1 A Static Model of Labour Supply

- Suppose workers can freely choose the amount of hours they work, wages are given, no unemployment. Interpretations:
  - Literally choosing hours (taxi drivers?)
  - Full-time vs. part-time work
  - Choosing jobs with shorter / longer hours
  - How many members of the household work
  - Choice of occupation
  - Age of retirement

- Assume a tax rate $\tau$ on labour income, and a lump-sum transfer from the government $T$
  - $T > 0$: social security, public education, etc.
  - $T < 0$: lump-sum taxes, rarely used in practice.

- Preferences

$$u(c) + v(l)$$

- $c$ is consumption
- $l$ is leisure

- Labour supply is $L = 1 - l$
  - Interpretation: you have one total unit of time, spend $1 - l$ working and $l$ enjoying leisure
– You could also describe preferences in terms of a disutility of labour instead of a utility of leisure, e.g. \( u(c) - z(L) \)

- Labour supply decision:
\[
\begin{align*}
\max_{c,l} & \quad u(c) + v(l) \\
\text{s.t.} & \quad c \leq w(1 - \tau)(1 - l) + T
\end{align*}
\]

- Lagrangian:
\[
L(c, l, \lambda) = u(c, l) - \lambda [c - w(1 - \tau)(1 - l) + T]
\]

- The FOCs are
\[
\begin{align*}
    u'(c) &= \lambda \\
    v'(l) &= \lambda w(1 - \tau)
\end{align*}
\]

\[\Rightarrow \frac{v'(l)}{u'(c)} = w(1 - \tau)\]

- Graph

- Income and substitution effects

- How does \( l \) respond to an inheritance?

- Interpretation:
  
  - Both consumption and leisure are normal goods
  
  - Holding the relative price \( w \) fixed, they both move together when there are income effects

- Example
\[
\begin{align*}
    u(c, l) &= \frac{c^{1-\sigma}}{1 - \sigma} - \frac{\theta \epsilon}{1 + \epsilon} (1 - l)^{\frac{1+\epsilon}{\epsilon}} \\
    u'(c) &= c^{-\sigma} \\
    v'(l) &= \theta (1 - l)^{\frac{1}{\epsilon}}
\end{align*}
\]

\[\Rightarrow \frac{\theta (1 - l)^{\frac{1}{\epsilon}}}{c^{-\sigma}} = w(1 - \tau)\]
• Now use budget constraint to solve out for $c$:

$$\frac{\theta (1 - l)^{1}}{(w (1 - \tau) (1 - l) + T)^{-\sigma}} = w (1 - \tau)$$

• This defines a labour supply curve (i.e. $L = 1 - l$ as a function of $w$)
  
  – Effect of $T$
  
  – Effect of $w (1 - \tau)$ ambiguous

• Special case with $T = 0$ and $\tau = 0$

$$1 - l = \left[ \frac{w^{1-\sigma}}{\theta} \right]^{\frac{1}{\theta + \sigma}}$$

• Whether income or substitution effects dominate depends on $\sigma$. Why?

• Graphs of evidence
  
  – US over time
  
  – International

Why do Europeans work less than Americans?

• Growth of US and Europe (graphs)

• Data on GDP per hour and hours (Blanchard)

• Graph of trends in hours
  
  – Difference includes participation, part-time share, hours per week and weeks per year.
  
  – (Graph is per-employee, so doesn’t capture participation margin)

Hypothesis 1 (Prescott): it’s the European welfare state

• Tax rates and government transfers (pensions, unemployment insurance, universal health benefits) are higher in Europe
  
  – Taxes have income and substitution effects in opposite directions
  
  – Transfers have pure income effect on leisure
  
  – Tax rates and transfers have increased in Europe but not the U.S.
• You’ll work out Prescott’s calculation for your problem set

• See graph of cross-country correlation

• Caveat on cross-country correlations: other things are also different across countries

• Some criticisms:

1. Labour supply elasticity?
   • Micro vs. macro evidence
   • Different groups in the population (age, gender, marital status)

2. Timing: tax increases in the 60s-70s, labour supply declines afterwards too

**Hypothesis 2 (Alesina, Glaeser, Sacerdote): unions are stronger in Europe**

• Graphs on unionization

• Why do unions fight for shorter hours? (as opposed to just higher wages)

**Hypothesis 3 (Blanchard): preferences are different**

• Europeans just prefer shorter working hours

• Blanchard: preferences

• Why now and not before?

• “Social multiplier”

• Cultural change in Europe but not US?

• Culture matters beyond an income threshold?

2 **Dynamic labour-supply decisions**

• Combine labour-leisure decision with consumption-savings decisions

• Allows one to distinguish between temporary and permanent changes in $w$
• Household solves

\[
\max_{c,L} u(c_1) + v(l_1) + \beta [u(c_2) + v(l_2)]
\]

s.t.

\[
a = w_1 (1 - \tau_1) (1 - l_1) + T_1 - c_1
\]

\[
c_2 = w_2 (1 - \tau_2) (1 - l_2) + T_2 + (1 + r) a
\]

• Replace \(a\) to get single budget constraint

\[
c_1 + \frac{1}{1 + r} c_2 = w_1 (1 - \tau_1) (1 - l_1) + \frac{1}{1 + r} w_2 (1 - \tau_2) (1 - l_2) + T_1 + \frac{1}{1 + r} + T_2
\]

• FOCs

\[
\frac{v'(l_t)}{u'(c_t)} = \frac{w_t (1 - \tau_t)}{u'(c_1) = \beta (1 + r) u'(c_2)}
\]

• An intratemporal labour-leisure condition plus an intertemporal Euler equation

• Response to temporary vs permanent wage changes

New York cab drivers (Camerer, Babcock, Loewenstein and Thaler, 1997)

• Truly temporary changes in wages
  - weather
  - subway breakdowns
  - day-of-the-week
  - holidays
  - conventions

• Truly unconstrained short-term labour supply

• Good data due to recorded trips

• Negative hours response to temporary increases in \(w\)

• Direct rejection of intertemporal model

• Psychological explanation: “one day at a time” approach, with target income levels
• Criticisms:
  – is it demand or supply that is moving? (draw graph) Correlation vs causation
  – participation margin
  – bias from mismeasurement of hours

• Oettinger, 1999 (baseball stadium vendors) and Fehr & Goette, 2005 (experiments with bike messengers in Switzerland) find results more consistent with standard dynamic model.

3 Labour demand

• Recall firm’s problem:

\[
\max_{k_i, l_i} F(k_i, l_i) - w l_i - r k_i
\]

FOC:

\[
F_k(k_i, l_i) - r = 0 \\
F_l(k_i, l_i) - w = 0
\]

• Take aggregate \( K \) as given (results from past capital accumulation) and aggregate across firms:

\[
F_L(K, L) = w
\]

• This defines a labour demand curve

• Cobb-Douglas case:

\[
F(K, L) = K^\alpha (AL)^{1-\alpha} \\
F_L(K, L) = (1 - \alpha) K^\alpha A^{1-\alpha} L^{-\alpha}
\]

\[
= A^{1-\alpha} \left( \frac{1 - \alpha}{w} \right)^{\frac{1}{\alpha}} K
\]

• Graph

• Role of \( A, K \) and \( w \)

• Does technology displace workers?
  – Employment across countries and across time
  – “Displace” might be the wrong term. How does technological progress affect wages?
– What happens if technological progress is not just an improvement in $A$ but a change in the shape of the production function?