Diamond and Dybvig [1983] and Jacklin [1987]

Econ 235, Spring 2013

- Banks as suppliers of “liquidity”
- (What is liquidity?) Insurance problem
- Demand-deposit contracts
- The case for deposit insurance
- How markets and contracts interact

1 The Diamond and Dybvig [1983] model

1.1 The environment

- A continuum of consumers with preferences
  \[ u(c_1, c_2) = u(c_1 + \theta c_2) \]

- \( \theta \in \{0, 1\} \) is an idiosyncratic shock, realized at \( t = 1 \).

- If \( \theta = 1 \) the consumer is “late” or “patient”, otherwise he is “early” or “impatient”

- \( \pi \equiv \Pr(\theta = 0) \). By LLN, \( \pi \) is also the fraction of early consumers in the population\(^1\)

- Each consumer has an endowment of 1 at \( t = 0 \)

- Two possible technologies:
  - Storage: transforms \( x \) goods at \( t \) into \( x \) goods at \( t + 1 \)

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\(^1\)As you know, this requires abusing the law of large numbers. See Judd [1985] on why this is a problem and Uhlig [1996] on how to fix it.
– Long-term investment: transforms $y$ goods at $t = 0$ into $Ry > y$ goods at $t = 2$. The investment can be terminated early (“liquidated”) at $t = 1$. If $z \leq y$ units are liquidated, then the investment yields $\lambda z$ goods at $t = 1$ and $R(y - z)$ goods at $t = 2$. Assume $\lambda \leq 1$.

• Note: original paper has $\lambda = 1$, so the storage technology is redundant $\Rightarrow$ the ex-ante choice of technology is not the main issue.

1.2 First-best planner problem

• Assume (it’s easy to prove) that the solution is symmetrical, that it involves early consumers consuming at $t = 1$ and late consumers consuming at $t = 2$ and that no long-term projects are liquidated

$$\max_{c_1, c_2, x, y} \pi u(c_1) + (1 - \pi) u(c_2)$$

$$\text{s.t. } \pi c_1 \leq x$$

$$\pi c_2 \leq Ry$$

$$x + y = 1$$

or equivalently

$$\max_{c_1, c_2, x, y} \pi u(c_1) + (1 - \pi) u(c_2)$$

$$\text{s.t. } \pi c_1 + (1 - \pi) \frac{c_2}{R} \leq 1$$

• FOC:

$$\pi u'(c_1) - \mu \pi = 0$$

$$\pi u'(c_2) - \mu (1 - \pi) \frac{1}{R} = 0$$

so

$$\frac{u'(c_1)}{u'(c_2)} = R$$  \hspace{1cm} (1)$$

• MRS = MRT

• What if types are not observable? Planner can set up a mechanism: declare your type (“early” or “late”) and you get a consumption bundle in return.

• The first best allocation is incentive compatible! $R > 1$ implies $c_2 > c_1$
1.3 Complete markets allocation

- Markets for consumption at $t = 1$ and $t = 2$ contingent on realization of $\theta$ for each individual.

- Assume (it’s easy to prove) again that in equilibrium the agent only buys positive amounts of the following contingent claims: “$c_1$ if impatient” (at price $p_1$) and “$c_2$ if patient” (at price $p_2$)

\[
\max_{c_1, c_2} \pi u(c_1) + (1 - \pi) u(c_2) \\
\text{s.t. } p_1 c_1 + p_2 c_2 \leq 1
\]

with FOC:

\[
\frac{u'(c_1)}{u'(c_2)} = \frac{1 - \pi}{\pi} \frac{p_1}{p_2}
\]

- By using the LLN, “firms” can transform one unit of $t = 0$ goods into $\frac{1}{\pi}$ units of the contingent claim “$c_1$ if impatient”, or into $\frac{R}{1 - \pi}$ units of the contingent claim “$c_2$ if patient”. Competition implies

\[
p_1 = \pi \\
p_2 = \frac{1 - \pi}{R}
\]

- Therefore

\[
\frac{u'(c_1)}{u'(c_2)} = R
\]

and we recover the first best allocation.

- (There’s nothing special about this, it just illustrates the Second Welfare Theorem)

- Anything other than this which arises in the model must be a result of some form of market incompleteness

1.4 Incomplete markets

- Suppose there are no insurance markets

- The only market is at $t = 1$, where agents can trade $t = 1$ goods against $t = 2$ goods (or, equivalently, ongoing projects)

- Let the price of $t = 2$ goods at $t = 1$ be $p$. 
• The consumer’s problem is
\[
\max_{c_1, c_2, x, y} \pi u(c_1) + (1 - \pi) u(c_2)
\]
\[
s.t. \quad c_1 \leq x + pRy
\]
\[
c_2 \leq Ry + \frac{x}{p}
\]
\[
x + y = 1
\]

• In any equilibrium where consumers invest in both technologies, it must be that
\[
p = \frac{1}{R}
\]

• If \( p > \frac{1}{R} \), then it is always preferable to invest in the long-term technology at \( t = 0 \), but then all early consumers will be trying to sell at \( t = 1 \) and nobody would buy

• If \( p < \frac{1}{R} \), then it is always preferable to invest in storage at \( t = 0 \), but then all late consumers would be willing to pay 1 for \( t = 2 \) goods, so \( p < \frac{1}{R} \) cannot be the equilibrium price.

• Therefore the consumer’s problem reduces to
\[
\max_{c_1, c_2} \pi u(c_1) + (1 - \pi) u(c_2)
\]
\[
s.t. \quad c_1 \leq 1
\]
\[
c_2 \leq R
\]

• The allocation
\[
c_1 = 1 \quad c_2 = R
\]
does not in general coincide with the complete markets allocation

• Allocations coincide for the special case of log preferences

• Assume that the consumer is more risk averse than log, i.e.
\[
-\frac{u''(c)c}{u'(c)} > 1 \quad \forall c
\]
Then $c_1^{FB} > 1$ and $c_2^{FB} < R$. Proof:

$$Ru'(R) = u'(1) + \int_1^R \frac{\partial cu'(c)}{\partial c} dc$$

$$= u'(1) + \int_1^R [cu''(c) + u'(c)] dc$$

$$= u'(1) + \int_1^R u'(c) \left[ \frac{cu''(c)}{u'(c)} + 1 \right] dc$$

$$< u'(1)$$

1.5 A bank

- Consumers get together and create a “bank”
- They each deposit their endowment with the bank, which invests the first best amount in each technology
- The bank sets up the following contract: each consumer can ask for $c_1^{FB}$ at $t = 1$, or can wait until $t = 2$ and get a pro-rata share of whatever is left.
- “Demand deposit”
- The bank commits to satisfying “sequential service”: satisfy consumer’s withdrawals in the order in which they arrive (which, assume, is random)
- If the stored goods are not enough to satisfy a consumer who demands payment, then the bank must liquidate the long-term investment until that, too, runs out.
- Consumers then play a game, where the actions are “withdraw” or “wait” (or you could allow for partial withdrawal, it doesn’t matter)

- Let
  - $f_j$ be the number of depositors who arrived in line before consumer $j$ and asked to withdraw and
  - $f$ be the total number of consumers that will eventually ask to withdraw

- The payoffs for an impatient consumer are:

<table>
<thead>
<tr>
<th></th>
<th>Withdraw</th>
<th>Wait</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1^{FB}$</td>
<td>$c_1^{FB}$ if $c_1^{FB} f_j &lt; x^{FB} + \lambda y^{FB}$</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>otherwise</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
• The payoffs for a patient consumer are:

<table>
<thead>
<tr>
<th>Withdraw</th>
<th>Wait</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1^{FB}$ if $c_1^{FB} f_j &lt; x^{FB} + \lambda y^{FB}$</td>
<td>$\max \left{ R \frac{1-\pi f_j^B-(f-\pi) \frac{1}{f} c_2^{FB}}{1-f}, 0 \right}$</td>
</tr>
<tr>
<td>0 otherwise</td>
<td></td>
</tr>
</tbody>
</table>

• The symmetric strategy profile “withdraw iff impatient” is a Nash equilibrium, with payoffs equal to the first best allocation.
  
  – Impatient consumers don’t want to deviate because they don’t care about future consumption
  – Patient consumers don’t want to deviate because $c_2^{FB} > c_1^{FB}$

• The symmetric strategy profile “withdraw no matter what” is also a Nash equilibrium
  
  – Given that $c_1^{FB} > 1 \geq x^{FB} + \lambda y^{FB}$, if everyone tries to withdraw the money will run out
  – Therefore anyone who waits will obtain zero
  – This justifies withdrawing
  – This equilibrium produces a very bad allocation!

### 1.6 Suspension of convertibility

• One variant of the contract can rule out the bad equilibrium

• The contract states that you can withdraw $c_1^{FB}$ at $t = 1$ as long as less than $\pi$ other consumers have withdrawn before you. After that, you are forced to wait

• Now the payoffs for an impatient consumer are:

<table>
<thead>
<tr>
<th>Withdraw</th>
<th>Wait</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1^{FB}$ if $f_j &lt; \pi$</td>
<td>0</td>
</tr>
<tr>
<td>0 if $f_j \geq \pi$</td>
<td></td>
</tr>
</tbody>
</table>

• and the payoffs for a patient consumer are:

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2 These payoffs are accurate for the case where $f \geq \pi$, which will be true in equilibrium because all impatient consumers withdraw. To be fully correct you have to take into account that if $f < \pi$ you need to use storage between $t = 1$ and $t = 2$

3 Again, you would need to take into account what happens if $f < \pi$
<table>
<thead>
<tr>
<th>Withdraw</th>
<th>Wait</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{1}^{FB}$ if $f_j &lt; \pi$</td>
<td>$c_{2}^{FB}$ if $\pi$</td>
</tr>
<tr>
<td>$0$ otherwise</td>
<td>$0$ otherwise</td>
</tr>
</tbody>
</table>

- So now waiting is dominant for patient consumers, and runs will not take place.
- There won’t be suspension of convertibility in equilibrium
- Problem: you need to know $\pi$ exactly in order to use these contracts!

1.7 Remarks

- Liquidity transformation as a form of insurance, not provided by “narrow banking” [Wallace, 1996]
- What is the reason for the sequential-service constraint? What does it mean [Wallace, 1988, Green and Lin, 2003, Ennis and Keister, 2009]?
- “Deposit insurance” as a way to undo the sequential service constraint
- “Chicken models” and the “why doesn’t the government just do everything” argument.
- Why have a bank rather than a direct mechanism?

2 The Jacklin [1987] critique

2.1 Who needs a bank?

- Suppose instead of a “bank” consumers set up a “firm”
- The firm invests the first-best amount and issues shares. Each consumer gets one share
- It declares the following dividend policy: a dividend of $d_1 = \pi c_{1}^{FB}$ will be paid at $t = 1$ and a dividend of $d_2 = (1 - \pi) c_{2}^{FB}$ will be paid at $t = 2$
- Consumers can trade shares in the firm at $t = 1$ (they trade “ex-dividend” - after paying dividends) at a price of $p$ goods per share.
- Supply of shares (impatient consumers sell):

\[ S = \pi \]
• Demand for shares (patient consumers perhaps buy):

\[ D = \begin{cases} 
\frac{(1-\pi)d_1}{p} & \text{if } p < d_2 \\
0 & \text{otherwise}
\end{cases} \]

• Market clearing:

\[ \pi = \frac{(1 - \pi) d_1}{p} \]
\[ p = \frac{(1 - \pi) d_1}{\pi} = (1 - \pi) c_1^{FB} \]

• Consumption attained by early consumers

\[ c_1 = d_1 + p = c_1^{FB} \]

• Consumption attained by late consumers

\[ c_2 = d_2 \left( 1 + \frac{d_1}{p} \right) = (1 - \pi) c_2^{FB} \left( 1 + \frac{\pi c_1^{FB}}{(1 - \pi) c_1^{FB}} \right) = c_2^{FB} \]

• No need for demand deposits, no risk of bank runs!

• Under more general preferences, e.g.

\[ u(c_1, c_2, \theta) = \theta u^E(c_1 + c_2) + (1 - \theta) u^L(c_1, c_2) \]

then it is the case that deposit contracts improve upon the dividend arrangement

• In general, demand deposits can implement:
  
  – The first best allocation, if it is incentive compatible
  
  – The constrained efficient allocation, if the first best is not incentive compatible

while shares can only do so under quite restrictive assumptions, which happen to hold in the original model
2.2 Free riding

- Suppose that, after a bank/firm is set up, at $t = 1$ a market opens where consumers can trade $t = 2$ goods against $t = 1$ goods

- What is the price?

- The bank/firm offers consumers either $c_1^{FB}$ at $t = 1$ or $c_2^{FB}$ at $t = 2$

- Therefore the price of $t = 2$ goods in terms of $t = 1$ goods must be $p = \frac{c_1^{FB}}{c_2^{FB}}$. (Do not confuse this price with the price of shares in the firm)

- Consider a free-riding strategy:
  - Don’t join the bank/firm
  - Invest in the long term technology
  - If you are impatient, sell your long term project at $t = 1$
  - If you are patient, hold on to your project and consume at $t = 2$

- This delivers

\[
c_1 = Rp = R \frac{c_1^{FB}}{c_2^{FB}} > c_1^{FB}
\]

\[
c_2 = R > c_2^{FB}
\]

so the bank\firm would unravel!

- “Pecuniary externality”:
  - Investing in the “bank” rather than in long-term projects increases $p$ by making $t = 1$ goods more abundant
  - This is good for insurance purposes (it makes up for the missing insurance market)
  - Without restrictions, individuals do not internalize the externality so they invest in long-term projects rather than in the bank

- Restrictions on trading are needed to sustain these insurance-like arrangements. Are they reasonable?
References


