• Solow: exogenous savings rate

• In general, what determines the consumption level? (for an individual household / in the aggregate)
  – Level of income
  – Accumulated wealth
  – Expectations about the future
  – Age (for an individual)
  – Family structure
  – Risk aversion
  – Government policies (e.g. Social Security, insurance)
  – Demographics (in the aggregate)
  – Access to credit
  – Interest rates
  – Self-control
  – Advertising for stuff

1 The Keynesian Consumption Function

• Keynes (1936):

  “The fundamental psychological law, upon which we are entitled to depend with great confidence both a priori from our knowledge of human nature and from the detailed facts of experience, is that men are disposed, as a rule and on the average, to increase their consumption as their income increases but not by as much as the increase in the income.”
\[ C_t = a + bY_t \]

- \( b \) is the “marginal propensity to consume”
- Keynes: \( b < 1 \)
- What are the long run implications (if \( Y_t \) is growing)?
  - Note on early Keynesians worrying about lack of demand in the long term
- Evidence on savings rates in the long run rejects this

2 A two-period world

- Assume
  - Perfectly rational forward-looking behaviour
  - Two-period life
  - No uncertainty
  - Additively separable utility. Discount rate \( \beta \)
  - Unconstrained lending and borrowing at interest rate \( r \)

- Consumer problem:
  \[
  \begin{align*}
  \max_{c_1,a,c_2} & \quad u(c_1) + \beta u(c_2) \\
  \text{s.t.} & \quad a = y_1 - c_1 \\
  & \quad c_2 = y_2 + (1 + r) a
  \end{align*}
  \]

- Rearrange budget constraint:
  \[
  c_2 = y_2 + (1 + r) (y_1 - c_1) \\
  c_1 + \frac{1}{1 + r} c_2 = y_1 + \frac{1}{1 + r} y_2
  \]

- Only Present Value of income matters, not timing
  \[
  W = y_1 + \frac{1}{1 + r} y_2
  \]

  is lifetime wealth.
• What is a Present Value? (or “Net Present Value”)

- Imagine you are selling your future income in the market
- “Next year, I will give you \( y_2 \). How much are you willing to give me today in exchange?”
  Call this amount \( x \)
- For somebody giving you \( x \), the alternative is to put it in the bank, and therefore obtain \((1 + r) x\) next year
- They will only be willing to give you \( x \) today if what you promise them tomorrow is at least \((1 + r) x\)
- Hence

\[
y_2 = (1 + r) x
\]

\[
x = \frac{y_2}{1 + r}
\]

- \( \frac{y_2}{1 + r} \) is the present value of \( y_2 \) goods delivered next year
- \( \frac{1}{1 + r} \) is the relative price of \( t = 2 \) goods

• Graph

- Saving-for-retirement example
- Optimism example
- Effect of interest rates
- Borrowing constraints

• Lagrangian:

\[
L (c_1, c_2, \lambda) = u (c_1) + \beta u (c_2) - \lambda \left[ c_1 + \frac{1}{1 + r} c_2 - y_1 - \frac{1}{1 + r} y_2 \right]
\]

• FOC:

\[
u' (c_1) - \lambda = 0
\]

\[
\beta u' (c_2) - \lambda \frac{1}{1 + r} = 0
\]

\[
\Rightarrow u' (c_1) = \beta (1 + r) u' (c_2)
\]

• “Euler equation”. Interpretation
Example with CRRA utility:
\[ u(c) = \frac{c^{1-\sigma}}{1-\sigma} \]
\[ u'(c) = c^{-\sigma} \]

so Euler equation becomes
\[ c_1^{-\sigma} = \beta (1 + r) c_2^{-\sigma} \]
\[ \frac{c_2}{c_1} = \left[ \beta (1 + r) \right]^\frac{1}{\sigma} \]

Role of \( \beta, r \) and \( \sigma \) in determining the time path of consumption

Explicit solution: use budget constraint
\[
\begin{align*}
    c_1 + \frac{1}{1+r} c_2 &= y_1 + \frac{1}{1+r} y_2 \\
    c_1 + \frac{1}{1+r} c_1 \left[ \beta (1 + r) \right]^{\frac{1}{\sigma}} &= y_1 + \frac{1}{1+r} y_2 \\
    c_1 \left[ 1 + \beta^{\frac{1}{\sigma}} (1 + r)^{\frac{1}{\sigma}-1} \right] &= y_1 + \frac{1}{1+r} y_2 \\
    c_1 &= \frac{y_1 + \frac{1}{1+r} y_2}{1 + \beta^{\frac{1}{\sigma}} (1 + r)^{\frac{1}{\sigma}-1}}
\end{align*}
\]

Exercises:
- increase in \( y_1 \) holding \( y_2 \) fixed. Relate to Keynesian consumption function
- Increase in both \( y_1 \) and \( y_2 \).
- Increase in \( y_2 \) holding \( y_1 \) constant
- change in interest rates. Relate equation to graphs.

Adding taxes: Ricardian equivalence and its limitations

A government wants to make purchases of goods and services \( G_1 \) and \( G_2 \) and needs to collect taxes to pay for them
- For now, we don’t ask why they choose \( G_1 \) and \( G_2 \): we take them as given
- Assume taxes are lump-sum

The government can also borrow and save at interest rate \( r \)
• Government budget constraint

\[ B = G_1 - T_1 \]
\[ T_2 = G_2 + (1 + r) B \]
\[ \Rightarrow T_2 = G_2 + (1 + r)(G_1 - T_1) \]
\[ G_1 + \frac{1}{1 + r} G_2 = T_1 + \frac{1}{1 + r} T_2 \]

• NPV of taxes must equal NPV of spending

• Consumer now has to pay taxes, so budget constraint

\[ c_1 + \frac{1}{1 + r} c_2 = (y_1 - T_1) + \frac{1}{1 + r} (y_2 - T_2) \]
\[ = y_1 + \frac{1}{1 + r} y_2 - \left( T_1 + \frac{1}{1 + r} T_2 \right) \]
\[ = y_1 + \frac{1}{1 + r} y_2 - \left( G_1 + \frac{1}{1 + r} G_2 \right) \]

• Budget constraint of consumer depends on NPV of taxes

• NPV of taxes has to equal NPV of government spending (due to government budget)

• Therefore consumer budget depends on NPV of government spending

• Consumer budget does NOT depend on the timing of taxes: “Ricardian equivalence”

• Examples:

  – Effect of a temporary tax rebate
  
  – Effect of a temporary tax rebate plus the announcement of lower government spending in the future

• Sources of non-equivalence in practice

  – Borrowing constraints
  
  – Lack of foresight
  
  – Expectations of future G changing
  
  – Distortionary taxation

• What Ricardian equivalence does NOT say:
An increase in $G$ implies a simultaneous decrease in $C$
$G$ has no effect on the economy
Anything about $G$

3 Many or infinite periods

- Household solves

$$\max_{c_t, a_{t+1}} \sum_{t=0}^{T} \beta^t u(c_t)$$

s.t.

$$a_{t+1} = y_t - c_t + (1 + r)a_t$$

some limitation on $a_{T+1}$

- Here I have a constant interest rate, you could also have interest rates changing over time

- "Life-cycle" model:
  - $T$ finite, e.g. $T = 85$
  - Some pattern of $y_t$, e.g. lower $y_t$ after $t = 65$
  - $a_{T+1} \geq 0$

- Infinite-horizon model
  - $T = \infty$
  - Interpretation as an infinitely-lived family
  - $\lim_{t \to \infty} \frac{a_{t+1}}{(1+r)^t} \geq 0$ (no Ponzi games condition)

- Uses of each type of model

- Lagrangian:

$$L(c_0, c_1, ..., a_1, a_2, ..., \lambda_0, \lambda_1, ...) = \sum_{t=0}^{T} \beta^t u(c_t) - \sum_{t=0}^{T} \lambda_t [a_{t+1} - y_t + c_t - (1 + r)a_t]$$
• FOC:

\[
\beta^t u'(c_t) - \lambda_t = 0 \\
\beta^{t+1} u'(c_{t+1}) - \lambda_{t+1} = 0 \\
- \lambda_t + (1 + r) \lambda_{t+1} = 0
\]

\[\Rightarrow \beta^t u'(c_t) = (1 + r) \beta^{t+1} u'(c_{t+1})\]

\[u'(c_t) = \beta (1 + r) u'(c_{t+1})\]

• Same Euler equation

• Sequence of budget constraints is equivalent to single PV budget constraint:

\[
a_1 = y_0 - c_0 + (1 + r) a_0 \\
a_2 = y_1 - c_1 + (1 + r) a_1 \\
= y_1 - c_1 + (1 + r) (y_0 - c_0) + (1 + r)^2 a_0 \\
a_3 = y_2 - c_2 + (1 + r) a_2 \\
= y_2 - c_2 + (1 + r) (y_1 - c_1) + (1 + r)^2 (y_0 - c_0) + (1 + r)^3 a_0 \\
\ldots
\]

\[a_{T+1} = \sum_{t=0}^{T} (y_t - c_t) (1 + r)^{T-t} + (1 + r)^{T+1} a_0\]

\[\frac{a_{T+1}}{(1 + r)^T} = \sum_{t=0}^{T} \frac{y_t - c_t}{(1 + r)^t} + (1 + r) a_0\]

• Imposing no-debt-after-death condition or no-Ponzi condition:

\[
\sum_{t=0}^{T} \frac{c_t}{(1 + r)^t} \leq \sum_{t=0}^{T} \frac{y_t}{(1 + r)^t} + (1 + r) a_0 \equiv W
\]

• Budget depends on total wealth: PV of lifetime income plus initial savings

• Consumption function

\[c(W)\]

instead of

\[c(y_t)\]

• Exact consumption function depends on \(\beta, r\) and \(\sigma\).

  – This was also true in the two-period model
• For $\beta = \frac{1}{1+r}$, exact solution is easy:
  
  - Euler equation implies
    
    $$ u'(c_t) = u'(c_{t+1}) $$
    $$ c_t = c_{t+1} $$
    
    constant consumption
  
  - Budget constraint:
    
    $$ \sum_{t=0}^{T} \frac{c}{(1+r)^t} \leq W $$
    
    $$ T = \infty: $$
    
    $$ \frac{c}{r} \leq W $$
    $$ c \leq \frac{r}{1+r} W $$
  
• What is the marginal propensity to consume out of wealth?

• What is the marginal propensity to consume out of income?
  
  - If income is temporary
  
  - If income is permanent

• The “Permanent Income Hypothesis”
  
  - What happens if you win the lottery?
  
  - What happens if you get an unexpected promotion?
  
  - What happens if you get an expected promotion?

• How does this explain the facts?