1 Model

• Three periods: 0, 1, 2

1.1 Consumers

• Utility

\[ u = c_0 + c_1 + c_2 \]

• Large endowments at each of the three periods

1.2 Entrepreneurs

• Utility

\[ u = c_0 + c_1 + c_2 \]

• Endowment \( A \) at \( t = 0 \)

• Can carry out a project of scale \( I \)

• Output is realized at \( t = 2 \) and can be \( yI \) or 0

• \( \Pr (y) = p_H \) or \( p_L \) depending on entrepreneur’s effort

• Lack of effort gives utility \( B \)

• At \( t = 1 \) the project needs an extra amount of money \( \rho I \) in order to continue (e.g. some machines broke down)

• \( \rho \sim F (\rho) \)

• You can call \( \rho \) a liquidity shock
• If the entrepreneur does not invest $\rho$ then the project terminates and pays 0. The private benefit of shirking also goes away if the project is terminated

• Assume

$$\int \max \{p_H y - \rho, 0\} \, dF(\rho) - 1 > 0 > \int \max \{p_L y + B - \rho, 0\} \, dF(\rho) - 1$$

• Timing:

1. Investment takes place
2. $\rho$ is realized
3. Either $\rho$ is reinvested or the project is terminated
4. Entrepreneur chooses effort
5. Output is realized

• The fact the effort is chosen after the reinvestment decision is really important. It means that even at that stage there is a limit to how much the entrepreneur can pledge to consumers

2 Second-best allocation

• Contracts between entrepreneur and consumers specify:

  – The size of the investment $I$ (equivalently the loan amount $I - A$)
  – When the project will be terminated $\lambda(\rho) \in \{0, 1\}$ (later we allow for fractional liquidation)
  – The dividends paid to each party $d_e, d_c$ if the project succeeds, possibly depending on the size of the liquidity shock

• Solve

$$\max_{I, d_e(\rho), d_c(\rho), \lambda(\rho)} I \int p_H d_e(\rho) \lambda(\rho) \, dF(\rho)$$

s.t. \quad I \int [p_H d_e(\rho) - \rho] \lambda(\rho) \, dF(\rho) \geq I - A
d_e(\rho) \Delta \rho \geq B
d_e(\rho) + d_c(\rho) = y$$
• Note that the implicit assumption is that the consumer agrees to pay for both the gap in the entrepreneur’s initial funds and the liquidity shock (in those cases where they agree that the shock will be met).

• Since entrepreneurs get surplus, objective is equivalent to just maximizing surplus

\[ I \left[ \int [pHy - \rho] \lambda(\rho) dF(\rho) - 1 \right] \]

• It’s immediate (from linearity) that the optimum will be given by a cutoff \( \hat{\rho} \) such that the shock is met if \( \rho < \hat{\rho} \) and the project is terminated otherwise.

• Define

\[ \rho_1 = pHy \]

• \( \rho_1 \) is the expected value of the project (per unit of investment).

• In the first-best allocation, the cutoff would be \( \rho_1 \).

• Define

\[ \rho_0 = pH \left[ \frac{y - B}{\Delta p} \right] \]

• \( \rho_0 \) is the maximum dividend the entrepreneur can promise the consumers while still satisfying the IR constraint.

• Why is this a useful thing to compute?

  – At \( t = 1 \), the parties will always find some way to meet the liquidity shock if it is less than \( \rho_0 \).

  – If they don’t meet the liquidity shock, both parties lose everything.

  – The entrepreneur can credibly promise \( \rho_0 \) to the consumers, so he will be able to persuade them to put up \( \rho_0 \) in order not to lose everything.

• Moral hazard model is just one way to justify nonpledgeability.

• What matters for the rest of the analysis is just the values of \( \rho_0 \) and \( \rho_1 \).

• (At MIT people knew this model in its various incarnations as the “\( \rho_0-\rho_1 \)model”)

• Holmström and Tirole [2011] is a book-length version of this paper, where \( \rho_0 \) and \( \rho_1 \) are taken as primitives of the model.
• Write the program as

\[
\max_{I, \hat{\rho}} I \left[ \int_{0}^{\hat{\rho}} [\rho_1 - \rho] dF(\rho) - 1 \right]
\]

s.t. \( I \int_{0}^{\hat{\rho}} [\rho_0 - \rho] dF(\rho) \geq I - A \)

• Assume that

\[
\int_{0}^{\rho_0} [\rho_0 - \rho] dF(\rho) < 1
\]

– Economically, this means that “maximum pledgeable income after meeting liquidity shocks” is not enough to pay for investment

– Otherwise investment would go to \( \infty \)

– The entrepreneur will need to put in his own money to be able to invest at all

• Let

\[
m(\hat{\rho}) \equiv \int_{0}^{\hat{\rho}} [\rho_1 - \rho] dF(\rho) - 1
\]

\[
k(\hat{\rho}) \equiv \frac{1}{1 - \int_{0}^{\hat{\rho}} [\rho_0 - \rho] dF(\rho)} = \frac{1}{1 + \int_{0}^{\hat{\rho}} \rho dF(\rho) - \rho_0 F(\hat{\rho})}
\]

• \( m(\hat{\rho}) \) is the marginal net social return on investment. It is maximized at \( \hat{\rho} = \rho_1 \)

• \( k(\hat{\rho}) \) is the “equity multiplier”. It measures how much investment can take place for each unit of the entrepreneur’s wealth. It is maximized at \( \hat{\rho} = \rho_0 \)

• The problem reduces to

\[
\max_{\hat{\rho}} U(\hat{\rho}) = m(\hat{\rho}) k(\hat{\rho}) A
\]

\[
= \frac{\int_{0}^{\hat{\rho}} [\rho_1 - \rho] dF(\rho) - 1}{1 + \int_{0}^{\hat{\rho}} \rho dF(\rho) - \rho_0 F(\hat{\rho})} A
\]

\[
= \frac{\rho_1 - \int_{0}^{\hat{\rho}} \rho dF(\rho) + 1}{F(\hat{\rho})} A
\]

\[
(1)
\]

• Therefore the optimal cutoff solves

\[
\rho^* = \arg \min_{\hat{\rho}} \frac{\int_{0}^{\hat{\rho}} \rho dF(\rho) + 1}{F(\hat{\rho})}
\]

(2)
• Interpretation: minimize the cost per unit of investment that reaches period 2
  – Numerator: Total cost
  – Denominator: Number of projects that reach $t = 2$

• What is the tradeoff?

• FOC:

$$\frac{\rho^* f (\rho^*) F(\rho^*) - f (\rho^*) \left( \int^\rho^* \rho dF (\rho) + 1 \right)}{[F (\rho^*)]^2} = 0$$

$$\rho^* = \frac{\int^\rho^* \rho f (\rho) d\rho + 1}{F (\rho^*)}$$

$$= \frac{\rho^* F (\rho^*) - \int^\rho^* F (\rho) d\rho + 1}{F (\rho^*)}$$

$$\int^\rho^* F (\rho) d\rho = 1$$

(3)

• Therefore:

$$U = \frac{\rho_1 - \rho^*}{\rho^* - \rho_0} A$$

(4)

– (This follows from integration by parts in equation (3):

$$\rho^* F (\rho^*) - \int^\rho^* \rho f (\rho) d\rho = 1$$

$$\rho^* = \frac{1 + \int^\rho^* \rho f (\rho) d\rho}{F (\rho^*)}$$

and replacing in (1)

• What happens if there is a mean-preserving spread in $F (\rho)$?
  – Lower $\rho^*$
  – Higher utility
  – Why?

3 Implementation

• Pure loan doesn’t work:
  – Lend $I - A$ at $t = 0$ in exchange for some promise at $t = 2$
At $t = 1$, entrepreneur needs $\rho$

- Lenders are willing to be diluted by the new lenders who provide $\rho$ because otherwise they will lose everything

- (They will of course require compensation ex-ante for this possibility)

- New lenders could be the same people as old lenders or different ones

- At $t = 1$, the most that the entrepreneur can pledge to new lenders is $\rho_0$

- So whenever $\rho \in (\rho_0, \rho^*)$, the second-best-optimal thing to do would be to continue the project, but we will need to terminate it!

• Initial loan plus “line of credit”

- Lend $I - A$ at $t = 0$ and grant an irrevocable line of credit for $\rho^* I$

- (For simplicity, suppose that $\rho$ is observable, or that the entrepreneur cannot consume at $t = 1$. Otherwise you have to worry about entrepreneurs claiming they’ve had a bad liquidity shock and running away with they money)

- In exchange for this, the entrepreneur promises $\rho_0 I$ to the lender.

- Notice that the consumers need to be able to commit to the line of credit

- For $\rho \in (\rho_0, \rho^*)$, the extra money they put up at $t = 1$ is less than what they will recover at $t = 2$, so they are making an expected loss!

- That’s the point of a line of credit!

- “Material adverse change” clauses

- Would the parties want to renegotiate? To lower credit line? To higher credit line?

• Large initial loan plus liquid assets

- Lend $I - A + \rho^* I$

- Contractually limit investment to exactly $I$ and agree that the entrepreneur will keep $\rho^* I$ in “liquid assets”

- (Requires that such liquid assets exist!)

- When the liquidity shock comes, the entrepreneur meets it with the liquid assets

- The entrepreneur promises $\rho_0 I + (\rho^* - \rho) I$ to the consumers

- Notice that the entrepreneur would in general not want to choose the right combination of investment/liquid assets at $t = 0$. The investment level has to be enforceable
4 What if consumers cannot commit (and there is no storage technology)?

- No commitment ⇒ no credit line
- No “storage” ⇒ what are the “liquid assets”? Do we have enough of them?
- Liquid assets cannot be claims on consumers, because consumers cannot commit
- They cannot be stored goods because goods are not storable
- Firms can acquire claims on other firms

4.1 Idiosyncratic $\rho$ shocks

- Paper argues that intermediary is necessary: direct stakes in firms are not enough. This is actually wrong (problem set)
- The total amount of resources needed at $t = 1$ to implement the second best allocation is
  \[ D = I \int_0^{\rho^*} \rho dF(\rho) \]
- An intermediary is set up at $t = 0$
- The intermediary obtains $I - A$ from investors in exchange for shares and lends it to firms
- (There are many firms and the law of large numbers applies: there is no aggregate risk)
- The intermediary also offers a credit line to firms for an amount of $\rho^* I$
- In exchange, the intermediary gets a claim to $\rho_0 I$ at $t = 2$ from each firm (but the firm of course will not pay if the project is terminated)
- The intermediary can commit to the credit line, but this doesn’t solve the problem, because the intermediary does not have an endowment at $t = 1$: it must persuade consumers to give up some endowment.
- At $t = 0$ the intermediary has to find $D$ to honour the credit lines.
- It can sell its claims on $t = 2$ output to consumers, obtaining up to $F(\rho^*) \rho_0 I$.
- Is this enough?
  \[ F(\rho^*) \rho_0 I - I \int_0^{\rho^*} \rho dF(\rho) = I - A > 0 \]
so yes, by selling its claims on firms to consumers the intermediary can satisfy the credit line and have some left over, which it will then use to repay the original investment $I - A$

- There is enough “inside liquidity” to implement the second-best allocation
- “Liquidity” here is used to mean something like “credible promises of goods at a specific date”

4.2 Aggregate $\rho$ shocks

- Now suppose that all firms have the same $\rho$ shock
- Now whenever $\rho \in (\rho_0, \rho^*)$ the value of the intermediary’s portfolio (assuming that the second best allocation is implemented) will be $\rho_0 I$
- But in order to face the liquidity shock it needs to raise $\rho I > \rho_0 I$

which is not possible even by selling all its holdings to consumers

- Therefore this doesn’t work
- The economy does not generate sufficient “inside liquidity”

4.3 Government bonds

- Suppose that, although consumers can’t commit, the government can commit for them!
- i.e. the government can issue “debt” and credibly promise to tax consumers in order to pay back its debt
- Now government debt plays the role of a storage technology!
- “Outside liquidity”
- We can go back to implementing the second-best allocation
- Is this a chicken model?
4.4 Liquidity premium on bonds

- Suppose that shocks are aggregate / there is just one firm
- There is a limited supply of government bonds, so that in equilibrium they trade at a price $q > 1$ (i.e. you pay $q$ at $t = 0$ to get $1$ at $t = 1$)
- How does that change the optimal decision of firms?
  - Invest $I$
  - Meet liquidity shock until some cutoff $\hat{\rho}$
  - In order to do so, hold $(\hat{\rho} - \rho_0) I$ government bonds
    - (because you can always raise $\rho_0$ at $t = 1$ by diluting the original investors)
- Program is:

$$
\max_{I,d,e,d,c,\lambda(\rho)} I \left[ \int_0^{\hat{\rho}} [\rho_1 - \rho] dF(\rho) - 1 - (q - 1) (\hat{\rho} - \rho_0) \right] \\
\text{s.t. } F(\hat{\rho}) \rho_0 I \geq I - A + I \int_0^{\hat{\rho}} \rho dF(\rho) + (q - 1) (\hat{\rho} - \rho_0) I
$$

- Now

$$
k(\hat{\rho}, q) = \frac{1}{1 + \int_0^{\hat{\rho}} \rho dF(\rho) - \rho_0 F(\hat{\rho}) + (q - 1) (\hat{\rho} - \rho_0)}
$$

$$
m(\hat{\rho}, q) = \int_0^{\hat{\rho}} [\rho_1 - \rho] dF(\rho) - 1 - (q - 1) (\hat{\rho} - \rho_0)
$$

- And, again, the problem can be rewritten as

$$
\max_{\hat{\rho}} k(\hat{\rho}, q) m(\hat{\rho}, q) A
$$

- As before, the optimal cutoff will satisfy

$$
\rho^* = \arg \min_{\hat{\rho}} \frac{1 + \int_0^{\hat{\rho}} \rho dF(\rho) + (q - 1) (\hat{\rho} - \rho_0)}{F(\hat{\rho})}
$$

4.5 The Jacklin [1987] critique rears its ugly head

- Suppose all firms are doing the optimal policy of holding bonds
- A single firm can deviate and choose to buy shares in other firms instead of bonds
• (The expected return on shares is $1 > \frac{1}{q}$, so they are better than bonds in that sense)

• What is the value of shares at $t = 1$ when $\rho \in (\rho_0, \rho^*)$?

\[(\rho^* - \rho) I\]

(i.e. the value of excess bonds that are not needed because the shock is only $\rho$)

• By buying enough shares in other firms the firm can implement the $q = 1$-second-best allocation

• And investors will be willing to lend it enough money to buy these shares at $t = 0$ because they are pledgeable and have a return of 1

4.6 Partial liquidation

• Is a simple cutoff still the correct solution of the problem? Suppose we could do partial termination. We buy $z$ bonds and use these to either partially or fully meet the liquidity shock

\[
\max_{\lambda(\rho,z)} I \int_0^\infty [\rho_1 - \rho_0] \lambda(\rho,z) \, dF(\rho) \\
\text{s.t. } I \int_0^\infty [\rho_0 - \rho] \lambda(\rho,z) \, dF(\rho) \geq I - A + I (q - 1) z
\]

\[
\lambda(\rho,z) \leq \min \left\{ 1, \frac{z}{\rho - \rho_0} \right\}
\]

• Notice here we express the objective as the entrepreneur’s surplus rather than total surplus. Because IR binds, this is equivalent. The paper switches back and forth in order to be harder to follow.

• Let $\delta$ be the multiplier on the investor’s IR constraint.

• The entrepreneur will choose the maximum possible continuation (i.e. $\lambda(\rho,z) \leq \min \left\{ 1, \frac{z}{\rho - \rho_0} \right\}$) iff

\[
\rho_1 - \rho_0 + \delta [\rho_0 - \rho] > 0
\]

\[
\Leftrightarrow \rho < \frac{\rho_1 - \rho_0 (1 - \delta)}{\delta} \equiv \bar{\rho}
\]

• The choice of $z$ is governed by

\[
\int_{\rho_0 + z}^{\bar{\rho}} \frac{\partial \lambda(\rho,z)}{\partial z} \cdot [\rho_1 - \rho_0 + \delta (\rho_0 - \rho)] \, dF(\rho) - \delta (q - 1) = 0 \quad (5)
\]
• Note that $q > 1$ this implies that partial liquidation will indeed be used! (if we had enough bonds to not use partial liquidation, we would have $\frac{\partial \lambda(\rho,z)}{\partial z}$ a.s., which is not optimal according to (5))

4.7 The demand for liquidity

• Go back to the case without partial liquidation (the same holds in the case with partial liquidation)

• We had that

$$\rho^* = \arg\min_\rho \frac{1 + \int_0^\rho \rho dF(\rho) + (q - 1)(\rho_0 - \rho)}{F(\rho)}$$

• Demand for bonds will be decreasing in $q$

• At some $q^{\text{max}}$ liquidity will be so expensive that projects will just be infeasible

• At $q = 1$ consumers become willing to buy government bonds.

• Shape of demand curve for bonds

• Evidence from Krishnamurthy and Vissing-Jorgensen [2010]
5 More issues

- Asset prices
- Optimal hedging
- Comparing this idea of liquidity to Diamond and Dybvig [1983]
- Distortionary taxation and optimal government debt

References


