Comment on Gabaix and Laibson, "The 6D bias and the equity premium puzzle"

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1 Discussion

An economy populated by a representative agent with power utility predicts an equity premium which is far below the realized equity premium in postwar data, at least for “reasonable parameters” for the endowment process and the coefficient of relative risk aversion γ. This is the equity premium puzzle stated by Mehra and Prescott (1985). Simply increasing γ (and somehow arguing that this is “reasonable”) does not solve the puzzle, because a high γ counterfactually leads to a high risk-free rate. The few models in the literature today that may be considered puzzle-free still rely on high γ's. An example is Campbell and Cochrane (1999) who use an average γ of 50. An argument that relies on estimation bias for γ alone, as suggested by the title of the paper, can therefore not be enough to reconcile the standard model with the data. But there is more to the model of Gabaix and Laibson than the title indicates, because it is populated by agents whose heterogeneity matters.

The high equity premium and the low riskfree rate are, literally speaking, no puzzles in the model: asset prices are specified exogenously. The endogenous variables in this model are the consumption processes of individual investors. By summing these over a group of investors, the paper obtains a measure of aggregate consumption. The interpretation of this portfolio choice model, or Merton model, as a production economy with exogenous production technologies (or as a small open economy) leads to another endogenous variable: net borrowing of this group of investors (or the current account). The behavior of these endogenous variables (consumption and net borrowing) is what is puzzling in models with exogenous returns (such as Constantinides (1990)). My discussion will thus concentrate on the model-implied behavior of these endogenous variables.

The model is a continuous-time version of Lynch (1996). Agents are indexed by a first adjustment time i and an interval length D_i between adjustments, which together define an (exogenous) adjustment sequence $\{i, i + D_i, i + 2D_i, ...\}$. Between adjustment times $[i + jD_i, i + (j + 1)D_i), j \in N$, agents do not know the returns of risky assets and do not trade them. This feature makes assets illiquid. Similar to a limited participation model, the Euler equations for only a subset of agents hold at any point in time t in this economy. With adjustment delays, the first order conditions for risky asset holdings at time t are only satisfied for those agents that are adjusting at time t. The intuition from a closed-economy version of this model tells us that in this case agents need to be compensated to hold these illiquid assets. The resulting equity premium is not so much a risk premium in the usual sense but a *liquidity premium*.

We need to be careful, however, in applying closed-economy intuition to this setup, because it is not clear whether the implications of the model will survive in a closed economy setting. The reason is that agents in the model continuously observe the riskless rate, which is assumed to be constant. In a closed economy, the *riskless rate responds to stock market movements* and therefore reveals information from other agents in the economy (who get to adjust their consumption earlier in response to these movements). This means that even agents who do not directly observe stock returns can infer from the riskless rate whether the stock market just tanked and thus can adjust their consumption immediately. The closed-economy version of the model with learning will be more difficult to solve, but future research will hopefully tell us how it behaves.

The puzzles lie in the numbers, so I will compare the model's implication to the joint time series of quarterly U.S. aggregate consumption and real stock returns. I will show that adjustment delays *alone* cannot provide an explanation for the equity premium. The model fails along three main dimensions: (*i*) consumption growth from the model is too autocorrelated, (*ii*) the normalized covariance of returns with consumption monotonically increases with horizon in the model, while it is hump-shaped in the data with a peak at 2 years, and (*iii*) returns are assumed to be *i.i.d.*, while they are predictable in the data.

The reason for (*i*) is that stock market shocks trigger a series of individual consumption adjustments in the same direction by agents who only get to adjust later to the shock. The resulting aggregate consumption growth process thus looks autocorrelated and predictable by stock returns. The model does not seem to generate too much predictability for consumption, but it does imply too much autocorrelation for consumption growth.

The reason for (*ii*) is that as we lower the frequency at which we observe data relative to the frequency at which consumption decisions are made, the model looks more and more like a standard model without adjustment delays. In standard models the covariance between consumption growth and stock returns divided by horizon increases with horizon. This feature is counterfactual, which is known as the 'equity premium puzzle at long horizons' documented by Cochrane and Hansen (1992).

There is a long list of variables that successfully predict stock returns in (*iii*). The list includes term spreads (Campbell (1987)), the dividend-earnings ratio (Lamont (1996)) and the consumption/wealth ratio (Lettau and Ludvigson (2001)). I show that even lagged consumption growth (which is a variable directly taken from the model) is a predictor (but of course less successful than other variables).

In addition to these three problems, the model may be relying on large and counterfactual net borrowing from 'foreigners' (agents whose consumption is not used to define aggregate consumption) to sustain the exogenously fixed low riskfree rate, but I have not looked at the behavior of net borrowing.

In the process of documenting the properties of the model, I also show that the first three autocorrelations of consumption growth are significantly different from zero in the data. Moreover, consumption growth is heteroscedastic in the data. For example, a Garch(1,1) is significant. These two properties mean that consumption growth is certainly not *i.i.d.*, an assumption often made by recent consumption-based asset pricing models (following Hall (1978)). Heteroscedasticity may be important for explaining the time-variation in expected returns which is not captured in this paper. Models that replicate this time-variation typically rely on features of preferences which produce time-varying risk aversion (Campbell and Cochrane (1999), Barberis, Huang and Santos (2000), Veronesi (2001)).

I also show that the cross-correlation of consumption growth and stock returns data seems to be seasonal. This seasonality appears even though the consumption data is seasonally adjusted. At first sight this adjustment looks successful, because the autocorrelation function of consumption growth does not show any obvious seasonal patterns. The cross-moments with returns, however, seem to indicate that it may matter for stock pricing that real-life investors are consuming a seasonal consumption process. This raises the question whether the predictability of consumption growth is a feature of the data that should be matched by an asset pricing model.

The following discussion will thus concentrate on the autocorrelation and predictability of consumption growth, the predictability of returns, and the equity premium at long horizons. Here, 'consumption' always refers to aggregate consumption. I will then return to the interpretation of adjustment delays in terms of cognitive costs that is offered in this paper and suggest extensions.

1.1 Data and Calibration

The comparison of the model with the data relies on two series, consumption and real stock returns. Consumption is for nondurables and services excluding shoes and clothing, seasonally adjusted in 1996 chain weighted dollars. The returns are for all stocks traded on NASDAQ, AMEX and the NYSE. The calculation of real returns relies on the consumer price index. The sample consists of quarterly data from 1953:1 to 2000:3.

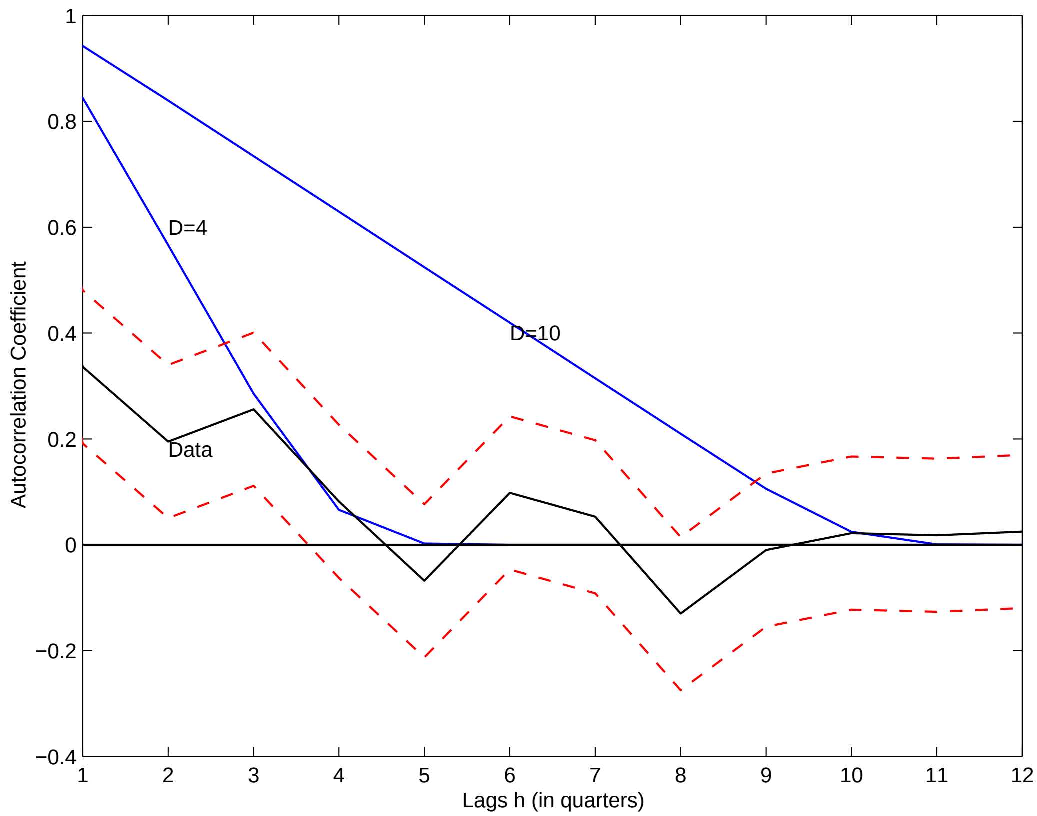
Since I use different consumption and returns data than the paper, I also use slightly different parameter values to calibrate the model: $r + \pi = 0.08, \sigma = 0.16, \gamma = 4, D_i = 4$ or $D_i = 10, \forall i$. I assume that initial adjustment times i are uniformly distributed over $[0, D]$.

1.2 Autocorrelation of consumption growth

Figure 1 shows the autocorrelation of consumption growth at different lags h together with 95% confidence bounds. The autocorrelation is significant up to the third quarter, which means that consumption growth is definitely not *i.i.d.* The figure also shows the autocorrelations implied by the model for $D = 4$ and $D = 10$. The general pattern is that the autocorrelation in a model with interval length D between two decisions dies off after D periods. The autocorrelation in the data seems to be best matched by choosing $D = 4$. The first two autocorrelations of 0.85 and 0.57 produced by the model for $D = 4$ are clearly too high compared to the data.

As an aside, I would like to add that autocorrelation is not the only dimension in which consumption growth is not *i.i.d.*. Consumption growth is also heteroscedastic, a property which may be important for explaining the time-variation in expected returns (which is not captured by the model). This can be seen from Table 1 which reports the maximum likelihood estimates of an

Figure 1: Autocorrelation of consumption growth



AR(3) combined with a Garch(1,1). The estimate of the Garch parameter α_2 is 0.95 and is strongly significant. The autoregressive parameters are partial correlations, so they differ from Figure 1 which shows autocorrelations.

Parameter Estimates

c_0	c_1	c_2	c_3	α_0	α_1	α_2
0.01	0.31	0.03	0.23	0.00	0.03	0.95
(7.04)	(4.01)	(0.33)	(2.93)	(0.64)	(1.19)	(29.56)

Table 1. Maximum Likelihood Estimates of

$$\Delta \log c_t = c_0 + c_1 \Delta \log c_{t-1} + c_2 \Delta \log c_{t-2} + c_3 \Delta \log c_{t-3} + \varepsilon_t,$$

where ε_t is conditionally normal with mean 0 and variance

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \sigma_{t-1}^2.$$

The estimation uses quarterly data on U.S. consumption of nondurables and services without shoes and clothing from 1953:1 to 2000:3. T-statistics are in brackets.

1.3 Equity premium at long horizons

To see how the model behaves as we vary the observation horizon h for a fixed decision interval length $D \geq 1$, consider the following equation that determines the equity premium in the model

$$\pi = \frac{\operatorname{cov}\left(\log\left(c_{t+h+1}/c_t\right), \log R_{t,t+h}\right)}{h} \cdot \gamma \cdot 6D. \tag{1}$$

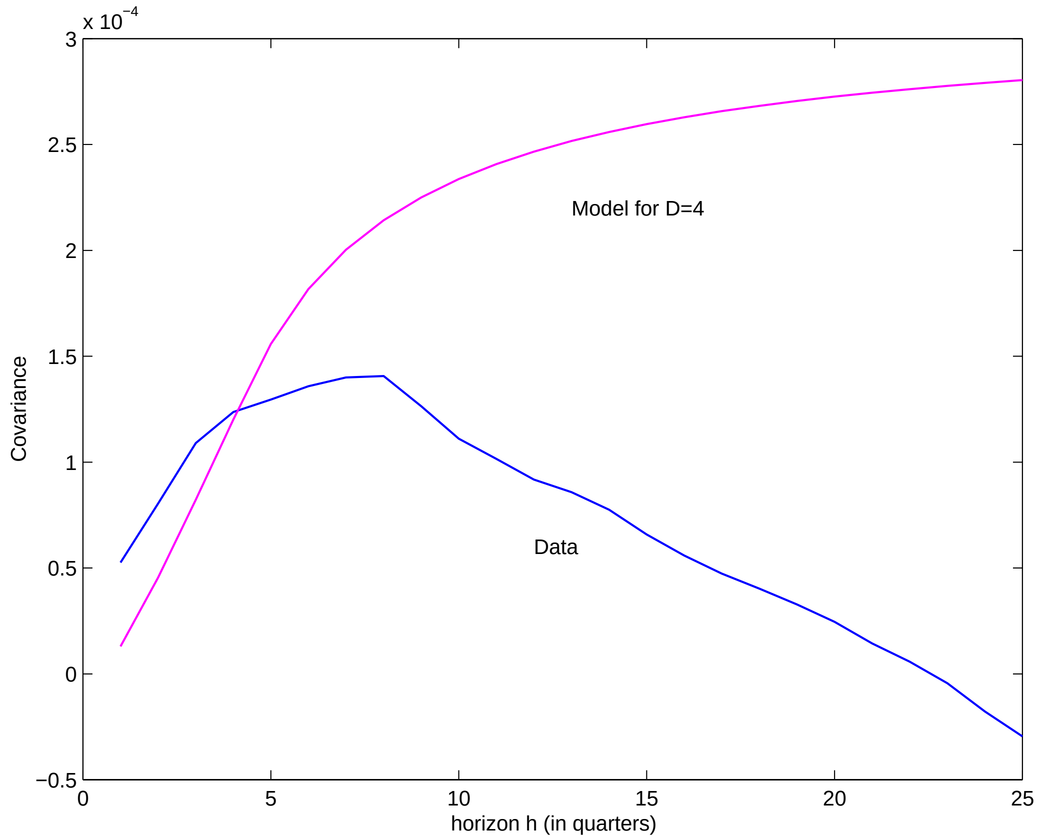
Figure 2 computes the covariance term on the right hand side of this equation, the covariance of consumption growth and stock returns divided by horizon. In the data, this covariance is hump-shaped as a function of horizon: increasing up to 2 years and then decreasing. The model predicts a monotonically increasing covariance. The reason is that as we lower the observation frequency relative to the decision interval length D, the model behaves more and more like the original Merton model without adjustment delays. Therefore the model predicts a high covariance of consumption growth and stock returns at long horizons, which is counterfactual.

The equity premium at long horizons was noted by Cochrane and Hansen (1992) and was seen as causing a problem for the time aggregation literature because aggregation problems matter less as we lower the frequency at which we observe the data. The same now applies to a model with adjustment delays. Figure 2 does not show the standard errors around the covariance estimates which get large with horizon to the extent that the hump in the empirical covariance is not significant. The equity premium, however, is not much of a puzzle if we take into account standard errors in this case (which can be seen from the cross-correlation at $h = 0$ in Figure 5 later).

1.4 Predictability of consumption growth

To look at the predictability of consumption with stock returns, Gabaix and Laibson compute the cumulative covariance of log stock returns $\log R_{t,t+1}$

Figure 2: Covariance of $\log c_{t+h}/c_t$ and $\log R_{t,t+h}$ divided by h



from time t to time $t+1$ with consumption growth $\log c_{t+1+h}/c_t$ from time t to time $t+1+h$, for different quarterly horizons h. By decomposing this covariance measure into its individual elements, we get

$$\begin{aligned} &\operatorname{cov}\left(\log\left(c_{t+h+1}/c_t\right), \log R_{t,t+1}\right) \\ &= \sum_{i=0}^{h} \operatorname{cov}\left(\log\left(c_{t+i+1}/c_{t+i}\right), \log R_{t,t+1}\right). \end{aligned}$$

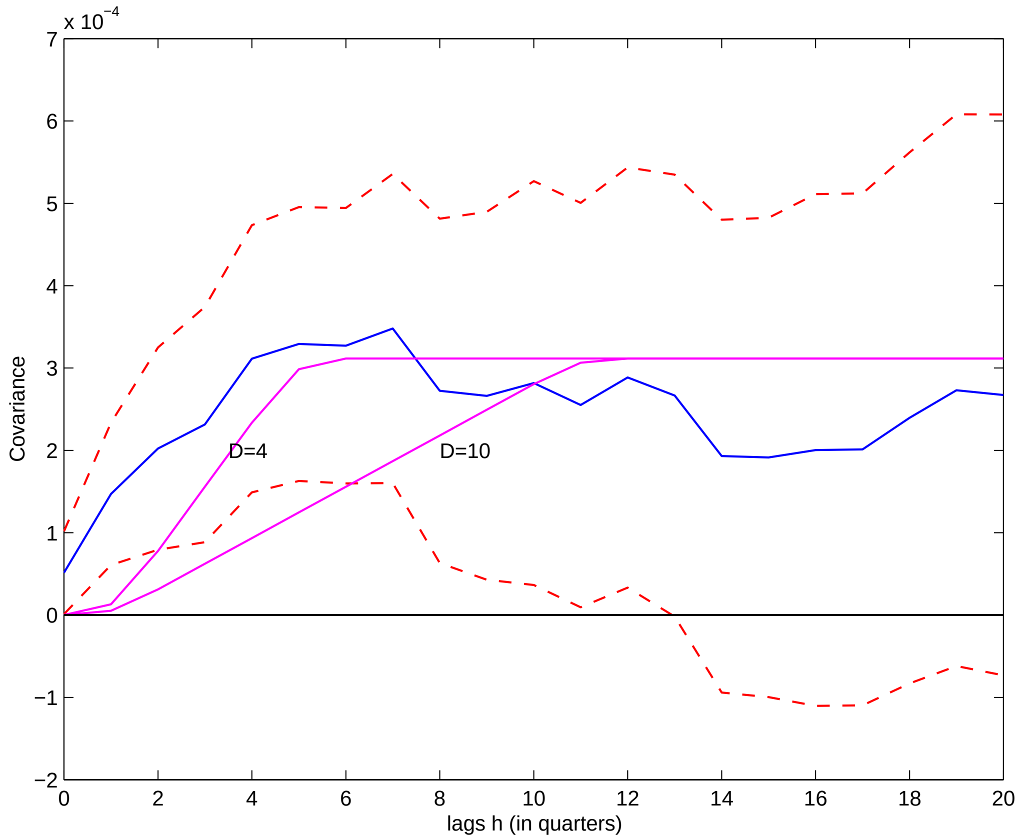
From the last equation, we can see that this cummulative covariance measure does not only reflect whether stock returns predict consumption growth, because part of the covariance is due to the contemporanous covariance $\text{cov}\left(\log\left(c_{t+1}/c_t\right), \log R_{t,t+1}\right)$ between returns and consumption growth.

Figure 3 plots this cummulative consumption measure (like Figure 7 in the paper), while Figure 4 plots the individual components in the sum on the right-hand side of the last equation. Both figures are based on U.S. data for nondurables and services instead of the total consumption series from different countries used in the paper. The dotted lines are 95% confidence bounds based on Newey-West standard errors. Figure 3 shows that the contemporanous covariance estimate in the data is already nonzero, and then the covariance measure increases up to 7 quarters. Beyond that, the covariance slightly decreases with horizon, but confidence bounds become large. The figure shows that the covariance pattern in the data is well replicated by the model if the interval length D between decisions is set to 4 quarters.

The covariance of the total consumption data (used in the paper) with returns seems to increase with horizon. To replicate this, Gabaix and Laibson use a distribution for D over [0,30] years to compute the covariance measure from the model. This is not necessary for data on nondurables and services. Section 6.2 of the paper compares Figure 7 with the plain-vanilla Merton model with *i.i.d.* consumption growth. This is not really an appropriate comparison, because it is clear that a model where consumption growth is assumed to be *i.i.d.* does not imply any predictability. Models with exogenous returns like Constantinides (1990) tend to produce too much predictability, and so they provide a more natural benchmark.

The individual covariances in Figure 4 represent the slope of the cumulative covariance function in Figure 3. We can see that the slope is significant and positive for horizons 1, 2, and 4, while it becomes negative at horizon 8. This pattern looks somewhat seasonal, even though the consumption series is seasonally adjusted. This pattern suggests that the covariance increase until $h = 7$ in Figure 3 may be due to seasonalities. In this case, it is not clear whether this predictability is a feature that the model should match. More generally, the pattern raises doubts about the use of seasonally adjusted data for tests of consumption-based asset pricing models.

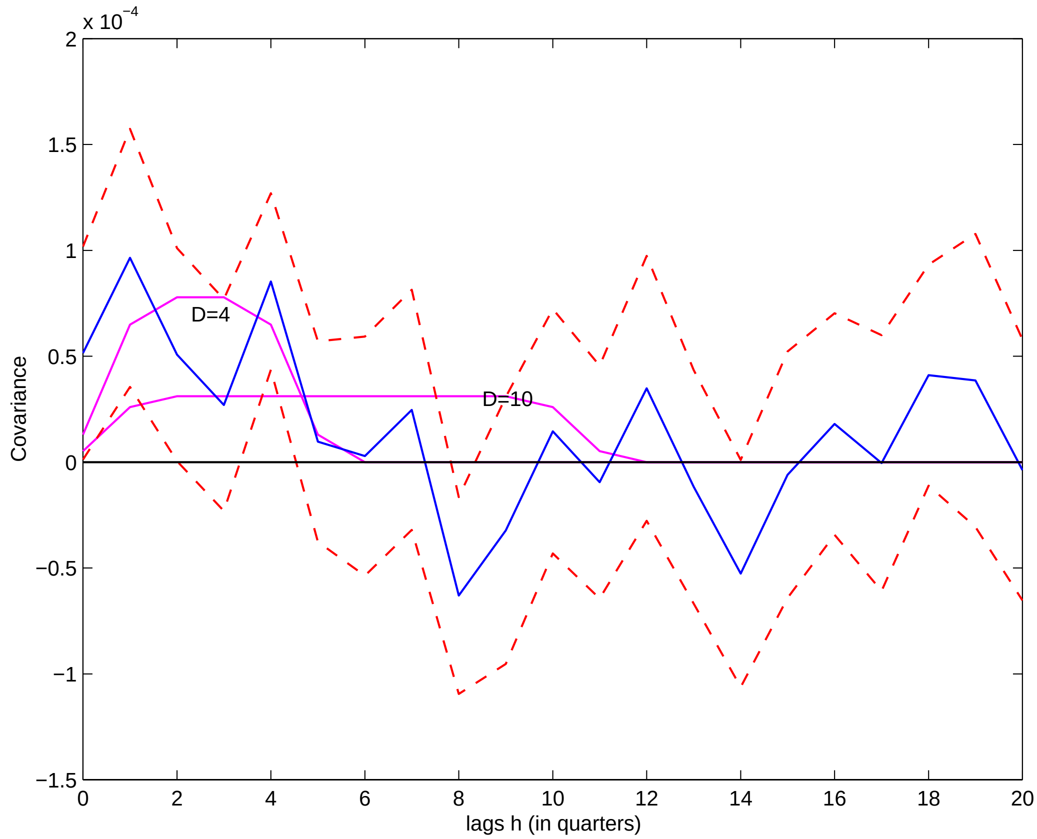
Figure 3: Covariance of $\log c_{t+1+h}/c_t$ and $\log R_{t,t+1}$



1.5 Predictability of returns

Stocks returns can be predicted with a large number of variables. Figure 5 shows the cross-correlation between current consumption growth $\log c_{t+1}/c_t$ and returns from time $t+h$ to $t+h+1$ for varying horizons h together with approximate 95% confidence bounds (computed as $\pm 2\sqrt{T}$, where T is the number of observations in the sample). The pattern of this cross-correlation for $h = 0, -1, -2, -4, -8$ shows again that the equity premium is measured with a lot of noise (supposing the standard Euler equation holds) and that consumption growth is predictable with stock returns as documented in Section 1.4. The interesting stylized fact that emerges from this graph is that the

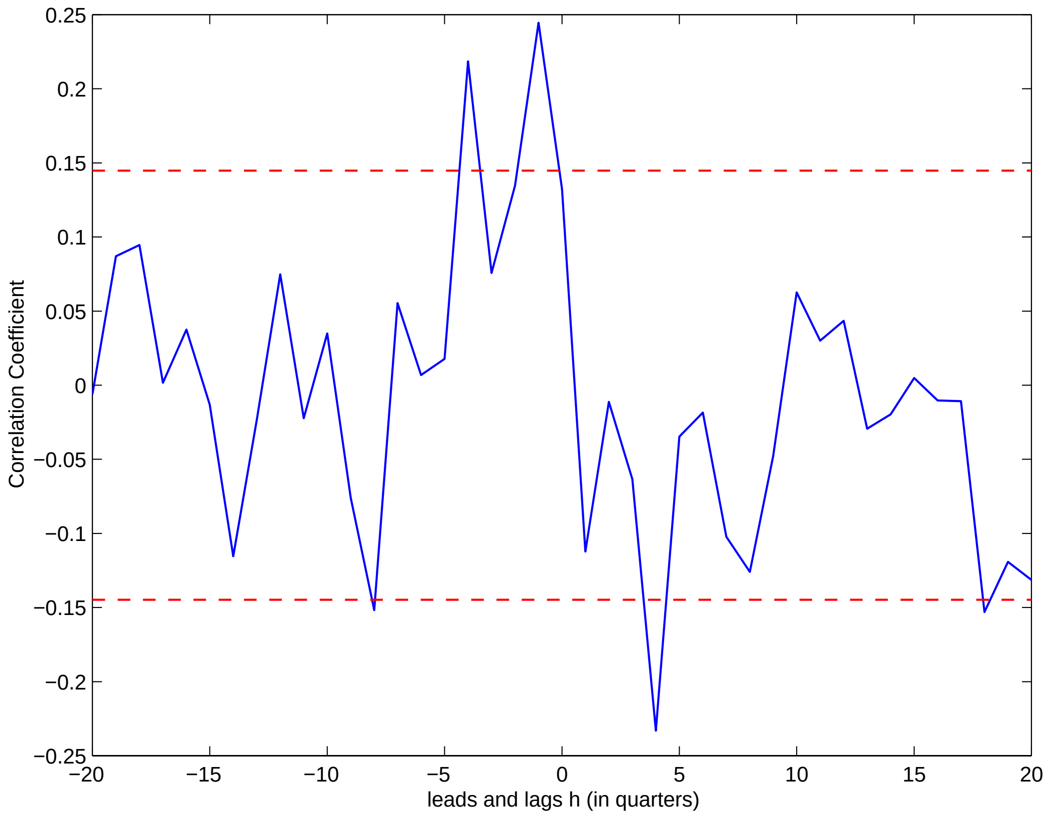
Figure 4: Covariance of $\log c_{t+1+h}/c_{t+h}$ and $\log R_{t,t+1}$



cross-correlation is also significant at $h = 4$! This means we can use current returns to predict consumption growth one year from now.

The model by Gabaix and Laibson is not consistent with this feature of the data because it assumes that these returns are *i.i.d.* and thus not predictable. Future research will hopefully show whether adjustment delays can be combined with something else, such as habit formation, so that the extended model can capture this important stylized fact.

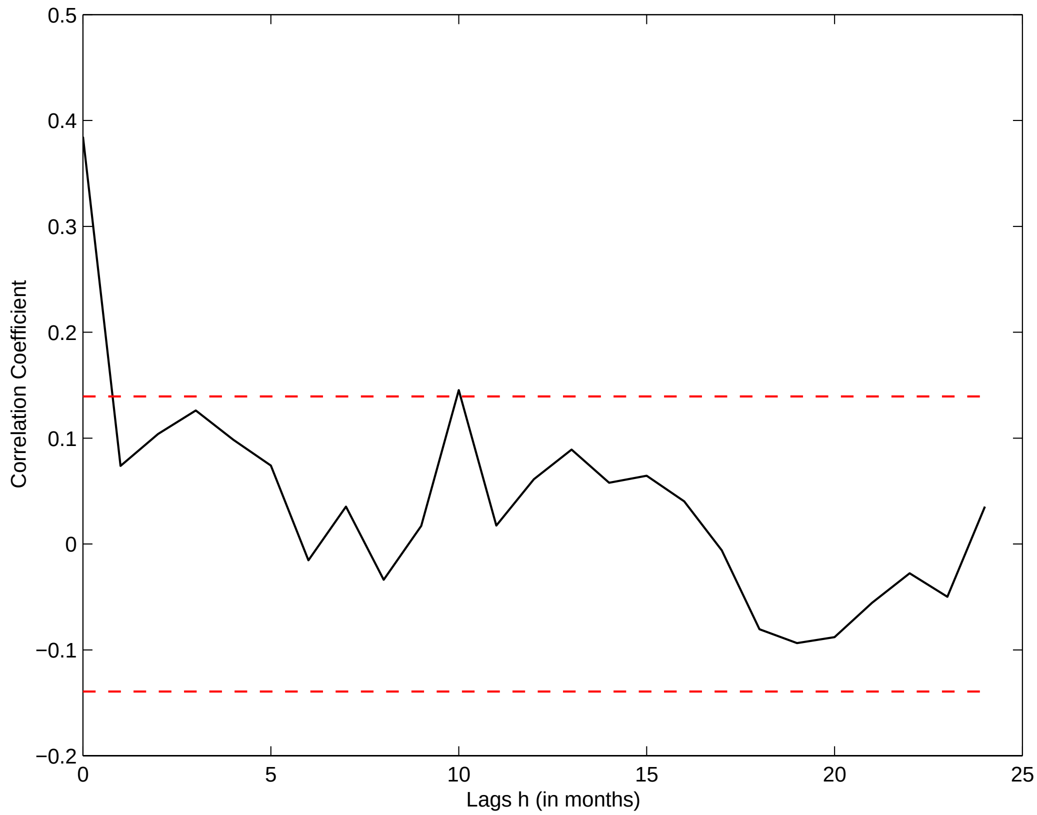
Figure 5: Crosscorrelation of $\log c_{t+1}/c_t$ with $\log R_{t,t+1+h}$



1.6 Some evidence about cognitive costs

The paper assumes that agents do not receive or process stock market information between any two periods. If this assumption is a good description of individual behavior, real net mutual fund inflows should react to past stock return information. To check this implication of the model, I collect monthly data on net inflows into stock funds from 1984:1 to 2001:2. This data can be obtained from the website of the Investment Company Institute. Real inflows are computed based on the consumer price index. I also subtract a linear trend from the real inflows. Figure 6 shows that only the contemporaneous correlation between real net inflows and returns is significant, not

Figure 6: Correlation between Inflows(t) and $\log R_{t-h,t-h+1}$



the correlation between inflows and past returns. While this is certainly not conclusive evidence against cognitive costs, the graph still provides some evidence that investors do not react to past return information when choosing their portfolio.

1.7 Extension to general equilibrium

The riskless rate is exogenous in this model and therefore does not reveal any information that agents have who have only recently adjusted their portfolio. I doubt this feature of the model will still be true in a closed economy version of this model, where the riskless rate is allowed to move in response to a stock market crash. This version is not easy to compute, because the wealth distribution matters even without idiosyncratic shocks. It would be interesting to link it to models in the incomplete market literature (e.g., Krusell and Smith (1997)) which also try to increase individual consumption volatility similar to Gabaix and Laibson, but with a different mechanism. There is some hope that a combination of the two will be successful at explaining the equity premium puzzle.

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