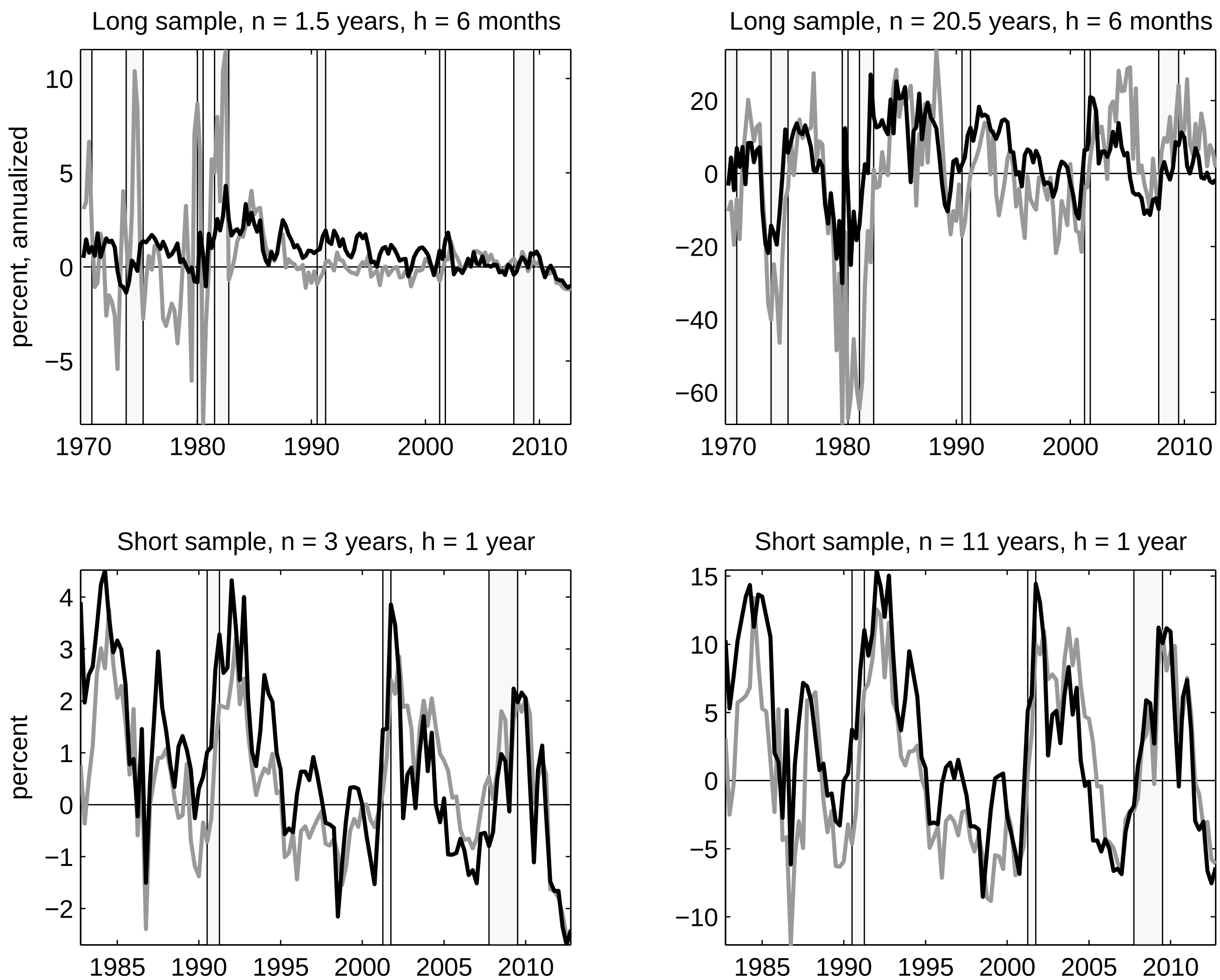

Figure 1: Each panel shows expectations of excess returns derived from a VAR in black (the left-hand side of equation (4)) and the difference between subjective and VAR interest-rate expectations in gray (the second term on the right-hand side of the equation) for the indicated bond maturity n and holding period/forecast horizon h. Shaded areas indicate NBER recessions. The numbers are annualized and in percent. The upper panels show data over a longer sample than the lower panels.

see the business-cycle patterns more directly by looking at realized survey forecast errors $E_t^* i_{t+h}^{(n-h)} - i_{t+h}^{(n-h)}$ instead of forecast differences relative to the VAR. Table 1 reports results from regressions of these realized survey forecast errors, scaled by $(n-h)$ as on the right-hand side of equation (4), on time t variables. The row labeled "YS" reports the slope coefficients of a regression on the time t yield spread between the 5-year bond and the 1-quarter bond. The point estimates in these regressions are positive, so that survey forecast errors $E_t^* i_{t+h}^{(n-h)} - i_{t+h}^{(n-h)}$ are countercyclical and thus systematic over the business cycle. In particular, survey interest rate forecasts tend to be above the subsequent realized values in periods of high spreads, and thus recessions. The slope coefficient estimates increase with the maturity of the bond whose interest rate we are forecasting and are significant for long bonds (10 and 30 years.) The R^2 in these regressions range from 2% to 14%. We also report the R^2 from a regression on five forward rates, as in Cochrane and Piazzesi (2005), indicated "CP R^2". Naturally, these R^2 are higher, and range from 8% - 19%.

A potential concern with Bluechip forecast data is that the survey is not anonymous, and so career concerns of survey respondents may matter. To address this concern, we also measure interest-rate expectations using the Survey of Professional Forecasters. Starting in 1992, the SPF reports median interest-rate forecasts for the 10-year Treasury bond over various forecast horizons. We find that median forecasts from the SPF are similar to those from the Bluechip survey. Importantly, the differences between SPF forecasts and VAR expectations show the same patterns as those documented in Figure 1.

To sum up, the evidence presented in this section suggests that subjective interest-rate expectations deviate from the expectations that we commonly measure from statistical models. Figure 1 suggests that these deviations may also be responsible for the time-variation in statistical bond premia.

Table 1: Forecast Error Regressions from Surveys

	maturity						
	1 year	2 year	3 year	5 year	7 year	10 year	30 year
YR	0.22	0.53	0.89	1.72	2.70	4.08	12.98
t-stat	0.60	0.84	1.05	1.46	1.85	2.09	2.18
R^2	0.02	0.03	0.04	0.06	0.08	0.10	0.14
CP R^2	0.14	0.10	0.08	0.09	0.11	0.13	0.19

Note: This table reports results from regressing h-period ahead forecast errors $(n-h)\left(E_t^* i_{t+h}^{(n-h)} - i_{t+h}^{(n-h)}\right)$ on time-t variables over the Bluechip sample. The forecast $E_t^* i_{t+h}^{(n-h)}$ is measured as median forecast in the Bluechip survey. The "YS" row reports the slope coefficient from a regression on a constant and the spread between the 5-year yield and the 3-month short rate, and its t-statistic computed with Hansen-Hodrick standard errors with 4 quarterly lags. The "CP R^2" uses five forward rates as right-hand side variables.

3 Joint Dynamics of Rates and Survey Forecasts

We describe the joint distribution of interest rates, inflation and median survey forecasts by a state space system with a small number of factors. For interest rates, this approach is known to work well because rates of different maturities are highly correlated. For example, in a typical cross section of interest rates at the quarterly frequency, the first two principal component (the "level" and "slope" of the yield curve) explain more than 95% of the variation. Since survey forecasts of interest rates also exhibit comovement, it should be possible to write a parsimonious model for both yields and forecasts.

To parametrize our state space system, we make use of two types of cross equation restrictions. First, in an arbitrage-free bond market, bond prices can be written as conditional expected payoffs under a "risk neutral" probability $\mathcal{Q}$, say. This risk neutral probability is typically different from the "physical" probability $\mathcal{P}$ that describes interest rates from the perspective of the econometrician. The difference reflects the presence of risk premia in market prices – in the bond context the key example is the failure of the expectations hypothesis of the term structure. The difference between $\mathcal{P}$ and $\mathcal{Q}$ is identified by comparing current long interest rates to conditional expectations of future short interest rates. We follow a standard approach for writing parsimonious state space models of the yield curve by parametrizing directly the Radon-Nikodym derivative of $\mathcal{Q}$ with respect to $\mathcal{P}$, where the parameters describe "market prices of risk". The large number of coefficients on yields in our state space system are then functions of a small number of parameters describing market prices of risk.

The second type of restriction says that survey forecasts are conditional expectations from a "subjective" probability $\mathcal{P}^*$. The difference between $\mathcal{P}$ and $\mathcal{P}^*$ is identified by the difference between our own (econometrician) forecasts and survey forecasts. We can write a parsimonious state space model of survey forecasts by parametrizing the Radon-Nikodym derivative of $\mathcal{P}^*$ with respect to $\mathcal{P}$, where the parameters describe systematic forecast bias in the surveys (relative to the econometrician). The large number of coefficients on survey forecasts in our state space model are then functions of a small number of parameters describing forecast bias.

We assume that there are three factors that follow a vector AR(1) process. For the purposes of estimation, it is convenient to identify two of the factors directly with observables from the yield curve. We select the first factor as the demeaned short term (1 quarter) interest rate and the second factor as the demeaned spread between the 5 year and 1 quarter interest rates. These variables capture movements in the level and slope of the yield curve, respectively. We identify the third factor with expected inflation. With these assumptions, the dynamics of the short rate, the spread and inflation can be summarized by a "small" state space system. Under the probability $\mathcal{P}$, the distribution of these variables, stacked in a vector h_t, can be summarized by the innovation representation

$$
\begin{aligned}
h_{t+1} &= \mu_h + \phi_h x_t + e_{t+1}, \qquad (5)\\
x_{t+1} &= \phi_x x_t + \sigma_x e_{t+1}.
\end{aligned}
$$

where x_t is a three-dimensional vector of factors driven by i.i.d. zero-mean normal shocks with $E\left(e_t e_t^\top\right) = \Omega$, respectively.[3] In order to derive observation equations for our other observables – other interest rates and survey forecasts – we now discuss how we parametrize the changes of measures between $\mathcal{P}$, $\mathcal{P}^*$ and $\mathcal{Q}$. Since h contains interest rates, the changes of measure will also imply additional restrictions on μ_h, ϕ_h and Ω discussed below.

Observation equations for interest rates: from physical to risk neutral probability

Since riskless arbitrage opportunities would be quickly eliminated by bond market participants, it makes sense to rule them out from the beginning. This is done by assuming that there exists a "risk neutral" probability Q under which bond prices are discounted present values of bond payoffs. In particular, the prices $P^{(n)}$ of zero-coupon bonds with maturity n satisfy the recursion

$$P_t^{(n)} = e^{-i_t} E_t^Q \left[P_{t+1}^{(n-1)} \right] \tag{6}$$

with terminal condition $P_t^{(0)} = 1$.

As illustrated in Figure 2, we specify the Radon-Nikodym derivative ξ_t^Q of the risk neutral probability Q with respect to the probability $\mathcal{P}$ by $\xi_0^Q = 1$ and

$$\frac{\xi_{t+1}^Q}{\xi_t^Q} = \exp\left(-\frac{1}{2} \lambda_t^\top \Omega \lambda_t - \lambda_t^\top e_{t+1} \right),$$

where λ_t is a vector which contains the "market prices of risk" associated with innovations e_{t+1}. This means that the (log) expected excess returns at date t on a set of assets with payoffs proportional to $\exp(\sigma_x e_{t+1})$ is equal to $\sigma_x \Omega \sigma_x' \lambda_t$. Again, the innovations e_{t+1} are i.i.d. zero-mean normal with variance Ω so that ξ_t^Q is a martingale under $\mathcal{P}$.

We assume that risk premia are linear in the state vector, that is,

$$\lambda_t = l_0 + l_1 x_t,$$

for some 3×1 vector l_0 and some 3×3 matrix l_1. With this specification, $e_{t+1}^Q = e_{t+1} + \Omega \lambda_t$ is an i.i.d. zero-mean normal innovation under Q. The state vector dynamics under Q are

$$x_{t+1} = -\sigma_x \Omega l_0 + (\phi_x - \sigma_x \Omega l_1) x_t + \sigma_x e_{t+1}^Q := \mu_x^Q + \phi_x^Q x_t + \sigma_x e_{t+1}^Q.$$

The recursion for bond prices (6) implies that the yield to maturity of an n-period zero-coupon bond is :

$$i_t^{(n)} = a_n + b_n^\top x_t, \tag{7}$$

where a_n is a scalar and b_n is a 3×1 vector of coefficients. Under our model, interest rates of all maturities are thus deterministic functions of the same small number of factors. If the interest rate coefficients a_n and b_n were unrestricted in the estimation, the resulting model

[3] Since (5) is an innovation representation, the same shocks e_t enter the observation equation as the transition dynamics. This is not restrictive, because any estimation can only identify three forecast errors e_t for the three observables h_t.

would generally imply that there are opportunities for riskless arbitrage in the bond market. Instead, the coefficients in our model solve the recursion

$$\begin{aligned} a_{n+1} &= \left(\frac{n}{n+1}\right)\left(a_n + b_n^\top\ \mu_x^Q - \frac{n}{2} b_n^\top \sigma_x \Omega \sigma_x^\top b_n\right) + \frac{1}{n+1} a_1, \\ b_{n+1}^\top &= \left(\frac{n}{n+1}\right) b_n^\top\ \phi_x^Q + \left(\frac{1}{n+1}\right) b_1^\top, \end{aligned} \tag{8}$$

starting at the unconditional mean of the short rate for a_1 and the $b_1^\top = (1\ \ 0\ \ 0)$. These recursions insure the absence of arbitrage and thereby impose cross-equation restrictions on interest rates (7). They also make the model more parsimonious.

Given these formulas for interest rates, the expected excess (log) return on an n-period bond is

$$E_t\left[rx_{t+1}^{(n)}\right] + \frac{1}{2} var_t\left[rx_{t+1}^{(n)}\right] = -\left(n-1\right) b_{n-1}^\top \sigma_x \Omega \lambda_t. \tag{9}$$

The constant thus depends on the market price of risk parameter l_0, while the slope coefficient is driven by the parameter l_1. The expression shows that the expectations hypothesis holds under the physical probability if $l_1 = 0$.

So far we have discussed only nominal yields. For inflation indexed bonds, we can develop a recursion analogous to (6) for pricing real payoffs. In particular, the nominal prices $\hat{P}^{(n)}$ of an indexed zero-coupon bonds with maturity n satisfy the recursion

$$\hat{P}_t^{(n)} = e^{-i_t} E_t^Q\left[\hat{P}_{t+1}^{(n-1)} e^{\pi_{t+1}}\right], \tag{10}$$

again with terminal condition $\hat{P}_t^{(0)} = 1$. Since inflation is also part of our system, the affine exponential structure of prices is maintained and we obtain a set of observation equations for real interest rates analogous to (7).

Observations for survey forecasts: from physical to subjective probability

We assume that subjective beliefs are also described by our state space system, but with different coefficients. To define subjective beliefs, we represent the Radon-Nikodym derivative of the subjective probability $\mathcal{P}^*$ with respect to the probability $\mathcal{P}$ by a stochastic process ξ_t^*, with $\xi_0^* = 1$ and

$$\frac{\xi_{t+1}^*}{\xi_t^*} = \exp\left(-\frac{1}{2}\kappa_t^\top \Omega \kappa_t - \kappa_t^\top e_{t+1}\right). \tag{11}$$

Since e_t is i.i.d. mean-zero normal with variance Ω under the probability $\mathcal{P}$, ξ_t^* is a martingale under $\mathcal{P}$. Since e_t is the error in forecasting z_t, the process κ_t captures the subjective bias in their forecast of z_t. The forecast bias is linear in state variables, that is

$$\kappa_t = k_0 + k_1 x_t.$$

Standard calculations now deliver that $e_{t+1}^* = e_{t+1} + \Omega\kappa_t$ is i.i.d. mean-zero normal with variance matrix Ω under the subjective belief $\mathcal{P}^*$, so that the dynamics of the observables h_t

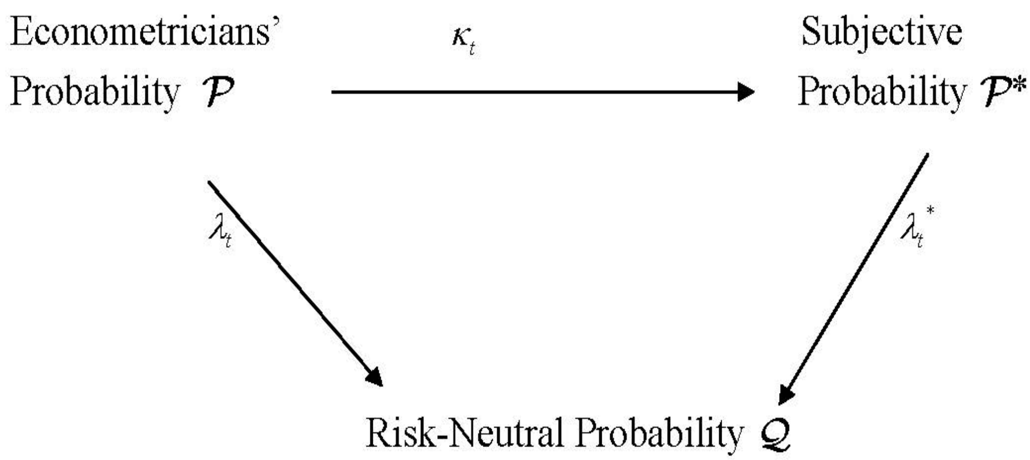

Figure 2: Changing between the econometrician's probability $\mathcal{P}$, the investors' probability $\mathcal{P}^*$ and the risk-neutral probability $\mathcal{Q}$.

under $\mathcal{P}^*$ can be represented by

$$\begin{aligned} h_{t+1} &= \mu_h - \Omega k_0 + (\phi_h - \Omega k_1)x_t + e^*_{t+1} := \mu^*_h + \phi^*_h x_t + e^*_{t+1} \\ x_{t+1} &= -\sigma_x \Omega k_0 + (\phi_x - \sigma_x \Omega k_1)x_t + \sigma_x e^*_{t+1} := \mu^*_x + \phi^*_x x_t + \sigma_x e^*_{t+1} \end{aligned} \tag{12}$$

The vector k_0 thus affects the mean of h_{t+1} under subjective beliefs and also the mean of the state variables x_{t+1}, whereas the matrix k_1 determines how their forecasts of h_{t+1} deviate from the econometrician's forecasts as a function of the state x_t.

Investors have beliefs $\mathcal{P}^*$ rather than $\mathcal{P}$, and so their subjective market prices of risk are not equal to λ_t. Instead, their market price of risk process is $\lambda^*_t = \lambda_t - \kappa_t$, so that the bond prices computed earlier are also risk-adjusted present discounted values of bond payoffs under their belief $\mathcal{P}^*$:

$$P^{(n)}_t = e^{-i_t} E^Q_t \left[P^{(n-1)}_{t+1} \right] = e^{-i_t} E^*_t \left[\exp\left(-\frac{1}{2} \lambda^{*\top}_t \Omega \lambda^*_t - \lambda^{*\top}_t e_{t+1} \right) P^{(n-1)}_{t+1} \right].$$

Investors have excess (log) returns expectations

$$E^*_t \left[x^{(n)}_{t+1} \right] + \frac{1}{2} var^*_t \left[x_{t+1} \right] = -\left(n-1 \right) b^\top_{n-1} \sigma_x \Omega \lambda^*_t.$$

More generally, we impose that subjective expectations are *on average* correct. The subjective forecasts of any observable is then

$$\begin{aligned} E^*_t h_{t+k} &= \mu_h + \phi^*_h \left(\phi^*_x \right)^{k-1} x_t, \\ E^*_t i^{(n)}_{t+k} &= a_n + b^\top_n \left(\phi^*_x \right)^k x_t. \end{aligned} \tag{13}$$

3.1 Estimation

The estimation proceeds in two steps. We first estimate the joint distribution of the factors in equation (5), expected inflation and two interest rates. In a second step, we use data on many other interest rates as well as survey forecasts to estimate the coefficients governing the changes of measure between in Figure 2.

Data

We work with quarterly observations over the sample 1952:2–2012:4. Our inflation series π_t is the GDP deflator. The data on zero-coupon interest rates are as in section 2.1. We include yields with maturity 1 quarter as well as 2, 5, 10 and 20 years in the estimation. We also use data on real zero coupon rates from the TIPS curve starting in 2000. While TIPS have been trading since 1997, the market was not very liquid for the first few years. We use TIPS maturities of 1, 2, 5, 10 and 20 years.

Short horizon interest rate forecasts (2 and 4 quarters ahead) come from the Goldsmith-Nagan and Bluechip surveys, and are available from 1970 onwards. Until 1984, we have forecasts of 1 quarter and 20 year rates for the 2 quarter horizon. After 1984 (that is, in the BlueChip era), we can add the 1 year horizon and maturities of 1, 2, 5 and 10 years.

Since 1984, the BlueChip survey also semiannually reports long term forecasts – participants are asked to predict average inflation and interest rates over several future calendar years between 1.5 and 5.5 years ahead (details depend on the survey date), as well as average inflation and interest rates 6-10 years ahead. We include long forecasts for inflation as well as interest rates of maturities 1 quarter and 1, 5 and 10 years. Finally, we use measures of inflation expectations from the Survey of Professional Forecasters (which specifically asks to forecast the rate of growth in the GDP deflator.) This survey is conducted at a quarterly frequency during the years 1968:4-2012:4.

Step 1: Statistical state space system

The observation equation for the vector $h_t = (i_t^{(1)}, i_t^{(20)} - i_t^{(1)}, \pi_t)$ is

$$\begin{pmatrix} i_{t+1}^{(1)} \\ i_{t+1}^{(20)} - i_{t+1}^{(1)} \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} a_1 \\ a_{20} - a_1 \\ \mu_\pi \end{pmatrix} + \begin{pmatrix} b_1^\top \phi_x \\ (b_{20} - b_1)^\top \phi_x \\ \phi_\pi \end{pmatrix} x_t + \begin{pmatrix} b_1^\top \sigma_x \\ (b_{20} - b_1)^\top \sigma_x \\ \sigma_\pi \end{pmatrix} e_{t+1}.$$

Here we have added three sets of restrictions on the system (5). First, to identify the first two factors with the demeaned short rate and the demeaned spread, we set a_1 and a_{20} equal to the unconditional mean of the short rate and the 5-year rate, $b_1^\top = (1,0,0)$ and $b_{20}^\top = (1,1,0)$. As a result, the first two observation equations become simply copies of the first two state equations. We also set the first two rows of σ_x equal to $\begin{pmatrix} I_{(2\times 2)} & 0_{(2\times 1)} \end{pmatrix}$. Second, we identify the third factor with expected inflation by setting $\phi_\pi = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$ and $\sigma_\pi = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$. In contrast to the first two factors, the third factor is latent. Even after these restrictions, the system is more flexible than a first order VAR in the three observables. This is because the presence of the latent factors allows for MA(1) style dynamics which are important for capturing the dynamics of inflation.

We estimate the unconditional means $\mu_z = (a_1, a_{20} - a_1, \mu_\pi)$ from the sample means of the short rate, the spread, and inflation. Table 2 reports the estimates in percent, so that $a_1 \times 100 = 1.18$ is a 4.72% average short rate in annualized terms. The remaining 18 parameters $(\phi_x, \sigma_x, \Omega)$ are estimated by maximum likelihood. The parameter estimates also deliver a sequence of estimates $\hat{x}_t$ for the realizations of the state variables, starting from their unconditional mean $x_0 = 0$. Of course, the first two components of $\hat{x}_t$ are equal to the demeaned short rate and spread. However, the third factor is latent, and thus its entry of $\hat{x}_t$ represent its conditional expectation given the data.

Table 2: Estimated Dynamics

Panel A: Maximum Likelihood of State Space System

		$\mu_h \times 100$			$\mathrm{chol}(\Omega) \times 100$	
short rate		1.18		0.236	0	0
		–		(0.011)	–	–
yield spread		0.26		-0.112	0.118	0
		–		(0.009)	(0.005)	–
exp. inflation		0.82		0.035	0.012	0.245
		–		(0.016)	(0.016)	(0.011)
		ϕ_x			σ_x	
short rate	0.907	0.147	0.129	1	0	0
	(0.029)	(0.065)	(0.043)	–	–	–
yield spread	0.037	0.747	-0.057	0	1	0
	(0.020)	(0.045)	(0.030)	–	–	–
exp. inflation	-0.014	-0.044	0.963	0.173	-0.042	0.547
	(0.022)	(0.053)	(0.033)	(0.095)	(0.134)	(0.068)

Note: This table contains the maximum likelihood estimates for the state space system (5)

$$\begin{aligned} h_{t+1} &= \mu_h + \phi_h x_t + e_{t+1} \\ x_{t+1} &= \phi_x x_t + \sigma_x e_{t+1} \end{aligned}$$

where e_{t+1} is an i.i.d. normal shock with mean zero and $var\left(e_{t+1}\right) = \Omega$. The observables $h_t = \left(i_t^{(1)}, i_t^{(20)} - i_t^{(1)}, \pi_t\right)$ consist of the short rate, the yield spread, and inflation. The state vector x_t consists of the short rate, the yield spread, and expected inflation, starting from $x_0 = 0$. Standard errors are in brackets. The sample is quarterly, 1952:2-2012:4.

Step 2: Changes of measure

We estimate the parameters (l_0, l_1, k_0, k_1) that govern the changes of measure by minimizing two types of squared fitting errors. The first type of fitting errors are squared differences between yield data and model-implied yields. At the second step, we take as given the estimate of the mean short rate a_1 and the factors $\hat{x}_t$ derived in step 1. For every value of the pair (l_0, l_1), we can form samples of predicted zero coupon yields according to (7). We then form squared fitting errors between actual yields and predicted yields computed from (**??**). We use yields of maturities 1, 2, 5, 10, 15, 20 and 30 years. We also impose the constraint that the spread between the 5-year and 1-quarter rate that serves as a factor be matched exactly. Panel B of Table 2 reports the estimates of l_0 and l_1.

The second type of fitting error is the difference between survey forecasts and the subjective forcasts (13). For given the factor estimates $\widehat{x}_t$ and means of the macro variables $\hat{\mu}_h$ from step 1 as well as interest rate coefficients (a_n, b_n), we can form samples of subjective forecasts for every (k_0, k_1):

$$\begin{aligned} E_t^* h_{t+h} &= \widehat{\mu}_h + \phi_h^* \left(\phi_x^*\right)^{k-1} \hat{x}_t, \\ E_t^* i_{t+h}^{(n)} &= \hat{a}_n + \hat{b}_n^\top \left(\phi_x^*\right)^{k} \hat{x}_t. \end{aligned} \tag{14}$$

Over our sample, subjective forecasts are approximately correct *on average*. To simplify the estimation, we thus set $k_0 = 0$, and estimate only k_1. Panel C in Table 2 reports the estimated coefficients k_1. Under our parametrization we would find $\phi_x^* = \phi_x$ and $\phi_h^* = \phi_h$ if k_1 were equal to zero.

Table 2: Estimated Dynamics

Panel C: Subjective state space system

		k_1	
short rate	-0.125	-0.265	0.033
yield spread	0.071	0.097	0.0433
exp. inflation	0.069	0.036	0.254

Note: We estimate k_1 by minimizing the squared difference between subjective forecasts (14) and survey forecasts. The table reports the estimated k_1, while we set $k_0 = 0$. Standard errors are computed by GMM, taking into account the three-step nature of the estimation.

3.2 Results

The appendix reports our estimates for subjective beliefs and beliefs derived from the statistical model. To understand the estimated statistical dynamics, we report covariance functions which completely characterize the Gaussian state space system. Figure 3 plots covariance functions computed from the state space system and the raw data. At 0 quarters, these represent variances and contemporaneous covariances. The black lines from the system match the gray lines in the data quite well. To interpret the units, consider the upper left panel. The quarterly variance of the short rate is 0.58 in the data which amounts to $\sqrt{0.58 \times 4} = 1.52$ percent annualized volatility. Figure 3 shows that all four variables are positively autocorrelated. For example, the covariance $\text{cov}\left(i_t^{(1)}, i_{t-1}^{(1)}\right) = \rho \text{ var}\left(i_t^{(1)}\right) = \rho \times 0.58 = 0.55$ which implies that the first-order autocorrelation is roughly 0.95.

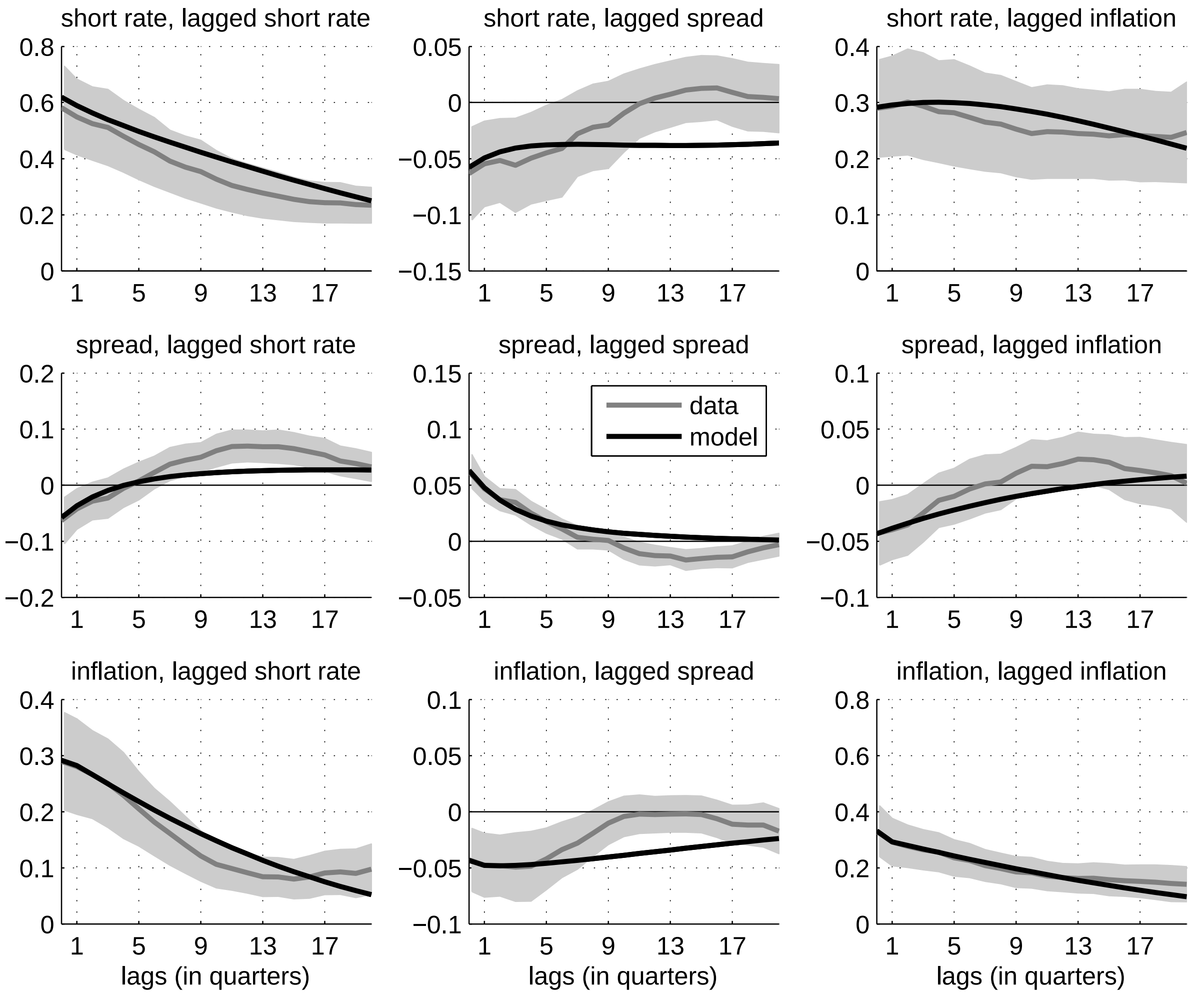

Figure 3: Covariance functions for the observables computed from the estimated state space system and from the raw data. Shaded areas indicate 2 $\times$ standard errors bounds around the covariance function from the data computed with GMM. For example, the graph titled "short rate, lagged short rate" shows the covariance of the current short rate with the short rate lagged τ quarters, where τ is measured in quarters on the horizontal axis.

To understand the implications of the estimated parameters l_0 and l_1, we investigate how expected excess returns depend on the state variables. This dependence can be derived from the state space system together with the yield coefficients. Table 3 reports the loadings of these conditional expected values on the state variables. For a 1-quarter holding period, these loadings are $-(n-1)\,b_{n-1}^{\top}\sigma_x\Omega l_1$. For longer holding periods, we can compute $E_t\left[rx_{t,t+h}^{(n)}\right]+\frac{1}{2}var_t\left(rx_{t,t+h}^{(n)}\right)$ using the recursions for the coefficients a_n and b_n. The coefficients in Table 3 indicate that the expected excess return on a 2-year bond is high in periods with high spreads. For example, a 1-percent increase in the spread (and everything else constant) leads to a 2.07 percent increase in the statistical premium. This dependence on the spread captures is most important driving force of statistical premia and captures their countercyclical nature. The premium on the 10-year bond has larger loadings on all state variables. Roughly speaking, the coefficient for the 10 year bond are roughly 10 times higher than for the 2-year bond. Panel B of Table 3 reports by how much the model-implied yields differ from observed yields on average. By construction, the model hits the 1-quarter and 5-year interest rates exactly, because these rates are included as factors. For intermediate maturities, the error lies within the $24-45$ basis points range. We will see below that these errors are sufficiently small for our purposes.

Table 3: Estimation of Statistical Model

Panel A: Loadings of expected excess returns on state variables

	n	horizon $h=1$ year: short rate	spread	exp inflation
maturity of the bond	2 year	1.13	2.07	-1.36
	10 year	6.71	17.99	-9.55

Panel B: Fitting errors for bond yields (annualized)

	maturity: 1 qrt	1 year	5 year	10 year	15 year	20 year	30 year
mean absolute errors (in %)	0	0.30	0	0.24	0.36	0.42	0.45

NOTE: Panel A reports the model-implied loadings of the function $E_t rx_{t,t+h}^{(n)}+\frac{1}{2}var_t\left(rx_{t,t+h}^{(n)}\right)$. Since the conditional variance of excess returns is constant, we can derive these loadings from

$$
\begin{aligned}
E_t rx_{t,t+h}^{(n)} &= E_t p_{t+h}^{(n-h)} - p_t^{(n)} - h i_t^{(h)} \\
&= -(n-h)\left(a_{n-h}+b_{n-h}^{\top}E_t\left[x_{t+h}\right]\right)+n\left(a_n+b_n^{\top}x_t\right)-h\left(a_h+b_h^{\top}x_t\right)
\end{aligned}
$$

on the current factors x_t for a holding period of $h=1$ year and bond maturities of $n=2$ years, 10 years. Panel B reports mean absolute model fitting errors for yields.

The statistical dynamics of the state variables are persistent. The largest eigenvalues of the matrix ϕ_x are complex with a modulus of 0.95, while the third eigenvalue is 0.70. In Figure 3, the autocovariance functions of the short rate and inflation are flatter than that of the spread, which indicates that they are more persistent. The short rate and the spread are contemporaneously negatively correlated and the spread is negatively correlated with the short rate lagged less than year, and positively correlated with longer lags of the short rate. The short rate is negatively correlated with the lagged spread, even at long lags.

Factors Dynamics under Subjective Beliefs

By construction, subjective forecasts are on average equal to the forecasts from our statistical model. At the same time, the factors are more persistent under the subjective system. The largest eigenvalues of the two matrices

$$\phi_x = \begin{pmatrix} 0.91 & 0.15 & 0.13 \\ 0.04 & 0.75 & -0.06 \\ -0.01 & -0.04 & 0.96 \end{pmatrix}, \quad \phi_x^* = \begin{pmatrix} 0.98 & 0.09 & -0.09 \\ -0.01 & 0.93 & 0.06 \\ 0.08 & 0.07 & 0.82 \end{pmatrix}.$$

are 0.95 and 0.98, respectively. The next highest eigenvalues are 0.70 for the statistical system and 0.86 for subjective forecasts. Other things equal, a one-percent increase in the short rate (spread) increases the subjective forecast of the short rate (spread) next period by 98% (93%), as opposed to 91% (75%) under the statistical model.

The estimated subjective dynamics imply that subjective risk premia are less cyclical than statistical premia. The subjective loadings on the spread in Table 4 are smaller than those in Table 3. For long bonds, the loading on the short rate increases under subjective beliefs, while the loading on expected inflation is now also positive. This indicates that subjective premia on long bonds will reflect some of the low-frequency movements in expected inflation and nominal interest rates.

Panel B of Table 4 reports mean absolute distances between the survey forecasts and model-implied forecasts, for both the subjective belief and the statistical model. Comparison of these errors provides a measure of how well the change of measure works to capture the deviation of survey forecasts from statistical forecasts.

Table 4: Estimation of Subjective Model

Panel A: Loadings of expected excess returns on state variables

		horizon $h = 1$ year		
	n	short rate	spread	exp. inflation
maturity	2 year	0.46	1.36	0.78
	10 year	4.64	3.42	-0.49

Panel B: Mean absolute fitting errors for survey forecasts (% p.a.)

	Bluechip sample, maturity of forecasted yield									
maturity	1 qrtr	1 year			5 year			10 year		
horizon	1 yr	1yr	3 yr	6-10 yr	1 yr	3yr	6-10 yr	1 yr	3 yr	6-10 yr
subj. model	.35	.41	.52	.40	.26	.56	.36	.35	.57	.39
obj.model	.63	.62	1.01	.71	.54	1.00	.90	.60	.93	.71
random walk	.53	.56	1.48	1.56	.43	1.10	1.18	.36	.79	.88

	Combined sample, maturity of forecasted yield	
maturity	1 year	20 year
horizon	2 qtrs	2 qtrs
subj. model	.84	.49
obj. model	..91	.60
random walk	.73	.31

The results show that the improvement is small for short-horizon forecasts of short yields. However, there is a marked reduction of errors for 1-year forecasts, especially for the 10-year bond. Figure 4 shows where the improvements in matching the long-bond forecasts come from. The top panel shows one-year ahead forecasts of the 10-year zero coupon rate constructed from survey data in Section 2.2.1, together with the corresponding forecasts from our subjective and statistical models, for the sample 1982:4-2007:3. All forecasts track the actual 10-year rate over this period, which is natural given the persistence of interest rates. The largest discrepancies between the survey forecasts and the subjective model on the one hand, and the statistical model on the other hand, occur during and after the recessions of 1990 and 2001. In both periods, the statistical model quickly forecasts a drop in the interest rate, whereas investors did not actually expect such a drop. The subjective model captures this property.

For our asset pricing application, we are particularly interested in how well the subjective model captures deviations of survey forecasts of long interest rates from their statistical forecasts over the business cycle. As discussed in Section 2.2.1, this forecast difference is closely related to measured expected excess returns. The bottom panel of the figure focuses again on forecasting a 10-year rate over one year, and plots the difference between the survey forecast and the statistical model forecast, as well as the difference between the subjective and statistical model forecasts. It is apparent that both forecast differences move closely together

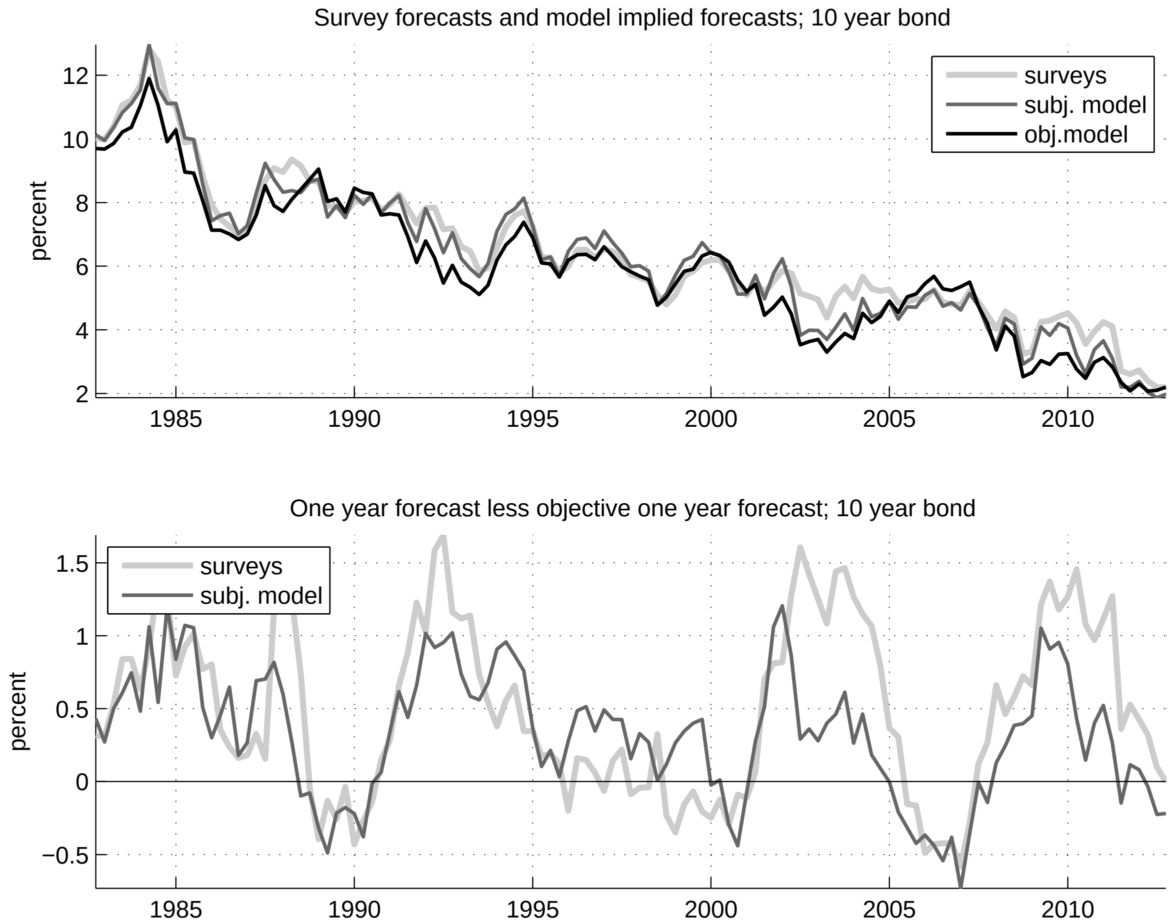

Figure 4: The top panel shows one-year ahead forecasts of the 10-year zero coupon rate constructed from survey data in Section 2.2.1, together with the corresponding forecasts from our statistical and subjective models. The bottom panel shows the difference between the survey forecast and the statistical model forecast, as well as the difference between the subjective and statistical model forecasts.

at business cycle frequencies, increasing during and after recessions. We thus conclude that the subjective model is useful to capture this key fact about survey forecasts that matters for asset pricing.

3.3 Subjective risk premia

We now compare our estimated subjective risk premia to common statistical measures of risk premia. The motivation is that statistical measures of premia provide stylized facts that rational expectations asset pricing models try to match. In particular, empirical evidence of predictability of excess returns from standard predictability regressions has led to a search for sources of time varying risk or risk aversion. The preliminary results of section 2.2.1 suggest that less time variation in risk premia is required once subjective forecast errors are taken into account. Here we quantify how much time variation in expected excess returns is left once we move to subjective beliefs.

We focus on 1-year holding period returns on bonds with 2 and 10 years maturity. We compare our subjective premia to three statistical measures of statistical premia. The first is the fitted value of a regression of excess returns on a single yield spread, the 5-year-1-quarter spread, denoted the YS measure. In other words, we regress the excess return $rx_{t,t+4}^{(n)}$ on $i_t^{(20)} - i_t^{(1)}$ and a constant. This regression is closely related to that in the classic Fama-Bliss study of bond return predictability, which uses the forward-spot spread. The second measure (labelled CP measure) is the fitted value from a regression on five yields with maturities 1,2,3, 4 and 5 years. This follows Cochrane and Piazzesi (2005) who showed that this approach leads to higher R^2s. The third measure is, for each subjective model specification, the forecast from the corresponding statistical model which provides the conditional expectation of the excess return.

Table 5 summarizes the properties of the regression based measures of statistical premia. According to these measures, the volatility of the predictable part of 1-year holding period returns is below 1% per year for the 2-year bond, and around 5.1% for the 10-year bond. The regression based on five yields naturally delivers a higher R^2, around 20% on both bonds. We are also interested in the frequency properties of premia. We use a band pass filter to decompose premia into three orthogonal components, a low frequency "trend" component (period > 8 years), a "cycle" component (period between 1.5 and 8 years), as well as high frequency noise. The columns labelled "trend" and "cycle" show the percentage of variance contributed by the respective components. Since the yield spread is a key business cycle indicator, the YS measure is particularly cyclical. The CP measure improves in part by including a larger trend component.

Table 5: Statistical Premia from Regressions

maturity 2 years				maturity 10 years			
Regression on yield spread (YS measure)							
volatility	% trend	% cycle	R^2	volatility	% trend	% cycle	R^2
0.55	13	62	0.10	5.12	13	62	0.24
Regression on five yields (CP measure)							
volatility	% trend	% cycle	R^2	volatility	% trend	% cycle	R^2
0.73	34	43	0.18	4.93	24	62	0.22

Table 6 presents a set of comparison statistics. The first line shows the properties of subjective premia itself. For the baseline model, the standard deviation of the one-year premium on a 2-year bond is 65 basis points; it is 3% on the 10-year bond. These volatilities are substantially smaller than those of regression measures of premia. The frequency properties are also different: subjective premia tend to have larger trend components and smaller cyclical components than regression based premia. This is particularly pronounced for the longer 10 year bond: in regression models at least one half of the time variation in premia is cyclical, whereas for subjective premia the share of cyclical variation is only 12%.

Table 6: Subjective Risk Premia

	maturity 2 years			maturity 10 years		
	Baseline Model					
	volatility	% trend	% cycle	volatility	% trend	% cycle
	0.65	80	29	3.20	76	12
	volatilities relative to measures of statistical premia					
	total	trend	cycle	total	trend	cycle
YS measure	1.18	2.91	0.46	0.62	1.50	0.28
CP measure	0.89	1.36	0.42	0.65	1.15	0.29
state space system	0.83	1.00	0.47	0.53	0.68	0.30

The other rows in the table consider directly the change in volatility as one moves from statistical to subjective premia. All numbers are ratios of standard deviations, subjective divided by statistical. The columns labelled "total" report the volatility of a the subjective premium as a fraction of the volatility of the statistical premium for the different measures. They range between 30 and 60 percent. Since the numerator is always the same, the volatility ratios are lower the better the comparison statistical model predicts excess returns. The largest ratios arise for the YS measure which generates the least time variation in statistical premia. The columns labelled "trend" and "cycle" report volatility ratios for the trend and cycle components across models. For the regression based measures, the reduction in volatility is driven primarily by a reduction in the volatility of the cyclical component. In fact, for the YS measure, the trend component is less volatile than that of the subjective premium.

The row labelled "state space system" compares the subjective premium to the premium from the estimated state space system. For the 10-year bond, this system is a better predictor of expected excess returns than even the CP regression based measure. The reason is that the expected inflation state variable helps forecast returns. For the system, the move to subjective premia implies a substantial reduction in the volatility of the trend component. The system differs from the regression based measures in that it generates premia with somewhat larger trend components; the shares of the trend components is 100% for the 2 year bond and 68% for the 10 year bond. These components are due to the presence of expected inflation in the system.

Figure 5 plots subjective premia on the two bonds together with the respective CP measures as well as the statistical premia from the state space system. The properties from the table are also visible to the naked eye. Consider first the long, (10 year) bond in the bottom panel. Both measures of statistical premia show substantial cyclical movements: most recessions during the sample period can be identified as upward spikes in statistical premia, for example in 1970, 1991, 2001 and 2009. The CP measure also spikes in 1974 and 1979. Here the state-space system measure responds less to the business cycle; this is because it is driven more by expected inflation which lowers premia during this period. In contrast to both measures of statistical premia, the subjective premium on the long bond responds only weakly to recessions. The bulk of the movement in the subjective premium is at low frequencies: it was high in the late 1970s and early 1980s when the level of the yield curve was high, and low towards the beginning and end of our sample.

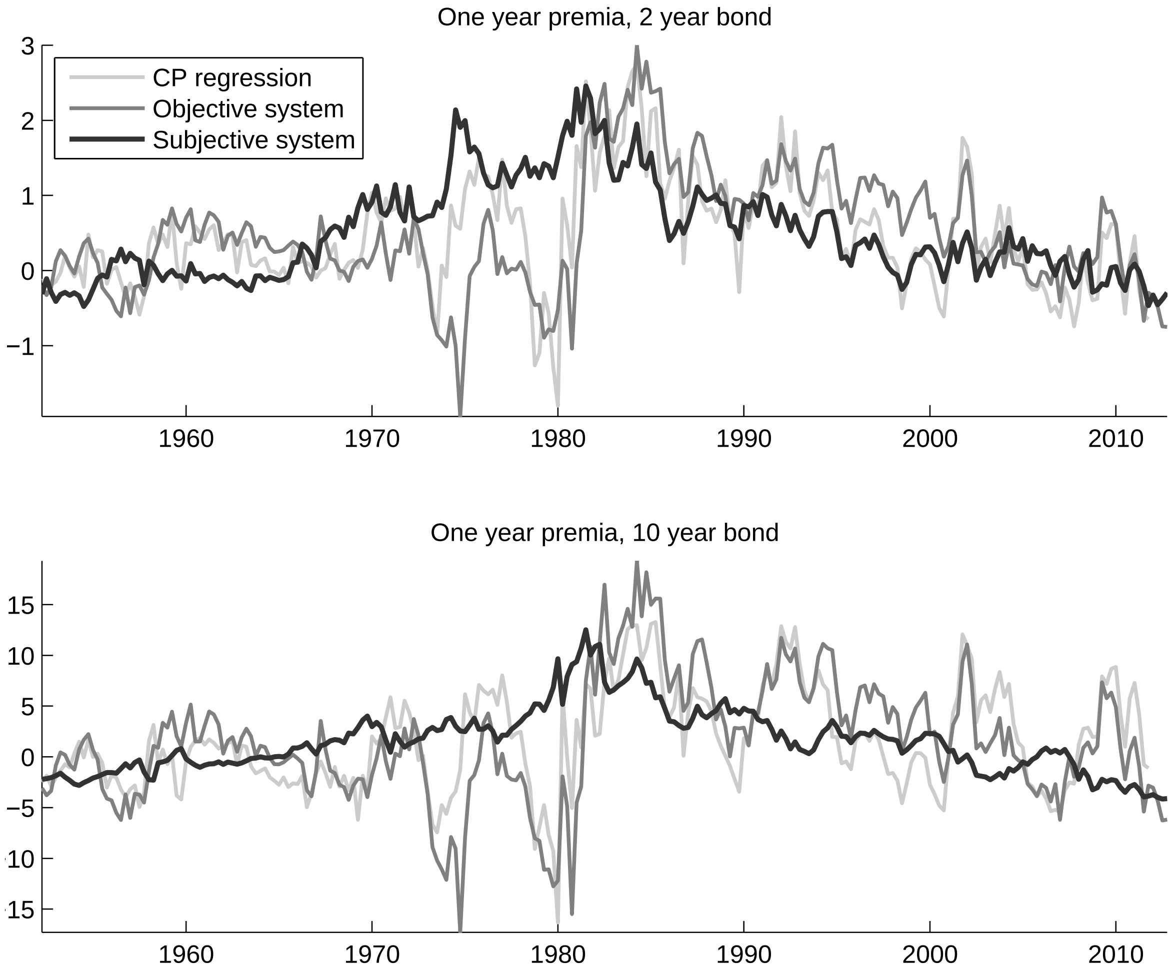

Figure 5: Investor premia compared with measures of statistical premia. Premia are expected excess holding period returns over one year, for a 2 year bond (top panel) and a 10 year bond (bottom panel). In both panels, black lines are estimated investor premia, light gray lines are fitted values from CP regression, dark gray lines are statistical premia from the state space system.

Consider next the medium (2 year) bond. It is clear again that the subjective premium is less volatile than the measures of statistical premia. At the same time, recession periods now register as upward spikes in all of the displayed lines. The main difference between statistical and subjective premia for the medium bond is in the trend: the subjective premium is much smaller in the late 1970s and early 1980s then the statistical premia. Comparing the statistical premia across maturities (across panels), it is also apparent that statistical premia on the long bond are more cyclical and exhibit less trend than statistical premia on the medium bond. For subjective premia, the situation is the reverse.

The intuition for these results comes from the differences between survey forecasts and forecasts derived from a statistical model. Under a statistical model, both the slope and the level are indicators of high expected excess returns. For example, Figure 5 shows that measures of statistical premia comove positively with both slope and level. We have seen in the previous section that survey forecasters treat both the level and the slope of the yield curve as more persistent than what they are under a statistical model. This difference between survey forecasts and forecasts derived from a statistical model of future bond prices then weakens the effect of both indicators.

If the level of the yield curve is high, survey forecasters, who view the level as more persistent than does an statistical model, expect higher interest rates, and hence lower prices, than the statistical model. Lower expected prices means lower expected excess returns. Similarly, if the slope is high, survey forecasters, who view the slope as more persistent, expect higher spreads, and hence higher long interest rates, and lower long bond prices, than the statistical model. Again, lower expected prices means lower expected excess returns. Both situations (high level, high spread) which lead statistical models to indicate high expected excess returns thus induce survey forecasters to predict lower prices and returns than the statistical models. This generates the overall reduction in volatility.

The higher persistence of level and slope perceived by survey forecasters also helps explain the different frequency properties of premia on medium (for example 2 year) and long (for example 10 year) bonds. The level of the yield curve is always relatively more important for short bonds rather than for long bonds. This is true not only for yields themselves, but also for measures of statistical premia: those measures are driven relatively more by the slope for long bonds and relatively more by the level for medium bonds. A move to subjective premia weakens the effect of both indicators of high premia, the slope and the level. For a given maturity, it tends to weaken more the effect of the indicator that is more important. The move thus makes the premium on long bonds less responsive to the slope and it makes the premium on medium bonds less responsive to the level. This is the effect displayed in the figure.

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